Fermi National Accelerator Laboratory

FERMILAB-Pub-83/37-THY
April 1983

TWIST-FOUR EFFECTS IN ELECTROPRODUCTION:

MODEL DEPENDENCE

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ABSTRACT

The twist-four, spin-two effects on the second Nachtmann moment in electroproduction have been calculated using the operator product expansion. To explore the model dependence of the nucleon matrix elements of the operators, proton and neutron wave functions from several different models of quark confinement were considered. While the twist-four, spin-two effects were indeed found to be substantially model-dependent, they were small ($<2\%$) in all cases, confirming the conclusions of Jaffe and Soldate, based on the MIT bag model.

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The comparison of the perturbative predictions of Quantum Chromodynamics (QCD) with the scaling violations seen in deep inelastic lepton scattering is complicated by the nonperturbative higher-twist effects, which are model dependent. In principle there is a systematic procedure for calculating the higher-twist effects on the Nachtmann moments of the structure functions:[1] Use the Wilson operator product expansion[2] (OPE) to expand the bilocal product of currents in a basis of local operators and evaluate the coefficient functions[3,4] using renormalization group techniques. And finally evaluate the nucleon matrix elements of the local operators proton and neutron using wave functions based on some model of quark confinement.

This procedure has been very clearly described by Jaffe and Soldate[4] who have also found a very useful basis of local operators. Applying this approach to the simplest, non-trivial case; viz., the twist-four, spin-two corrections to the second Nachtmann moment of the non-singlet electroproduction structure function, they used the MIT Bag Model[5] to evaluate the nucleon matrix elements of the operators and found the effects to be small (< 1%).

Here we have explored the model dependence of this very attractive approach by considering several other quark confinement models for the nucleon wave functions. The nucleon matrix elements were indeed found to be rather different in the various models; however, the resulting twist-four, spin-two effects on the second Nachtmann moment were still quite small in all cases. For these purposes, the MIT Bag Model[5] used by Jaffe and Soldate[4] seems typical, there being a similar suppresion of the matrix elements at work in each model. In addition to verifying the MIT Bag Model results, we therefore confirm the conclusion that, in the region of present experimental interest, the twist-four, spin-two effects on the second Nachtmann moment of the non-singlet electroproduction structure function is quite small (< 1%).

The spin-two second Nachtmann moment is

$$M_2(Q^2) = M_2^{T=2,N=2}(Q^2) + M_2^{T=4,N=2}(Q^2) \tag{1}$$

where, in the parton model, the twist-two, spin-two contribution $M_2^{T=2,N=2}$ is 1/3 for protons and 2/9 for neutrons, without QCD perturbative corrections. The twist-four, spin-two contributions can be written in the form

$$M_2^{T=4,N=2}(Q^2) = -\Delta^2/Q^2 \ , \tag{2}$$

since the effect is of order $(-1/Q^2)$; and Δ^2 involves the nucleon matrix elements of the quark operators in the OPE.

We shall consider quark confinement models in which the four-component proton and neutron wave functions are of the usual spherically symmetric form

$$q(r) = \begin{pmatrix} f(r) \\ \vec{\sigma}.\hat{r}g(r) \end{pmatrix} \chi \tag{3}$$

where χ is a two-component spinor and

$$\int dr^3\{|f\ (r)|^2+|g\ (r)|^2\} = 1 \tag{4}$$

One finds for protons and neutrons, respectively:

$$\Delta_p^{\ 2} = \frac{g_s^{\ 2}}{4M_p}\left(\frac{154}{27} I_1 - \frac{224}{27} I_2\right) \tag{5}$$

$$\Delta_n^{\ 2} = \frac{g_s^{\ 2}}{4M_n}\left(4\ I_1 - \frac{208}{27} I_2\right) \tag{6}$$

The QCD fine structure constant $\alpha_s = g_s^2/4\pi$ depends only logarithmically on Q^2 and in our numerical computations below we shall consider a typical region of experimental interest.

The integrals I_1 and I_2 are, in any model,

$$I_1 = \int d^3r \{ |f(r)|^2 + |g(r)|^2 \} \tag{7}$$

and

$$I_2 = \int d^3r \, |f(r) \, g(r)|^2 \tag{8}$$

and evidently have dimensions of $(\text{length})^{-3}$. Crudely speaking, I_1, measures the square of the wave function averaged over the confinement volume V, while I_2 measures the overlap of the "large" and "small" components of the wave function, f(r) and g(r). Therefore, one can anticipate that $I_1 \sim 1/V$, while $I_2/I_1 \ll 1$. However, as we shall see below, I_2 is not entirely negligible.

