



Energy Loss of Energetic Partons in Quark-Gluon Plasma:
Possible Extinction of High p_T Jets in Hadron-Hadron Collisions.

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Abstract

High energy quarks and gluons propagating through quark-gluon plasma suffer differential energy loss via elastic scattering from quanta in the plasma. This mechanism is very similar in structure to ionization loss of charged particles in ordinary matter. The dE/dx is roughly proportional to the square of the plasma temperature. For hadron-hadron collisions with high associated multiplicity and with transverse energy dE_T/dy in excess of 10 GeV per unit rapidity, it is possible that quark-gluon plasma is produced in the collision. If so, a produced secondary high- p_T quark or gluon might lose tens of GeV of its initial transverse momentum while plowing through quark-gluon plasma produced in its local environment. High energy hadron jet experiments should be analysed as function of associated multiplicity to search for this effect. An interesting signature may be events in which the hard collision occurs near the edge of the overlap region, with one jet escaping without absorption and the other fully absorbed.

I. INTRODUCTION

One of the most prominent features of the recent early data¹ from the CERN SPS proton-antiproton collider (SP \bar{P} S) has been the relatively large yield of events with very high transverse energy. These events are dissimilar from what is expected from hard parton-parton collisions yielding high- p_T "jets." Instead the large observed E_T is contributed essentially by a large number of low- p_T hadrons rather uniformly distributed in phase-space. This phenomenon, observed also in fixed target experiments at the SPS² and Fermilab,³ seems to become, if anything, more significant as the energy increases. It is undoubtedly connected with the existence of large fluctuations in multiplicity about the mean, i.e. the KNO-scaling phenomenon.⁴ However, the underlying dynamical explanation is far from clear.⁵

Define

$$\left. \frac{dE_T}{dy} \right|_{y=0} = \begin{array}{l} \text{energy produced per unit rapidity} \\ \text{at } 90^\circ \text{ center-of-mass production angle} \end{array} \quad (1.1)$$

(similar definitions for general y are obtained by boosting that y to zero). Then the observed distribution of dE_T/dy is roughly exponential, with a mean increasing from ~ 1 GeV for $\sqrt{s} \sim 25$ GeV to ~ 2 GeV for $\sqrt{s} \sim 540$ GeV. However, events with values of $(dE_T/dy) \sim 30$ GeV have already been seen at the SP \bar{P} S. This implies ~ 60 GeV of energy emerging more-or-less isotropically from the collision region into a 10 steradian central detector ($40^\circ < \theta < 140^\circ$). If one assumes only that this energy travels in a straight line outward from the initial collision region, of volume

$\sim 1 \text{ f}^3$, it follows that the initial energy density in the collision region is very large, of order $10\text{-}50 \text{ GeV/f}^3$. This is a few hundred times larger than nuclear matter density, and an order of magnitude above what is needed to produce quark-gluon plasma.

It is clear that this produced system cannot, on a time scale \leq a few fermi after impact of the projectiles, be considered a collection of hadrons. (We estimate, in fact, $5\text{-}10\text{f.}$ are necessary.) It is tempting to instead regard the system as quark-gluon plasma in thermal equilibrium. Although this is what we shall do, it is very unclear whether the time scales are in fact long enough to establish and maintain thermal equilibrium. But let us, for definiteness, suppose we do produce plasma in between the outgoing Lorentz-contracted hadron projectiles. We have, in the context of ion-ion collisions, already described⁶ the space time evolution of such produced plasma. For times small compared to the nucleon radius the plasma, to a reasonable approximation, expands longitudinally and homogeneously. For larger times it expands radially outward in a spherical shell at more or less the speed of light.

It is not our purpose here to discuss the properties or evolution of the plasma per se, but to study only one specific consequence of this picture. This has to do with the influence of this phenomenon upon hard-collision processes. From the space-time picture of the collision, we may infer that the two initial-state partons are, because of causality, unaffected by the plasma which is subsequently produced. However, the secondary high- p_T quarks or gluons are affected. This is best seen in the Lorentz frame where the secondary parton moves at 90° to the beam direction (Fig. 1). It encounters the quanta produced at

other impact parameters which also are moving mainly transverse to the beam directions. The high- p_T parton may elastically scatter from the quarks and gluons in the plasma, thereby degrading its energy and heating the plasma. In Section II we calculate this process and find that it is very similar to ionization loss of charged particles traversing ordinary matter. The mean energy loss dE/dx is well-defined, but with a "Landau-straggling" tail at large energy losses. The dE/dx turns out to be proportional to the square of the temperature of the plasma, hence (for ideal plasma) to the square root of the energy density. The plasma energy density (at an elapsed time of order the hadron radius) measures the (isotropic) transverse energy density dE_T/dy at $y=0$ which we have already defined. Thus we have the result that the p_T degradation of a jet $\langle \Delta p_T \rangle$ is roughly proportional to the square root of the associated transverse energy:

$$\langle \Delta p_T \rangle_{\text{jet}} \sim \left(\frac{dE_T}{dy} \right)^{1/2} \quad (1.2)$$

The coefficient of proportionality, while uncertain, suggests that for $(dE_T/dy) > 10-30$ GeV the p_T loss of the jet might be ≥ 10 GeV.

