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Detailed Calculation of the Complete Two Loop Higgs-Yukawa Beta Function in an Arbitrary α-Gauge

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ABSTRACT

We present a complete calculation of the two loop contributions to the Higgs-Yukawa beta function, giving the evaluations of individual Feynman diagrams. We calculate in an arbitrary α-gauge, and in a range of subtraction schemes that includes MS and $\overline{\text{MS}}$. We compute the beta function explicitly for the Weinberg-Salam theory, but our results should be readily adaptable to the computation of the beta function in other theories.

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INTRODUCTION

The renormalization group evolution of Yukawa couplings is important in many contexts and in some cases one needs to know this evolution quite precisely. For instance we have recently carried out a complete two loop calculation of the bottom mass within the framework of the SU(5) model.[1] By computing to this accuracy we were able to set limits on the number of generations, and we found that three and four, but not more than four generations gave predictions consistent with the observed value of M_b. This conclusion disagreed with the results of an earlier partial two loop calculation.[2]

Limits on the number of generations have also been derived in this way for supersymmetric unified theories.[3]

Another example is the prediction of the masses of heavy fermions ($\lesssim$ 300 GeV). We have calculated these quantities to two loops by making use of properties of the Yukawa renormalization group equations--the existence of so-called infrared pseudo fixed points.[4] Two loop precision is desirable here because the fermions are heavy, which means that their Yukawa couplings can be large.

More recently in supergravity theories the Yukawa evolution has been found to be important in determining the low energy spontaneous symmetry breaking scale.[5]

The evolution of a Yukawa coupling is of course described by the corresponding beta function. For our SU(5) prediction of M_b we calculated the complete two loop beta function for this coupling in the Weinberg-Salam theory. This calculation included all strong, electroweak, Yukawa and scalar self coupling terms, and was performed in an arbitrary α-gauge, and in a range of subtraction schemes comprising MS and $\overline{\mathrm{MS}}$. We have briefly reported the results elsewhere.[6]

In this paper we present the details of our calculation of the Higgs-Yukawa beta function, giving the evaluation of individual Feynman diagrams. Though we compute the beta function within the context of the Weinberg-Salam theory, our results should be readily adaptable to the computation of β_{H-Y} in other theories. Our explicit calculation of the gauge dependence should be particularly useful in any such computation as a check. For the Weinberg-Salam theory the requirement that the gauge dependence eventually cancel gave us a total of sixty-six additive checks on the correctness of our results.

We also calculate the two loop anomalous dimensions for the Higgs and the fermions. The Higgs anomalous dimension, though gauge variant, will be useful in calculating β_λ for the scalar self coupling. The anomalous dimensions have been in part calculated previously; we agree with the earlier results.[7]

The organization of the paper is as follows. In Section I we describe the overall framework of our calculation, and give the Lagrangian we employ. In Section II we briefly explain how the beta function is extracted from a computation of the infinite parts of the diagrams. In Section III we describe our subtraction schemes and the way in which the scheme dependence cancels diagram by diagram.

Finally in Section IV we present the contributions of the individual diagrams to the beta function, and our results for the Higgs and fermion anomalous dimensions.

I. FRAMEWORK

We are interested in computing the Higgs-Yukawa beta function for the standard Weinberg-Salam theory. To calculate we work with a slightly generalized version of this theory, so as to gain additional checks on our results as well as broader applicability.

1. We take the color gauge group to be $SU(N_c)$, $N_c \geq 3$, with fermions in either the fundamental or singlet representation (for quarks and leptons respectively).
2. Left-handed fermions and also the Higgs are taken to be in N_2 dimensional representations of $SU(2)_{weak}$, with $N_2 \geq 2$. However we include just the usual <u>pairs</u> of SU(2) singlet right-handed fermions (e.g. top and bottom). [Thus N_2-2 of the left-handed fermions in a multiplet

have no right-handed partner and remain massless.]

3. The left-handed fermions are assigned arbitrary hypercharge y. Their right-handed partners then have hypercharge y+1 (y-1) for an upper (lower) flavor. The Higgs have as usual hypercharge +1.
4. We assume that there are N_H Higgs, only one of which couples to the fermions.
5. N_g denotes the number of generations.

Also important are the following features of our calculation. It is carried out in an arbitrary α gauge, where the gauge propagator is $-i/k^2(g_{\mu\nu}-(1-\alpha)\ k_\mu k_\nu/k^2)$. We use dimensional regularization, discussing a range of MS-type schemes. The second order term in β is both gauge and scheme independent for these schemes, but the explicit cancellation of the dependence in individual sectors gives us stringent checks on the validity of our results. There are in all 66 additive checks due to gauge invariance, and the sum of each diagram separately with its counterterms is scheme independent (for the range of schemes we consider.)

Lastly, because we are concerned with the high energy evolution of coupling constants, we work with the high energy symmetric form of the theory. There are no unphysical Higgs particle and the W bosons are massless. Also we take the Higgs massless. As usual we ignore the hierarchy problem, assuming that quadratic divergences can simply be dropped.

For completeness we give below our working Lagrangian. We write down terms for a single generation only. The gauge coupling constants, g_1, g_2 and g_3, refer to the U(1), weak and colored groups respectively. λ_2^a and λ_3^a are matrices, elements of the Lie algebra, for the weak and colored groups. They are normalized so that in the fundamental representation $Tr(\lambda^2)=1/2$.

The Higgs-Yukawa coupling constants are labelled g_t and g_b for an upper and lower flavor. The matrix $i\sigma_2$ is a generalization of the usual Pauli Matrix, and allows the complex conjugate Higgs field ϕ^+ to give mass to the t-quark.

$$
\begin{aligned}
\mathcal{L} = {} & \bar{\psi}_L i\gamma_\mu (\partial^\mu + ig_1 \tfrac{y}{2} B^\mu + ig_2 W^\mu \cdot \lambda_2 + ig_3 G^\mu \cdot \lambda_3)\psi_L \\
& + \bar{T}_r \, i\gamma_\mu (\partial^\mu + ig_1 \tfrac{(y+1)}{2} B^\mu \qquad + ig_3 G^\mu \cdot \lambda_3) T_r \\
& + \bar{B}_r \, i\gamma_\mu (\partial^\mu + ig_1 \tfrac{(y-1)}{2} B^\mu \qquad + ig_3 G^\mu \cdot \lambda_3) B_r \\
& + |(\partial_\mu + ig_1 \tfrac{1}{2} B_\mu + ig_2 W_\mu \cdot \lambda_2)\phi|^2 - \tfrac{\lambda}{6} |\phi^+\phi|^2 \\
& - g_B(\bar{\psi}_L \phi \, B_r + \text{H.C.}) - g_T(\bar{\psi}_L (i\sigma_2)\phi^+ T_r + \text{H.C.}) \\
& - \tfrac{1}{4} (F^1_{\mu\nu} F^{\mu\nu}_1 + F^2_{\mu\nu} F^{\mu\nu}_2 + F^3_{\mu\nu} F^{\mu\nu}_3) - \tfrac{1}{2\alpha} [(\partial_\mu B^\mu)^2 + (\partial_\mu W^\mu)^2 + (\partial_\mu G^\mu)^2] \\
& + (\partial^\mu \eta_2^{a+})(\delta_{ab}\partial_\mu - g_2 \, C_2^{abc} W^b_\mu)\eta_2^c \\
& + (\partial^\mu \eta_3^{a+})(\delta_{ab}\partial_\mu - g_3 C_3^{abc} G^b_\mu)\eta_3^c
\end{aligned}
\tag{1}
$$

With this Lagrangian, then for a three-point diagram with an incoming B_r and an outgoing B_L, the neutral higgs is incoming.

Note that we choose the same gauge for all gauge fields.

II. EXTRACTING THE β FUNCTION

The quantities actually computed are the $1/\varepsilon$ poles in the 2-point (higgs and fermion self-energies) and the three-point (Yukawa) green functions. (Here $\varepsilon=4-d$, where d is the number of dimensions.) In an MS-type scheme these define the renormalization factors Z_{ψ_L}, Z_{ψ_r}, Z_ϕ and Z_{coupling} where:

$$\psi^{L,r}_{un} = Z^{1/2}_{\psi_{L,r}} \psi^{L,r}_{ren}$$

$$\phi_{un} = Z^{1/2}_{\phi} \phi_{ren}$$

$$g_{un}\bar{\psi}_{un}\phi_{un}\psi^{r}_{un} = Z_{\text{coupling}} g_{ren}\bar{\psi}^{L}_{ren}\phi_{ren}\psi^{r}_{ren} \quad . \tag{2}$$

For an MS-type scheme β can be obtained very simply from these quantities.

Suppose we are interested in β for g_T. The relevant Z is Z_{g_T}, defined by

$$g^{un}_T = Z_{g_T} g^{ren}_T \quad . \tag{3}$$

If Z_{g_T} is expanded as

$$Z_{g_T} = 1 + Z^{s}_{g_T} \frac{1}{\varepsilon} + \ldots \tag{4}$$

Then β is given by [8]

$$\beta_T = \frac{1}{2} g_T \left(g_T \frac{d}{d_{g_T}} + g_B \frac{d}{g_B} + \sum_{i=1}^{3} g_i \frac{d}{d_{g_i}} + 2\lambda \frac{d}{d\lambda} \right) Z^s_{g_T} . \quad (5)$$

(The expression in brackets is intended to represent a sum over <u>all</u> coupling constants, including those for fermions of other generations.)

Now since $Z_{g_T} = Z_{coupling}/\sqrt{Z_{\psi_L} Z_{\psi_r} Z_\phi}$, expanding the Z's in $1/\varepsilon$ gives:

$$Z^s_{g_T} = Z^s_{coupling} - \frac{1}{2} \left(Z^s_{\psi_L} + Z^s_{\psi_r} + Z^s_\phi \right) . \quad (6)$$

for the leading terms.

The superscript s indicates the coefficient of the simple pole. Also, the operator in brackets in Eq. (5) acts very simply on Z^s_g, so that

$$\beta^{1,s}_{g_T} = g_T \left(Z^{1,s}_{coupling} - \frac{1}{2} (Z^{1,s}_{\psi_L} + Z^{1,s}_{\psi_r} + Z^{1,s}_\phi) \right)$$

$$\beta^{(2)}_{g_T} = 2 g_T \left(Z^{2,s}_{coupling} - \frac{1}{2} (Z^{2,s}_{\psi_L} + Z^{2,s}_{\psi_r} + Z^{2,s}_\phi) \right) \quad (7)$$

where Z^1 and $\beta^{(1)}$, Z^2 and $\beta^{(2)}$, are the first and second order terms of Z and β. Thus β has been related to a simple combination of the $1/\varepsilon$ poles of the two- and three-point green functions.

By a similar argument, one can show that the two loop anomalous dimensions are:

$$\gamma_{\psi_{L,r}} = - Z^{S}_{\psi_{L,r}} \ , \ \gamma_{\phi} = - Z^{S}_{\phi} \tag{8}$$

Referring to equation 7, one sees that the anomalous dimensions are exactly the appropriate self-energy contributions to the β function.

