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Fermion Masses, Rare Processes and CP Violation in a Class of Extended Technicolor Models

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ABSTRACT

A class of m generation models based on $SU(N+m) \otimes SU(N+m)$ Extended Technicolor groups and $SU(N)$ Technicolor groups is discussed. Under certain conditions the models admit hermitian mass matrices in the up and down quark sectors and give acceptable quark masses and acceptable weak mixing angles. The $D^0\bar{D}^0$ mixing in these models is well below experimental limits due to a new suppression mechanism which is discussed in detail. The models accommodate weak CP violation and have a strong CP angle which is naturally small. They fail however to account for the smallness of the $\bar{K}^0K^0$ mixing.

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The calculation of the weak mixing angles and of the quark and lepton masses remains as one of the most outstanding challenges for theoretical particle physicists. In the otherwise successful standard model fermion masses are represented by arbitrary fermion-Higgs Yukawa couplings, and consequently their values are put by hand. In this situation the scheme of dynamical symmetry breaking[1] in which the elementary Higgs scalars are replaced by composite ones remains as an interesting alternative. In addition to solving the Gauge Hierarchy Problem, which plagues the theories with elementary scalars, it offers the possibility of the calculation of quark and lepton masses and of weak mixing angles in terms of a few gauge coupling constants.

A concrete realization of these ideas are the Extended Technicolor models[2] in which the gauge symmetry group is

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes G_{ETC} , \tag{1}$$

where the first three factors constitute the standard model and G_{ETC} denotes an <u>extended technicolor</u> gauge group. These models contain a set of new heavy (0(1 TeV)) fermions (technifermions) which are put together with the standard quarks and leptons into representations of G_{ETC}. The group G_{ETC} is assumed to be spontaneously broken at a scale 0(10-100TeV) down to a <u>Technicolor group</u>[3] G_{TC}, under which the technifermions transform nontrivially but the standard fermions are singlets. The group G_{TC} is chosen to be asymptotically free with a coupling which becomes strong at a scale 0 (1TeV). This strongly interacting system of technifermions replaces the standard Higgs system and when the $SU(2)_L \otimes U(1)_Y$ quantum numbers of technifermions are properly chosen the breakdown of weak gauge symmetries with the usual weak gauge boson mass spectrum and the relation $M_W \simeq M_Z \cos\theta_W$ can be achieved.

The current quark and lepton masses in these models are generated through diagrams like the one shown in Fig. 1. In a crude approximation one finds

$$m_f \simeq g^2_{ETC} \frac{\mu^3_{TC}}{2M^2_{ETC}} \tag{2}$$

where μ_{TC} is fixed by the $W^{\pm}$ gauge boson masses and M_{ETC} is the mass of a heavy ETC gauge boson (shown in Fig. 1) which couples fermions to technifermions. For $g_{ETC} \sim 0(1)$, $\mu_{TC} \simeq 400$ GeV and $M_{ETC} \simeq 10\text{-}100$ TeV the known quark and lepton masses can be obtained. Unfortunately in addition to the ETC gauge bosons of Fig. 1, there exist in the models in question heavy neutral ETC gauge bosons which couple the standard fermions of the same charge to each other. If the masses of these gauge bosons are chosen to be equal[4] to the masses of the gauge bosons in Fig. 1 then generally a good fit to the fermion mass spectrum results in the violation of the bounds on flavor changing neutral (FCN) transitions such as $K^0\bar{K}^0$ or $D^0\bar{D}^0$ mixing by several orders of magnitude.

Generally however the masses of these two sets of ETC gauge bosons are different and this difference depends on the breaking pattern of the G_{ETC} group. The art of the ETC game is then to find a breaking pattern of the G_{ETC} group (and the fermion representations under G_{ETC}) which on one hand would give the correct fermion spectrum through the formulae like (2) and on the other hand would not lead to the violation of experimental bounds on the rare processes.

