Fermi National Accelerator Laboratory

FERMILAB-Pub-82/46-THY
July, 1982

Family Gauge Symmetry From a Composite Model

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ABSTRACT

A family (or horizontal) gauge symmetry $SU^F(2)$ will naturally appear in a composite model of quarks and leptons in which the hypercolor and color gauge group are $SU(4)_H \otimes SU(3)_C$ under some appropriate assumptions of the chiral superflavor symmetry-breaking pattern. The massless quark-lepton generations can be assigned to $SU^F(2)$-triplet or/and -singlet representations.

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Operated by Universities Research Association Inc. under contract with the United States Department of Energy

The family (or horizontal) gauge symmetry is one of attempts to explain the "generation puzzle" of quarks and leptons [1-4]. However, its origin has been an open problem. The idea that this symmetry might naturally exist in a composit model of quarks and leptons is very attractive [1]. The point is whether we could have a realistic compsite model in which the massless quarks and leptons (in comparision with the energy scale Λ_H [5] of the hypercolor interaction which binds the constituents -preons- into quarks and leptons) would naturally possess family gauge symmetry added to the standard model's symmetry $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ or its left-right symmetric extension $SU(3)_C \otimes SU(2)_L \times SU(2)_R \times U(1)_{B-L}$.

Some authors have proposed a few models of such kind in which the dimension of a family group is related to hypercolor anomaly cancellation [6]. In this letter, we will show that a family gauge symmetry may also appear in a simple left-right symmetric model as long as some assumptions about chiral symmetry-breaking pattern are made, and the dimension of the family gauge group will be determined by masslessness of some preons and anomaly consistent conditions [10].

This model is an extension of the three-fermion compsite model proposed in the reference [7] in which the hypercolor group supposed to be confined at large distances is $SU(4)_H$. We assume that quarks and leptons are composed of massless preons α, β, x_i $(i=1,\dots,n)$ whose representations are arranged as in Table 1. The electroweak doublets of quarks and leptons will be $((\alpha xx),(\beta xx))$ and $((\alpha x^C x^C),\ (\beta x^C x^C))$ respectively, where x^C represents the antiparticle of x.

With electromagnetism switched off, the basic lagrangian of the model has a global $SU(2)_L \otimes SU(2)_R$ symmetry acting on the preons α and β and a global $SU^F(n)_L \otimes SU^F(n)_R$ symmetry acting on the preons x_i $(i=1,\ldots,n)$. The former will reproduce the left-right symmetric model of the weak interaction (the weak interaction either appears as the residual hypercolor interaction [8-9] or comes from being gauged of the global symmetry), and the latter, as will be seen later, could lead to a family gauge symmetry.

In order to see how many and which among the composite fermions in the model possibly are massless, we must consider the 't Hooft anomaly consistent conditions [10].

The superflavor symmetry group of the model at the preon level can be written as

$$G_{SF} = SU(3)_C \times SU(2)_L \times SU(2)_R \times SU^F(n)_L \times SU^F(n)_R \times U(1)_{B-L} \times U(1)_{B+L} \tag{1}$$

where $SU(3)_C$ is gauge symmetry and the other symmetries are considered either as global one or as gauged one. In addition, two unwritten axial $U_X(1) \otimes U_{X'}(1)$ in (1) (corresponding axial currents may be $J_\mu^{5\alpha} + J_\mu^{5\beta}$ and $\sum_{i=1}^{n} J_\mu^{5x_i}$) have been broken by the hypercolor and the color instantons [11][*1].

[*1] When a family group is gauged the axial symmetry will be broken further by the family charge instantons.

G_{SF} means that it is allowed to specify a definite baryon number B and lepton number L for each preon in the model*2.

The $SU(4)_H \otimes G_{SF}$ representations of the chiral preons

$$V_{\substack{L\\R}} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}_{\substack{L\\R}} \quad \text{and} \quad P_{\substack{L\\R}} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}_{\substack{L\\R}} \tag{2}$$

are shown in Table 2.

The anomaly equations are closely related to the unbroken superflavor symmetry when composite states are formed on the scale of Λ_H.

