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Supercomplementarity in Massless Supersymmetric QCD

T. R. TAYLOR*
Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, IL 60510

ABSTRACT

The presence of at least one supersymmetric phase is established for the strongly interacting SU(N) gauge theory with n flavors of massless matter in the $\underset{\sim}{N}+\underset{\sim}{\bar{N}}$ representation of the gauge group.

*On leave of absence from the Institute of Theoretical Physics, Warsaw University, Warsaw, Poland.

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In a recent paper [1] by the present author the complementarity property [2] of massless supersymmetric gauge theories (supercomplementarity) has been explored in order to prove the existence of some confining phases with unbroken supersymmetry in a wide class of models. In particular, the method of ref.[1] directly applies to the strongly interacting SU(N) gauge theory with n flavors of massless matter in the $\underset{\sim}{N} + \underset{\sim}{\overline{N}}$ representation [SU(N) massless SQCD with n flavors], for $N \leq n < 3N$. The upper limit on the number of flavors comes from the condition of asymptotic freedom, which is believed to be a necessary one for a confining theory. The lower limit reflects the properties of the scalar potential in the weak coupling regime. For $n \geq N$ the potential posesses some flat zero-value directions, which break the gauge group completely, allowing perturbative determination of the massless spectrum[F1]. By supercomplementarity this spectrum can be then continued to the confining phase. Although a completely broken phase does exists for $n=(N-1)$ as well, we consider this case separately, since unlike the $n \geq N$ case, there are no baryon-like composite chiral supermultiplets for $n<N$.

In the present paper the approach of ref.[1] is extended to the case of $n<N$. The presence of at least one supersymmetric phase will be established. We will show that even with a gauge symmetry partly unbroken, the matter in

some Higgs phases does not contain any light particles transforming nontrivially under the unbroken subgroup of the gauge group. The theory evolves then into the infrared as a pure supersymmetric Yang-Mills model, and consequently by Witten's index theorem [3] supersymmetry remains unbroken.

In this letter the SU(3) example will be discussed in detail. The generalization to SU(N) is straightforward and will be given at the end of the paper.

Let us consider SU(3) gauge theory with n massless flavors. The "matter" of this model is contained in the left-handed chiral superfields

$$Q_i^\alpha = (\varphi_i^\alpha, \Upsilon_i^\alpha) \tag{1}$$

$$\bar{Q}_\alpha^i = (\eta_\alpha^i, \chi_\alpha^i), \qquad \alpha = 1,2,3; \quad i = 1,2,..,n$$

where the first component in a bracket denotes a complex scalar and the second one a chiral fermion. The "radiation" - gauge fermions λ and bosons A_μ - belong to the massless real vector superfields V^a, with $a=1,2,..,8$, and may be alternatively described in terms of the chiral spinorial superfields

$$W^a = (\lambda^a, F_{\mu\nu}^a) \tag{2}$$

in an adjoint representation. This model possesses

$SU(n)_L xSU(n)_R xU(1)_V xU(1)_X$ global symmetry, where $U(1)_V$ is the baryon number symmetry and the charges under non-anomalous $U(1)_X$ symmetry [4] are the following

$$x(\varphi) = x(\eta) = N-n = 3-n$$
$$x(\Upsilon) = x(\chi) = N = 3$$
$$x(\lambda) = -n \tag{3}$$

Let us first consider the case of one flavor. In the weak coupling regime the scalar potential has a zero-value direction along

$$\langle \varphi^\alpha \rangle = \langle \eta^{*\alpha} \rangle \tag{4}$$

Without loosing generality we consider perturbation theory around a fixed real vacuum expectation value (v.e.v.) v

$$\langle \varphi^\alpha \rangle = \langle \eta^{*\alpha} \rangle = v\,\delta^\alpha_3 \tag{5}$$

This v.e.v. breaks the gauge group down to SU(2), while preserving supersymmetry. The massless spectrum of the model consists of three SU(2)-gauge vector superfields V^a, a=1,2,3, and one chiral superfield $Q^3 + \bar{Q}_3$. The latter one, which contains a Goldstone boson of the spontaneously broken

$U(1)_X$ symmetry in its scalar component, is a singlet under the unbroken SU(2). The remaining particles form one massive SU(2)-singlet vector superfield V^8 and two doublets

$$A^j = \sum_{\alpha=1}^{8} V^\alpha (t^\alpha)^j_3$$

$$\bar{A}_j = \sum_{\alpha=1}^{8} V^\alpha (t^\alpha)^3_j \qquad (6)$$

where j=1,2, and t^α denote the SU(3) generators.