The mean value of dE_T/dy is much smaller than 10 GeV, more like 2-3 GeV. In this regime, the calculation depends not only on the properties of non-ideal plasma at relatively low temperature but also on the low-momentum tail of the quark and gluon distribution functions in the plasma. Thus even upon assuming applicability of hydrodynamic ideas in this regime, the numerical estimates are especially uncertain. Thus the main message of this study is to urge experimentalists to carefully examine jet phenomena as a function of associated multiplicity and

(dE_T/dy) . Clearly the best guidance will come from experiment.

In Section II we calculate the differential energy loss of a high energy quark or gluon in quark-gluon plasma. In Section III we apply these results to estimate the conditions under which high- p_T jets might be "extinguished" by absorption by the plasma in its local environment, and also discuss some experimental signatures.

II. CALCULATION OF DIFFERENTIAL ENERGY LOSS

We consider the propagation of a high-energy quark or gluon through an ideal uniform quark-gluon plasma which is in thermal equilibrium at temperature $T \equiv \beta^{-1}$ and which has zero chemical potential. The gluon density is thus

$$dn_g = \frac{16 d^3k}{(2\pi)^3} (e^{\beta k} - 1)^{-1} \equiv \rho_g d^3k \quad (2.1)$$

and the quark-plus-antiquark density is (neglecting mass)

$$dn_q = 12N_f \frac{d^3k}{(2\pi)^3} (e^{\beta k} + 1)^{-1} \equiv \rho_q d^3k \quad (2.2)$$

Consider now the elastic scattering of the incident parton of momentum p from a parton of momentum k in the plasma. The cross-section is, for the (dominant) small momentum transfers $t \ll s$

$$\frac{d\sigma}{dt} \approx \frac{2\pi\alpha_s^2}{t^2} \cdot \begin{cases} \left(\frac{2}{3}\right)^2 & qq \\ 1 & qg \\ \left(\frac{3}{2}\right)^2 & gg \end{cases} \quad (2.3)$$

This form is factorizable; hence if we take out a factor 2/3 for quark and 3/2 for gluon, and define an effective plasma density ρ

$$\rho = \frac{2}{3}\rho_q + \frac{3}{2}\rho_g \quad (2.4)$$

we may write for the cross-section of an energetic parton on effective plasma:

$$\begin{aligned} \frac{d\sigma}{dt} &= \frac{2\pi\alpha_s^2}{t^2} \times \begin{cases} 2/3 & \text{incident quark} \\ 3/2 & \text{incident gluon} \end{cases} \\ &= \left(\frac{2}{3}\right)^{\pm 1} \cdot \frac{2\pi\alpha_s^2}{t^2} \end{aligned} \quad (2.5)$$

The interaction of the incident parton with the plasma is analogous to ionization loss; we want to estimate the mean ionization loss dE/dx (in GeV/f.). Let E' be the energy of the emergent parton and $\nu = E - E'$. Then

$$\frac{dE}{dx} = \int d^3k \rho(k) \cdot [\text{Flux factor}] \int dt \frac{d\sigma}{dt} \cdot \nu \quad (2.6)$$

If all participating partons are massless, then there is, in the limit $E, E' \gg k$, various kinematical simplifications:

$$s = 2kE(1-\cos\theta)$$

$$|t| = s\left(1 - \frac{E'}{E}\right) = s\frac{v}{E} \quad (2.7)$$

Flux factor $\equiv (1-\cos\theta)$

Here θ is the laboratory angle between the incident partons. Thus:

$$\frac{dE}{dx} = \left(\frac{2}{3}\right)^{\pm 1} \int d^3k \rho(k)(1-\cos\theta) \cdot 2\pi\alpha_s^2 \cdot \int \frac{dv}{v} \frac{E}{s}$$

$$= \pi\alpha_s^2 \left(\frac{2}{3}\right)^{\pm 1} \int \frac{d^3k}{k} \rho(k) \log \frac{v_{\max}}{v_{\min}} \quad (2.8)$$

We must decide upon values for v_{\max} and v_{\min} . In addition it may be unrealistic to integrate over all k ; in the infrared region there may be screening and other kinds of corrections.⁷

For v_{\max} , we may take $v_{\max} = E/2$. For v_{\min} , we choose $|t_{\min}|^{1/2} = 0.5 - 1$ GeV $\equiv M$. This should be conservative. Therefore

$$v_{\min} \equiv \frac{EM^2}{s} = \frac{M^2}{2k(1-\cos\theta)} \sim \frac{M^2}{2k} \quad (2.9)$$

Thus

$$\log \frac{v_{\max}}{v_{\min}} \sim \log \frac{2kE}{M^2} \quad (2.10)$$

We note that the differential energy loss is, in accordance with the t^{-2} behavior of the basic cross-section,

$$\frac{d\sigma}{dE} \sim \frac{1}{v^2} \quad (v \text{ large}) \quad (2.11)$$

like the Landau straggling in ordinary ionization-loss phenomena. Thus the probability of catastrophic loss becomes small with very high energy.

