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Composite Model with Confining SU(N) x $SU(2)_L$ x $SU(2)_R$ Hypercolor

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ABSTRACT

A model of composite quarks and leptons is constructed with confining SU(N) x $SU(2)_L$ x $SU(2)_R$ hypercolor interactions such that only standard quark and lepton families appear in global $SU(2)_L$ x $SU(2)_R$ doublets. Several generations are admitted by the anomaly-matching conditions and labeled by a discrete axial symmetry. The SU(N) interactions are N independent and play the role of technicolor. Three conserved $U(1)_V$ charges identified with Q, B − L and B + L prohibit $qqq \rightarrow \ell^c$ transitions.

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With little or no support from the experimental realm, an extensive literature[f1] on the subject of composite quarks and leptons has emerged over the past few years. Many of the papers are based on the pioneering work of 't Hooft[3] who formulated several criteria which composite models should satisfy in order to explain why quarks and leptons are nearly massless on the large energy scale where the hypercolor forces become sufficiently strong to bind the massless preons together. General searches for candidate preon models have been carried out, or specfific models themselves have been proposed, in which the fundamental preons are either all spinors or spinors and scalars and the weak gauge fields are either fundamental or composite. With only fermionic preons and fundamental gauge fields, 't Hooft-type models have been found by Bars and Yankielowicz,[4] while models with some chiral symmetry breaking have been explored by many authors.[5] Models with composite gauge fields and/or scalar preons are illustrated by those of ref. 6.

Here we take special note of the Abbott-Farhi model[6] in which only the left-handed quarks and leptons are composite while the right-handed ones are fundamental. The $SU(2)_L$ interaction is strong and confining, while the observed weak $SU(2)_L$ interaction appears as a residual force between global symmetry doublets. Elementary scalars were eliminated from this model in papers by Abbott, Farhi and Schwimmer[7] and by Bordi, Casalbuoni, Dominici and Gatto.[8]

More recently, B. Schrempp and F. Schrempp[9] have extended the Abbott-Farhi model by considering confining $SU(2)_L \times SU(2)_R$ interactions, so that both left- and right-handed quarks and leptons are composite with a simple choice of just a few fermionic preons.

Since no particular preon model has emerged as the "standard" model, it seems worthwhile to pursue models in all possible categories. In this paper we report the results of an attempt to construct a model with three confining gauge groups: $SU(N) \times SU(2)_L \times SU(2)_R$. Left-right symmetric weak interactions appear as residual interactions with the composite quarks and leptons forming $SU(2)_L \times SU(2)_R$ global doublets.

Several interesting features emerge in the model to be presented. If the original hyperflavor group is taken to be $SU(4)_L \times SU(4)_R$, anomaly matching[3] at the preon and composite levels is achieved once this group is broken down to chiral SU(3) via a natural multipreon condensate. The N independent hypercolor interactions at first sight produce just one complete family of quarks and leptons in global $SU(2)_L \times SU(2)_R$ doublets, but 3 or 5 generations can be recovered in two interesting special cases. The SU(N) group plays the role of technicolor[10] in forming scalars from which the composite W gauge fields are created by the confining $SU(2)_L \times SU(2)_R$ interactions. A $U(1)_A$ group is identified which prevents quarks and leptons from pairing off and becoming massive. The pseudo-Goldstone bosons[11] which arise from the chiral SU(4) symmetry breaking all carry color and can be used to give masses to the quarks and leptons. Three conserved $U(1)_V$ groups emerge along with the chiral SU(3) and $U(1)_A$ hyperflavor groups to which can be assigned charge Q, B - L and B + L generators. An effective $qqq \to \ell^c$ transition leading to proton decay is thus forbidden by preon rearrangement at the hypercolor scale. Details of the model follow.

The preons in our model are assigned to the reducible representation of the three confining groups $SU(N) \times SU(2)_L \times SU(2)_R$ according to $(N;2,2)_L + (N;2,2)_R + (N;2,1)_L + (N;1,2)_R + 4(1;2,1)_L + 4(1;1,2)_R$. The $U(4)_L \times U(4)_R \times (U(1))^4$ hyperflavor symmetry is broken by instanton effects down to

$$G_{HF} = SU(4)_L \times SU(4)_R \times U(1)_V \times U(1)_{V'} \times U(1)_A \tag{1}$$

whereby the (left-handed) preons appear in the representations

$$\begin{aligned} T &= (N;2,2;1,1)_{1,0;2} & T' &= (\bar{N};2,2;1,1)_{-1,0;2} \\ U &= (N;2,1;1,1)_{0,4;-4} & U' &= (\bar{N};1,2;1,1)_{0,-4;-4} \\ V &= (1;2,1;4,1)_{0,-N;-N} & V' &= (1;1,2;1,\bar{4})_{0,N;-N} \end{aligned} \tag{2}$$