III. SCHEME AND GAUGE DEPENDENCE

The β function through second order is gauge and scheme independent for the range of schemes we employ. Furthermore our Lagrangian contains several arbitrary parameters, and our result must be gauge invariant regardless of what values they have. Therefore individual terms of β, e.g. the coefficients of N_c, N_2, y, y^2, y^3, etc., will be gauge invariant by themselves. The 80 odd diagrams can be grouped into overlapping subclasses, where each subclass contributes to a different term and is separately gauge invariant for the correct combination of its diagrams. As mentioned earlier, we have then a total of 66 additive checks on our calculation.

The scheme independence of the second order term of β gives additional checks. It turns out that the sum of each diagram with its counterterms is individually scheme independent for our schemes.

The schemes that we utilize[9] can be described as minimal subtraction with a modified definition of d-dimensional integration. The counterterms are chosen as usual to exactly cancel the poles in $1/\varepsilon$, but the standard formulas for integrals are multiplied by some smooth function of d that goes to 1 at d=4. For example we define:

$$\int \frac{d^dk}{(2\pi)^d} \frac{1}{(k^2+m^2)^\alpha} = \frac{1}{N(d)} \text{ [standard definition] } =$$

$$\frac{1}{N(d)} \frac{1}{(4\pi)^{d/2}} \frac{\Gamma(\alpha-d/2)}{\Gamma(\alpha)\,(m^2)^{\alpha-d/2}} \tag{9}$$

where N(d) is such a function. In this approach MS corresponds to chosing N(d)=1, and $\overline{\text{MS}}$ to $N(d) = (4\pi)^{4-d}/\Gamma(d-3)$.

The great advantage of these schemes is that the simple expression for β derived earlier in Eq. (7) remains valid.

Scheme independence then follows from the fact that when one computes in one of these schemes, say in MS, the coefficient of $1/\varepsilon^2$ for each diagram is -1/2 times the corresponding counterterm coefficient. It is easy to see how this relationship arises computationally. Crudely, it is due to the fact that when calculating a counterterm one is really computing two independent integrals: one to determine the counterterm insertion, and the second to evaluate a diagram with this insertion. On the other hand, for a two loop diagram the two integrations are linked by their dependence on a common momentum.

The relationship of $1/\varepsilon^2$ coefficients leads to the cancellation of scheme dependence as follows. The two loop diagram in our scheme has one $1/N(d)$ factor for each loop integration--two in all. But for the counterterm one of these integrals is truncated. The pole part only of the counterterm insertion is used, and this is proportional to $1/N(4)$ or 1. Thus the insertion is independent of the behavior of N away from d=4, and the whole counterterm is proportional to $1/N(d)$ only. Then if a diagram is evaluated to be

$$\frac{1}{N^2(\varepsilon)}\left(\frac{A}{\varepsilon^2}+\frac{B}{\varepsilon}+0(1)\right) = \frac{A}{\varepsilon^2}+\frac{B-2N_1A}{\varepsilon}+0(1) \tag{10}$$

with $N=1+N_1\varepsilon+0(\varepsilon^2)$ and the sum of its counterdiagrams is

$$\frac{1}{N(\varepsilon)}\left(\frac{-2A}{\varepsilon^2}+\frac{C}{\varepsilon}+0(1)\right) = \frac{-2A}{\varepsilon^2}+\frac{C+2N_1A}{\varepsilon}+0(1) \tag{11}$$

the total is

$$\frac{-A}{\varepsilon^2}+\frac{B+C}{\varepsilon}+0(1). \tag{12}$$

The simple pole, which alone contributes to β, is independent of N.

One would not expect cancellation of scheme dependence by this mechanism in higher loops.

IV. CONTRIBUTIONS OF THE DIAGRAMS TO β_B

In this section we give the contributions of individual diagrams to β_B the beta function for g_B (rather than the actual value of the diagrams.) Here g_B represents the coupling of the Higgs to any down-type fermion. β for g_T can be obtained by letting $g_B \leftrightarrow g_T$ and $y \rightarrow -y$ in each generation.

Note that a listed contribution represents the sum of the illustrated diagram with its associated counterterm diagrams, where the latter are obtained by replacing a divergent subdiagram with minus its $1/\varepsilon$ pole part.

We use the following notation:

1. g_H represents any Higgs-Yukawa coupling; g_i any gauge coupling.
2. $N_3=N_c$ for a quark, $N_3=1$ for a lepton. $D_{3(2)}$ is the dimension of the adjoint representation for $SU(N_c)$ $(SU(2))$. Thus $D_3=N_c^2-1$ and $D_2=3$.
3. We denote Casimirs as follows:

   $R_{3(2)}$ is the Casimir for the fundamental representation of $SU(N_c)$ $(SU(2))$. Thus e.g. $\lambda_3^a\lambda_3^a=R_3\,\mathbb{1}$ for λ_3 in the fundamental representation of $SU(N_c)$.

   $G_{3(2)}$ is the Casimir for the adjoint representation of $SU(N_c)$ $(SU(2))$.

   For the standard theory, with $N_c=3$:

$$R_3 = \frac{4}{3}\ ,\ R_2 = \frac{3}{4}\ ,\ G_3 = 3,\ G_2 = 2.$$

For a lepton, we set $R_3=0$.

4. For diagrams with fermion loops, fermions of any generation can contribute. We use barred coupling constants, e.g. $\bar{g}_B$, to indicate that a sum over <u>all</u> fermions with the corresponding T_3 should be performed. Similarly, $\bar{y}$ or $\bar{N}_3$ signifies that in this sum the appropriate values of y or N_2 should be substituted. For example:

$$\bar{g}_B^2\ \bar{N}_3\ \bar{y} \equiv g_B^2\ N_c\ \frac{1}{3} + g_\tau^2(1)(-1)$$

$$+\ G_S^2\ N_c\ \frac{1}{3} + g_\mu^2(1)(-1)\ldots \tag{13}$$

5. The gauge propagator is $-i/k^2(g_{\mu\nu}-(1-\alpha)k_\mu k_\nu/k^2)$.

To obtain β for g_T, take $g_B \leftrightarrow g_T$, $\bar{g}_B \leftrightarrow \bar{g}_T$ $y \rightarrow -y$, $y \rightarrow -\bar{y}$.

In part A. we give the Higgs self-energy contributions and their total, the Higgs anomalous dimension. In B. we give the fermion self-energy contributions and anomalous dimensions, separating right-handed from left-handed, and in C. the three point results.

A. Higgs Self Energy Contributions

The relevant diagrams appear in figures 1, 5, 7 and 10(a and b). All expressions should be multiplied by $g_B/(16\pi^2)^2$ to get the true contribution to β_B. The g_H^5 contributions to β_B derive from the diagrams in Fig. 1:

$$\text{1a)} \quad [\tfrac{1}{2}(\bar{g}_B^4+\bar{g}_T^4)(N_2+1) + \bar{g}_B^2\bar{g}_T^2]\ \bar{N}_3\left\{\frac{-3}{2}\right\}$$

$$\text{1b)} \quad [-\bar{g}_B^2\bar{g}_T^2]\ \bar{N}_3\ \{-2\} \tag{14}$$

The $g_H^3 g_i^2$ terms come from the diagrams in Fig. 5:

$$\text{5a)} \quad \Big\{-[g_3^2R_3(\bar{g}_B^2+\bar{g}_T^2)]\ \bar{N}_3 - \tfrac{1}{2}[g_2^2R_2(\bar{g}_B^2+\bar{g}_T^2)]\ \bar{N}_3$$

$$- \tfrac{1}{4}\ g_1^2(\bar{g}_B^2[\bar{y}^2-\bar{y}+\tfrac{1}{2}] + \bar{g}_T^2[\bar{y}^2+\bar{y}+\tfrac{1}{2}])\bar{N}_3\Big\}\ \{\alpha\}$$

$$\text{5b)} \quad \Big\{-\tfrac{1}{2}[g_3^2R_3(\bar{g}_B^2+\bar{g}_T^2)]\ \bar{N}_3 - \tfrac{1}{8}[g_1^2(\bar{g}_B^2[\bar{y}^2-\bar{y}]+\bar{g}_T^2[\bar{y}^2+\bar{y}])]\ \bar{N}_3\Big\}$$

$$\times\ \{-10-2\alpha\}$$

$$\text{5c)} \quad \Big\{-\tfrac{1}{2}[g_2^2R_2(\bar{g}_B^2+\bar{g}_T^2)]\ \bar{N}_3 - \tfrac{1}{8}[g_1^2(\bar{g}_B^2+\bar{g}_T^2)]\ \bar{N}_3\Big\}\{-7+5\alpha\}$$

5d) $\left\{-[g_2^2 R_2(\bar{g}_B^2+\bar{g}_T^2)]\ \bar{N}_3 - \frac{1}{4}\ [g_1^2(\bar{g}_B^2+\bar{g}_T^2)]\ \bar{N}_3\right\}\{1-3\alpha\}$ (15)

Figure 7 contributes to terms in β_B proportional to $g_B g_i^4$

7a) $-2\ [g_2^4 R_2 G_2]\left\{\frac{53}{48} + \frac{11}{8}\alpha + \frac{\alpha^2}{2}\right\}$

7b) $-2\ [g_2^4 R_2 G_2]\left\{\frac{11}{48} - \frac{\alpha^2}{8}\right\}$

7c) $-2\ [g_2^4 R_2 G_2]\left\{\frac{63}{16} - \frac{15}{8}\alpha - \alpha^2\right\}$

7d) $\left\{-[g_2^4 R_2 \bar{R}_2 N_2/D_2] N_H - \frac{1}{16}\ [g_1^4 N_2 N_H]\right\}\left\{-\frac{11}{6}\right\}$

7e) $\left\{\frac{1}{2}\ [g_2^4 R_2 \bar{R}_2 N_2/D_2]\ \bar{N}_3 + \frac{1}{32}\ [g_1^4(\bar{y}^2(N_2+2)+2)]\ \bar{N}_3\right\}\left\{\frac{10}{3}\right\}$

7f) $\left\{-[g_2^4 R_2^2] - \frac{1}{2}\ [g_1^2 g_2^2 R_2] - \frac{1}{16}\ [g_1^4]\right\}\left\{\frac{9}{2} - 3\alpha + \frac{\alpha^2}{2}\right\}$

7g) $\left\{4[g_2^4(R_2^2 - \frac{1}{4}\ R_2 G_2)] + 2[g_1^2 g_2^2 R_2] + \frac{1}{4}\ [g_1^4]\right\}\left\{\frac{9}{8} - \frac{\alpha}{2} - \alpha^2\right\}$

7h) 0

$$7i)\quad \left\{-2\ [g_2^4(R_2^2 - \tfrac{1}{4} R_2G_2)] - [g_1^2g_2^2R_2] - \tfrac{1}{8}\ [g_1^4]\right\} \left\{-\tfrac{3}{2} - \tfrac{\alpha^2}{2}\right\}$$

$$7j)\quad \left\{-[g_2^4(R_2^2 - \tfrac{1}{2} R_2G_2)] - \tfrac{1}{2}\ [g_1^2g_2^2R_2] - \tfrac{1}{16}\ [g_1^4]\right\} \left\{\tfrac{3}{2} + \alpha - \tfrac{7}{2}\alpha^2\right\}$$

