Fermi National Accelerator Laboratory

FERMILAB-Pub-82/63-THY
September, 1982

Two Loop Calculations of M_b/M_τ and Heavy Fermion Masses in the SU(5) Model

J. OLIENSIS
Fermi National Accelerator Laboratory
P.o. Box 500, Batavia, Illinois 60510

and

M. FISCHLER
Department of Physics and Astronomy
University of Pittsburgh
Pittsburgh, PA 15260

(Received

ABSTRACT

Two loop calculations of M_b/M_τ and M_s in the SU(5) model for 3-5 generations are presented, based on the complete SU(3)×SU(2)×U(1) two loop Higgs-Yukawa beta function. We find that the observed M_b/M_τ is consistent with 3 and 4 generations, in contrast to earlier work, but probably not with 5. We find upper limits on the top mass from the requirement that M_b/M_τ not be too large. We redo earlier calculations of M_x and $\sin^2\theta_w$, assuming the existence of heavy fermions, and essentially are in agreement with them. Lastly we consider the possibility of predicting the masses of heavy fermions by means of "pseudo fixed points" of the Higgs-Yukawa renormalization group equations.

The original SU(5) G.U.T. has engendered predictions that agree with experiment surprisingly well. The prediction for $\sin^2\theta$ in particular is very clean, being relatively independent of the other parameters of the model, and should provide an excellent test of SU(5) as experiment improves. The prediction of the ratio M_b/M_τ [1] has also been a success, though of course its testability is limited by the usual problems with the definition of a quark mass.

Because the model's success has been so striking, many of its consequences have been carried through to two loop order. In this paper we consider the effect of second order terms on M_b/M_τ, and address the important question of the dependence of this ratio on the number of generations. We find that this dependence is not strong enough to rule out a fourth generation, in disagreement with an earlier attempt at a second order calculation [2]. We give new upper limits on M_{top}, based again on the requirement that M_b/M_τ not be too large.

We recalculate $\sin^2\theta_W$ and M_X to two loops, allowing for the possible existence of heavy fermions, and are essentially in agreement with earlier work [3]. Lastly we reconsider to two loops recent predictions for the masses of heavy quarks, based on the existence of "pseudo-fixed points" of the renormalization group equations [10].

The authors of Ref. [2] have argued that more than three generations yields too large a prediction for M_b/M_τ in the SU(5) model. This was based on a two loop calculation of the evolution of the masses down from the unified scale M_x, where they are equal. They found that two loop effects significantly increased the dependence of M_b/M_τ on the number of generations. However the evolution was performed by making use of the anomalous dimension of the mass operator, which is non-gauge invariant in $SU(3)\times SU(2)\times U(1)$, and the results of Ref. [2] were consequently gauge dependent and incomplete. In addition Ref. [2] chose to fix $\sin^2\theta_W=0.2$ a priori, rather than allowing it to be determined by the type of self consistent procedure we shall describe.

We repeat this calculation making use of our recent computation [4] of the complete two loop beta function for the Higgs-Yukawa coupling g_f. This calculation includes all strong, electroweak and Yukawa terms, but not terms involving the coupling $\lambda\phi^4$. (The latter terms will not significantly affect the m_b/m_τ ratio.[13]) It was calculated in an arbitrary α-gauge, and we were able to see explicitly the cancellation of gauge dependence in the final answer. It is of course the Higgs-Yukawa couplings that properly determine fermion masses at low energy after spontaneous symmetry breaking occurs.

Thus we consider the evolution of the Yukawa couplings g_b and g_τ from the scale M_x where they are (approximately) equal down to low energies where they determine masses. We assume M_τ is known, and use the relation of the couplings at M_x to determine M_b. We ignore Cabibbo mixings of generations, and the possibility of heavy neutrinos.

The situation is complicated by the fact that M_x is not known beforehand, but is defined as the point where the gauge couplings join together, reflecting their common origin in a single SU(5) gauge coupling constant. It is necessary therefore to employ an iterative procedure, in which one firsts evolves up, fixing M_x, and then evolves down to find M_b and M_τ.

