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MONOPONUCLEOSIS - The wonderful things that monopoles can do to nuclei if they are there.

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1. Mixing of deuteron triplet and singlet states
2. Monopole nuclear matter
3. Catalysis of spontaneous fission
4. Catalysis of nuclear fusion
   (with implications for solar neutrinos)
5. Enhancement of forbidden beta decays.

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In the proceedings of the 1965 Coral Gables Conference, Behram Kursonoglu included appropriate folk tales about the Turkish folk hero, Nasreddin Hoja, before every talk. One of these tales seems particularly appropriate for this conference.

> One morning a woodcutter saw Hoja by the edge of a lake, throwing quantities of yeast into the water. "What the devil are you doing, Hoja?" he asked. Hoja looked up sheepishly and replied, "I am trying to make all the lake into yogurt." The wooducutter laughed and said, "Fool, such a plan will never succeed." Hoja remained silent for a while, and stroked his beard. Then he replied, "But just imagine if it should work!"

A modern version of this story would have Hoja sitting by the edge of a monopole detector saying, "But just imagine if the monopoles are there!" In this talk we consider the wonderful things that they can do to nuclei by examining nuclear physics in strong magnetic fields. We have seen that monopoles can bind nuclei.[1-3] We shall investigate the following other possible processes:

1. Mixing of singlet and triplet states of deuteron-like positronium.
2. Production of a new kind of nuclear matter with nucleon moments oriented in the field.
3. Catalysis of nuclear fission.
4. Catalysis of nuclear fusion (with implications for solar neutrinos).
5. Enhancement of forbidden decays like triplet positronium, e.g. fission products.

1. Mixing of deuteron triplet and singlet states

The magnetic energy of a nuclear spin in the magnetic field produced by a monopole at a distance of several fermis is of the order of nuclear binding energies and level spacings.[4] This field can produce appreciable mixing of nuclear wave functions and may give observable effects. For a rough estimate consider a deuteron in a very strong magnetic field. In the same way that a field splits and mixes the triplet and singlet spin states of positronium, the spin triplet deuteron ground state is split into three energy levels and its central member is mixed with the (unbound) singlet spin state to produce the eigenstates $|n\uparrow p\downarrow\rangle$ and $|n\downarrow p\uparrow\rangle$. The splitting between the two states by the field at a distance of $\underline{r}$ from a monopole of charge g is

$$\Delta E = g(1.91 + 2.79)(e\hbar/Mc)(1/r^2) \tag{1}$$

where M is the proton mass. If we set $eg/\hbar c = 1$, the splitting is equal to the binding energy of the deuteron, 2 MeV when

$$\hbar c/r = 20 \text{ MeV}. \tag{2a}$$

$$r \simeq 10 \text{ fermis} \tag{2b}$$

This suggests that a monopole might break up a deuteron at a distance of 10 fermis.

However, a slowly-moving heavy monopole does not have sufficient kinetic energy in the monopole-deuteron center of mass system to break up a deuteron, and the two nucleons either remain bound to the monopole or escape as a normal deuteron. This differs from positronium, where both the triplet and singlet states are metastable and decay by annihilation. The triplet decay is inhibited by selection rules and is third order in α (3γ) while the singlet is second order (2γ). An external magnetic field catalyzes the decay of the triplet state by mixing in a singlet component for which the second order decay is allowed.

The nuclear analog of positronium is a mestastable nuclear state whose decay can be enhanced or catalyzed by the presence of the monopole magnetic field; e.g. beta unstable odd-odd nuclei like Al^{26} whose ground states have the proton spin j_p and the neutron spin j_n coupled to the maximum possible spin $J = j_p + j_n$ and whose beta decay to a J=0 even-even nucleus is highly forbidden because of the large spin change. A strong magnetic field decouples j_p and j_n and mixes in all lower spin couplings down to $J = |j_p - j_n|$. As in triplet positronium, decays from the admixed states have a much lower order of forbiddenness and the decay rate is enhanced.