(16)

Contributions involving the four point coupling:

$$10a)\quad -\frac{\lambda^2}{9}(N_2 + 1)\ \left\{-\frac{1}{2}\right\}$$

$$10b)\qquad 0 \tag{17}$$

The total contribution to β_{g_B} from the two loop Higgs self-energy diagrams is $g_B/(16\pi^2)^2$ times:

$$-\frac{3}{4}(\bar g_B^4+\bar g_T^4)(N_2+1)\bar N_3 + \frac{1}{2}(\bar g_B^2\bar g_T^2)\bar N_3 + 5g_3^2R_3(\bar g_B^2+\bar g_T^2)\bar N_3$$

$$+\frac{5}{2}g_2^2R_2(\bar g_B^2+\bar g_T^2)\bar N_3 + \frac{5}{8}g_1^2[\bar g_B^2(2\bar y^2-2\bar y+1)\bar N_3+\bar g_T^2(2\bar y^2+2\bar y+1)\bar N_3]$$

$$+g_2^4R_2G_2\left\{-\frac{35}{3} + 2\alpha + \frac{\alpha^2}{4}\right\} + \frac{11}{6}g_2^4\frac{R_2^2N_2}{D_2}N_H + \frac{10}{6}g_2^4\frac{R_2^2N_2\bar N_3}{D_2}$$

$$+g_2^4R_2^2\left(\frac{3}{2}\right) + g_1^2g_2^2R_2\left(\frac{3}{4}\right) + \frac{10}{96}g_1^4(\bar y^2(N_2+2)+2)\bar N_3$$

$$+\frac{11}{96}g_1^4N_2N_H + g_1^4\frac{3}{32} + \lambda^2\frac{(N_2+1)}{18} \tag{18}$$

Equation 18 also gives the two-loop Higgs anomalous dimension.

B. Fermion Self Energy Contributions

The fermion self-energy diagrams appear in Figures 2, 4 and 8. We give the results separately for the right- and left-handed self energies where these are different. A single listing represents as usual the sum of equal right-handed and left-hand contributions. A factor $g_B/(16\pi^2)^2$ has been suppressed throughout.

The g_H^5 contributions to β_B derive from figure 2:

2a) $\qquad 0$

2b) $\quad -[2g_B^4 + g_B^2 g_T^2 + g_T^4]\ N_2 \left\{\frac{1}{8}\right\}$

2c) $\quad \frac{1}{2}\ (\bar{g}_B^2 + \bar{g}_T^2)\ \ (g_B^2(N_2+1)\ + g_T^2)\bar{N}_3\ \left\{-\ \frac{3}{2}\right\}$

The $g_H^3 g_i^2$ contributions derive from Figure 4:

4a)

Left	Right
$\left\{[g_3^2 R_3(g_B^2+g_T^2)]\right.$	$\left\{[g_3^2R_3+g_2^2R_2+g_1^2y^2/4]g_B^2N_2\right\}$
$\left.+\ \frac{1}{4}\ [g_1^2(g_B^2(y-1)^2+g_T^2(y+1)^2)]\right\}\left\{\frac{\alpha}{4}\right\}$	$\times\left\{\frac{1}{4}\ \alpha\right\}$

4b)

Left	Right
$\left\{[g_3^2R_3+g_2^2R_2+g_1^2y^2/4](g_B^2+g_T^2)\right\}$	$\left\{[g_3^2R_3+g_1^2(y-1)^2/4]g_B^2N_2\right\}$
$\times\left\{-\frac{3}{4}\alpha\right\}$	$\times\left\{-\frac{3}{4}\alpha\right\}$

4c)

$$[g_2^2R_2+\frac{1}{4}g_1^2](g_B^2+g_T^2) \qquad [g_2^2R_2+\frac{1}{4}g_1^2]g_B^2N_2$$

$$\times\left\{\frac{5}{4}+\frac{\alpha}{4}\right\} \qquad \times\left\{\frac{5}{4}+\frac{\alpha}{4}\right\}$$

4d)

$$\frac{1}{4}g_1^2[g_T^2(y+1)-g_B^2(y-1)]\left\{3-\frac{1}{2}\alpha\right\} \qquad [g_2^2R_2+\frac{1}{4}g_1^2y]g_B^2N_2\left\{3-\frac{1}{2}\alpha\right\}$$

4e)

$$\left\{-2[g_2^2R_2(g_B^2+g_T^2)]+\frac{1}{2}[g_1^2(g_T^2-g_B^2)y]\right\} \qquad \frac{1}{2}[g_1^2g_B^2N_2(y-1)]\left\{-\frac{3}{4}-\frac{\alpha}{4}\right\}$$

$$\times\left\{-\frac{3}{4}-\frac{\alpha}{4}\right\}$$

4f)

Left	Right
$\left\{2[g_3^2R_3(g_B^2+g_T^2)]\right.$	$\left\{2[g_3^2R_3g_B^2N_2]\right.$
$\left.+\frac{1}{2}[g_1^2(g_B^2(y^2-y)+g_T^2(y^2+y))]\right\}$	$\left.+\frac{1}{2}[g_1^2g_B^2N_2(y^2-y)]\right\}$
$\times\left\{-1+\frac{\alpha}{4}\right\}$	$\times\left\{-1+\frac{\alpha}{4}\right\}$

$g_Bg_1^4$ terms arise from figure 8:

8a)

$$-[g_2^4\frac{R_2^2}{D_2}+\frac{1}{16}g_1^4y^2]N_2N_H\left\{\frac{1}{2}\right\} \qquad -\frac{1}{16}[g_1^4N_2(y-1)^2]N_H\left\{\frac{1}{2}\right\}$$

8b)

Left	Right
$\left\{\frac{1}{2}[g_3^4\bar{R}_3R_3(N_2+2)/D_3]\bar{N}_3\right.$	$\left\{\frac{1}{2}[g_3^4R_3\bar{R}_3(N_2+2)/D_3]\bar{N}_3\right.$
$+\frac{1}{2}[g_2^4R_2^2N_2/D_2]\bar{N}_3$	$\left.+\frac{1}{32}[g_1^4(\bar{y}^2(N_2+2)+2)(y-1)^2]\bar{N}_3\right\}$
$\left.+\frac{1}{32}[g_1^4(\bar{y}^2(N_2+2)+2)y^2]\bar{N}_3\right\}\left\{-2\right\}$	$\times\left\{-2\right\}$

8c)

$$\{[g_3^4R_3G_3]+[g_2^4R_2G_2]\}\left\{\frac{1}{8}+\frac{\alpha^2}{4}\right\} \qquad [g_3^4R_3G_3]\left\{\frac{1}{8}+\frac{\alpha^2}{4}\right\}$$

8d)

Left | Right

$$\left\{\frac{1}{2}\ [g_3^4 R_3 G_3] + \frac{1}{2}\ [g_2^4 R_2 G_2]\right\} \times \left\{\frac{25}{4} - \frac{3}{2}\ \alpha - \frac{1}{2}\ \alpha^2\right\}$$

$$\frac{1}{2}\ [g_3^4 R_3 G_3] \left\{\frac{25}{4} - \frac{3}{2}\ \alpha - \frac{1}{2}\ \alpha^2\right\}$$

8e)

$$\frac{1}{2}\ [g_3^4 R_3 G_3 + g_2^4 R_2 G_2] \left\{\frac{9}{2} + \frac{11}{2}\ \alpha + \alpha^2\right\}$$

$$\frac{1}{2}\ [g_3^4 R_3 G_3] \left\{\frac{9}{2} + \frac{11}{2}\ \alpha + \alpha^2\right\}$$

8f)

$$\left\{- g_3^2 (R_3^2 - \frac{1}{2}\ G_3 R_3) \quad -2 [g_2^2 g_3^2 R_2 R_3] - \frac{1}{2}\ [g_1^2 g_3^2 R_3 y^2] - [g_2^4 (R_2^2 - \frac{1}{2}\ G_2 R_2)] - \frac{1}{2}\ [g_1^2 g_2^2 R_2 y^2] - \frac{1}{16}\ [g_1^4 y^4]\right\} \left\{\frac{3}{2} - \frac{\alpha^2}{2}\right\}$$

$$\left\{- g_3^2 (R_3^2 - \frac{1}{2}\ G_3 R_3) - \frac{1}{2}\ [g_1^2 g_3^2 R_3 (y-1)^2] - \frac{1}{16}\ [g_1^4 (y-1)^4]\right\} \times \left\{\frac{3}{2} - \frac{\alpha^2}{2}\right\}$$

8g)

Left Right

$$\left\{-[g_3^4R_3^2]-2[g_2^2g_3^2R_2R_3]-\frac{1}{2}[g_1^2g_3^2R_3y^2]\right. \quad \left\{-[g_3^4R_3^2]-\frac{1}{2}[g_1^2g_3^2R_3(y-1)^2]\right.$$

$$\left.-[g_2^4R_2^2]-\frac{1}{2}[g_1^2g_2^2R_2y^2]-\frac{1}{16}[g_1^4y^4]\right\} \quad \left.-\frac{1}{16}[g_1^4(y-1)^4]\right\}\left\{\frac{1}{2}\alpha^2\right\}$$

$$\times\left\{\frac{1}{2}\alpha^2\right\} \tag{21}$$

The total left-handed fermionic self energy contribution is $g_B/(16\pi^2)^2$ times:

$$- \frac{1}{2} N_H[g_2^4 R_2 \bar{R}_2/D_2] N_2 - \frac{1}{32} N_H[g_1^4 y^2] N_2 - [g_3^4 R_3 \bar{R}_3 (N_2+2)/D_3] \bar{N}_3$$

$$- [g_2^4 R_2 \bar{R}_2 N_2/D_2] \bar{N}_3 - \frac{1}{16} [g_1^4 (\bar{y}^2 (N_2+2)+2) y^2] \bar{N}_3$$

$$+ g_3^4 \left[- \frac{3}{2} R_3^2 + \left(\frac{25}{4} + 2\alpha + \frac{\alpha^2}{4} \right) R_3 G_3 \right] - 3 [g_2^2 g_3^2 R_2 R_3] - \frac{3}{4} [g_1^2 g_3^2 R_3 y^2]$$

$$+ g_2^4 \left[- \frac{3}{2} R_2^2 + \left(\frac{25}{4} + 2\alpha + \frac{\alpha^2}{4} \right) R_2 G_2 \right] - \frac{3}{4} [g_1^2 g_2^2 R_2 y^2] - \frac{3}{32} [g_1^4 y^4]$$

$$-2 g_3^2 R_3 (g_B^2 + g_T^2) + \frac{11}{4} g_2^2 R_2 (g_B^2 + g_T^2) + g_1^2 \Big[g_B^2 \left(- \frac{1}{2} y^2 + \frac{1}{8} y + \frac{17}{16} \right)$$

$$+ g_T^2 \left(- \frac{1}{2} y^2 - \frac{1}{8} y + \frac{17}{16} \right) \Big]$$

$$- \left(\frac{1}{8} g_B^4 + \frac{1}{16} g_B^2 g_T^2 + \frac{1}{16} g_T^4 \right) N_2 - \frac{3}{8} (\bar{g}_B^2 + \bar{g}_T^2)(g_B^2 (N_2+1) + g_T^2) \bar{N}_3 \quad (22)$$