We now describe our procedure more precisely. The inputs are:

1) $\Lambda_{\overline{MS}}$ We take a range of values corresponding to the quoted [5] world average $\Lambda_{\overline{MS}}=0.16\,^{+0.1}_{-0.08}$.
2) $\alpha_{E.M.}$ The extrapolation of $\alpha_{E.M.}$ from the Compton limit to $\mu_{\overline{MS}}=M_W$ has been performed by Marciano [3], including higher order corrections and assuming M_{top}=20 GeV. We employ his value of 127.54, correcting it to first order for our various choices of fermion masses below M_x.
3) The τ and top masses, and their equivalents for a fourth generation. Quark masses are here defined by $M_q=M(\mu_{\overline{MS}}=M_q)$ where M is the running mass, and its argument the usual mass scale for $\overline{MS}$. Masses so defined are gauge invariant, but not physical. Relating this definition to a more physical one such as in a momentum subtraction scheme involves a gauge dependent extrapolation. This is the usual problem with the definition of a quark mass [14]. However in practice we do not expect our definition to be significantly in

error for heavy quarks, with mass $>>\Lambda_{QCD}$. In fact reasonable changes in our mass definition do not significantly affect our predictions.

The desired output is M_x, $\sin^2\theta_w$, the mass of the bottom-type quarks, and the values of the Yukawa couplings at M_x (to be discussed later).

Evolution of the couplings is governed by different beta functions in different regions of energy. Below M_w, for instance, the betas corresponding to the broken SU(3)×U(1) theory are appropriate, while above M_w the unbroken Weinberg-Salam (W-S) theory describes the running of the couplings. The effect of such thresholds can be dealt with by properly matching the running couplings in one region onto those of the other. This we do in the standard way [6].

We take M_x to be the threshold energy corresponding to SU(5) symmetry breaking. To two loop accuracy, the W-S and SU(5) gauge couplings are related across this threshold by

$$\alpha_1^{-1}(M_x) = \alpha_2^{-1}(M_x) - \frac{1}{6\pi} = \alpha_3^{-1}(M_x) - \frac{1}{4\pi} = \alpha_{GUT}^{-1}(M_x) - \frac{5}{12\pi} \quad (1)$$

where $\alpha_i = g_i^2/(4\pi)$ [3,6]. In our iterative scheme, the scale at which the W-S running couplings satisfy the above condition defines M_x.

Similarly, whereas g_b and g_τ are equal at M_x to one loop order, to two loops they are related by

$$g_b - g_\tau = -\frac{1}{(4\pi)^2} f \left\{ g_{GUT}^2 \left[3.5 - \frac{M_S^2}{M_x^2 - M_S^2} \ln\left(\frac{M_x^2}{M_S^2}\right) \right] + g_{top}^2 \left[\frac{7}{8} + \frac{3}{4} \ln\left(\frac{M_x^2}{M_S^2}\right) \right] \right\} \quad (2)$$

where f, g_b, g_{top}, g_τ are the Yukawa couplings to the SU(5) 5 and $\overline{10}$, the bottom quark, the top quark, and the τ lepton respectively [7]. M_S is the mass of a heavy scalar. We are assuming here the simplest standard Higgs structure for SU(5). We will take $M_S^2 = M_x^2/4$ in our calculations--the results are relatively insensitive to the choice of this parameter.

An analogous relation holds for the fourth generation couplings.

At low energies we must consider the effect of the symmetry breaking threshold at M_w, and also of heavy fermion ($M_f > M_w$) thresholds. For fermions of mass less than 1 TeV it is possible to treat these effects together [6]. The W-S couplings that are appropriate to the high energy region can be directly determined <u>at M_w</u> from the lower energy parameters such as $\sin^2\theta_w$, $1/\alpha_{E.M.}$, M_b, etc. If the threshold effects are properly taken into account at M_w, the W-S couplings can be evolved from M_w to M_x without worrying further about thresholds. One has:

$$\frac{1}{\alpha_1(M_W)} = \frac{3}{5}\,(1-\sin^2\theta_W)\left[\frac{1}{\alpha_{E.M.}(M_W)} + \frac{1}{6\pi} + \frac{2}{3\pi}\,N_c \sum_{Fi} Q_i^2 \ln\left(\frac{M_{Fi}}{M_W}\right)\right]$$