The twist-four, spin-two corrections have been numerically computed for several models in the literature and the results for both protons and neutrons are presented in the Table. For convenience in comparing the size of the twist-four effect to the twist-two parton model result, we have included in the Table the ratio

$$\delta = M_2^{T=4,N=2} / M_2^{T=2,N=2} = -\left(\frac{\Delta^2}{Q^2}\right) / M_2^{T=2,N=2} \tag{9}$$

where, recall M_2 is 1/3 for protons and 2/9 for neutrons. In obtaining the numerical results shown in the Table we have taken $Q^2 = 5\ \text{GeV}^2$ and $\alpha_s(Q^2) = 0.25$. The integrals I_1 and I_2 depend on the particular quark confinement model and, of course, are independent of Q^2. The twist-four spin-two corrections to the second Nachtmann

$M_2^{N=4,T=2} = -\Delta^2/Q^2$ are directly proportional to $\alpha_s(Q^2)/Q^2$ and the corrections for different values of Q^2 can simply be obtained from the Table by appropriately scaling the entries shown.

A variety of models in the literature were chosen to explore the model dependence of the twist-four effects. The usual MIT Bag model[5] has been designated MIT Bag (A) in the Table and corresponding entries verify the results of Jaffe and Soldate.[4] In the MIT Bag (B) and MIT Bag (C) models[6] an additional term in the hadron energy, proportional the number of quarks minus antiquarks, is included to account for differences between baryons and mesons.

MODEL	$I_1 \times 10^4$ (GeV^3)	$I_2 \times 10^4$ (GeV^3)	Δ_p(MeV)	δ_p(%)	Δ_n(MeV)	δ_n(%)
MIT BAG (A)[5]	20.36	3.21	87	0.44	69	0.42
MIT BAG (B)[6]	64.41	10.14	154	1.42	123	1.36
MIT BAG (C)[6]	54.69	0.65	46	0.13	38	0.12
RELATIVISTIC[7] POTENTIAL	41.16	4.03	130	1.01	106	1.01
HARMONIC[8] OSCILLATOR SHELL MODEL	75.93	0.64	189	2.14	158	2.25

TABLE: Twist-Four, spin-two effects for various quark confinement models assuming $Q^2 = 5\ GeV^2$ and $\alpha_s(Q^2) = 0.25$.

In both of these models the parameters are determined by fitting certain hadron masses and the protron magnetic moment. They differ in that a non-zero quark mass is assumed in the MIT Bag (C) model. In the Relativistic Potential model[7] the wave function is an approximate solution of the Dirac equation assuming a linear confining potential, while the Harmonic Oscillator Shell Model[8] is based on using nonrelativistic harmonic oscillator shell model functions for the "large" components f(r) and approximating the "small" components of q(r) by $g(r) = (\vec{\sigma} \cdot \vec{p}/2m)f(r)$.

It is clear from the Table that the twist-four effects are considerably model dependent, the variation in Δ^2 being greater than an order of magnitude; but, in no case are they very large, being at most about 2%. It is also quite noteworthy that the twist-four effects are rather similar for protons and neutrons in each model and consequently the contributions to the non-singlet second Nachtmann moment are exceedingly small.

In summary, we have found the twist-four, spin-two effects on the second Nachtmann moments of the electroproduction structure function to be very dependent on the quark confinement model used for the nucleon wave functions. However, while the variation was greater than an order of magnitude amongst several models in the current literature, in all cases the effects were small (< 2%), confirming the conclusions of Jaffe and Soldate[4] based on the MIT Bag model[5]. Quite interestingly, the twist-four effects were rather comparable for both protons and neutrons in all models and, therefore, the twist four effects on the non-singlet second Nachtmann moment is extremely small.

We thank Chris Quigg for his kind hospitality in the Theoretical Physics Department at Fermilab where these calculations were done. One of us (R.J.O.) is grateful to Pete Carruthers for the opportunity to visit the Theoretical Physics Division of the Los Alamos National Laboratory where the manuscript was prepared. We have also benefited from discussions with Dubravko Tadić regarding quark confinement.

This work was supported in part by the National Science Foundation under NSF Grants pHY 78-11607 A01 and PHY 82-09145.

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