The right-handed contribution is: $g_B/(16\pi^2)^2$ times:

$$-\left(\frac{1}{8}\, g_B^4 + \frac{1}{16}\, g_B^2 g_T^2 + \frac{1}{16}\, g_T^4\right) N_2 - \frac{3}{8}\,(\bar{g}_B^2+\bar{g}_T^2)\,(g_B^2(N_2+1)+g_T^2)\bar{N}_3$$

$$-2\,[g_3^2 R_3 g_B^2 N_2] + \frac{17}{4}\,[g_2^2 R_2 g_B^2 N_2] + [g_1^2 g_B^2 N_2]\,\left(-\frac{1}{2}\,y^2 + \frac{7}{8}\,y + \frac{11}{16}\right)$$

$$-\frac{1}{32}\,[g_1^4 N_2 (y-1)^2]\,N_H - [g_3^4 R_3 \bar{R}_3 (N_2+2)/D_3]\,N_3$$

$$-\frac{1}{16}\,[g_1^4 (\bar{y}^2 (N_2+2)+2)(y-1)^2]\,\bar{N}_3$$

$$+\, g_3^4 R_3 G_3 \left[\frac{1}{4}\,\alpha^2 + 2\alpha + \frac{25}{4}\right]$$

$$-\frac{3}{2}\,[g_3^4 R_3^2] - \frac{3}{4}\,[g_1^2 g_3^2 R_3 (y-1)^2]$$

$$-\frac{3}{32}\,[g_1^4 (y-1)^4] \qquad (23)$$

The expressions in (22) and (23) represent also the two-loop fermion anomalous dimensions.

C. Three Point Contributions

These arise from the diagrams of figures 3, 6, 9 and 10c). A factor $g_B/(16\pi^2)^2$ has been suppressed throughout. The g_H^5 contributions of figure 3 are:

3a)

$$-2\,[g_B^2 g_T^2]\ \{1\}$$

3b)

$$2[g_T^4(N_2+1)+g_T^2g_B^2]\ \left\{\frac{1}{2}\right\}$$

3c)

$$0$$

3d)

$$-(\bar{g}_B^2+\bar{g}_T^2)g_T^2\bar{N}_3\{-2\}$$

3e)

$$-2[g_B^4]\{-1\}$$

The $g_H^3 g_1^2$ contributions of figure 6 are:

6a)

$$0$$

6b)

$$\left\{2[g_3^2R_3(g_B^2(N_2+1)+g_T^2)]+\frac{1}{2}\ [g_1^2(g_B^2[N_2y^2+(y-1)^2]+g_T^2[y^2-1])]\right\}\left\{\frac{1}{2}-\frac{\alpha}{2}\right\}$$

6c)

$$\left\{-4[g_3^2R_3g_T^2]+2[g_2^2R_2g_T^2]-\frac{1}{2}\ [g_1^2g_T^2(2y^2-1)]\right\}\ \{3-\alpha\}$$

6d)

$$\left\{-[g_2^2R_2(\bar{g}_B^2+\bar{g}_T^2)]\bar{N}_3-\frac{1}{4}\ [g_1^2(\bar{g}_B^2+\bar{g}_T^2)]\bar{N}_3\right\}\ \{2\alpha\}$$

6e)

$$\left\{2[g_3^2R_3+\frac{1}{4}g_1^2(y^2-y)](g_B^2(N_2+1)+g_T^2)\right\}\left\{\frac{5}{2}+\frac{1}{2}\alpha\right\}$$

6f)

$$\left\{-2[g_3^2R_3g_T^2]-\frac{1}{2}[g_1^2g_T^2(y^2-y)]\right\}\ \{5+\alpha\}$$

6g)

$$\left\{-2[g_3^2R_3g_T^2]-\frac{1}{2}[g_1^2g_T^2(y^2+y)]\right\}\ \{1+\alpha\}$$

6h)

$$\{-4[g_3^2R_3g_T^2]-[g_1^2g_T^2y^2]\}\ \{-2\}$$

6i)

$$\left\{2[g_2^2R_2(g_B^2+g_T^2)]-\frac{1}{2}[g_1^2(g_B^2N_2(y-1)-(g_B^2+g_T^2)y)]\right\}\ \left\{\frac{1}{2}\alpha\right\}$$

6j)

0

6k)

0

6l)

$$\left\{-\frac{1}{2}g_1^2[-g_B^2N_2y+g_B^2(y-1)+g_T^2(y+1)]\right\}\left\{\frac{\alpha}{2}\right\}$$

6m)

$$\left\{-2\,[g_2^2R_2g_T^2]-\frac{1}{2}\;g_1^2g_T^2\right\}\{1+\alpha\}$$

6n)

$$\left\{2\,[g_2^2R_2(g_B^2+g_T^2)]-\frac{1}{2}\;[g_1^2(-g_B^2(N_2+1)+g_T^2)]\right\}\;\left\{-\frac{\alpha}{2}\right\}$$

6o)

$$\left\{\frac{1}{2}\;[g_1^2(g_B^2N_2y+(g_T^2-g_B^2)(y-1))]\right\}\;\left\{\frac{5}{2}+\frac{\alpha}{2}\right\}$$

6p)

$$\left\{+2\,[g_2^2R_2g_T^2]-\frac{1}{2}\;[g_1^2g_T^2(2y-1)]\right\}\;\{\alpha\}$$

6q)

$$\left\{-2\;[g_2^2R_2g_T^2]-\frac{1}{2}\;[g_1^2g_T^2(2y+1)]\right\}\quad\{3-\alpha\}$$

6r)

$$0$$

6s)

$$\left\{-[g_2^2R_2(\bar{g}_B^2+\bar{g}_T^2)]\bar{N}_3-\frac{1}{4}\;[g_1^2(\bar{g}_B^2+\bar{g}_T^2)]\bar{N}_3\right\}\;\{-2\alpha\}$$

The diagram of figure 9 contribute terms of the form $g_Bg_1^4$:

9a)

$$[g_3^4R_3G_3]\;\left\{-\frac{32}{3}-4\alpha-\alpha\right\}$$

9b)

$$2[g_3^4R_3G_3]\ \left\{-\frac{7}{12}-\frac{1}{4}\alpha^2\right\}$$

9c)

$$2[g_3^4R_3G_3]\ \{-12+\alpha+\alpha^2\}$$

9d)

$$\left\{-4[g_3^4R_3^2]-2[g_2^2g_3^2R_2R_3]-\frac{1}{2}[g_1^2g_3^2R_3(4y^2-4y+1)]-\frac{1}{2}[g_1^2g_2^2R_2(y^2-y)]\right.$$

$$\left.-\frac{1}{8}[g_1^4(y^3(y-1)+(y-1)^3y)]\right\}\ \{-2\alpha\}$$

9e)

$$\left\{-4[g_3^4(R_3^2-\frac{1}{2}R_3G_3)]-2[g_2^2g_3^2R_2R_3]-\frac{1}{2}[g_1^2g_3^2R_3(4y^2-4y+1)]\right.$$

$$\left.-\frac{1}{2}[g_1^2g_2^2R_2(y^2-y)]-\frac{1}{8}[g_1^4(y^3(y-1)+(y-1)^3y)]\right\}\ \{-6+2\alpha\}$$

9f)

$$\left\{-2[g_3^4R_3^2]-[g_1^2g_3^2R_3(y^2-y)]-\frac{1}{8}[g_1^4y^2(y-1)^2]\right\}\ \{9+6\alpha+\alpha^2\}$$

9g)

$$\left\{-2[g_3^4(R_3^2-\frac{1}{2}R_3G_3)]-[g_1^2g_3^2R_3(y^2-y)]-\frac{1}{8}[g_1^4y^2(y-1)^2]\right\}\ \{3-6\alpha-\alpha^2\}$$

9h)

$$-\frac{1}{8}\ [g_1^4 N_2 N_H (y^2-y)]\ \left\{-\frac{7}{3}\right\}$$

9i)

$$\left\{g_3^4 \bar{R}_3 R_3\ \frac{(N_2+2)}{D_3}\ \bar{N}_3 + \frac{1}{16}\ [g_1^4 (\bar{y}^2 (N_2+2)+2)(y^2-y)]\bar{N}_3\right\}\left\{\frac{16}{3}\right\}$$

9j)

$$[g_2^4 R_2 G_2]\ \left\{\frac{1}{2}\ \alpha^2\right\}$$

9k)

$$[2g_2^4 R_2 G_2]\ \left\{-\frac{1}{4}\ \alpha^2\right\}$$

9l)

$$[g_2^4 R_2 G_2]\ \left\{-\frac{1}{2}\ \alpha^2\right\}$$

9m)

$$0$$

9n)

$$\left\{-2[g_2^4 R_2^2] - [g_1^2 g_2^2 R_2] - \frac{1}{8}\ [g_1^4]\right\}\ \{\alpha+\alpha^2\}$$

9o)

$$0$$

9p)

$$\left\{-2[g_2^2g_3^2R_2R_3]-\frac{1}{2}[g_1^2g_3^2R_3]-\frac{1}{2}[g_1^2g_2^2R_2(y^2-y)]-\frac{1}{8}[g_1^4(y^2-y)]\right\}$$

$$\times\ \{2\alpha\}$$

9q)

$$\left\{-2[g_2^2g_3^2R_2R_3]-\frac{1}{2}[g_1^2g_3^2R_3]-\frac{1}{2}[g_1^2g_2^2R_2(y^2-y)]-\frac{1}{8}[g_1^4(y^2-y)]\right\}\{-2\alpha\}$$

9r)

$$0$$

9s)

$$0$$

9t)

$$\left\{-[g_1^2g_2^2R_2(y-1)]-\frac{1}{4}[g_1^4(y^2-y)]\right\}\ \{\alpha^2\}$$

9u)

$$\left\{-2[g_2^4R_2^2]-[g_1^2g_2^2R_2y]-\frac{1}{8}[g_1^4(2y^2-2y+1)]\right\}\ \{3\alpha-\alpha^2\}$$

9v)

$$\left\{-2g_2^4(R_2^2-\frac{1}{2}R_2G_2)-[g_1^2g_2^2R_2]-\frac{1}{8}[g_1^4]\right\}\ \{\alpha^2\}$$

9w)

$$[g_2^4R_2G_2]\ \left\{-\frac{\alpha}{2}-\frac{\alpha^2}{2}\right\}$$

9x)

$$\left\{-2[g_2^4(R_2^2-\tfrac{1}{2}R_2G_2)]-[g_1^2g_2^2R_2y]-\tfrac{1}{8}[g_1^4(y^2+(y-1)^2)]\right\}\{-3\alpha\}$$

