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# THE STATUS OF PERTURBATIVE QCD

R K Ellis

INFN, Sezione di Roma, Italy and Fermilab, Batavia, USA

## INTRODUCTION

In this talk I discuss the status, within the framework of perturbative QCD, of four topics which have received theoretical attention in the last year. They are,

i. Jets at the CERN Sp$\bar{p}$S collider,
ii. Power corrections to leptoproduction,
iii. Hard scattering off nuclear targets,
iv. The photon structure function.

## JETS AT THE Sp$\bar{p}$S COLLIDER

The striking jets observed at the Sp$\bar{p}$S collider[1] by triggering on events with large total transverse energy explore a new region in jet physics. The important parameter is $x_T=2E_T/\sqrt{S}$, which, for jets with transverse energy spanning the range $20 \leq E_T(\text{Jet}) \leq 150$ GeV, lies between 0.07 and 0.56. In QCD the jet cross sections have been calculated using the formula,

$$E\frac{d^3\sigma}{d^3P} = \sum_{i,j}\int dx_1 dx_2\, F_i(x_1,Q^2)F_j(x_2,Q^2)\left[p^0\frac{d^3\sigma}{d^3p}\left(\alpha_S(Q^2),p_1,p_2\right)\right]_{\substack{p_1=x_1P_1\\ p_2=x_2P_2}} \quad (1)$$

Present investigations[2,3,4] evaluate the parton distribution functions $F_i$ using leading log evolution equations and use parton cross sections calculated in lowest order ($\alpha_S^2$). In the absence of higher order corrections, the correct choice of the scale $Q^2$ is unknown, but experience from qq scattering for which higher corrections have been calculated[5] suggests the value $Q^2 \sim p_T^2/2$. Theoretical predictions exhibit some degree of stability (about a factor 3 or 4) under changes of the input parameters, which are principally the shape of the ill-determined gluon distribution function and the choice of the scale Q(or $\Lambda$). Increasing $\Lambda$ increases the strength of the strong coupling constant in the parton cross-sections and increases the rate of evolution of the distribution functions (i.e. depletion of the gluons in the interesting $x_T$ range). The two effects partially compensate one another. Uncertainty due to the shape of the gluon distribution function is more serious.

All authors agree that at low values of $x_T$ the important parton subprocesses are $qG\rightarrow qG$ and $GG\rightarrow GG$ with the former dominating. At some value of $E_T\sim 60$ GeV (the precise value is dependent on the exact form of the gluon distribution function) the $qq\rightarrow qq$ process wins over $qG\rightarrow qG$ because of the stiffer form of the quark distribution. As a consequence of the dominance of these gluon rich processes the one particle inclusive cross section

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primarily triggers on a fragmenting gluon in the low $E_T$ range. In passing from the ISR to the collider, the physics has shifted from the valence region to the gluon or sea dominated region. This trend will continue as we shift to yet higher energies. The degree of uncertainty in the shape of the gluon distribution xG(x) is illustrated in Fig. 1 which contrasts the shape of the counting rule gluon distribution function with that of Ref. 6. Both curves are normalised to unit area. In the hatched region where gluons dominate at the collider the two curves differ substantially. It is important to use the collider to measure the gluon distribution function. This should be possible because of the greater sensitivity to the form of the gluon distribution function than to Λ.

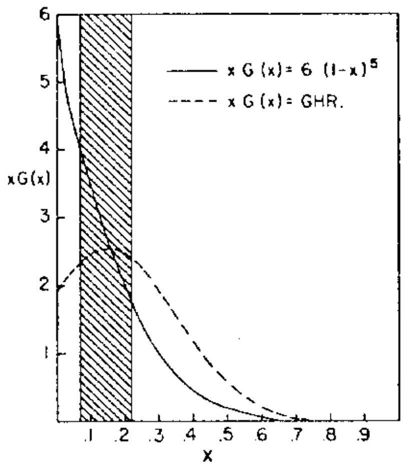



Fig. 1 Uncertainty in the gluon distribution function.

Sizeable event rates for three jet events are also expected. The matrix elements for these processes have all been calculated[4,7] and give event structures with typical bremsstrahlung shapes dependent on the presence of the three gluon coupling.