2. Monopole nuclear matter

If a monopole is placed at the center of a nucleus, the nucleons will gain energy if their magnetic moments are oriented parallel to the field. Changing the spin orientation will lose nuclear binding energy. However, if the magnetic energy gained is greater than the nuclear binding energy lost, the magnetically polarized nucleus will be the ground state of the system and there will be a different kind of nuclear matter.

For a crude estimate of this effect, let us consider a monopole at the center of a sphere of nuclear matter of radius R with nuclear density ρ (R). The magnetic energy gained by orienting the magnetic moments of all nucleons parallel to the field is

$$E_{mag} = \int_o^R \frac{\bar{\mu} e g \hbar}{M\, c r^2} \rho(r) dr \cdot 4\pi r^2 \tag{3}$$

where $\bar{\mu}$ is the mean value of the magnitude of the nucleon magnetic moment. For systems with equal numbers of neutrons and protons, we take $\bar{\mu} = 2.35$ nuclear magnitons. For a sphere of uniform density containing A nucleons

$$\rho(r) = \frac{A}{\frac{4}{3}\pi R^3} \tag{4a}$$

It is convenient to parameterize the radius R as

$$R = \frac{e^2}{mc^2} \frac{A^{1/3}}{\xi} = 2.8 \frac{A^{1/3}}{\xi} \text{ fermis} \tag{4b}$$

where ξ is a parameter of order unity and is about 2 for conventional densities. Then, for a monopole strength g given by the minimum Dirac value

$$\frac{eg}{\hbar c} = \frac{1}{2} \tag{4c}$$

equation (3) becomes

$$\Delta E_{mag} = \frac{3}{2} \bar{\mu} \frac{\hbar^2 c^2}{e^4} \frac{m^2 c^2}{M} \xi^2 A^{1/3} \approx 18\, \xi^2 A^{1/3} \text{MeV} \tag{5a}$$

$$\frac{\Delta E_{mag}}{A} \approx 18\, \xi^2 A^{-2/3} \text{ MeV} \tag{5b}$$

We see that for $\zeta = 2$ equations (5) give magnetic energies which are comparable to nuclear binding energies. Whether or not monopole nuclear matter is stable compared to normal nuclear matter cannot be determined by such a crude calculation. It is necessary to calculate also the effects of reorienting the nucleon spins on the nuclear interaction and to include the changes in kinetic energy if the density is varied to give a minimum energy.

Note also that even if a magnetic field is not strong enough to change the polarization states of an entire nucleus, the states in the higher shells are more likely to be affected than the inner shells. A monopole field can split the degeneracy of the states in the spherical shell model and cause the levels whose magnetic moments are oriented parallel to the field to move downward in energy while those with magnetic moments oriented anti-parallel to the field move upward. In this way some of the levels in the highest filled shell move upward while some of the levels in the lowest unfilled shell move downward. At some value of the magnetic field strength, these levels will cross and the ground state configuration for the nucleus will change. One might consider the analogue of the Nilsson model used to consider the effects of deformation on nuclear levels. Instead of plotting the level energies as a function of deformation, they can be plotted as a function of the external magnetic field. Thus, an equilibrium may be reached in which the center of the nucleus is normal nuclear matter, whereas the outer shells have become monopole nuclear matter.

3. Catalysis of Spontaneous Fission

The rearrangement of nuclear levels produced by a magnetic monopole may make the nucleus much more susceptible to spontaneous fission. Consider a nucleus which could gain energy by splitting into two nuclei because the energy gained from the Coulomb repulsion is greater than the energy lost from the nuclear attraction. However, because the nuclear force is a short range force, a potential barrier is created as the nucleus is deformed. A small deformation reducing the Coulomb energy only slightly increases the nuclear energy very sharply. However, if an appreciable part of the binding energy no longer comes from the nuclear force, but comes instead from the magnetic interaction with the monopole, this situation can change. The magnetic interaction has a longer range than the nuclear interaction and the deformation of the nucleus will cost less when part of the energy is magnetic than when all of the energy is nuclear. Thus, the fission barrier may be reduced appreciably and nuclei which do not fission spontaneously in the absence of monopoles may fission rapidly when monopoles are present.