$$\frac{1}{\alpha_2(M_W)} = \sin^2\theta_W\left[\frac{1}{\alpha_{E.M.}(M_W)} + \frac{1}{6\pi} + \frac{2}{3\pi}\,N_c \sum_{Fi} Q_i^2 \ln\left(\frac{M_{Fi}}{M_W}\right)\right]$$

$$\frac{1}{\alpha_3(M_W)} = \frac{1}{\alpha_{QCD}(M_W)} + \frac{4}{3} \sum_{Fi} \ln\left(\frac{M_{Fi}}{M_W}\right) . \qquad (3)$$

N_c is the number of colors, and the sum is to be performed over all fermions Fi with masses $M_{Fi} > M_W$. $\alpha_{E.M.}(M_W)$ and $\alpha_{QCD}(M_W)$ are respectively the electromagnetic and strong coupling extrapolated up to M_W from the energies at which they are measured. The definition of the W-S gauge couplings α_i then includes all threshold effects and the evolution of each of these couplings from M_W to M_X is described by a single beta function. The number of generations, for instance, that enters into the calculation of this beta should be the total number, even though M_W may be below the fourth or fifth generation threshold. We are assuming of course the existence of a desert between low energies and M_X.

In principle one must also consider the two loop corrections to the familiar formula

$$M_f(M_W) = \frac{V}{\sqrt{2}}\, g_f(M_W) \text{ where } \frac{V}{\sqrt{2}} \simeq 175 \text{ GeV} \qquad (4)$$

that relates masses of the low energy theory to the W-S

Higgs-Yukawa couplings. However these corrections will affect M_b/M_τ by less than a percent, and so we ignore them.

Next we mention the evolution of the couplings below M_W. We calculate the pure SU(3) two loop evolution of α_3 from low energy ($\sim\sqrt{10}$ GeV) to M_W. Quark thresholds are dealt with as usual by demanding that the coupling be continuous at the threshold. We have already discussed the extrapolation of $\alpha_{E.M.}$. The Higgs-Yukawa couplings evaluated at M_W are obtained by evolving the masses of the light fermions upward to M_W from the energies at which they are defined. We perform this evolution using the one loop beta function, except that two loop terms of SU(3) are included. We take the running masses continuous at thresholds.

Thus we have described the evolution of the couplings in all regions of energy.

The beta functions that govern the running of the couplings from M_W to M_X have been given elsewhere [4,8,9]. They are the form

$$\frac{dg_i}{dT} = \frac{1}{16\pi^2}\left(-\beta_0^{(i)} g_i^3\right) + \sum_j \frac{\beta^{ij}}{(16\pi^2)^2}(g_j)^2 (g_i)^3 + \sum_f \frac{C_f^i (g_f)^2}{(16\pi^2)^2}(g_i)^3 \qquad (5)$$

$$\frac{dg_f}{dT} = \frac{1}{16\pi^2} g_f \left[\sum_f \beta_{ff'} g_{f'}^2 + \sum_i D_{fi} g_i^2\right] + \frac{1}{(16\pi^2)^2} g_f \left[\sum_{f,f''} E^f_{f'f''} g_{f'}^2 g_{f''}^2 + \sum_{f',i} F^f_{f'i} g_{f'}^2 g_i^2\right] \qquad (6)$$

Here g_i represents a gauge coupling, and $g_{f,f',f''}$ a Higgs-Yukawa coupling. We have included in the gauge beta both the first and second order [9] contribution due to Higgs loops, and also terms in which Higgs-Yukawa couplings explicitly appear due to fermion loops [8]. The beta function for the g_f includes all strong, electroweak and Yukawa contributions, but no ϕ^4 coupling terms [4]. It is important that there are terms in g_f on the RHS of Eq. 6 to which all generations contribute.

Finally we turn to the numerical integrations of these equations. We use a scheme similar to that of Ref. [8].

1) At M_W we guess values for $\sin^2\theta_W$ and $M_b(M_W)$. From $\sin^2\theta_W(M_W)$ and $\alpha_{E.M.}(M_W)$ we determine α_1 and $\alpha_2 \cdot \alpha_3(M_W)$, $\alpha_\tau(M_W)$, $\alpha_{top}(M_W)$, and fourth generations equivalents are inputs.