POWER CORRECTIONS TO LEPTOPRODUCTION

Power corrections to leptoproduction are due to the transverse momentum of partons inside the parent hadron and to contributions involving more than one active parton per hadron. At the tree graph level we may write the $1/Q^2$ corrections to the non-singlet structure function $F_2$ as

$$F_2(x,Q^2) = q(x) + \frac{1}{Q^2}\left\{ 4T_1(x) - x\int dx_2\, dx_1 \frac{\delta(x-x_2)-\delta(x-x_1)}{(x_2-x_1)} T_2(x_2,x_1)\right\} + \frac{1}{Q^2}\left\{\text{FOUR FERMION CONTRIBUTIONS}\right\} \qquad (2)$$

$T_1$ and $T_2$ are parton correlation functions, the generalisations of the quark distribution function q of the scaling parton model; they parametrize the infinite momentum distributions of partons inside the target hadron. They can be expressed in terms of bilocal, trilocal and quadrilocal operators on the light cone in two complementary ways. Simple expressions for $T_1$, $T_2$ are obtained in terms of covariant derivatives along the direction transverse to the hadron momentum and an auxiliary vector n (the transverse basis)[8,9]. However since the dependence on the auxiliary vector n is spurious, it can be eliminated by relating transverse momentum to off-shellness and hence, using the classical equations of motion, $T_1$ and $T_2$ can be represented in terms of operators involving no derivatives of quark fields (the collinear basis).[10] In either basis the parton correlation functions have support for momentum fractions $0 \le x_i \le 1$.[11] As yet there is no information on power corrections to singlet structure functions.

The important issue of the magnitude of these power corrections is still unclear. The matrix elements of lower spin operators have been investigated using the MIT bag model and other simple models.[12] For the Gross-Llewellyn Smith sum rule there is only one twist four operator which contributes,[13]

$$\int_0^1 dx\left(F_3^{\nu H}(x,Q^2)+F_3^{\bar{\nu} H}(x,Q^2)\right) = 3\left[1-\frac{\alpha_s(Q^2)}{\pi}-\frac{4}{27Q^2}\langle\tilde{G}_1\rangle+O\left(\frac{1}{Q^4}\right)\right] \tag{3}$$

where

$$\langle\tilde{G}_1\rangle P^\alpha = g\,\epsilon^{\mu\nu\alpha\beta}\langle P|\bar{q}(0)\gamma_\beta\gamma_5 F_{\mu\nu}\,q(0)|P\rangle$$

is the gluonic operator of spin one. $\langle\tilde{G}_1\rangle$ has been calculated[14] in the MIT bag model to be 0.57 GeV$^2$, confirming the order of magnitude estimate[13] of $\langle G_1\rangle \sim m_N(m_\Delta - m_n) \sim 0.55$ GeV$^2$. In Ref. 9, the bound $\langle G_1\rangle > 0$ is derived. On the other hand the matrix elements of low-spin operators containing four quarks are smaller,[10,12] for example ($\Delta^2 \sim .03$ GeV$^2$),

$$\int_0^1 dx\, F_2^{ep-en}(x,Q^2) = M_2^{\mathrm{TWIST\ 2}}(Q^2)\left(1-\frac{\Delta^2}{Q^2}+\frac{(\text{gluonic operators})}{Q^2}\right) \tag{4}$$

The gluonic operators grow faster than the leading twist operators in the phenomenologically important $x\to 1$ region.[15] The conclusion is that the gluonic operators are likely to be the dominant twist four corrections and require further investigation.

## HARD SCATTERING OFF NUCLEAR TARGETS

The discovery of differences between the structure function per nucleon in iron and in deuterium[16] has been confirmed by reanalysis[17] of experiments at SLAC on iron and aluminum compared with deuterium. This effect is not due to the power corrections described above, because for $x>0.1$ the results of the two experiments are compatible, despite their differing $Q^2$ ranges. In addition the x dependence is quite different from that expected from Fermi motion corrections.

The measurement of $\Lambda$ is unchanged by this phenomenon. Defining $x=Q^2/2M_N(E-E')$ as usual we have $0 \le x \le A$ where A is the number of nucleons. If the nuclear structure function per nucleon $F^A$ is related to the nucleon structure function $F^N$ as follows[18]