At larger distances the coupling constant increases and the model evolves into the infrared, confining all SU(2) non-singlets. Since except for massless gauge particles the only gauge non-singlets A^j and $\bar{A}_j$ are heavy, the dynamics at large distances should be exactly like those of a supersymmetric Yang-Mills theory. From Witten's index theorem [3] it follows immediately that the nonperturbative effects do not break supersymmetry in such a model. They confine all strongly-interacting particles in some SU(2)-singlet supermultiplets of supersymmetry.

Supercomplementarity relates the properties of a partly broken phase to the physics of a fully SU(3) gauge-symmetric phase. In the symmetric phase the condensate

$$\left\langle \sum_{\alpha=1}^{3} \varphi^\alpha \eta_\alpha \right\rangle = v^2 , \qquad (7)$$

"dual"[F2] to the v.e.v.s of eqs. (4) and (5), is expected to form. The massless meson-like superfield

$$T = \sum_{\alpha=1}^{3} Q^{\alpha} \bar{Q}_{\alpha} \tag{8}$$

corresponds to the superfield $Q^3 + \bar{Q}_3$ of the partly broken phase. The scalar component of T contains a Goldstone boson of the dynamically broken $U(1)_X$ symmetry. Some massive particles have their counterparts as well, for instance the superfield V^8 may be interpreted as

$$V^8 = \sum_{\alpha,\beta=1}^{3} \left(Q^{+}_{\alpha} (e^{V})^{\alpha}_{\beta} Q^{\beta} - \bar{Q}_{\alpha} (e^{-V})^{\alpha}_{\beta} \bar{Q}^{+\beta} \right). \tag{9}$$

In such a way we arrive to the conclusion that in the present model there exists at least one confining phase with supersymmetry unbroken.

The case of n=2 flavors is even simpler to study, because of the existence of a Higgs phase with a completely broken SU(3) gauge symmetry. The following configuration of the scalar v.e.v.s

$$\langle \varphi^{\alpha}_{i} \rangle = \langle \eta^{*\alpha}_{i} \rangle = v \delta^{\alpha}_{i} \tag{10}$$

corresponds in the symmetric phase to the "dual" condensate

$$\langle \sum_{\alpha=1}^{3} \varphi^{\alpha}_{i} \eta^{j}_{\alpha} \rangle = v^2 \delta^{j}_{i} \tag{11}$$

In the symmetric phase the global symmetry $SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_X$ breaks down to $SU(2)_V \times U(1)_V$, where $SU(2)_V$ is the usual isospin subgroup of the chiral group. The massless composite superfields

$$T^a = \mathrm{Tr}\,(T\,t^a)\,, \qquad a = 1,2,3 \tag{12}$$

contain three Goldstone bosons of chiral symmetry ("pions"), whereas the superfield

$$T_X = \mathrm{Tr}\,T \tag{13}$$

contains a Goldstone boson of $U(1)_X$ symmetry.

