Fermi National Accelerator Laboratory

FERMILAB-CONF-83/102-THY
December, 1983

Conservation Laws In The Monopole-Fermion System*

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ABSTRACT

It is shown that the monopole induced baryon number non-conservation is a necessary consequence of the exact conservation laws of the full four dimensional fermion-gauge field-Higgs system and properties of the J=0 partial wave fermions.

It is also shown that the charge associated with the unbroken gauge symmetry is exactly conserved in the monopole-fermion interaction.

*Presented at the monopole '83 conference held in the University of Michigan, Ann Arbor, Michigan, October 6-9.

The subject of monopole induced baryon number violation, first introduced by Rubakov [1] and subsequently by Callan [2], has been of great interest in the recent past [3-11]. In this lecture I shall try to give a simple account of the subject, based on the conservation laws in the monopole fermion system. I shall show that the conservation laws of the full four dimensional gauge theory uniquely forces us to baryon number violating processes in the monopole fermion system. These conservation laws also help us determine the origin of the monopole induced baryon number violation, and, in particular, the role of anomaly in such processes. Finally, I shall discuss the conservation of electric charge in such systems, since it has also been a subject of great controversy in the last year [6-9].

I shall start the discussion with an SU(2) gauge theory with the massless Dirac doublet of fermions:

$$\begin{matrix} \Psi_{1\uparrow} \\ \Psi_{1\downarrow} \end{matrix} \qquad \text{and} \qquad \begin{matrix} \Psi_{2\uparrow} \\ \Psi_{2\downarrow} \end{matrix}$$

which I shall identify with

$$\begin{matrix} U_1 \\ U_2^c \end{matrix} \qquad \text{and} \qquad \begin{matrix} e^+ \\ d_3 \end{matrix}$$

respectively, keeping in mind the lowest charge monopole in SU(5). We also need a triplet of Higgs ϕ whose vacuum expectation value (VEV) breaks SU(2) to U(1). The Lagrangian for such a system is,

$$\mathcal{L} = -1/4\ \mathrm{Tr}\ (F_{\mu\nu} F^{\mu\nu}) + \sum_{i=1}^{2} \bar{\psi}_i i \not{D} \psi_i + (D_\mu \phi)^\dagger (D_\mu \phi) - V(\phi)$$

In terms of the four Dirac fields $\psi_{1\uparrow}$, $\psi_{1\downarrow}$, $\psi_{2\uparrow}$, $\psi_{2\downarrow}$, we may define eight charges:

$$Q_{i\uparrow} = \int \bar{\psi}_{i\uparrow} \gamma^0 \psi_{i\uparrow} d^3x \qquad Q_{i\downarrow} = \int \bar{\psi}_{i\downarrow} \gamma^0 \psi_{i\downarrow} d^3x$$

$$Q^5_{i\uparrow} = \int \bar{\psi}_{i\uparrow} \gamma^0 \gamma^5 \psi_{i\uparrow}\ d^3x \qquad Q^5_{i\downarrow} = \int \bar{\psi}_{i\downarrow} \gamma^0\ \gamma^5 \psi_{i\downarrow}\ d^3x \tag{2}$$

These eight charges completely determine the fermion content of the system, as well as the helicities of the fermions, up to fermion-antifermion pairs. The SU(2) Lagrangian, given in (1), has three exact global symmetries, giving rise to three conserved charges:

Symmetry	Conserved Charge
$\begin{pmatrix}\psi_{1\uparrow}\\ \psi_{1\downarrow}\end{pmatrix} \to e^{i\Theta_1}\begin{pmatrix}\psi_{1\uparrow}\\ \psi_{1\downarrow}\end{pmatrix}$	$S_1 = Q_{1\uparrow} + Q_{1\downarrow}$
$\begin{pmatrix}\psi_{2\uparrow}\\ \psi_{2\downarrow}\end{pmatrix} \to e^{i\Theta_2}\begin{pmatrix}\psi_{2\uparrow}\\ \psi_{2\downarrow}\end{pmatrix}$	$S_2 = Q_{2\uparrow} + Q_{2\downarrow}$
$\left.\begin{matrix}\begin{pmatrix}\psi_{1\uparrow}\\ \psi_{1\downarrow}\end{pmatrix} \to e^{i\Theta_3\gamma^5}\begin{pmatrix}\psi_{1\uparrow}\\ \psi_{1\downarrow}\end{pmatrix}\\ \begin{pmatrix}\psi_{2\uparrow}\\ \psi_{2\downarrow}\end{pmatrix} \to e^{-i\Theta_3\gamma_5}\begin{pmatrix}\psi_{2\uparrow}\\ \psi_{2\downarrow}\end{pmatrix}\end{matrix}\right\}$	$S_3 = Q^5_{1\uparrow} + Q^5_{1\downarrow} - Q^5_{2\uparrow} - Q^5_{2\downarrow}$

Besides these three symmetries, the total electric charge carried by the fermions:

$$S_4 = Q_{1\uparrow} - Q_{1\downarrow} + Q_{2\uparrow} - Q_{2\downarrow} \tag{3}$$

is conserved if we ignore all other charge degrees of freedom of the system (e. g. the dyonic excitation of the monopole).