We now may integrate over k . Neglecting the variation of k in the logarithm, and choosing a minimum momentum $k_{\min} \approx M$ gives

$$\frac{dE}{dx} = 4\pi^2 \alpha_s^2 \left(\frac{2}{3}\right)^{\pm 1} \log \frac{2\langle k \rangle E}{M^2} \int_M^\infty k dk \rho(k) \quad (2.12)$$

For temperatures $\beta^{-1} \gg M$, we get

$$\begin{aligned} \frac{dE}{dx} &\approx \frac{4\pi^2 \alpha_s^2}{8\pi^3} \left(\frac{2}{3}\right)^{\pm 1} \log \frac{2\langle k \rangle E}{M^2} \left\{ \frac{3}{2} \cdot \frac{16}{\beta^2} \sum_{n=1}^\infty \frac{1}{n^2} + \frac{2}{3} \cdot \frac{12N_f}{\beta^2} \sum_{n=1}^\infty \frac{(-)^{n+1}}{n^2} \right\} \\ &= \frac{2\pi\alpha_s^2}{\beta^2} \left(\frac{2}{3}\right)^{\pm 1} \log \frac{2\langle k \rangle E}{M^2} \left(1 + \frac{N_f}{6}\right) \end{aligned} \quad (2.13)$$

It will be convenient to relate this to the energy density in the plasma (so that we ultimately correlate the energy loss in the jet to the observed isotropic produced transverse energy). The energy density ϵ is, from Stefan and Boltzmann

$$\epsilon = \epsilon_q + \epsilon_g \quad (2.14)$$

with

$$\epsilon_q = \frac{(12N_f)}{(2\pi)^3} \cdot \left(\frac{24\pi}{\beta^4}\right) \cdot \frac{7}{8} \cdot \sum_{n=1}^{\infty} \frac{1}{n^4}$$

$$\epsilon_g = \frac{16}{(2\pi)^3} \cdot \left(\frac{24\pi}{\beta^4}\right) \cdot \sum_{n=1}^{\infty} \frac{1}{n^4} \quad (2.15)$$

or

$$\epsilon = \frac{8\pi^2}{15\beta^4} \left[1 + \frac{21N_f}{32} \right] \quad (2.16)$$

Thus

$$\frac{dE}{dx} = \left(\frac{2}{3}\right)^{\pm 1} \cdot \frac{\sqrt{30}}{2} \cdot \alpha_s^2 \epsilon^{1/2} \cdot \log \frac{2\langle k \rangle E}{M^2} \cdot \frac{\left(1 + \frac{N_f}{6}\right)}{\sqrt{1 + \frac{21N_f}{32}}} \quad (2.17)$$

This is the basic formula. The final flavor-dependent factor is an irrelevant nuisance; we may choose ≈ 0.85 for that. With $\langle k \rangle \approx 2\beta^{-1}$ and $\alpha_s \approx 0.2$ (conservatively small??), we get

$$\frac{dE}{dx} \approx \left(\frac{2}{3}\right)^{\pm 1} \epsilon^{1/2} \left(\log \frac{4ET}{M^2} \right) \text{GeV}/f \quad (2.18)$$

where ϵ is measured in GeV/f^3 , and one chooses +1 for gluon, -1 for quark.

We have made the approximation $T = \beta^{-1} \gg M$. If the temperature T is comparable to M , as may well be more realistic, the correction factor is roughly

$$\text{correction} \cong \frac{\int_0^{\infty} k dk e^{-\beta k}}{\int_0^{\infty} k dk e^{-\beta k}} \cong e^{-M/T} (1+M/T) \quad (2.19)$$

We use, therefore, as the working formula

$$\frac{dE}{dx} \approx \left(\frac{2}{3}\right)^{\pm 1} \epsilon^{1/2} \left(\log \frac{4ET}{M^2}\right) \cdot e^{-M/T} (1+M/T) \text{ GeV/f} \quad (2.20)$$

III HIGH- p_T COLLISIONS

Consider a high energy hadron-hadron collision for which there is a high- p_T parton-pair produced, and go into the frame in which the parton of interest emerges at 90° . We shall envisage that shortly after the collision ($t \ll 1f.$) ideal quark-gluon plasma is formed in between the outgoing hadron pancakes, and that ideal hydrodynamics applies. The evolution of the system is shown in Fig. 1. The entropy in a comoving volume element remains constant. Hence during the longitudinal expansion the entropy (number) density of quanta decreases roughly as t^{-1} and energy density as $t^{-4/3}$. We are assuming that at given longitudinal coordinate z and time t that the initial energy density ϵ and temperature T are, within the overlap region, independent of transverse coordinates. During the longitudinal evolution, the high- p_T parton of interest must plow through the produced plasma. We start the evolution at an early time t_1 and stop it (i.e. ignore all subsequent absorption) at a time $t_f \approx R_0$ when the energy density is ϵ_f , and where R_0 is of order the nucleon radius. At this time the plasma is contained in a volume $\sim \frac{4}{3}\pi R_0^3$. At later times we assume that the relevant plasma