2) Evolve upward until $\alpha_1^{-1}=\alpha_3^{-1}-1/4\pi$. This will determine a "first guess" for M_x. α_2 will not necessarily match the other two gauge couplings at M_x, nor will g_b and g_τ match up.

3) At this M_x, we fix $\alpha_2^{-1}=\alpha_1^{-1}-1/6\pi$ and fix g_b according to Eq. 6.

4) Evolve back down to M_w. α_1 and α_2 now give a better value for $\sin^2\theta_w$, $g_b(M_w)$ a better value for $M_b(M_w)$. However $\alpha_3(M_w)$ $\alpha_\tau(M_w)$, and $\alpha_{top}(M_w)$ will not necessarily have their input values.

5) Reset the above three couplings to their original input values, and repeat from 2).

This procedure generally converges within 3 or 4 iterations to give the desired matching of couplings at M_x.

In all cases the fourth generation is handled in the same manner as is the third.

Following the determination of a consistent set of parameters for M_x, $\sin^2\theta(M_w)$, $M_b(M_w)$, $g_b(M_x)$, etc., we integrate down from M_w to determine the mass of the bottom quark, $M_b(M_b)$. Also the masses of fermions heavier than M_w are extrapolated to the energy at which they are defined.

We proceed to the discussion of our results. Table I displays our predictions of M_b and $\sin^2\theta$ for three generations. $\sin^2\theta$ depends primarily on $\Lambda_{\overline{MS}}$; M_b depends on M_{top} as well.

For both three and four generations, the value of M_b for $m_{top} < 200$ GeV is relatively stable to changes in $\alpha_3(M_W)$, or $\alpha_{E.M.}(M_W)$, or adding another light Higgs, or using a different M_S, and the results are probably accurate to within a percent. An additional light Higgs, for instance, decreases M_b by a half percent. Of course the unknown relation of our mass definition to the physical mass introduces an additional uncertainty.

One can represent the $\Lambda_{\overline{MS}}$ dependence of M_b for the smaller values of M_{top} by approximately $M_b \sim (\Lambda_{\overline{MS}})^{0.12}$.

Note that the prediction of M_b increases with M_{top}, and becomes inconsistent with experiment when M_{top} is large. This has been noticed before [11]. Since the top mass will in fact be large if it is determined by a "pseudo-fixed point" of the beta functions, we rule out this possibility. If we take 5.5 GeV as the upper limit on the b mass, then the upper limit on the top mass is about 180, 145, and 20 GeV for Λ=0.08, 0.16, and 0.26. However these limits depend sensitively on the b mass upper limit 5.5 GeV and the possibility of Cabibbo mixing. For four generations, using the same criterion, we get somewhat lower limits on the top mass: $M_{top} < 145$ for Λ=0.08. Λ=.16 and Λ=0.26 give $M_b > 5.5$ for M_{top}=20 GeV.

These numbers differ from those given earlier [11] though the general behavior of M_b/M_τ with increasing top mass is the same.

In three generations, we find by evolving M_s/M_μ that the strange quark mass is 0.42, 0.51, and 0.62, when $\Lambda_{\overline{MS}}$ = 0.08, 0.16, 0.26 respectively. The values are independent of the top mass at the 1% level. However there are likely to be large QCD corrections to these numbers.

Some of our fourth generation results are shown in Table 2. Assuming M_{top} = 20 GeV, we have M_b = 5.06, 5.55, and 5.94 for $\Lambda_{\overline{MS}}$ = 0.08, 0.16 and 0.26. For larger values of M_{top}, M_b increases consistent with our expectations from the 3 generation case. These values change by less than a percent as we vary the masses of the fourth generation fermions. This stability is due to the fact that the beta functions for b and τ have exactly the same dependence on the fourth generation masses, so that the influence of the masses cancels when we consider the ratio of the two. We have checked that the ratio does not change significantly when we simply eliminated the fourth generation terms in the two beta functions. Thus even in four generations it is possible to obtain $M_b \lesssim 5.1$ or 5.2 GeV, for $M_{top} \lesssim 100$ GeV and $\Lambda_{\overline{MS}}$ at the low end of its experimental range. Such a value (and thus the existence of a fourth generation) can not be ruled out by experiment.