[In writing down (3), I have assumed that the unbroken generator of the SU(2) group is T_3. When we are considering the interaction of fermions with the monopole, then, if we work in the spherically symmetric $A_0 = 0$ gauge, the total U(1) charge carried by the fermions is given by

$$\int \sum_{i=1}^{2} \bar{\psi}_i \gamma^0 \, \hat{r}.\vec{\sigma}\psi_i d^3x \quad .$$

This can be cast in the form give in (2) by taking the fields $\psi_{i\uparrow}$ and $\psi_{i\downarrow}$ to be eigenstates of $\hat{r}.\vec{\sigma}$ with eigenvalues +1 and -1 respectively. This does not change the expression for the conserved charges S_1, S_2 and S_3.]

Let us now concentrate on the monopole-fermion scattering. For a given initial state containing incoming fermions, we may compute all the charges Q_i and Q_i^5 given in (2). Of these, S_1, S_2, S_3 and S_4 are conserved in the scattering. Thus we need four more constraints on the Q's to completely determine the final state for a given initial state. These are obtained by restricting the fermions to the J = 0 partial wave. This is a plausible assumption, since only the J = 0 partial wave has a non-vanishing particle density at the core (the wave-function blows up as 1/r as $r \to 0$).

The restriction J = 0 implies that,

$$\hat{r}.\vec{J} = 0 = \hat{r}.(\vec{L} + \vec{S} + \vec{T}) \tag{4}$$

Since $\hat{r} \, . \, \vec{L} = 0$, $\hat{r} \, . \, \vec{S}$ measures the radial component of the spin of the particle (which may be identified with the helicity for outgoing particles) and $\hat{r}.\vec{T}$ measures the unbroken U(1) charge carried by the particle, we have, for outgoing particles, Helicity = - the U(1) charge (5)

This constraint says, for example, that only $\psi_{i\uparrow L}$ and $\psi_{i\downarrow R}$ can be in the outgoing state (where R and L refer to positive and negative helicity respectively in our convention), but not $\psi_{i\downarrow L}$ or $\psi_{i\uparrow R}$. Expressed in terms of the Q_i's, it reads as,

$$Q_{i\uparrow} = - Q^5_{i\uparrow} \ , \ Q_{i\downarrow} = Q^5_{i\downarrow} \ i = 1,2 \tag{6}$$

These four constraints, together with the conservation laws for S_1, S_2, S_3 and S_4, uniquely determine the final state for a given initial state. For example, for an initial state,

$$U_{1R} + d_{3L}$$

the reader can easily verify that the only possible final state consistent with all the conservation laws and the constraints given in (6) is $u^c_{2R} + e^+_L$. Similarly, the only possible final state for an initial state of $U_{1R} + U_{2R}$ is $d^c_{3L} + e^+_L$. Thus we see that the conservation laws imposed by the unbroken global and local symmetries of the theory, together with the restriction that all the fermions must belong to the J = 0 partial wave, uniquely leads to baryon number violating processes.

We may also use the conservation laws to find the origin of baryon number violation in the monopole-fermion scattering. To do this, we must look at the linear combination of the Q's other than the four conserved charges S_1, S_2, S_3, and S_4. Some of these linear combinations are conserved in the presence of the unbroken U(1) field, but not in the presence of the full SU(2) fields. These are associated with the approximate symmetries of the Lagrangian when we ignore the W^+ and the W^- fields. They are,

Approximate Symmetry	Associated Charge
$\left.\begin{array}{l} \psi_{1\uparrow}\to e^{i\Theta_1}\psi_{1\uparrow}\ ,\ \psi_{1\downarrow}\to e^{-i\Theta_1}\psi_{1\downarrow} \\ \psi_{2\uparrow}\to e^{-i\Theta_1}\psi_{2\uparrow}\ ,\ \psi_{2\downarrow}\to e^{i\Theta_1}\psi_{2\downarrow} \end{array}\right\}$	$N_1 = Q_{1\uparrow} - Q_{1\downarrow} - Q_{2\uparrow} + Q_{2\downarrow}$
$\psi_{1\uparrow}\to e^{i\Theta_2\gamma^5}\psi_{1\uparrow}\ ,\ \psi_{1\downarrow}\to e^{-i\Theta_2\gamma^5}\psi_{1\downarrow}$	$N_2 = Q^5_{1\uparrow} - Q^5_{1\downarrow}$
$\psi_{2\uparrow}\to e^{i\Theta_3\gamma^5}\psi_{2\uparrow}\ ,\ \psi_{2\downarrow}\to e^{i\Theta_3\gamma^5}\psi_{2\downarrow}$	$N_3 = Q^5_{2\uparrow} - Q^5_{2\downarrow}$