4. Catalysis of Nuclear Fusion by Magnetic Monopoles

If two nuclei with magnetic moments are near a monopole, the attraction of both nuclei by the monopole can compensate for the Coulomb repulsion between the nuclei and greatly reduce the potential barrier which inhibits nuclear fusion. Such an effect can catalyze the He^3-He^3 reaction in the sun while not affecting the He^3-He^4 reaction because He^4 has no magnetic moment. Since the He^3-He^4 reaction leads to the Li-Be-B chain which produces the high energy solar neutrinos investigated in Davis' experiment, monopole catalyis could explain Davis' failture to observe solar neutrinos.[5,6]

Quantitative estimates of this effect are difficult because barrier penetration factors are exponential and very sensitive to small effects, while the three-body problem of two nuclei and a monopole is not easily solved to the precision required. For the thermonuclear reactions in the sun, the reaction rate depends on an integral over nuclear kinetic energies of the product of a Boltzmann factor which decreases exponentially with increasing energy and a barrier penetration factor which decreases exponentially with decreasing energy. The maximum of the integrand, called the Gamow peak, occurs at energies ten times larger than thermal energy where the Boltzmann factor is e^{-10} and barrier penetration factors of e^{-60} are common.[7] The problems arising are illustrated by the following simple example.

We assume that a bound state of a nucleus of charge z and a monopole of magnetic charge g exists and consider a collision between this bound state and another nucleus of charge Z, mass number A and magnetic moment μ in nuclear magnetons. At distances large compared to the size of the bound state the interaction between the two bodies at a distance r is the sum of the Coulomb interaction and the magnetic interaction

$$V = \frac{Zze^2}{r} - \frac{\mu e\hbar}{M_p c}\frac{g}{r^2} = \frac{Zze^2}{r} - \frac{\mu\hbar^2}{2M\, r^2} \tag{6}$$

where we have assumed that the magnetic moment is oriented parallel to the magnetic field to give an attractive interaction and used the Dirac value (4c).

At large distances the potential is the normal Coulomb repulsion. However at small distances the attractive magnetic interaction takes over and the potential goes through zero and becomes attractive at the distance

$$R_o = \frac{\mu}{2}\frac{\hbar^2}{M\ Zze^2} \tag{7a}$$

The maximum value of the interaction (6) occurs at the distance

$$R_{max} = 2R_0 = \frac{\mu\hbar^2}{M\ Zz^2} = \frac{\mu\hbar^2\ mc^2}{M\ Zz^4}\cdot\left(\frac{e^2}{mc^2}\right) = \frac{(137)^2}{1840}\times\frac{\mu}{Z}\left(\frac{e^2}{mc^2}\right) \tag{7b}$$

Then

$$V(R_{max}) = \frac{Zze^2}{2R_{max}} = \frac{Z^2 z^2}{2\mu}\left(\frac{e^2}{\hbar c}\right)^2 M\ c^2 \tag{7c}$$

The cross section for a reaction between the two systems includes a barrier penetration factor $e^{-\gamma}$ where γ is given by the usual Gamow expression

$$\gamma = \frac{2}{\hbar} \sqrt{2AM} \int_R^b (V-T)^{1/2} \, dr \tag{8}$$

The limits of the integral R and b are the two points where the integrand vanishes, the classical turning points, and T is the kinetic energy.

For the case where the monopole is absent, the dominant contribution to the result (8) comes from the upper limit and the lower limit can be taken as zero for a first approximation. Let us write

$$\gamma = \gamma_0 - \gamma_R \tag{9}$$

where γ_0 denotes the value of the expression (3) with R = 0 and γ_R denotes the correction due to the finite value of R. For the case where there is no magnetic monopole present, the values of γ_0 and γ_R are given to a good approximation as

$$\gamma_0 = \frac{2\pi Zze^2}{\hbar v} \tag{10a}$$

$$\gamma_R = \frac{4}{\hbar} (2Zze^2 AM\ R)^{1/2} \tag{10b}$$

where v is the relative velocity.