The subgroup $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ will be assumed to keep unbroken so as to reproduce the left-right symmetric standard model and protect V-preons and some composite fermions from acquiring masses.

The chiral group $SU^F(n)_L \otimes SU^F(n)_R$ could contain the family symmetry as its subgroup which will be gauged. The lower bounds for the masses of the family gauge bosons must lie in the 10-100 TeV region on the basis of the analyses of rates for rare processes [3]. Hence the symmetry-breaking scale Λ_F of the chiral symmetry $SU^F(n)_L \otimes SU^F(n)_R$ must be above 10-100 TeV. This breakdown can be produced dynamically, for example, through some scalar condensates of either certain new technipreons (if $\Lambda_H < \Lambda_F$) or preons themselves (if $\Lambda_H \gtrsim \Lambda_F$) [12].

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*2 If we take the "economic principle" [6], i. e. the massless composite fermions are from the binding of the configurations containing only three "valence" preons, then such color triplets and color singlets as $(3\alpha+2x)$ $(B=2/3, L=1)$ and $(5\alpha+4x)$ $(B=7/6, L=3/2)$ (refer to Table 2) which have exotic B-and L- numbers will be excluded out of the massless composite fermions.

However, in order to constitute massless composite fermions, we hope to have some discrete chiral symmetry left which is enough to protect the P-preons from acquiring masses.

We note that the symmetry $SU^F(n)_L \otimes SU^F(n)_R \otimes U(1)_P$ ($U(1)_P$ is a subgroup of $U(1)_{B-L} \otimes U(1)_{B+L}$, the corresponding vector current is $4J_\mu^{B-L} + 2J_\mu^{B+L} = \bar{P}\gamma_\mu P$) does contain a axial discrete subgroup $Z_{2n}^{X_p}$ from $U(1)_P$ and the center of one of the two $SU^F(n)$ factors. The chiral charge X_p is conserved modulo 2n if we define $X_p(P_L)=+1$ and $X_p(P_R)=-1$. Therefore, the desired discrete chiral symmetry might be from the $Z_{2n}^{X_p}$. However, if we assumed (for example, considering the case of $\Lambda_H \gtrsim \Lambda_F$) v.e.v. $\langle P\bar{P}\rangle \neq 0$ as usual, then when the continuous chiral symmetry was broken there would be no discrete chiral symmetry left. Instead, if we assume that, following the idea given by Harari and Seiberg [13], v.e.v.$\langle P\bar{P}\rangle=0$ but $\langle (P_{iL}\bar{P}_{iR})^2\rangle \neq 0$, then there would be a discrete chiral symmetry Z_4 left which could protect P-preons from acquiring masses. We also note that the mass terms of quarks (VPP) and leptons (VP^cP^c) have $X_p=\pm 4$, hence, their masslessness will not be due to the Z_4 symmetry.

Because the surviving symmetry Z_4 must be from the original symmetry $Z_{2n}^{X_p}$, n can only be even number. This is an important limitation to the dimension of family gauge group.

The condensate $(P_{iL}\bar{P}_{iR})^2$ belongs to representations $(\square\square, \overline{\square\square})$ and $\left(\begin{smallmatrix}\square\\\square\end{smallmatrix}, \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}\right)$ of $SU^F(n)_L \otimes SU^F(n)_R$ but to singlet of the vectorial group $SU^F(n)_{L+R}$ (note that the sum of i from 1 to n has been made). Therefore, the chiral symmetry $SU^F(n)_L \otimes SU^F(n)_R$ will be broken to $SU^F(n)_{L+R} \otimes Z_4$, and when $SU^F(n) \equiv SU^F(n)_{L+R}$ is gauged and broken spontaneously, the n^2-1 goldstone bosons corresponding to the n^2-1 axial $SU^F(n)$ generators will acquire their masses of order $\sqrt{\alpha_F(\Lambda_F)}\, m_F$ from one family gauge boson exchange [14], where α_F and m_F represent the family gauge coupling and the masses of family gauge bosons respectively. This means that for n=2, the case to be resulted, there would be three pseudogoldstone bosons of masses larger than 2-20 TeV if we assume $\alpha_F(\Lambda_F) \sim$ weak gauge coupling $\alpha_W = \alpha/\sin^2\theta_W$ and take the lower bounds of m_F to be in the 10-100 TeV region [3]. It would be a test for the model to detect these pseudogoldstone bosons in future experiments.