Finally, there is an anomalous charge, associated with the anomalous symmetry transformation:

Anomalous Symmetry

$$\begin{pmatrix} \psi_{i\uparrow} \\ \psi_{i\downarrow} \end{pmatrix} \to e^{i\Theta\gamma^5} \begin{pmatrix} \psi_{i\uparrow} \\ \psi_{i\downarrow} \end{pmatrix} \quad i = 1,\ 2$$

Anomalous Charge

$$A = Q^5_{1\uparrow} + Q^5_{1\downarrow} + Q^5_{2\uparrow} + Q^5_{2\downarrow}$$

This charge has an anomaly in the presence of the U(1) gauge field and is non-conserved even in the absence of the $W^\pm$ fields.

Since the four exact conservation laws and the constraint imposed by $J = 0$ partial waves completely determine the final state for a given initial state, we cannot, in general, satisfy the conservation laws of N_1, N_2, N_3 and A, and some of them must necessarily be violated in the monopole-fermion interaction [10]. Of these, the violation of N_1, N_2, and N_3 takes place because of the presence of classical SU(2) gauge fields inside the monopole core, while the violation of A is due to the anomaly induced by the unbroken U(1) gauge field.

We may now try to investigate the origin of the baryon number violation by expressing it as,

$$B = \frac{1}{3}\,(\bar{u}_1\gamma^0 u_1 - \frac{1}{3}\,\bar{U}^c_2\gamma^0 U^c_2 - \frac{1}{3}\,\bar{d}^c_3\gamma^0 d^c_3) = \frac{1}{3}\,(Q_{1\uparrow} - Q_{1\downarrow} + Q_{2\downarrow}) \tag{7}$$

and noting that it may be expressed as a linear combination of S_1, S_2, S_4 and N_1. This clearly shows that the baryon number violation in the monopole-fermion interaction is essentially due to the presence of the non-trivial SU(2) gauge fields inside the monopole core, and not due to anomaly. For processes like $U_{1R} + U_{2R} \to d^c_{3L} + e^+_L$, both, the baryon number, and the anomalous charge A are violated. Hence, for this process both the anomaly and the non-trivial dynamics inside the monopole core play a vital role. This is apparent from the calculation of Rubakov [1] and Callan [2], where gauge field configurations with non-zero topological charge is needed to get a non-zero value of the condensate $\langle U_{1R}U_{2R}d_{3R}e_R^-\rangle$. However, for the process $U_{1R} + d_{3L} \rightarrow U_{2R}^c + e_L^+$, B is violated but A is conserved. Hence we expect the anomaly to play no role in these processes. This becomes apparent if we calculate the condensate $\langle U_{1R}d_{3L}U_{2L}e_R^-\rangle$ (i.e. $\langle \Psi_{1\uparrow R}\psi_{2\downarrow L}\psi_{1\downarrow L}^c\psi_{2\uparrow R}^c\rangle$ in our notation) in the Rubakov-Callan model. Such condensates receive contributions from the gauge field configuration with zero topological charge [4], which shows that the anomaly does not play any role in these processes.

The conservation laws for the charges S_1, S_2, S_3 and S_4 may also be derived from the effective two dimensional bosonized Hamiltonian introduced by Callan [2]. The boundary conditions on the boson fields at the monopole core play a vital role in deriving these conservation laws [5]. This shows that these boundary conditions are necessary consequences of the conservation laws of the full four dimensonal field theory, although they were originally derived by solving the one particle Dirac equation in the background of a classical monopole field. In the full SU(5) grand unified theory, the four conservation laws refer to the conservation of electric charge, color isospin, color hyper charge and weak isospin. In this case, however, the baryon number violation comes from two independent sources, the non-trivial gauge field configuration inside the monopole core, and the anomaly in the baryon number current due to the weak interaction gauge fields. A detailed analysis of the effect of the weak anomaly in a general grand unified theory has been made by A. N. Schellekens [11].