To include the effect of the magnetic monopole, we must choose the value of R to be the point where the integrand vanishes in the presence of the monopole potential; i.e. a value greater than R_0 given by eq. (7a). There will also be a considerable correction in the integrand in the region between R_0 and distances several times this radius. We can give a rough estimate of these two effects by setting $R = R_{max} = 2R_0$ in eq. (10b). In this case we obtain

$$\gamma_{2R_0} = 4\sqrt{2\mu A} \tag{11}$$

For the case of a He^3 nucleus relevant to fusion in the sun, $Z = 2$, $A = 3$ and $\mu \approx 2.13$. With these values we obtain γ_R approximately equal to 14. This means that the cross section is enhanced by a factor e^{14}.

A more refined calculation which evaluates the integral (8) explicitly gives an approximate result with the factor 4 in eq. (11) replaced by 2π. This changes the enhancement factor from e^{14} to e^{22}. With factors like these arising easily one can understand that large effects are possible and that it is necessary to be very careful before drawing quantitative conclusions.

Another estimate of this effect is obtained by noting that the barrier exponent γ must vanish when the kinetic energy T is equal to the barrier height,

$$T = \frac{1}{2} AM_p v^2 = V(R_{max}) = \frac{Z^2 z^2}{2\mu} \left(\frac{e^2}{\hbar c}\right)^2 M c^2 \approx 190 \text{ KeV} \tag{12a}$$

Then

$$v = \frac{Zze^2}{\hbar\sqrt{\mu A}} \tag{12b}$$

But the barrier exponent γ_0 in the absence of the monopole at this energy is given by eq. (10a) as

$$\gamma_0 = 2\pi\sqrt{\mu A} \tag{13}$$

For the He^3 nucleus, $\gamma_0 = 2\pi\sqrt{6.39} \sim 16$. Thus a barrier penetration factor of e^{-16} is present at this energy in the absence of the monopole and completely disappears when the monopole attraction is included.

With such large factors present which are sensitive to details of the calculation, and many unknown factors, it is very difficult to obtain quantitative results. One example of such an unknown factor is the effect of a condensate of electron-positron pairs that must be created in the monopole field at distances of the order of the electron Compton wavelength. The magnetic energy of an electron-positron pair in the magnetic field of a monopole at a distance r is

$$\frac{2e\hbar}{mc} \cdot \frac{g}{r^2} = mc^2 \left(\frac{\hbar}{mcr}\right)^2 . \tag{14}$$

Thus at distances of the order $\hbar/mc$ the vacuum seems to becomes unstable against creating pairs and orienting the moments parallel to the field. This naive picture is not correct. More sophisticated treatments of the charge density around a monopole are given elsewhere.[8]

The electron-positron pairs cannot screen a magnetic charge, because of Gauss' law applied to magnetic charges. However, the cloud of pairs could very well screen the Coulomb repulsion between the nuclei.

At the distance $r = \hbar/\sqrt{2}mc$ which makes the magnetic energy of a pair (9) equal to its rest energy, the Coulomb barrier has the value

$$\frac{Zze^2}{r} = \frac{Zze^2\sqrt{2}\,mc}{\hbar} = \frac{Zz\sqrt{2}\,mc^2}{137} . \tag{15}$$

This is about 20 keV for He^3.

If the pair condensate screens off the Coulomb barrier at this point, then there is no barrier penetration factor for energies above 20 keV, while at lower energies the penetration is enhanced by the factor e^{γ_R} with

$$\gamma_R = 4\left(\frac{\sqrt{2}\;Zze^2 AM}{\hbar mc}\right)^{1/2} = 4(19ZzA)^{1/2} . \tag{16}$$

For He^3, $\gamma_R = 60$, which is enormous.

The essential physics underlying these numbers is that the conventional calculation of barrier penetration includes enormous contributions to the Gamow integral (8) from distances smaller than $\hbar/\sqrt{2}$ mc. Any effect which reduces this contribution in the exponent produces a large enhancement of the cross sectiions.