9y)

$$\left\{[g_1^2g_2^2R_2(y-1)]+\tfrac{1}{4}[g_1^4(y^2-y)]\right\}\qquad\{3+\alpha^2\}$$

9z)

$$\left\{4[g_2^4(R_2^2-\tfrac{1}{4}R_2G_2)]+2[g_1^2g_2^2R_2y]+\tfrac{1}{4}[g_1^4(2y^2-2y+1)]\right\}\qquad\{-\tfrac{3}{2}-\tfrac{\alpha^2}{2}\}$$

9z')

$$\left\{4[g_2^4(R_2^2-\tfrac{1}{4}R_2G_2)]+2[g_1^2g_2^2R_2]+\tfrac{1}{4}[g_1^4]\right\}\qquad\{\tfrac{1}{2}\alpha+\alpha^2\}$$

9z")

$$0 \tag{26}$$

Diagram 10c) contributes

10c)

$$\frac{-2}{3}\lambda g_B^2(N_2+1)\ \{1\} \tag{27}$$

Finally, the complete two loop β function for g_B is $g_B/(16\pi^2)^2$ times:

$$-\bar{N}_3\left\{\frac{3}{4}(N_2+1)(\bar{g}_B^4+\bar{g}_T^4)-\frac{1}{2}\bar{g}_B^2\bar{g}_T^2+\frac{3}{4}(N_2+1)(\bar{g}_B^2+\bar{g}_T^2)g_B^2-\frac{5}{4}(\bar{g}_B^2+\bar{g}_T^2)g_T^2\right\}$$

$$-g_B^4\left(\frac{N_2}{4}-2\right)-g_B^2g_T^2\left(\frac{N_2}{8}+1\right)+g_T^4\left(\frac{7}{8}N_2+1\right)$$

$$+5g_3^2R_3(\bar{g}_B^2+\bar{g}_T^2)N_3-16g_3^2R_3g_T^2+4g_3^2R_3(g_B^2(N_2+1)+g_T^2)$$

$$+\frac{17}{4}g_2^2R_2g_B^2N_2+\frac{5}{2}g_2^2R_2(\bar{g}_B^2+\bar{g}_T^2)\bar{N}_3+\frac{11}{4}g_2^2R_2(g_B^2+g_T^2)$$

$$-2g_2^2R_2g_T^2+\frac{5}{4}g_1^2\left[\bar{g}_B^2(\bar{y}^2-\bar{y}+\frac{1}{2})+\bar{g}_T^2(\bar{y}^2+\bar{y}+\frac{1}{2})\right]\bar{N}_3$$

$$-\frac{1}{2}g_1^2\left[g_B^2N_2(-2y^2-\frac{7}{4}y-\frac{11}{8})+g_B^2(-2y^2+\frac{23}{4}y-\frac{41}{8})+g_T^2(-2y^2+\frac{1}{4}y+\frac{7}{8})\right]$$

$$-\frac{1}{2}g_1^2g_T^2(8y^2+2y+1)$$

$$-\frac{3}{8}g_1^4(\frac{1}{2}y^4-y^3-\frac{1}{2}y^2+y+1)-\frac{1}{8}g_1^4N_2N_H(-\frac{2}{3}+\frac{11}{6}y-\frac{11}{6}y^2)$$

$$+\frac{1}{16}g_1^4\bar{N}_3(\bar{y}^2(N_2+2)+2)(\frac{2}{3}-\frac{10}{3}y+\frac{10}{3}y^2)$$

$$-\frac{1}{2}g_1^2g_2^2R_2(\frac{9}{2}+6y-\frac{9}{2}y^2)-\frac{1}{2}g_1^2g_3^2R_3(3y^2-3y-\frac{9}{2})$$

$$+\frac{4}{9}g_2^4R_2\bar{R}_2\bar{N}_2N_H+\frac{2}{9}g_2^4\bar{R}_2R_2\bar{N}_2\bar{N}_3-2g_2^4[3R_2^2+\frac{47}{24}G_2R_2]$$

$$+9g_2^2g_3^2R_2R_3+g_3^4(-3R_3^2-\frac{97}{3}G_3R_3)$$

$$+\frac{10}{3}\,g_3^4\bar{R}_3 R_3\,\frac{\bar{N}_3}{D_3}\,(N_2+2)$$

$$-\frac{2}{3}\,\lambda g_B^2(N_2+1)+\frac{\lambda^2}{18}\,(N_2+1) \qquad (28)$$

When the sum over the loop fermions is performed, β_B is

$$g_B/(16\pi^2)^2\times\Bigg\{\left[-\frac{3}{2}\,N_c(N_2+1)-\frac{N_2}{4}+2\right]g_B^4+\left[N_c[1-\frac{3}{4}\,N_2]-\frac{N_2}{8}-1\right]g_B^2g_T^2$$

$$-\frac{3}{4}\,(N_2+1)g_B^2g_\tau^2-\frac{3}{4}\,N_c(N_2+1)g_B^2(g_C^2+g_S^2)$$

$$+g_B^2g_3^2R_3(5N_c+4N_2+4)+g_B^2g_2^2R_2(N_2\,\frac{17}{4}+N_c\,\frac{5}{2}+\frac{11}{4})$$

$$+g_B^2g_1^2\left(N_c\,\frac{25}{72}+N_2\,\frac{157}{144}+\frac{247}{144}\right)+g_T^4\left[N_c(\frac{1}{2}-\frac{3}{4}\,N_2)+\frac{7}{8}\,N_2+1\right]$$

$$+\frac{5}{4}\,g_T^2g_\tau^2+\frac{5}{4}\,g_T^2(g_S^2+g_C^2)N_c+g_T^2g_3^2R_3(N_c5-12)$$

$$+g_T^2g_2^2R_2(N_c\,\frac{5}{2}+\frac{3}{4})+g_T^2g_1^2(N_c\,\frac{85}{72}-\frac{79}{48})-\frac{3}{4}\,g_\tau^4(N_2+1)$$

$$+\frac{5}{2}\,g_\tau^2g_2^2R_2+\frac{25}{8}\,g_\tau^2g_1^2+\frac{1}{2}\,g_S^2g_C^2N_c$$

$$5(g_S^2+g_C^2)g_3^2R_3+\frac{5}{2}(g_S^2+g_C^2)g_2^2R_2N_c+\frac{25}{72}g_S^2g_1^2N_c+\frac{85}{72}g_C^2g_1^2N_c$$

$$-\frac{3}{4}(g_S^4+g_C^4)N_c(N_2+1)$$

$$+g_3^4\left[R_3^2\frac{(N_C(N_2+2)N_G)}{2D_3}\frac{20}{3}-3R_3^2+R_3G_3(-\frac{97}{3})\right]$$

$$+9g_3^2g_2^2R_3R_2+\frac{31}{12}g_3^2g_1^2R_3$$

$$+g_2^4\left[R_2^2(N_2N_H\frac{4}{9}+(N_c+1)N_2N_G\frac{2}{9}-6)-G_2R_2(\frac{47}{12})\right]$$

$$-3g_2^2g_1^2R_2-\frac{2}{3}\lambda g_B^2(N_2+1)+\frac{\lambda^2}{18}(N_2+1)$$

$$+\frac{g_1^4}{216}\left[7N_HN_2-N_G(\frac{N_c}{9}(N_2+20)+N_2+4)-101\right]\Bigg\} \quad (29)$$

N_g is the number of generations, and N_H the number of Higgs. The g_S, g_C, and g_τ terms are intended to represent terms involving upper, lower, and leptonic couplings of other generations. Similarly, β_T is $g_T/(16\pi^2)^2$ times

$$g_T^4\left[-\frac{3}{2}N_c(N_2+1)-\frac{N_2}{4}+2\right]+g_T^2g_B^2\left[N_c\left(1-\frac{3}{4}N_2\right)-\frac{N_2}{8}-1\right]$$

$$-\frac{3}{4}g_T^2g_\tau^2(N_2+1)-\frac{3}{4}g_T^2(g_C^2+g_S^2)(N_c)(N_2+1)+g_T^2g_3^2R_3(5N_c+4N_2+4)$$

$$+g_T^2g_2^2R_2\left(N_2\frac{17}{4}+N_c\frac{5}{2}+\frac{11}{4}\right)+g_T^2g_1^2\left(N_c\frac{85}{72}+N_2\frac{73}{144}+\frac{523}{144}\right)$$

$$+g_B^4\left[N_c\left(\frac{1}{2}-\frac{3}{4}N_2\right)+\frac{7}{8}N_2+1\right]+\frac{5}{4}g_B^2g_\tau^2+\frac{5}{4}g_B^2(g_C^2+g_S^2)N_c$$

$$+g_B^2g_3^2R_3(5N_c-12)+g_B^2g_2^2R_2\left(\frac{5}{2}N_c+\frac{3}{4}\right)+g_B^2g_1^2\left(N_c\frac{25}{72}-\frac{43}{48}\right)$$

$$-\frac{3}{4}g_\tau^4(N_2+1)+\frac{5}{2}g_\tau^2g_2^2R_2+\frac{25}{8}g_\tau^2g_1^2-\frac{3}{4}(g_C^4+g_S^4)N_c(N_2+1)$$

$$+\frac{1}{2}g_C^2g_S^2N_c+5(g_C^2+g_S^2)g_3^2R_3N_c+\frac{5}{2}(g_C^2+g_S^2)g_2^2R_2N_c$$

$$+ \frac{85}{72} g_C^2 g_1^2 N_c + \frac{25}{72} g_S^2 g_1^2 N_c$$

$$+ g_3^4 \left[R_3^2 \left(\frac{(N_G N_c (N_2+2)}{2 \quad D_3} -3 \right) + R_3 G_3 \left(- \frac{97}{3} \right) \right]$$

$$+9 \ g_3^2 g_2^2 R_3 R_2 + \frac{19}{12} g_3^2 g_1^2 R_3$$

$$+g_2^4 \left[R_2^2 \left(N_2 N_H \ \frac{4}{9} + N_2 N_G (N_c+1) \ \frac{2}{9} -6 \right) + G_2 R_2 \left(- \frac{47}{12} \right) \right]$$

$$-g_2^2 g_1^2 R_2$$

$$+ \frac{29}{216} g_1^4 N_G \left(\frac{N_c}{9} (N_2+20) + N_2 + 4 \right)$$

$$- \frac{2\lambda}{3} g_T^2 (N_2+1) + \frac{\lambda^2}{18} (N_2+1) \qquad (30)$$

Finally, β_τ for g_τ is $g_\tau/(16\pi^2)^2$ times

$$
\begin{aligned}
& g_\tau^4\left[-\frac{3}{2}(N_2+1)-\frac{N_2}{4}+2\right]-\frac{3}{4}g_\tau^2(g_B^2+g_T^2)N_c(N_2+1)-\frac{3}{4}g_\tau^2 g_\mu^2(N_2+1)\\
&+g_\tau^2 g_2^2 R_2\left(N_2\frac{17}{4}+\frac{21}{4}\right)+g_\tau^2 g_1^2\left(\frac{13}{16}N_2+\frac{153}{16}\right)\\
&-\frac{3}{4}(g_B^4+g_T^4)N_c(N_2+1)+\frac{1}{2}g_B^2 g_T^2 N_c+5(g_T^2+g_B^2)g_3^2 R_c N_c\\
&+\frac{5}{2}(g_T^2+g_B^2)g_2^2 R_2 N_c+\frac{25}{72}g_B^2 g_1^2 N_c+\frac{85}{72}g_T^2 g_1^2 N_c\\
&-\frac{3}{4}g_\mu^4+\frac{5}{2}g_\mu^2 g_2^2 R_2+\frac{25}{8}g_\mu^2 g_1^2\\
&+g_2^4\left[R_2^2\left(N_2 N_H\frac{4}{9}+(N_c+1)N_2 N_G\frac{2}{9}-6\right)+R_2 G_2\left(-\frac{47}{12}\right)\right]\\
&+3g_2^2 g_1^2 R_2+g_1^4\left[N_G\left(N_c\frac{(N_2+20)}{9}+4+N_2\right)\frac{11}{24}+N_2 N_H\frac{13}{24}-\frac{3}{8}\right]\\
&-\frac{2\lambda}{3}g_\tau^2(N_2+1)+\frac{\lambda^2}{18}(N_2+1)
\end{aligned}
\tag{31}
$$

g_μ represents the coupling for a lepton μ of a different generation than τ.