Finally, I shall come to the question of charge conservation in the monopole-fermion system, since this has been a subject of great controversy in the last year. This work was done in collaboration with Y. Kazama [7]. As was shown by Callan [2], the monopole-fermion system in the J = 0 partial wave may be represented by a two dimensional boson field theory with the Hamiltonian:

$$H = \int_{r_o}^{\infty} dr\Big[\sum_{i=i}^{2} (P_i^2 + \pi_i^2 + \Phi_i'^2 + Q_i'^2) + \frac{c}{r^2}(\Phi_1 + \Phi_2 + Q_1 + Q_2)^2\Big] \tag{8}$$

with the boundary conditions:

$$\Phi_i' + Q_i' = 0 \qquad \Phi_i = Q_i \quad i = 1,2 \tag{9}$$

where Φ_i, Q_i are two dimensional boson fields and π_i, P_i are their conjugate momenta. The charges Q given in (2) have simple expressions when expressed in terms of the bosonized fields Φ and Q:

$$Q_{i\uparrow} = \int_{r_o}^{\infty} \Phi_i' \, dr \qquad Q_{i\downarrow} = -\int_{r_o}^{\infty} Q_i' \, dr$$

$$Q^5_{i\uparrow} = \int_{r_o}^{\infty} \pi_i \, dr \qquad Q^5_{i\downarrow} = \int_{r_o}^{\infty} P_i \, dr \tag{10}$$

The charges S_1, S_2 and S_3, constructed from the boson fields, may easily be shown to commute with H as a consequence of the boundary conditions (9). The commutator of S_4 with H, on the other hand, is proportional to,

$$(\dot{Q}_1 + \dot{Q}_2 + \dot{\Phi}_1 + \dot{\Phi}_2)|_{r=r_0} \tag{11}$$

which is not zero as a consequence of the boundary conditions. However, as can be seen from (8), as $r_0 \to 0$, $\Phi_1 + \Phi_2 + Q_1 + Q_2$ must vanish at r_0, in order to satisfy the finiteness of the energy. Physically, this reflects the fact that for a finite r_0, it is possible for the fermions to dump charge into the monopole core, but as $r_0 \to 0$, it becomes more and more difficult, since the Coulomb energy associated with such dyonic excitations blows up.

The question then is, how do we see the conservation of the total charge in the monopole-fermion system for finite r_0? The problem is that in model of Rubakov and Callan, we summarize the full dynamics inside the monopole core by effective boundary conditions on the fermion fields at r_0, and then completely forget about the core. One way to see charge conservation in the monopole fermion system would be to introduce extra degrees of freedom which corresponds to the charge degree of freedom of the monopole, and then define a conserved charge as the sum of the charge carried by the fermions and the monopole core. Such treatments have been given by Balachandran et. al. [8] and Yan [9].

However, it is not really necessary to introduce new degrees of freedom to measure the charge inside the monopole core. The total charge inside the monopole core is measured by the radial electric field at r_0, which, in turn, may be expressed in terms of the Φ_i and Q_i fields at r_0. We may then define the total charge of the system as,

$$(r^2 E_r)_{r=r_0} + S_4 = (\Phi_1 + \Phi_2 + Q_1 + Q_2)|_{r_0} + S_4 \tag{12}$$

This can easily be shown to commute with H, hence the total charge is conserved.

The conservation of the total charge is reflected in the Green's functions as follows. In the present model, there are two ways to define a gauge invariant fermion field, e. g.

$$\psi_{\uparrow N}(r,t) = \exp\left(-ie \int_{r_0}^{r} A_r(r',t)\, dr'\right) \psi_{\uparrow}(r,t) \tag{13}$$

$$\tilde{\psi}_{\uparrow N}(r,t) = \exp\left(ie \int_{r}^{\infty} A_r(r',t)\, dr'\right) \psi_{\uparrow}(r,t) \tag{14}$$

and similarly for $\psi_{\downarrow}$.

$\Psi_{\uparrow N}$ creates a gauge invariant state by creating a fermion at the point r, and an equal and opposite charge at the monopole core. On the other hand, $\tilde{\Psi}_{\uparrow N}$ creates a fermion at a point r and an equal and opposite charge at infinity. When we calculate the Green's function involving the ψ_N fields, then, for a finite r_0, the Green's function is non-vanishing even if the total charge carried by all the fermion fields in the Green's function is non-zero. This is not surprising, since the operators ψ_N always create a charge neutral state by creating an equal and opposite charge at the monopole core together with the fermion. On the other hand, a Green's function involving the $\tilde{\Psi}_N$ fields vanish identically unless the total charge carried by all the fermion fields in the Green's function is zero. This shows that the Green's functions in the monopole-fermion system are indeed charge conserving.

To summarize, I shall state the two main points of the talk:

1) Conservation laws derived from the full four dimensional Lagrangian of the fermion-gauge field-Higgs system, together with the restriction on the fermions to be in the J = 0 partial wave, uniquely lead to baryon number violating processes. The boundary condition on the boson fields at the monopole core, used by Callan in the bosonized version of the theory, follow

2) Total charge of the fermion-monopole system is exactly conserved, although the monopole ground state may make virtual transition to states containing fermion fields carrying a net charge, and an equal and opposite charge at the core.

I wish to thank Y. Kazama for discussions, who was a collaborator in part of this work. I also thank S. Das, A. S. Goldhaber and A. N. Schellekens for useful conversations during various stages of this work.

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