What is this fermion charge around a monopole? Is it observable? Can you polarize the distribution by putting it between condenser plates? Is it a dielectric? Is it a conductor? Clearly, a better understanding of the underlying physics is needed in order to obtain reliable estimates of fussion catalysis by monopoles.

5. Enhancement of forbidden β decays

Spontaneous electromagnetic mixing has been considered [10,11] as a radiative correction to ordinary beta decay and found to be much too small to produce an observable effect. The induced mixing due to a monopole is similar to this radiative mixing, but the transition matrix element is of order unity instead of order α. The transition probability is thus increased by the large factor of α^2.

The enhancement factor in the transition matrix element M_B for the magnetically induced beta decay over the ordinary decay matrix element M_o for various combinations of electromagnetic transitions and one ordinary beta decay is given by standard perturbation theory as

$$\frac{\langle f|M_B|i\rangle}{\langle f|M_o|i\rangle} = \frac{\langle f|V|A\rangle}{\langle f|V|i\rangle} \cdot \frac{\langle A|H_B|i\rangle}{(E_A-E_i)} \tag{17a}$$

$$\frac{\langle f|M_B|i\rangle}{\langle f|M_o|i\rangle} = \frac{\langle B|V|i\rangle}{\langle f|V|i\rangle} \cdot \frac{\langle f|H_B|B\rangle}{(E_B-E_f)} \quad . \tag{17b}$$

$$\frac{\langle f|M_B|i\rangle}{\langle f|M_o|i\rangle} = \frac{\langle B|V|A\rangle}{\langle f|V|i\rangle} \cdot \frac{\langle f|H_B|B\rangle}{(E_B - E_f)} \cdot \frac{\langle A|H_B|i\rangle}{(E_A - E_i)} \qquad (17c)$$

where V denotes the transition operator for ordinary beta decay, H_B denotes the electromagnetic transition operator, $|i\rangle$, $|f\rangle$, $|A\rangle$ and $|B\rangle$ denote the initial and final nuclear ground states and the intermediate excited states and E_i, E_f, E_A and E_B denote their energies.

Particularly interesting cases might be odd-odd nuclei with the odd proton and odd neutron in the same L-shell and coupled to a spin of 2 or greater. The beta decay to a 0^+ ground state then involves recoupling the angular momenta of the two nucleons and is forbidden because orbital factors are needed for a change in total angular momentum larger than one. However, the monopole could break the couplings of the proton and neutron spins. In perturbation theory this appears as a cascade of M1 transitions via the other states of the same configuration down to the 1^+ state, from which the beta decay is an allowed GT transition. Thus very long-lived highly forbidden transitions might go much more rapidly via the magnetic transition through several intermediate states. The relevant matrix elements for such transitions can be crudely estimated using shell model wave functions and experimental values of magnetic moments and Gamow-Teller matrix elements within the same configurations.

Consider, for example Al^{26}, which has $J^P = 5^+$ and decays to the excited 2^+ state of Mg^{26} with a lifetime of 7.2×10^5 years and a log ft of 14.2. Al^{26} also has excited states with $J^P = 4^+$, 3^+, 2^+ and 1^+ at excitation energies of 2 MeV, 0.4 MeV, 1.8 MeV and 1.1 MeV respectively. The beta decays from the 1^+ state of Al^{26} to the 0^+ ground state of Mg^{26} and from the 3^+ and 4^+ states of Al^{26} to the 2^+ and 3^+ excited states of Mg^{26} are both allowed GT transitions. Thus a monopole-induced fifth-order transition via four intermediate states or a second or third-order transition via one or two intermediate states might have a much shorter lifetime than the observed decay.