Our results should be compared with those of Ref. 2, who for $\Lambda_{\overline{MS}}$ = 0.19 find M_b = 5.2 and 6.3 in three and four generations. The discrepancy may arise in part from the use in Ref. 2 of $\sin^2\theta_w = 0.2$ as a priori input, in lieu of determining $\sin^2\theta_w$ from the requirement that all three gauge couplings unify at M_x. Note that in one loop the increase in M_b from three to four generations is only 0 [1].5 GeV, consistent with our results [1].

Motivated by the stability of M_b, we have repeated our iterative procedure for 5 generations without explicitly putting in new fermions. The total number of generations, which appears as a parameter in the beta functions, is simply set equal to 5. We find that M_b = 5.6, 6.2 and 6.7. Again, these values are relatively independent of the way in which we let β_b and β_τ depend (in common) on the heavy generations.

Thus there is about an 8% increase in M_b from 3 to 4 generations, and another 13% or so from 4 to 5.

The strange quark mass increases by a similar fraction. We find for four generations M_s = 0.44, 0.53, and 0.64.

Our values for $\sin^2\theta_w$ are remarkably independently of the other parameters. Numerical accuracy is better than ± 0.0001. $\sin^2\theta_w$ seems most sensitive to $\Lambda_{\overline{MS}}$ and $\alpha_3(M_w)$. Adding a Higgs increases $\sin^2\theta_w$ by about 2%.

When all fermions are $\lesssim$ 100 GeV, we find that $\sin^2\theta_w$ is nearly the same for 4 generations as it is for 3. In both cases it increases for larger masses.

Our values of M_x have a numerical error of a few percent. They are in reasonable (~10%) agreement with values quoted by Marciano. An additional light Higgs decreases M_x by about 30%. M_x does not depend significantly on the existence of heavy fermions [8].

There is a 12 or 13% increase in M_x from 3 to 4 generations, in accord with Marciano [3], but not with the 80% increase of Ref. 12.

Using the value $M_S^2/M_x^2 = 0.05$ instead of 0.25 affects only the fourth generation masses, and then only by a few GeV for the fourth generation "top" quark mass very large.

We now turn to the question of fixing heavy fermion masses by means of "pseudo-fixed points."

A recent proposal, possibly more general as a principle than the particular SU(5) model, is that low energy observable quantities might arise as "pseudo-fixed points" of beta functions [10]. In other words, the values of currently observable parameters might be relatively independent of the details of the high energy theory.

They would be essentially determined by the beta functions of an effective low energy theory, so that for a large range of initial values at high energies, parameters would be swept in the course of evolution towards their "fixed point" values at low energies.

This idea is exemplified by an SU(5) model that includes very heavy quarks. It has been shown at the one loop level that a sufficiently heavy top quark most probably would have a mass around 240 GeV [10]. That is, there is a large range of values for the Yukawa coupling $g_{top}(M_x)$ that result in top masses close to 240 GeV.

Of course one is considering here the evolution of the Higgs-Yukawa coupling down through quite large values ($g_f(M_x) \leq 5$ or 6), so that one would expect second order terms of the beta functions to be important. We have therefore improved previous fixed point calculations to the two loop level.

For three generations, as has already been stated, the experimental value of M_b rules out a large top mass, and thus fixed point predictions are not relevant. Nevertheless we have considered this case for purposes of comparison to previous work. For $\Lambda_{\overline{MS}} = 0.16$, we find that the range $2 \leq g_{top}(M_x) \leq 6$ corresponds approximately to $224 \leq M_{top} \leq 232$. We can therefore "predict" the likelihood of a top quark in this range.

An upper limit on the top mass can be obtained by requiring that perturbation theory is valid up to the M_x scale, i.e. that $g_{top}(M_x)$ is small enough so that the second order terms are smaller than the first. For $\Lambda_{\overline{MS}} = 0.16$ this limit is about 235 GeV. For $\Lambda_{\overline{MS}} = 0.08$ and 0.26, the upper limits are respectively 230 and 240 GeV. Expected top masses are slightly less than these upper limits. These results are close to those obtained earlier by a one loop calculation [10].