The U(1) values are assigned by constructing from the preon number operators three conserved generators

$$
\begin{aligned}
Y_1 &= n_T - n_{T'} \\
Y_2 &= 4(n_U - n_{U'}) - N(n_V - n_{V'}) \\
X &= 2(n_T + n_{T'}) - 4(n_U + n_{U'}) - N(n_V + n_{V'})
\end{aligned} \tag{3}
$$

which are orthogonal to the three generators broken by instanton effects

$$
\begin{aligned}
Q_{HC} &= 2(n_T + n_{T'}) + (n_U + n_{U'}) \\
Q_L &= 2N(n_T + n_{T'}) + Nn_U + 4n_V \\
Q_R &= 2N(n_T + n_{T'}) + Nn_{U'} + 4n_{V'}
\end{aligned} \tag{4}
$$

The $U(1)_A$ generator X keeps all preons massless.

Composite fermions can be constructed which are singlets under the three confining groups according to

$$
\begin{array}{llll}
\ell_1: & V(TU') = (4,1) & \ell_{1'}: & V'(T'U) = (1,\bar{4}) \\
\ell_2: & V(TU')^+ = (4,1) & \ell_{2'}: & V'(T'U)^+ = (1,\bar{4}) \\
\ell_3: & V^+(T'U'^+) = (\bar{4},1) & \ell_{3'}: & V'^+(TU^+) = (1,4) \\
\ell_4: & V^+(T'U'^+)^+ = (\bar{4},1) & \ell_{4'}: & V'^+(TU^+)^+ = (1,4)
\end{array} \tag{5}
$$

where only the chiral SU(4) representations are indicated in parentheses. It is a simple matter to check that the anomaly matching conditions[3] of 't Hooft fail. However, a nonzero value for the multipreon condensate[f2]

$$
\langle TT'VV'(VV')^+ \rangle = (1;1,1;15,15)_{0,0;4} \neq 0 \tag{6}
$$

for example, results in a chiral symmetry breaking of the hyperflavor group of (1) into

$$G_{HF'} = SU(3)_L \times SU(3)_R \times U(1)_V \times U(1)_{V'} \times U(1)_{V''} \times U(1)_{A'} \tag{7}$$

such that the preons now transform according to

$$\begin{array}{ll}
T = (N;2,2;1,1)_{1,0,0;0} & T' = (\bar{N};2,2;1,1)_{-1,0,0;0} \\
U = (N;2,1;1,1)_{0,4,0;0} & U' = (\bar{N};1,2;1,1)_{0,-4,0;0} \\
Q = (1;2,1;3,1)_{0,-N,1;1} & Q' = (1;1,2;1,\bar{3})_{0,N,-1;1} \\
L = (1;2,1;1,1)_{0,-N,-3;-3} & L' = (1;1,2;1,1)_{0,N,3;-3}
\end{array} \tag{8}$$

where the first two $U(1)_V$ quantum numbers are Y_1 and Y_2, as before, and the new third $U(1)_{V''}$ and $U(1)_{A'}$ generators are

$$\begin{aligned}
Y_3 &= n_Q - n_{Q'} - 3(n_L - n_{L'}) \\
X' &= n_Q + n_{Q'} - 3(n_L + n_{L'})
\end{aligned} \tag{9}$$

respectively. The original $U(1)_A$ is broken down[f3] to the discrete symmetry $Z(2)_A$ by the condensate in (6). One finds that the anomaly matching conditions now can be satisfied for any value of N with the index restriction

$$\ell_A - \ell_B = 1 \tag{10}$$

where $\ell_A = \ell_1 = \ell_2 = \ell_{1'} = \ell_{2'}$; $\ell_B = \ell_3 = \ell_4 = \ell_{3'} = \ell_{4'}$. With $\ell_A = 1$ and $\ell_B = 0$, the composite states are just

$$\begin{array}{ll}
L(TU') = (1,1)_{1,-(4+N),-3;-3} & L'(T'U) = (1,1)_{-1,4+N,3;-3} \\
L(TU')^+ = (1,1)_{-1,(4-N),-3;-3} & L'(T'U)^+ = (1,1)_{1,-(4-N),3;-3} \\
Q(TU') = (3,1)_{1,-(4+N),1;1} & Q'(T'U) = (1,\bar{3})_{-1,4+N,-1;1} \\
Q(TU')^+ = (3,1)_{-1,(4-N),1;1} & Q'(T'U)^+ = (1,\bar{3})_{1,-(4-N),-1;1}
\end{array} \tag{11}$$

where the representations of the broken hyperflavor group $G_{HF'}$ in (7) are indicated. A global $SU(2)_L \times SU(2)_R$

symmetry appears for the composite states resulting in a global doublet structure which did not exist for the preons.