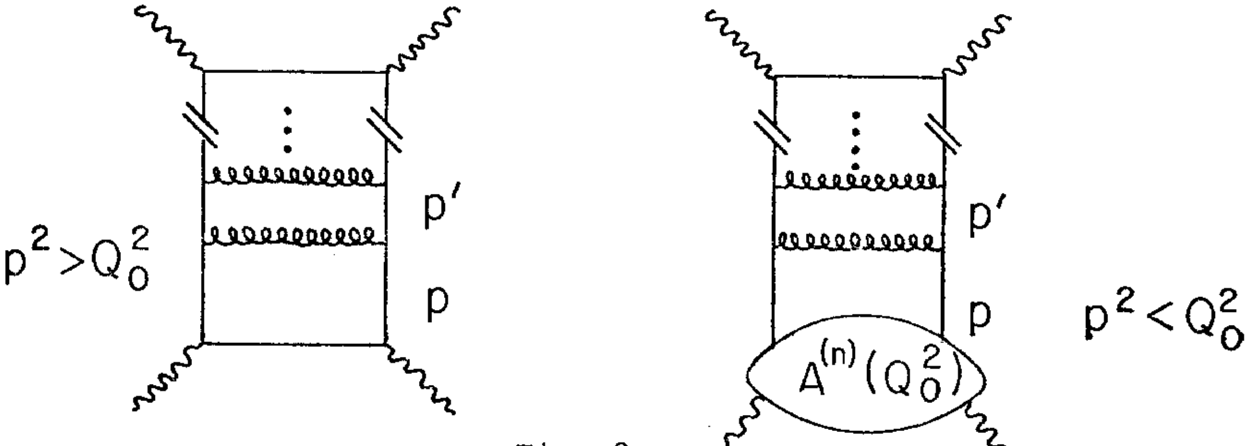
$$F^A(x,Q^2) = \int_{x/A}^1 dz\, f_N(z)\, F^N\!\left(\frac{x}{Az},Q^2\right) \tag{5}$$

both $F^A$ and $F^N$ satisfy the same Altarelli-Parisi eqns. In practice of course the extraction of $\Lambda$ may depend on the unmeasured part of the structure function which now extends from some $x_{MAX}$ to A.

The implications for the various parton distributions in iron are easily derived.[19] The difference $F_2^{Fe}-F_2^D$ is negative for $x\geq 0.35$. In this region sea quarks are negligible so the valence quarks in iron are depleted for $x>0.35$. Since the number of valence quarks is fixed this depletion at large x must be accompanied by an enhancement at lower x. This enhancement is insufficient to explain the observed behavior at low x, which therefore requires an increase in the sea distribution function, perhaps by as much as 60%.[19] These additional quarks carry momentum; the momentum carried by gluons is correspondingly decreased. If these estimates are correct the effect should also be seen in neutrino scattering. Moreover it should also be seen in the comparison of $pp\to\mu^+\mu^- X$ with $pA\to\mu^+\mu^- X$.[20] The effect is probably too small to be seen experimentally in Drell Yan processes with valence antiquarks in the beam particle or in the comparison of different heavy nuclear targets because of the approximate saturation property. In addition the change may be obscured at lower values of $q^2$ by shadowing effects.[21]

## THE PHOTON STRUCTURE FUNCTION

The two photon process in $e^+e^-$ annihilation provides a sensitive probe of the deep structure of the photon. In the parton model the photon structure function is given by the box diagram. This model displays three features (dependence on the fourth power of the quark charges, the hard x distribution and the logarithmic growth of the structure function with $Q^2$) which are also present in the full QCD treatment. These features are observed in the data although the growth with $Q^2$ is confused by the opening of the charm threshold for which there is still no satisfactory theoretical treatment. In addition to the point-like contribution there is a vector meson dominance contribution, significant at lower values of x, which does not grow with $Q^2$.




(a) Fig. 2 (b)

In QCD the photon structure function is exactly calculable asymptotically.[22,23] The derivation of the non-singlet leading log result using graphical techniques[24] displays the reason for this result and highlights some potential problems. As we proceed down the ladder diagram (Fig. 2) we have at every juncture a choice either to terminate the ladder with a point-like coupling to photons (Fig. (2a)) or to add another rung (Fig. (2b)). For the point-like contribution we have before integration over the last rung,

$$\int dx\, x^{n-2} F_2^{\gamma}(x,Q^2) = \int_{Q_0^2}^{Q^2} \frac{dp^2}{p^2}\, K^{(n)} \left[\frac{\alpha_s(Q^2)}{\alpha_s(p^2)}\right]^{d^{(n)}} \qquad (6)$$

Performing the integration and adding in the contribution for $p^2<Q_0^2$ we obtain

$$\ln\frac{Q^2}{\Lambda^2}\,\frac{K^{(n)}}{1+d^{(n)}}\left[1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{d^{(n)}+1}\right] + \left[\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right]^{d^{(n)}} A^{(n)}(Q_0^2) \qquad (7)$$

thus the asymptotic solution is

$$\int dx\, x^{n-2} F_2^{\gamma}(x,Q^2) = \ln\frac{Q^2}{\Lambda^2}\,\frac{K^{(n)}}{d^{(n)}+1}\ , \quad Q^2\to\infty \quad d_n > -1 \qquad (8)$$

The approach to asymptotia is non-uniform as a function of x.[22] If the quantity $d^{(n)}+1$ vanishes for some $n=n_0$, Eq. (8) suggests a behaviour