The specialization of the above results to the standard theory ($N_3=2$, $N_2=2$, etc.) has already been given in reference [6].

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REFERENCES

[1] J. Oliensis and M. Fischler, "Two Loop Calculations of M_B/M_τ and Heavy Fermion Masses in the SU(5) Model," to be published in Physical Review D, July 1983.

[2] D.V. Nanopoulos and D.A. Ross, Nucl. Phys. B157 (1979) 273.

[3] D.V. Nanopoulos and D.A. Ross, Phys. Lett. 118 (1982) 99.

[4] C.T. Hill, Phys. Rev. D24 (1981) 691; M. Machacek and M. Vaughn, Phys. Lett. 103B (1981) 427; B. Pendleton and G.G. Ross, Phys. Lett. 98B (1981) 291.

[5] See for example L. Hall, J. Lykken, and S. Weinberg, University of Texas Report No. UTTG-1-83 (to be published).

[6] M. Fischler and J. Oliensis, Phys. Lett. 119B (1982) 385. This calculation is also being carried out by M. Machacek and M. Vaughn, (private communication). For their results for the anomalous dimensions in a general field theory see Ref. 7.

[7] See M. Machacek and M. Vaughn, "Two-Loop Renormalization Group Equations in a General Quantum Field Theory I," Northeastern University preprint NUB-2590, and references therein. We received this preprint following the completion of our paper, and the publication of our results for the Weinberg-Salam beta function (Ref. 6).

[8] D.J. Gross, in Methods in Field Theory, Les Houches 1975, R. Balian and J. Zinn-Justin, ed.

[9] E. Braaten and J.P. Leveille, Phys. Rev. D24 (1981) 1369.

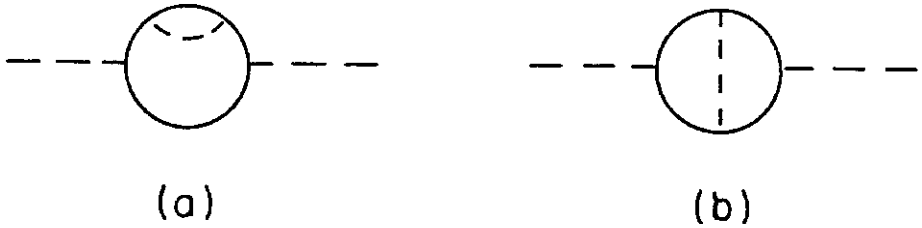
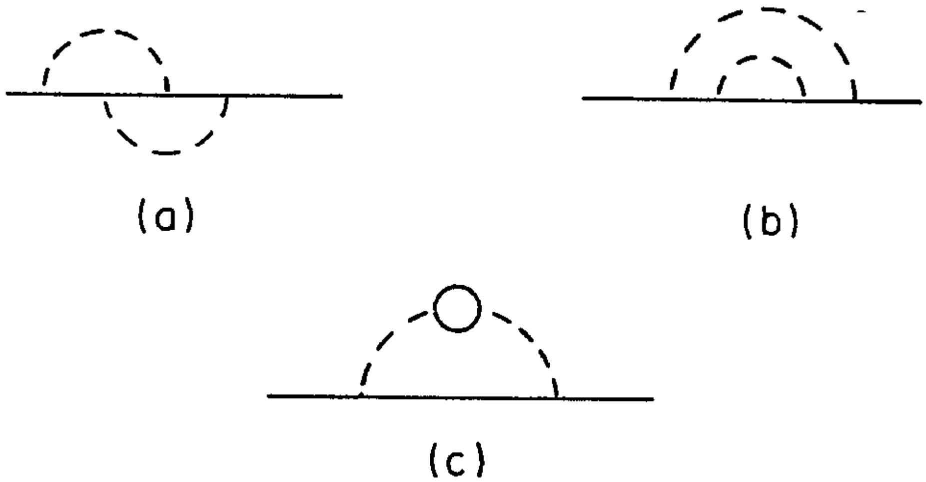


Fig. 1



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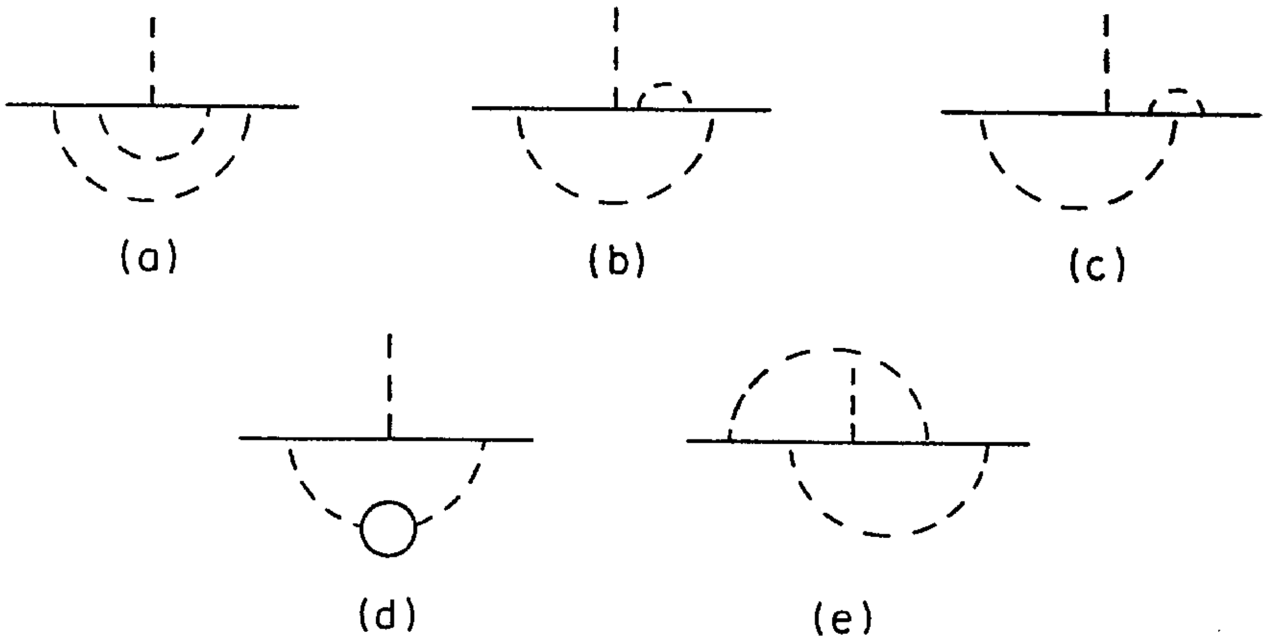

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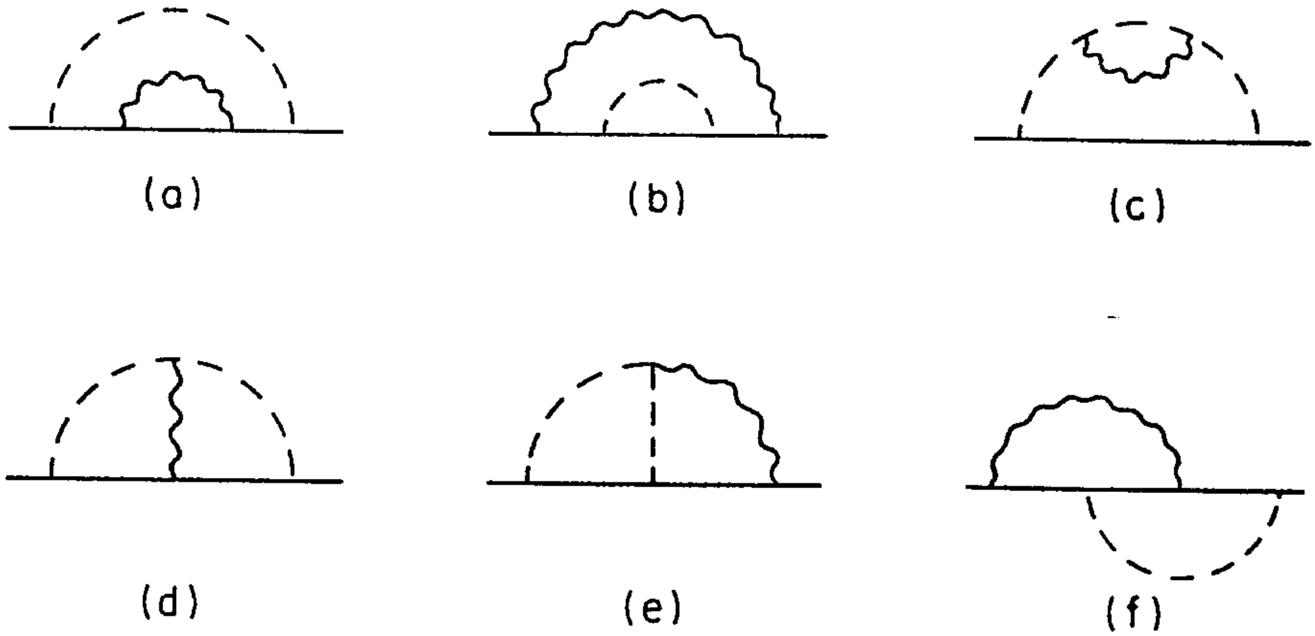