For four generations the situation is more complicated. Here we are assuming a small top mass, $\gtrsim 20$ GeV, and must investigate the possible masses of the three fourth generation fermions, which we call the high and low quarks, and the E lepton. These masses are determined by $g_H(M_x)$ and $g_E(M_x)$ (or equivalently $g_L(M_x)$). Following Ref. 10, we consider all integer values for the pair $(g_H(M_x),\ g_E(M_x))$ with $1 \leq g_H(M_x),\ g_E(M_x) \leq 4$, and look at the corresponding masses. The results, for $\Lambda_{\overline{MS}} = 0.16$ and $M_{top} = 20$ GeV, are displayed in Table 3. The masses displayed there do not cluster around a single fixed point, but in general we might expect $M_H = M_L = 195$ GeV, $M_E = 70$ GeV.

There are three regions of particular interest in (g_H, g_E): $g_H >> g_E$, $g_E >> g_H$, and $g_E \sim g_H$. The first two regions can be studied in the same way as the three generation case. Region one gives an upper limit on M_H, $M_H < 230$ GeV. This limit increases by about 5 or 10 GeV for larger M_{top} (50 or 70 GeV). The upper limits for $\Lambda_{\overline{MS}} = 0.08$ and 0.26 are 225 and 245.

Region two yields upper bounds on M_L and M_E. For all three values of $\Lambda_{\overline{MS}}$, these are about $M_E < 110$-120, $M_L < 220$-230. For heavier M_{top}, the limit on M_L increases slightly, and that on M_E decreases.

The middle region does reveal fixed point behavior; if $g_H = g_E$ is in the range of 2 to 4, then $M_H = 202 \pm 2$, $M_L = 197 \pm 1$, and $M_E = 68.5 \pm 0.5$.

In conclusion, we have considered the effect of two loop terms in the Higgs-Yukawa beta functions on low energy fermion masses in the minimal SU(5) model. We have also checked previous calculations of M_x and $\sin^2\theta(M_w)$. We find that the observed M_b/M_τ does not rule out four generations, but probably does rule out five. The observed M_b/M_τ also rules out very large values for the top mass, in particular "fixed point" values for this mass. We have given upper limits on the top mass assuming three or four generations. We find that our expectations for the mass of fourth generation fermions, based on randomly chosen values for Yukawa couplings at M_x, is on the order of 195 GeV for quarks and 70 for the lepton, in rough agreement with an earlier one loop calculation.

ACKNOWLEDGMENTS

We would like to thank C. Hill for useful comments and discussions.

REFERENCES

[1] M. Chanowitz, J. Ellis and M.K. Gaillard, Nucl. Phys. B128 (1977) 506.

A. Buras, J. Ellis, M.K. Gaillard and D. Nanapoulos, Nucl. Phys. B135 (1978) 66.

[2] D.V. Nanapoulos, D.A. Ross, Nucl. Phys. B157 (1979) 273.

[3] W. Marciano, invited talk at the 19th Orbis Scientiae Meeting, January 18-21, 1982, BNL 21036.

[4] M. Fischler and J. Oliensis, Phys. Lett. B, (1982), to be published.

[5] A. Buras, Proceedings of the 1981 Lepton Photon Symposium, August 1981, Fermilab preprint, FERMILAB-Pub-81/69-THY.

[6] L. Hall, Nucl. Phys. B178 (1981) 75.

[7] M. Machacek, in the Second Workshop on Grand Unification, University of Michigan, Ann Arbor, 1981, edited by J.P. Leveille, L. Sulak, and D. Unger, p. 265.

[8] M. Fischler, C. Hill, Nucl. Phys. B193 (1981) 53.

[9] D.R.T. Jones, Phys. Rev. D25 (1982) 581.

[10] C. Hill, Phys. Rev. D24 (1981) 691.