(everything which is not very close to the Lorentz-contracted projectiles) expands isotropically, so that the energy observed per unit solid angle is related to ϵ_f and R_0 as follows:

$$\frac{dE}{d\Omega} = \frac{dE_T}{d\Omega} \Bigg|_{90^\circ} = \frac{1}{2\pi} \frac{dE_T}{dy} = \frac{1}{4\pi} \left(\frac{4}{3} \pi R_0^3 \right) \epsilon_f \quad (3.1)$$

Thus

$$\epsilon_f = \frac{3}{2\pi R_0^3} \left(\frac{dE_T}{dy} \right)_{90^\circ} \quad (3.2)$$

In this way we may relate the final energy density ϵ_f to the observable isotropic component of the transverse energy density.

The total energy loss of the high- p_T parton is then

$$\begin{aligned} \Delta p_T &= \int_{t_i}^{t_f} dt \frac{dE}{dx} \\ &\sim \left(\frac{2}{3} \right)^{\pm 1} \epsilon_f^{1/2} \left(\log \frac{4ET_f}{M^2} \right) e^{-M/T_f (1+M/T_f)} \int_{t_i}^{t_f} dt \left(\frac{t_f}{t} \right)^{2/3} \\ &\cong 3R_0 \cdot \left(\frac{2}{3} \right)^{\pm 1} \cdot \left(\frac{3}{2\pi} \right)^{1/2} \frac{1}{R_0^{3/2}} \left(\log \frac{4ET_f}{M^2} \right) e^{-M/T_f (1+M/T_f)} \left(\frac{dE_T}{dy} \right)^{1/2} \left[1 - \left(\frac{t_i}{t_f} \right)^{1/3} \right] \end{aligned} \quad (3.3)$$

For $t_f \gg t_i$

$$\Delta p_T \cong 2 \left(\frac{2}{3} \right)^{\pm 1} \left(\log \frac{4ET_f}{M^2} \right) e^{-M/T_f (1+M/T_f)} \left(\frac{1}{R_0} \frac{dE_T}{dy} \right)^{1/2} \quad (3.4)$$

The units here are understood to be as follows: dE_T/dy in GeV, R_0 in f,

and Δp_T in GeV. Happily, many of the uncertainties drop out in the limit of very large dE_T/dy and hence T_f . There is no dependence on t_i provided $t_i \ll t_f$ (a factor of at least 10). Likewise all the M/T_f corrections eventually disappear. We may put in some rough numbers:

If

$$R = 0.7f, \quad M = 500 \text{ MeV},$$

$$\frac{dE_T}{dy} = 10 \text{ GeV} \quad (3.5)$$

Then

$$\epsilon_f \approx 15 \text{ GeV/fm}^3 \quad (3.6)$$

and

$$T_f \approx 300 \text{ MeV} \quad (3.7)$$

For $p_T = 20 \text{ GeV}$

$$\Delta p_T = \begin{cases} 30 \text{ GeV} & \text{gluon} \\ 13 \text{ GeV} & \text{quark} \end{cases} \quad (3.8)$$

This is quite sufficient to quench low- p_T jets!!

In Fig. 2a we plot the energy loss of the jet versus the transverse energy (dE_T/dy), assuming a $p_T \sim 50 \text{ GeV}$ initial jet (in order to evaluate the logarithm) and $R_0 = 0.7 \pm 0.4f$. It should be clear that there is little dependence on R_0 and initial p_T . But there is considerable dependence upon the infrared cutoff if these estimates are applied to "typical" associated (dE_T/dy) $\sim 1-10 \text{ GeV}$, and to fixed target jet phenomena with $p_T \sim 5-10 \text{ GeV}$. We believe it safest to explore these ideas at the highest p_T and/or highest (dE_T/dy) available at $p\bar{p}$ colliders.

Also plotted in Fig. 2b is the effect of relatively late establishment of the dense equilibrium plasma. There we choose $t_f \sim 3t_i$ for the period during which energy loss occurs.

IV. COMMENTS AND CONSEQUENCES

We have seen that, at least for events with high associated E_T ($dE_T/dy \gg 1$ GeV) and multiplicity, that there is an appreciable chance that a high- p_T jet will be completely absorbed by the environment in which it is produced. The uncertainties are considerable and become very great for events with dE_T/dy less than 10 GeV. This jet-extinction phenomenon is, however, sufficiently credible that experimental efforts should be made to look for it. In particular, it should be interesting to carefully study all jet phenomena as function of associated multiplicity. In addition, one might anticipate, even in the presence of quark-gluon plasma and "extinction," special classes of events associated with particular collision geometries (Fig. 3). Most spectacular would be events (Fig. 3b) containing one clean observable high- p_T jet, with no sign whatsoever of a recoiling jet, and where the p_T of the observed jet is (visibly) balanced by a large aggregation of low p_T particles.