As for subgroup $U(1)_{B+L}$, we will consider two possibilities: unbreaking and breaking.

For the possibility of unbroken $U(1)_{B+L}$, the conserved superflavor group on the scale of Λ_H is*3

$$H'_{SF} = SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes SU^F(n) \otimes U(1)_{B-L} \otimes U(1)_{B+L} \otimes Z_4 \qquad (3)$$

This will forbit the processes of changing B- and L- numbers at the composite level. The composite fermions as $SU(4)_H$-singlets may be from configurations (VPP) and (VP^cP^c) whose representations under H'_{SF} are listed in Table 3.

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*3 In spite of the fact that the family gauge group $SU^F(n)$ has been broken above or on the scale of Λ_H we have still shown it in (3) so as to classify generations of massless composite fermions according to $SU^F(n)$ representations.

Following 't Hooft [10], we specify an index for each representation. By calculating the anomalies from the three-point function of H'_{sF} symmetric currents $[SU(2)_L]^2 U(1)_{B-L}$ and $[SU(2)_L]^2 U(1)_{B+L}$, we obtain the anomaly equations

$$(\ell_{1+} \mp q_{1+} + 8\ell_{2+} + 10\ell_{3+} \mp 2q_{2+} \mp 5q_{3+}) \frac{n(n+1)}{2} +$$

$$(\ell_{1-} \mp q_{1-} + 8\ell_{2-} + 10\ell_{3-} \mp 2q_{2-} \mp 5q_{3-}) \frac{n(n-1)}{2} = \begin{cases} 6 & (4) \\ 12 & (5) \end{cases}$$

In order to seek for physically realistic solutions of (4) and (5), we will take two assumptions:

(i) For each configuration only the smallest color multiplet is approximately massless on the scale of Λ_H. This is precisely the same assumption as one in Rishon model by Harari and Seiberg [8,17]. It means that only color-triplets in (VPP) and color-singlets in (VP^cP^c) contribute anomalies, thus the indices

$$\ell_{2\pm} = q_{2\pm} = \ell_{3\pm} = q_{3\pm} = 0 \qquad (6)$$

(ii) The quark and lepton flavors in nature have one-to-one correspondence. Thus a realistic solution will have

$$|q_{1\pm}| = |\ell_{1\pm}| \qquad (7)$$

From the requirement that the indices have to be integers and the above even n limitation, n can only be 2, thus the dimension of the family gauge group is determined uniquely. Consequently, (4) and (5) have only the following two possible solutions:

A: $\ell_{1+}=-q_{1+}=1$ (triplet), $\ell_{1-}=q_{1-}=6$ (singlet)

B: $\ell_{1+}=q_{1+}=2$ (triplet) , $\ell_{1-}=-q_{1-}=3$ (singlet)

The words in parentheses imply the corresponding $SU^F(2)$-representations. The both solutions correspond to nine generations of massless quarks and leptons. In both cases, seeking for some new quantum numbers or constraints so as to distinguish the multiple $SU^F(2)$-singlet and $SU^F(2)$-triplet generations and finding out a realistic correspondence to present three generations of quarks and leptons are still open problems.

Next we turn to the possibility that $U(1)_{B+L}$ is broken. We prefer its explicit symmetry-breaking pattern to spontaneous one so as to avoid appearance of an undesirable massless goldstone boson. This explicit breaking could be caused by some new interaction which is embodied in a multiple preon field operator 0, invarant under $SU(4)_H \otimes SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \otimes Z_4$ but having B+L guantum number and being not singlet of $SU^F(n)$. For example, a possible form of the operator O (for the case of n=2) is

$$0 \sim \varepsilon_{ij}\varepsilon_{kl}(\bar{V}^c_{Li}V_{Rk}\bar{V}^c_{Lj}V_{Rl}+\bar{V}^c_{Ri}V_{Lk}\bar{V}^c_{Rj}V_{Ll})\ (\bar{P}^cP)^2 \tag{8}$$