$x^{-(n_0-1)}$, which reference back to the full solution (Eq. (7)) shows to be spurious. This phenomenon, which is relatively benign at leading log level, shows up in the singlet asymptotic formulae in the form of a negative cross-section spike in the next order.[25] This has led some authors to suggest that the only prediction of QCD is the non asymptotic formula (c.f. Eq. (7)) containing unknown distributions for quarks and gluons inside the photon.[26]

The terms which have been dropped in passing from the full solution Eq. (7) the asymptotic pointlike solution Eq. (8) certainly cancel the spurious poles in the latter.[27] The experimentally observed point-like behaviour indicates that this is their only numerically significant role. In this circumstance the regularised result (i.e., the asymptotic solution with the spurious poles cancelled) differs little from the asymptotic solution in the middle x range (0.4~0.8).[28] In this range the contamination from these terms introduces a modest theoretical uncertainty which still leaves the photon structure function competitive with other processes as a method of determining $\alpha_S$.

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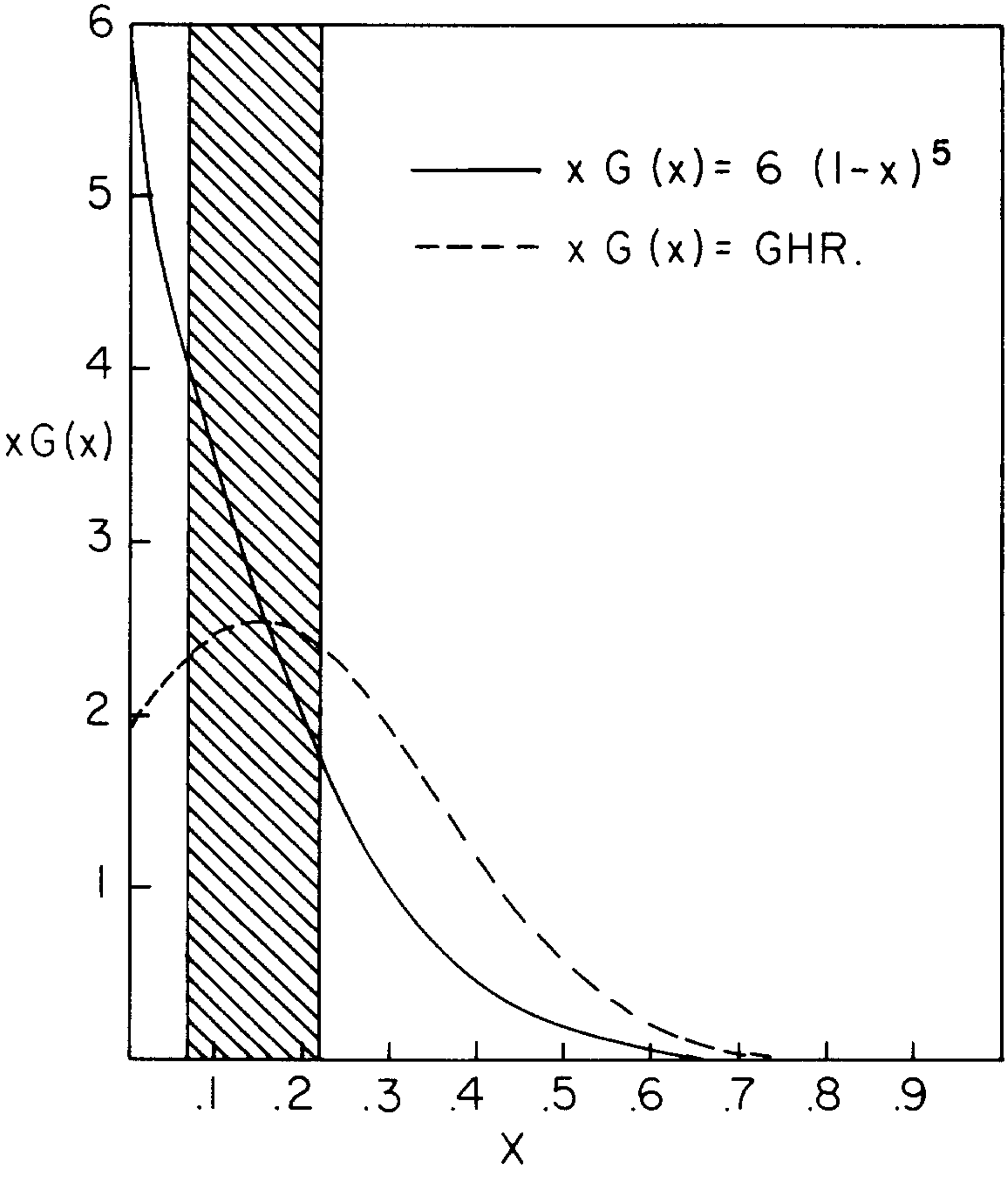

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