The transition matrix elements for the fifth and third order transitions are

$$\langle 0^+|M_B|5^+\rangle = \langle 0^+|V|1^+\rangle \frac{\langle 1^+|H_B|2^+\rangle}{E(1^+)-E(5^+)} \cdot \frac{\langle 2^+|H_B|3^+\rangle}{E(2^+)-E(5^+)} \\ \frac{\langle 3^+|H_B|4^+\rangle}{E(3^+)-E(5^+)} \cdot \frac{\langle 4^+|H_B|5^+\rangle}{E(4^+)-E(5^+)} \tag{18a}$$

$$\langle 2^+|M_B|5^+\rangle = \langle 2^+|V|3^+\rangle \cdot \frac{\langle 3^+|H_B|4^+\rangle}{E(3^+)-E(5^+)} \cdot \frac{\langle 4^+|H_B|5^+\rangle}{E(4^+)-E(5^+)} \ . \tag{18b}$$

The matrix element $\langle 0^+|V|1^+\rangle$ should be approximately equal to that of the mirror transition from the 0^+ ground state of Si^{26} to the 1^+ state of Al^{26}, which has an experimentally measured log ft of 3.5. The matrix element $\langle 2^+|V|3^+\rangle$ cannot be taken directly from another transition like $\langle 0^+|V|1^+\rangle$. Reasonable estimates are obtained by using log ft values of the neighboring decays of the 3^+ ground state of Na^{26} to the same 2^+ state of Mg^{26} with a log ft of 4.7 and of the 3^+ ground state of Al^{28} to the 2^+ state of Si^{28} with a log ft of 4.9.

Rough quantitative estimates of the expression (18) are obtainable from the shell model description of the states in Al^{26} as a neutron and a proton in the $d_{5/2}$ shell coupled to spins 1, 2, 3, 4 and 5. We can use the experimental magnetic moments of the $5/2^+$ ground states of the nuclei Mg^{25} and Al^{25}; namely -0.9 n.m. and +3.6 n.m. respectively, as values for the effective magnetic moments of the $5/2^+$ neutron and the $5/2^+$ proton configurations in Al^{26}. We therefore need assume only that the states of spins 1-5 in Al^{26} are described by different couplings of the neutron configuration of Mg^{25} and the proton configuration of Al^{25}, without assuming a particular model like a single-particle description for either.

The electromagnetic transition operator H_B and its relevant matrix elements can then be written

$$H_B = (g_p j_{pz} + g_n j_{nz})B_z = (g_p+g_n)J_z B_z/2 + (g_p-g_n)(j_{pz}-j_{nz})B_z/2 \tag{19a}$$

$$\langle J|H_B|J+1\rangle = \langle J|j_{pz}-j_{nz}|J+1\rangle \{0.9(e\hbar/2Mc)B_z\} \tag{19b}$$

where g_p and g_n denote the gyromagnetic ratios of the proton and neutron configurations, j_{pa} and j_{nz} the z-components of the angular momenta of these configurations, B_z the magnetic field strength, chosen to be in the z-direction and J_z the z-component of the total angular momentum. The values of g_p and g_n were taken from the experimental moments,

$$(g_p-g_n) = (2/5)(3.6 + 0.9)(e\hbar/2Mc) = 0.9(e\hbar/Mc) \tag{19c}$$

The angular momentum matrix elements are easily evaluated by standard methods. Assuming equal populations for the 11 J_z states and using the values log ft = 3.5 and 4.9 respectively for the two beta transitions we obtain

$$\log ft(5^+\to 0^+) = 7.4 + 16 \log r \tag{20a}$$

$$\log ft(5^+\to 2^+) = 6.8 + 8 \log r \tag{20b}$$

where r is in fermis.

The $(5^+ \to 0^+)$ transition is seen to have log ft values of 7.4 and 12.3 for values of r of 1 and 2 fermis respectively. The $(5^+ \to 2^+)$ transition (9b) has log ft values of 6.8, 9.2 and 10.6 for values of r of 1, 2 and 3 fermis respectively. These should be compared with the log ft of 14.2 for the competing observed decay.

These very crude estimates only indicate orders of magnitude. Better calculations can be made with time-dependent magnetic fields to account for the passage of a monopole by a nucleus, and with more complicated nuclear wave functions, but these are probably not worth the effort until more information is available about monopoles.

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