It contains the terms $(\bar{\alpha}^c\alpha)(\bar{\beta}^c\beta)(\bar{x}^c_1x_1)^2$ and $(\bar{\alpha}^c\beta)(\bar{\beta}^c\alpha)(\bar{x}^c_1x_1)^2$ (ignoring the chirality of α and β) which are responsible for processes $ud\to\bar{u}e^+$ and $uu\to\bar{d}e^+$ respectively, i.e. for the B-L-conserved proton decay $p^+\to e^+\pi^0$*4. By using the similar analysis to one in reference [15], we obtain the proton lifetime $\tau_p\sim(g^2)^{-6}\Lambda_H^{16}/M_p^{17}$. If we assume $g^2(\Lambda_H^2)/4\pi\sim1$, then the present lower limit of $\tau_p>10^{30}$ years [16] will restrict $\Lambda_H\gtrsim10$ TeV. The limitation to Λ_H is compatible with the Λ_H estimation from the other experiments [5].

In the case of $U(1)_{B+L}$ being broken, the superflavor symmetry at the preon level

$$H_{SF} = SU(3)_C\otimes SU(2)_L\otimes SU(2)_R\otimes SU^F(n)\otimes U(1)_{B-L}\otimes Z_4 \tag{9}$$

except $SU^F(n)$ will remain on the scale of Λ_H. We note that H_{SF}/Z_4 is exactly the symmetry of the left-right symmetric standard model to which only the family gauge group $SU^F(n)$ is added.

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*4 Operator (8) also contains the terms having factors $(\bar{x}^c_1x_2)^2$ and $(\bar{x}^c_2x_2)^2$, and from the following results, this will cause the similar processes to $ud\to\bar{u}e^+$ and $uu\to\bar{d}e^+$ in the other generations of quarks and leptons. However, the rates of these processes must be far less than ones of the usual weak decay processes of heavy quarks caused by Cabibbo-like mixture, hence are undetectable.

Now the anomaly appear only in the three-point function of superflavor symmetric currents $[SU(2)_L]^2 U(1)_{B-L}$, hence only equation (4) is left. Consequently the dimension n of the family gauge group will still be limited to 2 (n=6 is the other possible value, however, we will not consider that case because it leads to violation of the asymptotic freedom of $SU(4)_H$. The several simplest solutions for n=2 are:

C: $\ell_{1+}=-q_{1+}=1$ (triplet), $\ell_{1-}=q_{1-}=0$

D: $\ell_{1+}=-q_{1+}=0$, $\ell_{1-}=-q_{1-}=3$ (singlet)

E: $\ell_{1+}=-q_{1+}=1$ (triplet), $\ell_{1-}=q_{1-}=1$ (singlet)

Because the anomaly equation is merely a necessary condition satisfied by massless bound states, in general, there will be many non-physical solutions of it in which we are not interested. However, the solutions C,D and E are several possible realistic ones.

In fact, the $SU^F(2)$-triplet in solution C or the three $SU^F(2)$-singlets in solution D may just correspond to the present three generations of massless quarks and leptons. However, in the case of solution D we still have the problem to seek for some new quantum number so as to distinguish the three $SU^F(2)$-singlet generations. If we assume further that the indexes of massless composite fermions' representations $|\ell_i|$ (and $|q_i|$) $\leq$ 1 [6], then only the solution C survives as a realistic one. This means that the three generations of quarks and leptons belong only to a $SU^F(2)$-triplet.

If there are four generations of quarks and leptons in nature, then the solution E could also accommodate to them. In this case, the fourth generation belongs to a $SU^F(2)$-singlet and probably be much heavier than the first three generations belonging to a $SU^F(2)$-triplet when quarks and leptons acquire their masses from the symmetry-breaking of $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \otimes SU^F(n)$.

ACKNOWLEDGMENTS

We are grateful to E. Eichten for helpful criticism and discussions. One of the authors (B. R. Zhou) also thanks W. K. Tong, R. Huerta, S. Gottleib for disussions and assistances, W. A. Bardeen and the other members of Theoretical Physics Department of Fermilab for kind hospitality. One of the authors (C. H. Chang) also thanks R. N. Mohapatra for discussions.

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