However, we must remember that our basic assumptions are fragile. Existence of quark-gluon plasma during the evolution of a hadron-hadron collision requires a large number of equilibrating collision processes to occur on a very short time scale $\ll 1f$. It would seem that perturbative QCD could be invoked to decide whether in fact plasma can be produced so quickly. However, if there is an answer, we do not know it. And even if, strictly speaking, plasma is not produced, it still

seems almost unavoidable that comparably high energy densities must exist on the time scale 0.1-1f after impact. Thus our naive estimates of "extinction" may not be too badly wrong even in a more general context.

We also note that, while "extinction" may be an important phenomenon, it should not be dominant for hadron-jets from the anticipated W and Z electroweak bosons. And as one enters the high- p_T region of hundreds of GeV, it would require an increase in the height of the central-plateau by an order of magnitude to extinguish or greatly modify the produced jets.

Production of quark-gluon plasma in high- E_T events will have consequences other than the possibility of jet extinction or modification. The produced system may undergo transverse hydrodynamic expansion⁸ on a distance scale of $\sim 5-10f$, leading to modification of the inclusive-particle spectrum as function of associated multiplicity. Because of the elevated temperature there may be enhanced strangeness and even charm production in such events. The large size of the system which exists when pions are produced might be seen - and measured - via Bose-statistic (Hanbury, Brown, Twiss) enhancements⁹ of $\pi^\pm \pi^\pm$ correlations at very small relative momenta. Detailed considerations are, however, well beyond the scope of this note.

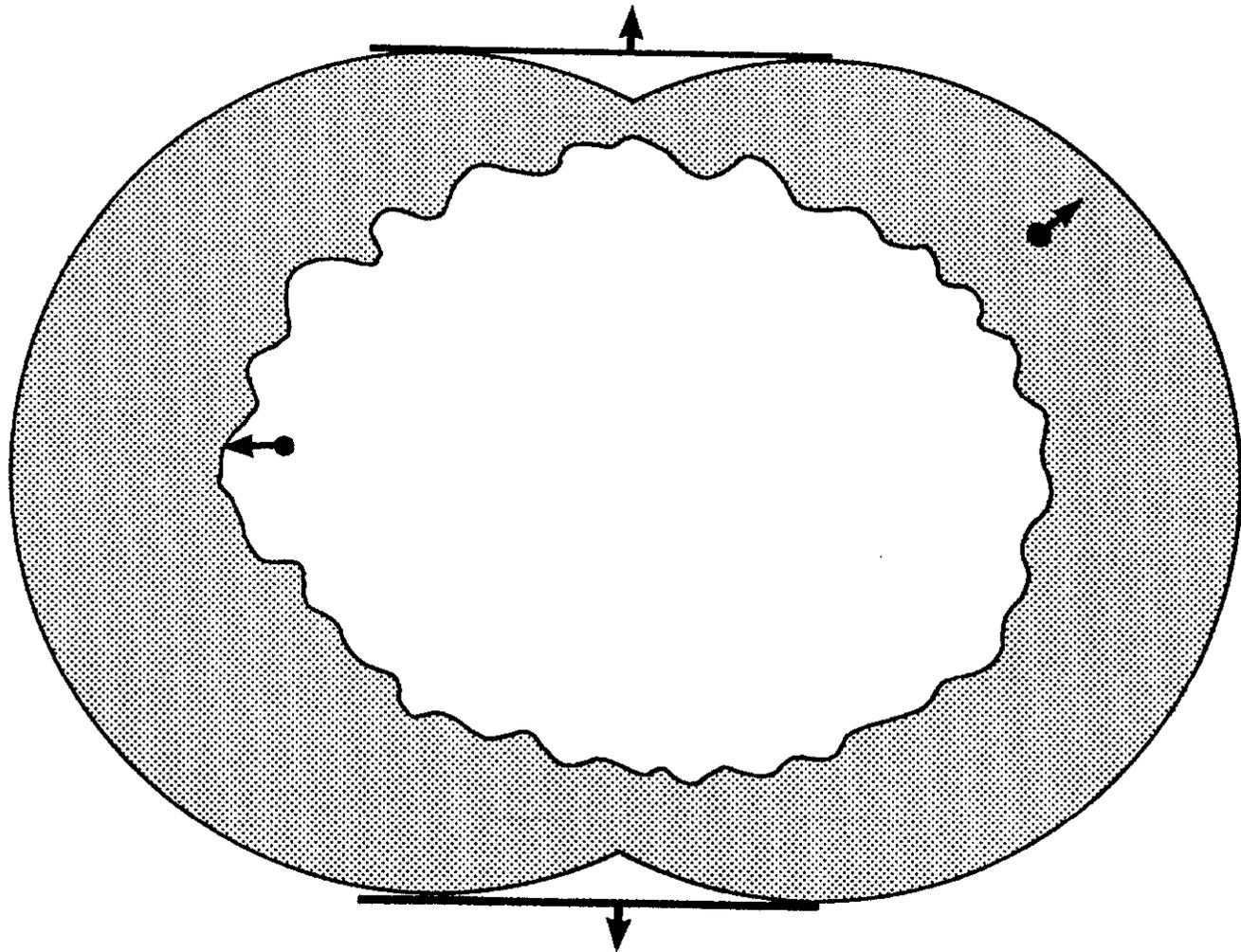
We thank J. Kapusta, L. McLerran, and A. Mueller for especially helpful suggestions, stimulation and criticism.