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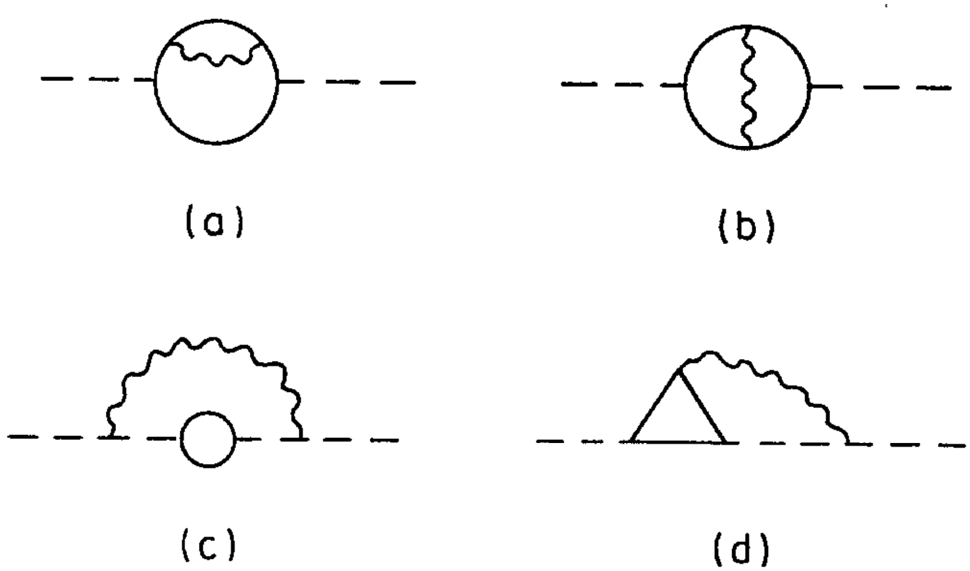

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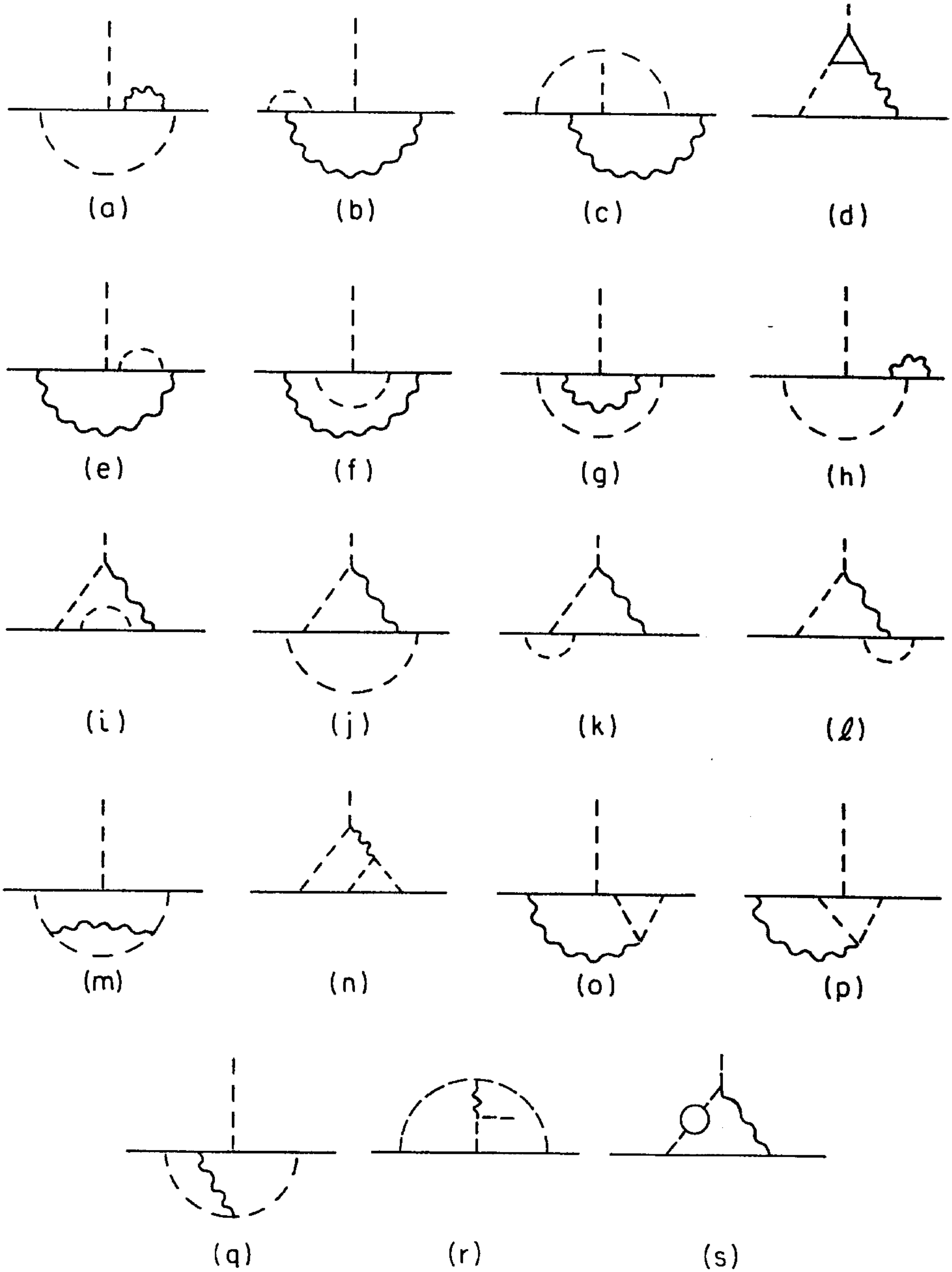

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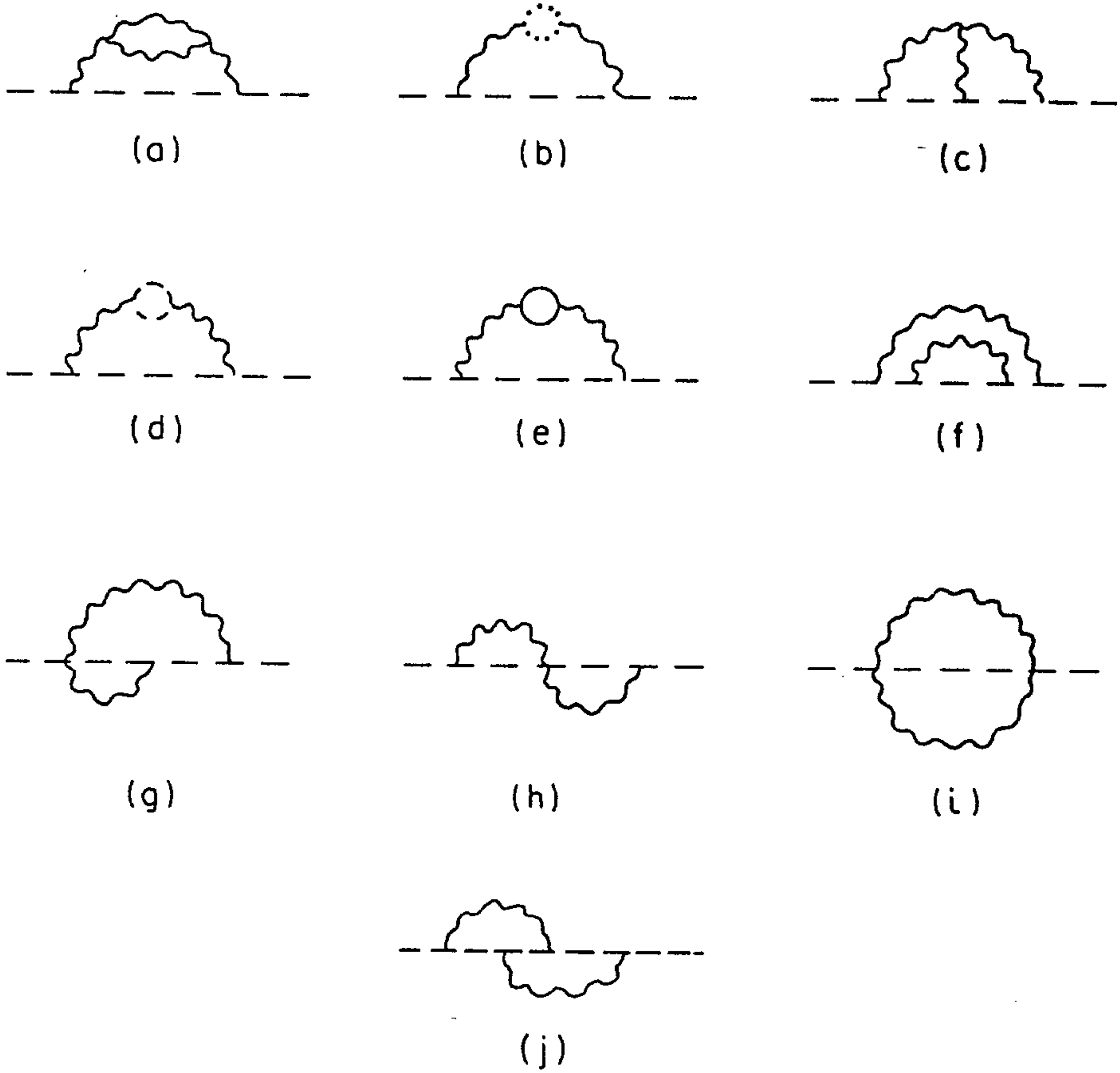