The case of n=3 has been explicitely discussed in ref.[1]. Some new chiral symmetry breaking patterns become possible, due to possible condensations of the type

$$\langle \varepsilon_{\alpha\beta\gamma}\, \varphi_i^\alpha \varphi_j^\beta \varphi_k^\gamma \rangle = v^3 f_{ijk}\,, \tag{14}$$

where the dimensionless constant f_{ijk} is totally antisymmetric in flavor indices. The massless spectrum, in addition to the meson-like supermultiplets, contains some baryon-like superfields

$$B = \varepsilon_{\alpha\beta\gamma}\, Q_i^\alpha Q_j^\beta Q_k^\gamma$$

$$\bar{B} = \varepsilon^{\alpha\beta\gamma}\, \bar{Q}_\alpha^i \bar{Q}_\beta^j \bar{Q}_\gamma^k\,. \tag{15}$$

The generalization of the above arguments to the SU(N) model with n flavors is straightforward. The model contains 2nN massless matter supermultiplets. For n<N the scalar v.e.v.s

$$\langle \varphi_i^\alpha \rangle = \langle \eta^{*\alpha}_i \rangle = v\,\delta_i^\alpha \tag{16}$$

break the gauge group SU(N) down to SU(N-n), leaving

$$2nN - \left(N^2 - (N-n)^2\right) = n^2 \tag{17}$$

massless supermultiplets, all SU(N-n) singlets. The model evolves then into the infrared with supersymmetry unbroken.

In the SU(N) gauge-symmetric phase the corresponding massless particles form n^2 chiral supermultiplets

$$T^a = \mathrm{Tr}\left(T t^a\right), \qquad a = 1, 2, \ldots, n^2-1$$

$$T_x = \mathrm{Tr}\,T \tag{18}$$

with the scalar components containing (n^2-1) Goldstone bosons of $SU(n)_L \times SU(n)_R / SU(n)_V$ and one Goldstone boson of $U(1)_X$, respectively. This standard pattern of global symmetry breaking is a consequence of the following condensation[F3]

$$\left\langle \sum_{\alpha=1}^{N} \varphi_i^\alpha \eta_\alpha^j \right\rangle = v^2 \delta_i^j\,, \tag{19}$$

which leaves supersymmetry unbroken.

For $n \geq N$ the arguments of ref.[1] directly apply, leading to the conclusion that some new realizations of chiral symmetry become possible. In particular, some baryon-like supermultiplets are expected to be present in the massless spectrum.

Our arguments rely on two assumptions on the dynamics of the Higgs phase. Firstly, it has been assumed that flat directions of the potential remain unchanged despite all non-perturbative effects, like instantons e.t.c. Moreover, it has been assumed that the heavy particles, which interact with respect to the unbroken subgroup of the gauge group, do decouple from the massless system at sufficiently low - i.e. much smaller than v - energies[F4]. Although both assumptions remain to be verified on some more rigorous basis, they are obviously correct in the asymptotic limit of an infinitely large v.e.v. v. Hence even if some non-perturbative effects were present in the Higgs phase, in the confining phase supersymmetry would remain unbroken, at least for the infinite values of the condensate (19). This would explain [5] the singular zero mass limit of SQCD, which has been found in the effective Lagrangian analysis [4].

So far our analysis is completely independent of any effective Lagrangians. It is interesting, however, to speculate how a flat direction could arise in the effective potential. One of the general features of supersymmetric phases is the existence of some composite massive vector supermultiplets, corresponding to the broken generators of the gauge group in Higgs phases. The interaction of the form

$$\mathcal{L}_{int} = \int d^4\theta \left(T^* V T + V^2 \right) \tag{20}$$

could give an arbitrary large v.e.v. to the scalar component of T, at cost of a large v.e.v. of the scalar component of V. In such a way some flat directions of the perturbative potential could be reproduced in the effective potential of a confining theory.

The results of the present paper and ref.[1] strongly indicate against dynamical supersymmetry breaking in massless SQCD. Nevertheless, the existence of a supersymmetric phase does not prove definitely that supersymmetry cannot be broken in such a model. In principle some other metastable vacua might exist [6]. They could be physically interesting, if separated by some potential barriers high enough to suppress possible tunneling to the present supersymmetric phases.

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FOOTNOTES

[F1]Some non-perturbative phenomena in the broken phase will be discussed further on.

[F2]For the rules of "duality" between gauge-symmetric condensates and their counterpart v.e.v.s in the Higgs phase, see ref.[1].

[F3]This condensate has naturally appeared in the effective Lagrangian analysis of massive SQCD in ref.[4].

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