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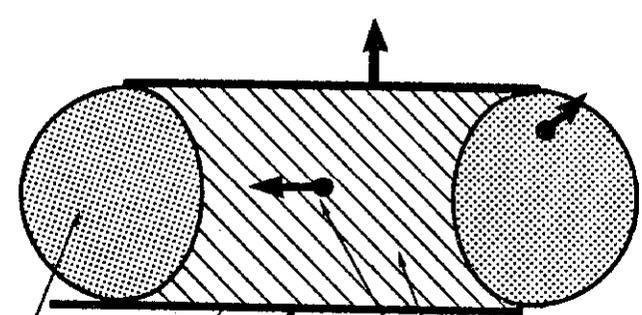
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FIGURE CAPTIONS

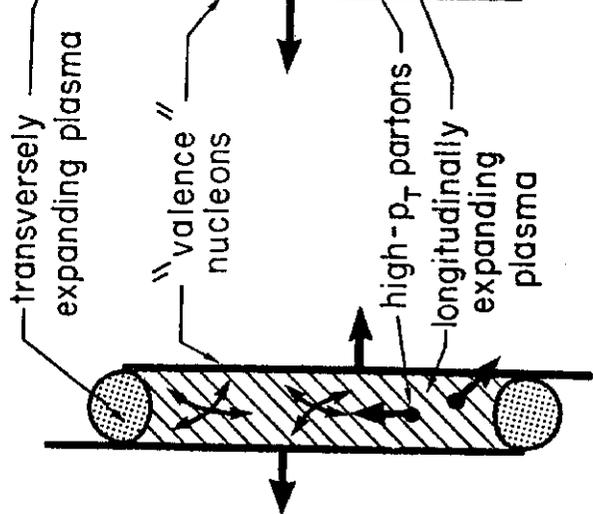
1. Evolution of hypothetical quark gluon plasma, and fate of high- p_T produced partons: (a) shortly after the collision, (b) a time ~ 0.5 fm. after the collision, and (c) several fm. after the collision. During the early stage of longitudinal expansions, a fluid element a distance z from the collision plane (along the beam axis) moves with velocity z/t , with t the elapsed time since the collision.
2. Energy degradation of high- p_T jet as function of associated isotropic transverse energy deposition. Also shown is the effect of an "infrared cutoff" of mass scale M .
 - (a) Fully hydrodynamic evolution, with early starting time $t_i < 10^{-1} t_f \sim 0.1 \text{ fm./c.}$
 - (b) Partially hydrodynamic evolution with $t_i \sim 0.3 t_f$.
3. Fate of secondary high- p_T jets for various collision geometries: (a) both degraded, (b) one degraded, and (c) none degraded.



(c)



(b)



(a)