Just one generation of standard quarks and leptons appears by restricting the indices to 0 or 1, if we make the following identification of the hypercharges (see Table I):

$$\begin{aligned} I_{3L} + I_{3R} &= \tfrac{1}{2} Y_1 \\ B - L &= \tfrac{1}{3} Y_3 \\ B + L &= -\tfrac{2}{N} Y_1 - \tfrac{1}{2N} Y_2 - \tfrac{1}{6} Y_3 \end{aligned} \tag{12}$$

with the electric charge given by

$$Q = I_{3L} + I_{3R} + \tfrac{1}{2}(B - L) = \tfrac{1}{2} Y_1 + \tfrac{1}{6} Y_3 \tag{13}$$

The appearance of a third conserved $U(1)_V$ enables one to define $B + L$ as well as $B - L$, so proton decay-type transitions $qqq \rightarrow \ell^c$ are forbidden at the confining scale Λ by simple preon rearrangement. In the case of $N = 4$ a condensate such as

$$\langle UU(U'U')^+\rangle = (1;1,1;1,1)_{0,16,0;0} \tag{14}$$

can form which would break the $B + L$ conserved symmetry and allow the $qqq \rightarrow \ell^c$ transition to proceed at a reduced rate.

The $SU(N)$ group acts as technicolor[10] in that the following $SU(N)$- singlet scalar mesons can be formed which are confined by the chiral $SU(2)$ interactions:

$$H = TU' = (1;2,1;1,1)_{1,-4,0;0}$$
$$K = T'U = (1;1,2;1,1)_{-1,4,0;0} \tag{15}$$

as well as their Hermitian conjugates, all with B = L = 0. The composite weak bosons which transmit the (residual) weak force between the global chiral SU(2) doublets are formed by the confining SU(2) interactions:

$$\begin{aligned} W_L^{+} &= H\partial_\mu H & \qquad W_R^{+} &= K^{+}\partial_\mu K^{+} \\ W_L^{o} &= H^{+}\overleftrightarrow{\partial}_\mu H & \qquad W_R^{o} &= K^{+}\overleftrightarrow{\partial}_\mu K \\ W_L^{-} &= H^{+}\partial_\mu H^{+} & \qquad W_R^{-} &= K\partial_\mu K \end{aligned} \tag{16}$$

The 12 pseudo-Goldstone bosons which result from the chiral symmetry breaking condensate in (6) transform as

$$\begin{aligned} TT'QL^{+}L'L'^{+} &= (3,1)_{0,0,4;4} & \qquad TT'LL^{+}Q'L'^{+} &= (1,\bar{3})_{0,0,-4;4} \\ (TT')^{+}LQ^{+}L'L'^{+} &= (\bar{3},1)_{0,0,-4;-4} & \qquad (TT')^{+}LL^{+}L'Q'^{+} &= (1,3)_{0,0,4;-4} \end{aligned} \tag{17}$$

and all carry chiral color with $B = \pm 1/3$, $L = \mp 1$ and $Q = \pm 2/3$. They can combine with ordinary quarks to produce (heavy) leptons and/or objects with both integer baryon and lepton number. The only true Goldstone boson is of the form

$$\mathrm{Im}\ TT' = (1,1)_{0,0,0;0} \tag{18}$$

One can break the chiral SU(3) group to $SU(3)_c$ by gauging just the vectorial part and thereby obviate the appearance of spectator objects in the model. The pseudo-Goldstone bosons should then contribute masses to the quarks and leptons via loop diagrams.

We have noted earlier that the anomaly matching condition (10) follows in a hypercolor N-independent fashion. Presumably certain values of N will be singled out if one successfully unifies the confining hypercolor interactions. Clearly several mass scales appear in the problem: Λ_{HC}, Λ_L, Λ_R and Λ_c which will effect the evolution of the couplings g_{HC}, g_L, g_R and g_c. Depending on the unification scheme, not all interactions need be asymptotically free, but low N values are favored.