B. Pendleton and G.G. Ross, Phys. Lett. 98B (1981) 291.

M. Machacek and M. Vaughn, Phys. Lett. 103B (1981) 427.

[11] Ref 10; ref.7, and D. Nanapoulos and D. Ross, Phys. Lett. 108B (1982) 351.

[12] T. Goldman and D. Ross, Nucl. Phys. B171 (1980) 273. J. Ellis, M.K. Gaillard, D. Nanopoulos and S. Rudaz, Nucl. Phys. B176 (1980) 61.

[13] J. Oliensis and M. Fischler, in preparation.

[14] P. Binẽtruy and T. Schücker, Nucl. Phys. B178 (1981) 193.

TABLE I. Three Generation Results

$\Lambda_{\overline{MS}}$	M_{top}	M_{bot}	$M_x(\times 10^{14}$GeV)	$\sin^2\theta_W$	$1/\alpha_{E.M.}(M_W)$	$\alpha_3(M_W)$	$\alpha_{top}(M_x)/4\pi$
.08	20	4.73	.944	.21823	127.54	.0967	
"	99	4.86	.963	.21865	128.00	.0955	.00034
"	146	5.11	.963	.21881	128.11	.0950	.0011
"	210	6.27	.963	.21890	128.21	.0944	.0097
"	225.5	7.68	.944	.21901	128.23	.0941	.077
"	229		.983	.21874	128.23	.0944	.27
.16	20	5.13	1.90	.21428	127.54	.1068	
"	99	5.26	1.94	.21472	128.00	.1052	
"	145	5.51	1.94	.21487	128.11	.1046	
	193	6.12	1.94	.21496	128.19	.1041	.0036
"	232	7.5	1.94	.21494	128.24	.1037	.24
.26	20	5.49	3.14	.21146	127.54	.1152	
"	99	5.61	3.20	.21191	128.00	.1133	.00027
"	145	5.84	3.20	.21206	128.11	.1126	.00083
"	207		3.20	.21222	128.21	.1120	.0052
"	233		3.07	.21241	128.24	.1112	.11

TABLE II. Fourth Generation Results

$\Lambda_{\overline{MS}}$	M_H	M_L	M_E	M_{bot}	$M_X(\times 10^{14}GeV)$	$\sin^2\theta(M_W)$	$1/\alpha_{E.M.}(M_W)$	$\alpha_3(M_W)$
.08	99	70.0	30	5.09	1.06	.21829	127.37	.0965
"	201	191	70	5.06	1.13	.21869	127.82	.0950
"	211	176	57		1.09	.21877	127.78	.0949
"	228	114.5	31.2		1.08	.21839	127.59	.0956
"	178	203	86		1.11	.21885	127.78	.0949
"	110.5	215.5	109		1.15	.21865	127.75	.0955
.16	99	75	30	5.57	2.19	.21427	127.37	.1063
"	194	86	30	5.55	2.19	.21451	127.58	.1055
"	214	94	30	5.55	2.19	.21455	127.61	.1053
"	204	197	68	5.55	2.28	.21479	127.82	.1045
"	118	221	106	5.55	2.37	.21482	127.77	.1050
"	154	213	94		2.28	.21482	127.78	.1046
"	189	185	70		2.23	.21481	127.78	.1041
.26	99	78	30	5.98	3.61	.21150	127.37	.1146
"	223	103	30	5.94	3.61	.21178	127.63	.1133
"	207	201	66	5.93	3.75	.21203	127.82	.1125
"	219	186	53		3.75	.21184	127.78	.1128
"	187	211	80		3.83	.21194	127.78	.1128
"	119.5	224	103		3.83	.21197	127.72	.1133

M_H, M_L, M_E are the masses of the fourth generation "top" and "bottom" quarks, and lepton, M_{top} =20 GeV.

TABLE III. (M_H, M_L, M_E) as a function of (g_H, g_E)

g_H \ g_E	1	2	3	4
4	(227,147,39)	(216,181,54)	(209,192,62)	(204,198,68)
3	(222,158,44)	(210,188,60)	(203,197,68)	(197,202,73)
2	(214,170,53)	(200,196,69)	(190,203,77)	(183,207,83)
1	(189,185,70)	(168,206,86)	(154,213,94)	(142,216,98)