Fig. 1

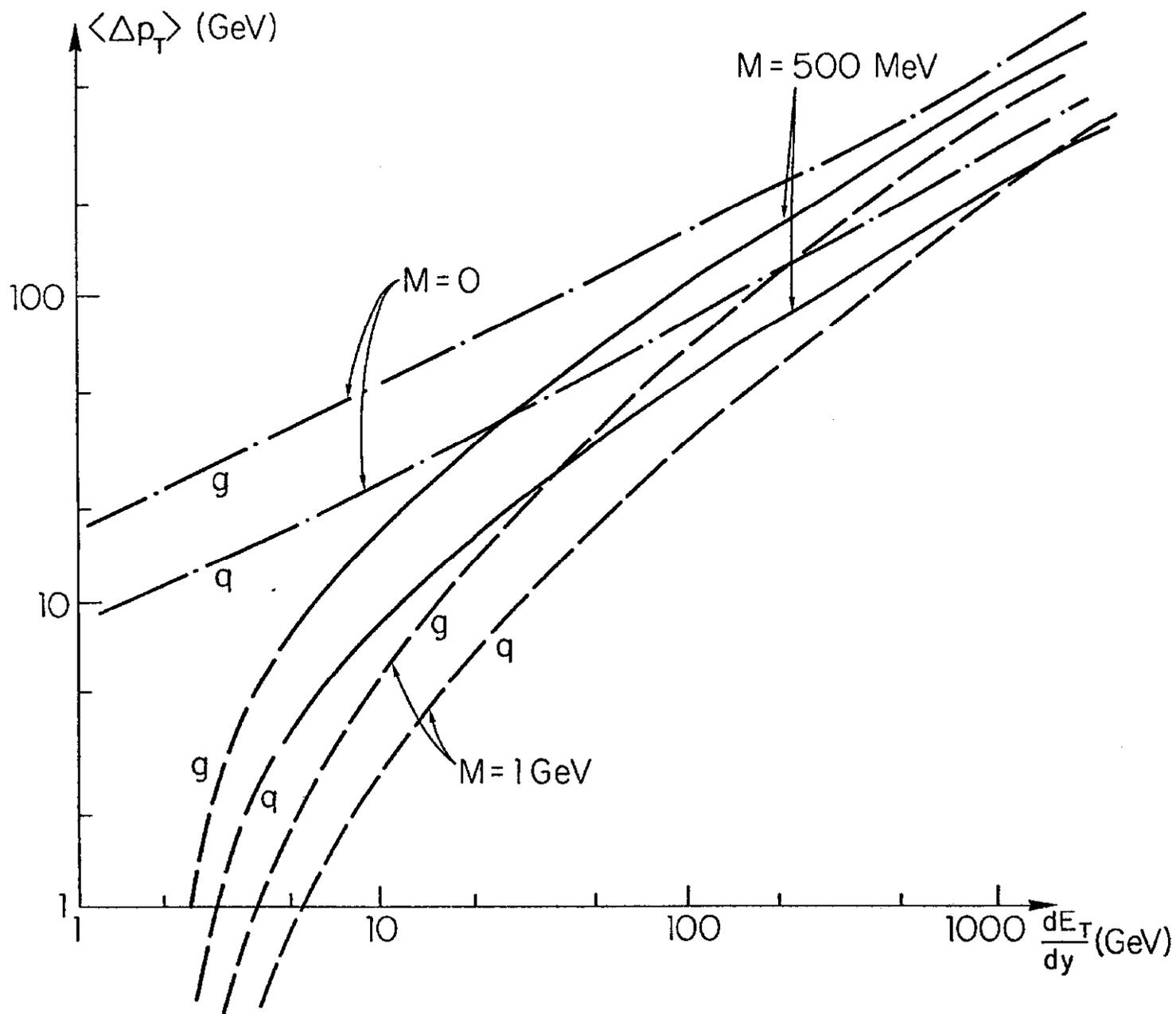


Fig. 2a

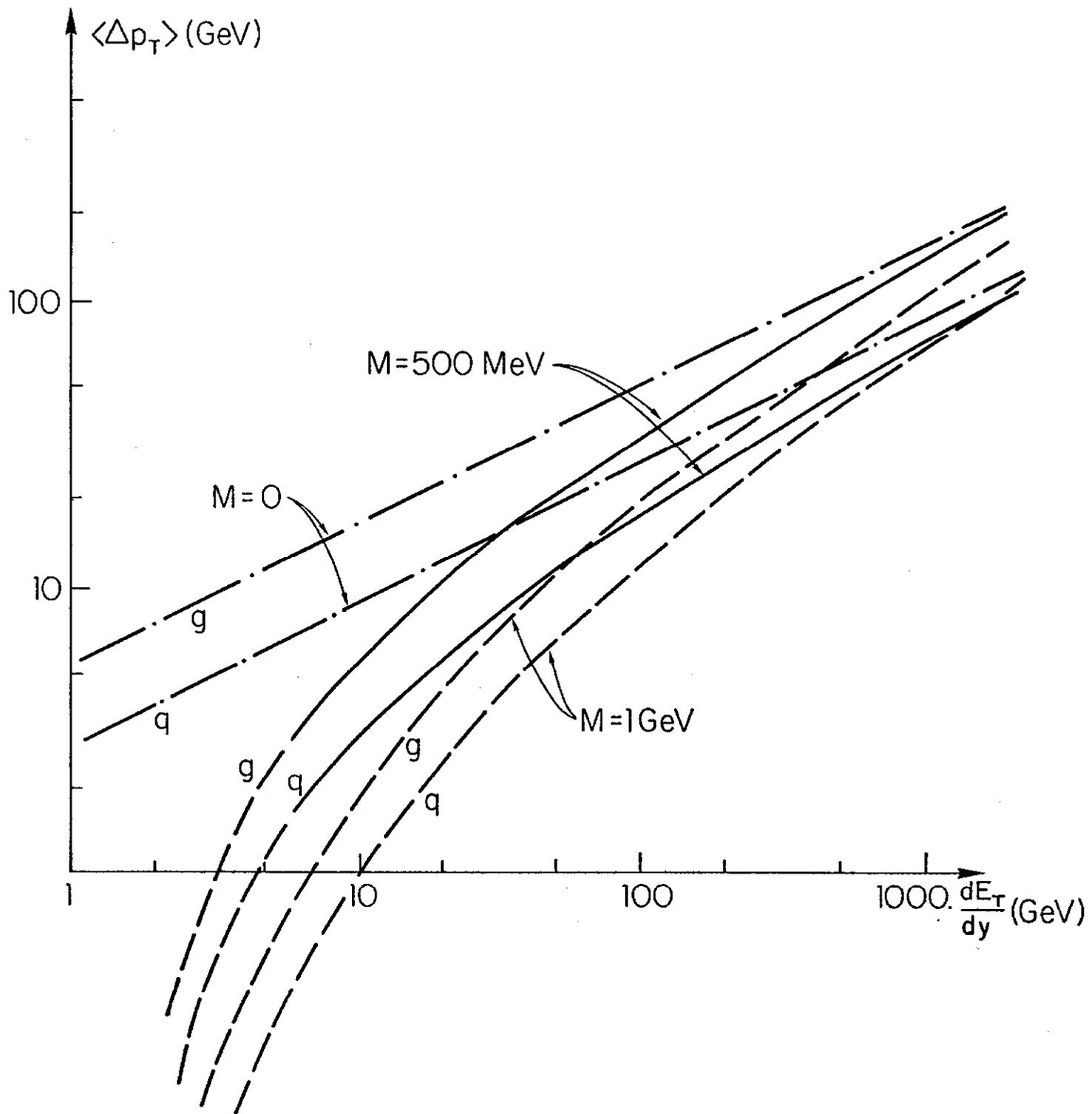
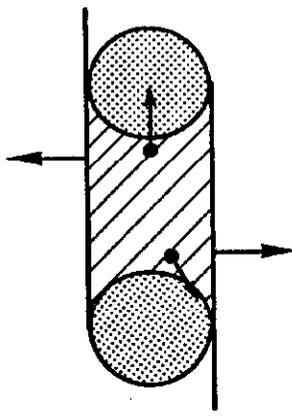
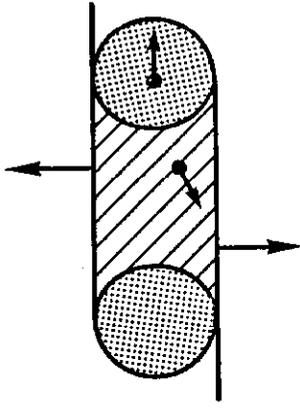