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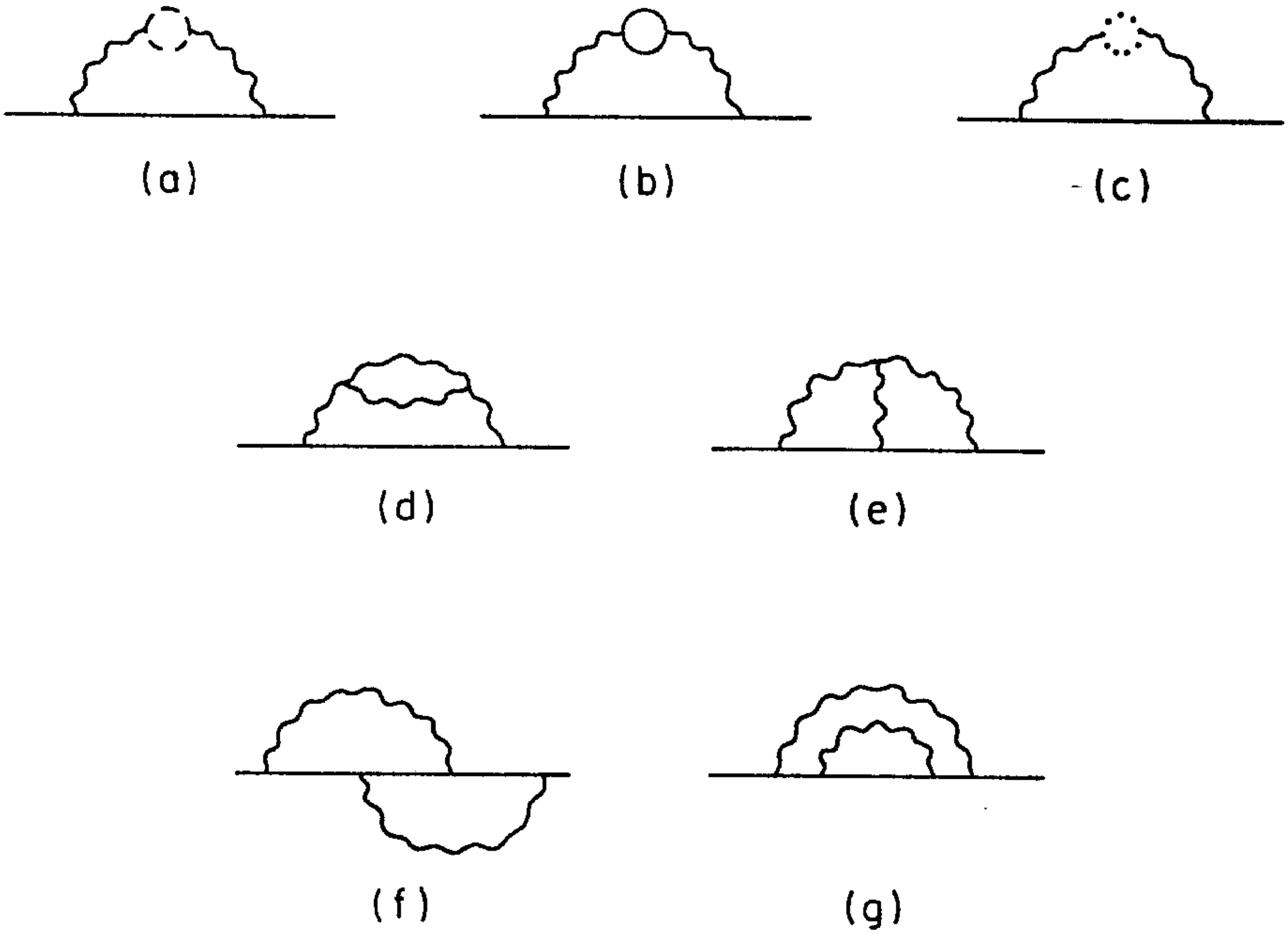

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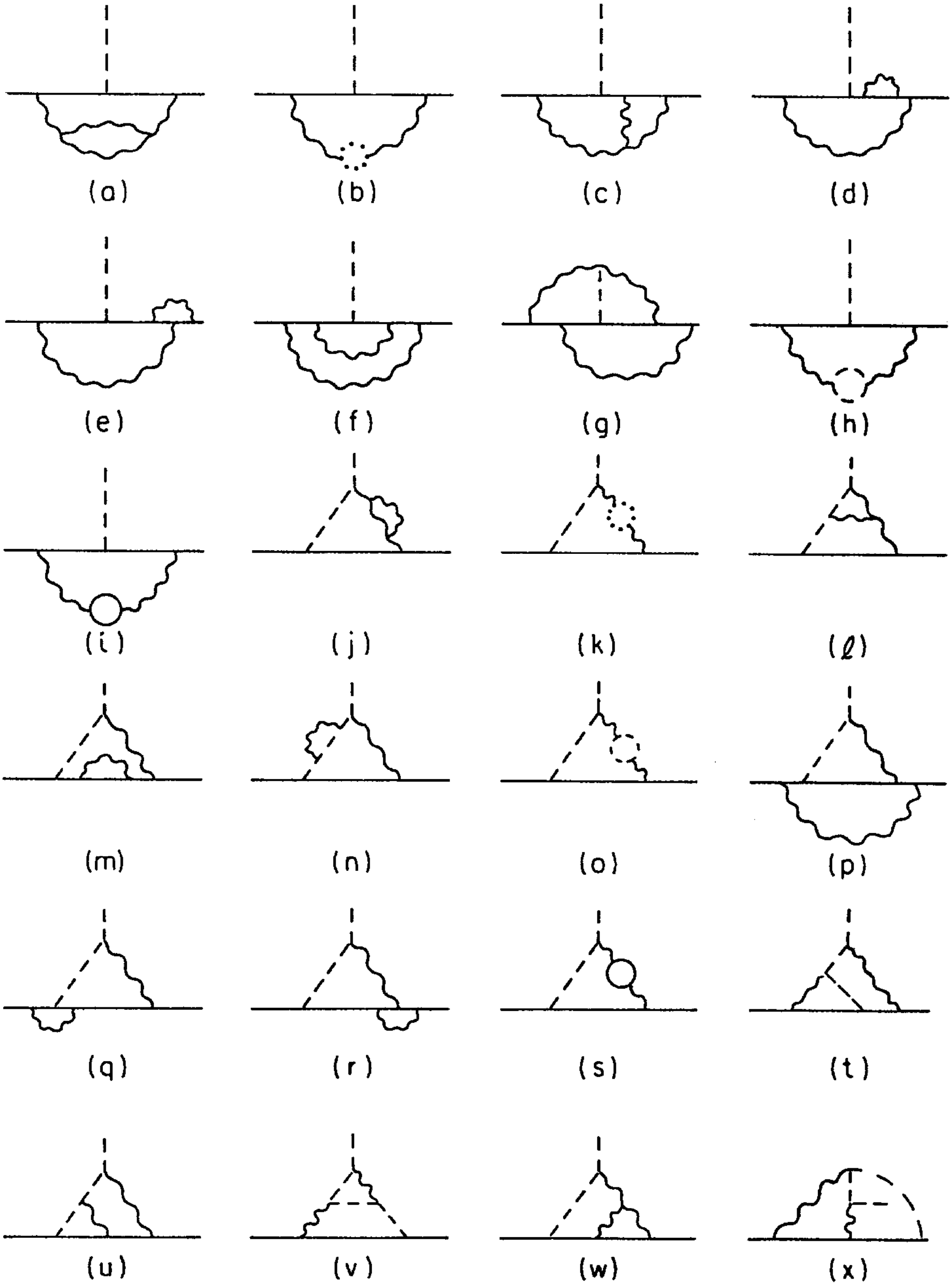
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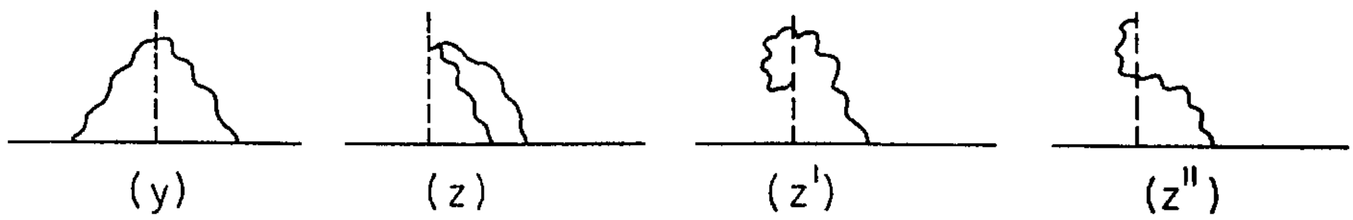

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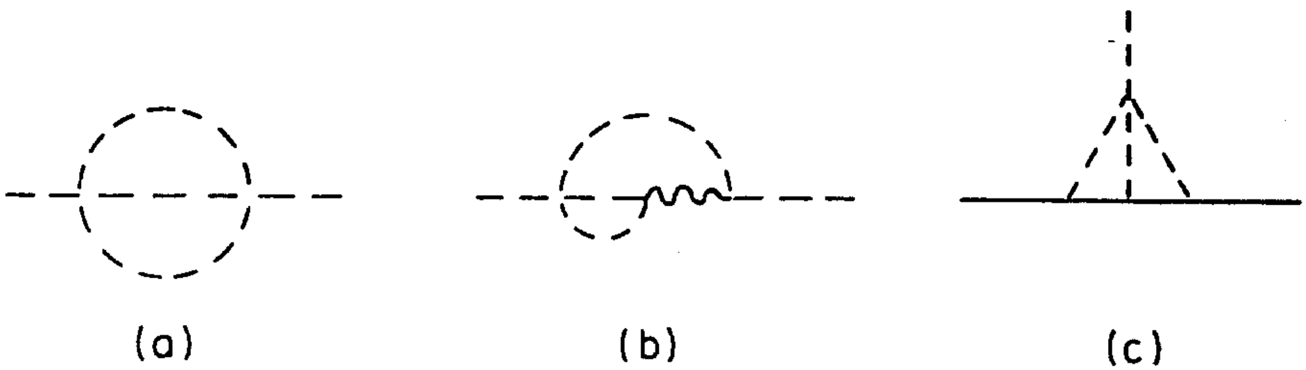

Fig. 10

FIGURE CAPTIONS

Fig. 1: g_H^4 contributions to the Higgs anomalous dimension.

Fig. 2: g_H^4 contributions to the fermion anomalous dimension.

Fig. 3: g_H^5 contributions to the Higgs-Yukawa β.

Fig. 4: $g_H^2 g_i^2$ contributions to the fermion anomalous dimension.

Fig. 5: $g_H^2 g_i^2$ contributions to the Higgs anomalous dimension.

Fig. 6: $g_H^3 g_i^2$ contributions to the Higgs-Yukawa β.

Fig. 7: g_i^4 contributions to the Higgs anomalous dimension.

Fig. 8: g_i^4 contributions to the fermion anomalous dimension.

Fig. 9: g_i^4 contributions to the Higgs-Yukawa β.

Fig. 10: Contributions to β involving the scalar self-coupling.

AUTHORS: PLEASE FILL IN THE FOLLOWING HANDWRITTEN SYMBOLS AND NUMBERS FOR CLARITY WHEN TYPING

Oliensis

	Typed Capitals	Handwritten Caps.	Typed L.C.	Handwritten L
Alpha	A	A	α	α
Beta	B	B	β	β
Gamma	Γ	Γ	γ	γ
Delta	Δ	Δ	δ	δ
Epsilon	E	E	ε	ε
Zeta	Z	Z	ζ	ζ
Eta T	H	H	η	η
Theta	Θ	Θ	θ	θ
Iota	I	I	ι	ι
Kappa	K	K	κ	κ
Lambda	Λ	Λ	λ	λ
Mu	M	M	μ	μ
Nu	N	N	ν	ν
Xi	Ξ	Ξ	ξ	ξ
Omicron	O	O	o	o
Pi	Π	Π	π	π
Rho	P	P	ρ	ρ
Sigma	Σ	Σ	σ	σ
Tau	T	T	τ	τ
Upsilon	Υ	Υ	υ	υ
Phi	Φ	φ	φ	φ
Chi	X	χ	χ	χ
Psi	Ψ	ψ	ψ	ψ
Omega	Ω	Ω	ω	ω

NUMBERS: 1 1, 2 2, 3 3, 4 4, 5 5, 6 6, 7 7, 8 8, 9 9, 0 0.

SCRIPT: A a B b C c D d E e F f G g H h I i J j K k L l M m N n O o P p Q q R r S s T t U u V v W w X x Y y Z z

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