This brings us to the generation problem, as only one generation appeared above with $\ell_A = 1$, $\ell_B = 0$. Three or five generations can be recovered if we set $\ell_A = 2$, $\ell_B = 1$ or $\ell_A = 3$, $\ell_B = 2$. In order to distinguish the quark and lepton generations, we can make use of the discrete symmetry arising from the broken $U(1)_A$ generator. This suggests that we replace the single condensate in (6) by three condensates such as

$$\begin{aligned} \langle TV(TV)^+\rangle &= (1;1,1;15,1)_{0,0;0} \neq 0 \\ \langle T'V'(T'V')^+\rangle &= (1;1,1;1,15)_{0,0;0} \neq 0 \end{aligned} \tag{19}$$

and either

$$\langle TT'UUU'U'\rangle = (1;1,1;1,1)_{0,0;-12} \neq 0 \tag{20a}$$

or

$$\langle TT'(UUU'U')^{+}\rangle = (1;1,1;1,1)_{0,0;20} \neq 0 \qquad (20b)$$

With (20a) the resultant discrete symmetry is $Z(6)_A$, while for (20b) it is $Z(10)_A$, which enables one to identify three or five generations, respectively. Generations of type A are prevented from pairing off and becoming massive by the conserved $U(1)_{A'}$ symmetry, while type A and B generations are only prevented from doing so by the remaining discrete symmetry.

The model presented clearly exhibits a number of interesting features, while some pertinent issues remain. It is of interest to unify the interactions , to determine the various energy scales and to trace the evolution of the strong coupling parameters. Although several generations can be accommodated by the discrete axial symmetry, it is crucial that flavor-changing neutral currents be suppressed to a sufficiently high degree. The relative scales of the 6-, 4- and 2-preon condensates can influence this issue as well as the suppression of proton-decay type transitions. Mass generation for the quarks and leptons will be effected by appropriate condensates in combination with the color interactions of the pseudo-Goldstone bosons. Finally, the generality of this type of model is also a question of interest. These and other issues are under investigation and will be reported elsewhere.

Note added in proof.

In this paper we have indicated that the pseudo-Goldstone bosons can contribute masses to the quarks and leptons through loop diagrams. An alternative and even more attractive mechanism for that purpose arises through the formation of technifermion condensates which appear at energy scale Λ_{HC} when the SU(N) force becomes strong; exchange of massive technibosons formed at the $SU(2)_L$ or $SU(2)_R$ confining scales Λ_L or Λ_R then breaks the chiral flavor symmetry and gives masses of order $\Lambda_{HC}^3/\Lambda_{L,R}^2$ to the quarks and leptons. By suitable arrangement of the confining scales the correct light fermion mass scale can be obtained.[14] Either mechanism plays a role like extended technicolor, but the usual difficulty with flavor-changing neutral currents can be suppressed. A manuscript on this subject is now in preparation.

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Footnotes

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[f1] A rather comprehensive recent survey of composite models has been made by L. Lyons in ref. 1. See also M. Peskin in ref. 2.

[f2] S. F. King in ref. 12 has encently presented plausibility arguments why multipreon condensates are favored over two-preon condensates in various chiral symmetry breaking schemes. See also R. Casalbuoni and R. Gatto in ref. 13.

[f3] The discrete axial symmetries resulting from the instanton effects can be used to keep all the preons massless.

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Table I. Quantum numbers for the preons and composite fermions of one generation

			Q	B	L
Preons					
T	=	$(N;2,2;1,1)_{1,0,0;0}$	1/2	- 1/N	- 1/N
U	=	$(N;2,1;1,1)_{0,4,0;0}$	0	- 1/N	- 1/N
Q	=	$(1;2,1;3,1)_{0,-N,1;1}$	1/6	1/3	0
L	=	$(1;2,1;1,1)_{0,-N,-3;-3}$	- 1/2	0	1
T'	=	$(\bar{N};2,2;1,1)_{-1,0,0;0}$	- 1/2	1/N	1/N
U'	=	$(\bar{N};1,2;1,1)_{0,-4,0;0}$	0	1/N	1/N
Q'	=	$(1;1,2;1,\bar{3})_{0,N,-1;1}$	- 1/6	- 1/3	0
L'	=	$(1;1,2;1,1)_{0,N,3;-3}$	1/2	0	- 1
Composites					
ν_ℓ:	$L(TU')$	$= (1,1)_{1,-4-N,-3;-3}$	0	0	1
ℓ :	$L(TU')^+$	$= (1,1)_{-1,4-N,-3;-3}$	- 1	0	1
u :	$Q(TU')$	$= (3,1)_{1,-4-N,1;1}$	2/3	1/3	0
d :	$Q(TU')^+$	$= (3,1)_{-1,4-N,1;1}$	- 1/3	1/3	0
ν_ℓ^c:	$L'(T'U)$	$= (1,1)_{-1,4+N,3;-3}$	0	0	- 1
ℓ^c:	$L'(T'U)^+$	$= (1,1)_{1,-4+N,3;-3}$	1	0	- 1
u^c:	$Q'(T'U)$	$= (1,\bar{3})_{-1,4+N,-1;1}$	- 2/3	- 1/3	0
d^c:	$Q'(T'U)^+$	$= (1,\bar{3})_{1,-4+N,-1;1}$	1/3	- 1/3	0