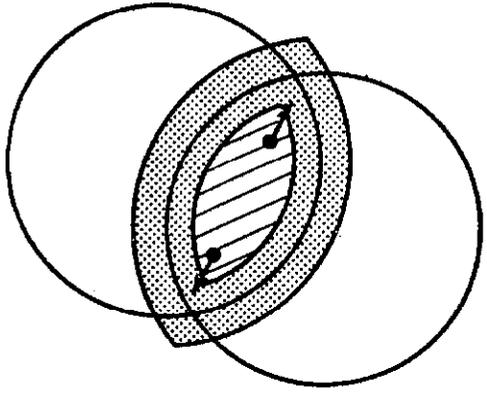


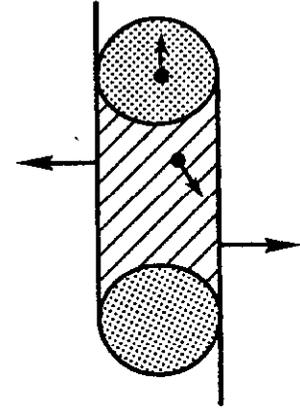
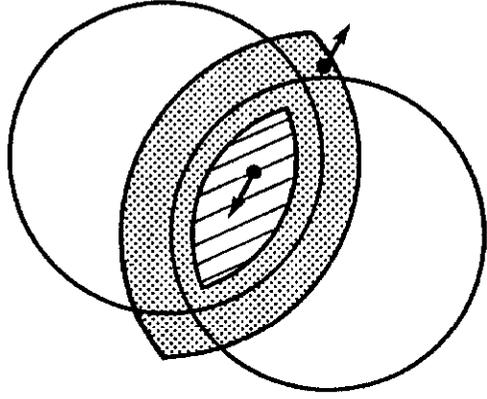
Fig. 2b



(a)



(b)



(c)

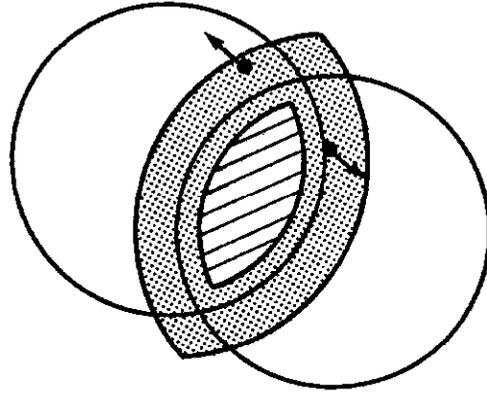


Fig. 3