In the literature the only explicit study of these questions has been done by Ellis and Sikivie[5] in the context of a two generation model based on an $SU(7)_{ETC}$ group broken by a set of Higgs scalars down to $O(5)_{TC}$. It has been found that the model while giving an acceptable quark mass spectrum and the correct value of the Cabibbo angle

i) fails to account for the smallness of the $K^0\bar{K}^0$ mixing by <u>two</u> orders of magnitude,

ii) predicts the $D^0\bar{D}^0$ mixing which is on the borderline of phenomenological acceptability, and

iii) having a non-hermitian down quark matrix suffers from a strong CP problem.

Ellis and Sikivie suggested however that a model with hermitian mass matrices in the up and down quark sectors could have a good chance to simultaneously solve the problems i) and iii). Motivated by this suggestion we have constructed a class of ETC models which under certain conditions admit hermitian mass matrices in the up and down quark sectors.

The Models

We shall consider gauge theories based on the symmetry group of Eq. (1) with $G_{ETC}=[SU(N+m)\otimes SU(N+m)]_{ETC}$, which is broken to the technicolor group $G_{TC}=SU(N)_{TC}$. Here m is the number of light fermion generations.

The fermion representations are given as follows

$$(3,2,\tfrac{1}{6},N+m,1)_L = \begin{pmatrix} \{U_{Li}\} & \{u_{La}\} \\ \{D_{Li}\} & \{d_{La}\} \end{pmatrix}$$

$$(3,1,\tfrac{2}{3},N+m,1)_R = \left(\{U_{Ri}\} \quad \{u_{Ra}\}\right)$$

$$(3,1,-\tfrac{1}{3},1,N+m)_R = \left(\{D_{Ri}\} \quad \{d_{Ra}\}\right) , \tag{3}$$

where i=1...N, a=1...m and the capital and small letters denote techniquarks and quarks respectively.

The models we consider are not realistic because they contain only quarks. Furthermore, with the representations of Eq. (3), the models contain anomalies involving the hypercharge Y which will be cancelled after the introduction of leptons, and there exist also ETC anomalies which can be cancelled by the introduction of additional heavy color singlet fermions. We ignore all these problems as our prime interest is to investigate the question of quark masses and mixing angles in conjunction with FCN currents and CP violation.

The breakdown of SU(N+m) ⊗ SU(N+m) down to SU(N) can be accomplished by m(N+m,1) Higgs representations H_a, m(1,N+m) representations H'_α, $(a,\alpha=1...m)$, and one $(N+m, \overline{N+m})$ representation G which develop the following vacuum expectation values:

$$H_a = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \phi_1^a \\ \vdots \\ \phi_m^a \end{bmatrix} \begin{matrix} \left.\vphantom{\begin{matrix}0\\ \vdots \\ 0\end{matrix}}\right\} N \\ \\ \\ \\ \end{matrix} \qquad\qquad H'_\alpha = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \phi_1^{\alpha\prime} \\ \vdots \\ \phi_m^{\alpha\prime} \end{bmatrix} \begin{matrix} \left.\vphantom{\begin{matrix}0\\ \vdots \\ 0\end{matrix}}\right\} N \\ \\ \\ \\ \end{matrix} \tag{4}$$

and

$$G = \begin{bmatrix} v & & & & & \\ & \ddots & & & 0 & \\ & & v & & & \\ & & & a_1 & & \\ & 0 & & & \ddots & \\ & & & & & a_m \end{bmatrix} \tag{5}$$

For the moment, all the vacuum expectations values are complex.

It should be emphasized that the Higgs scalars considered here are chosen to be singlets under $SU(2)_L$ which implies that their direct Yukawa couplings to fermions are forbidden by the $SU(2)_L$ symmetry. Consequently, the fermion current masses are generated only by the ETC mechanism. In other words, the only role of the Higgs system introduced here is to cause the breakdown of the ETC group.

In a more dynamical scheme (e.g. "tumbling"),[6] the vacuum expectation values in Eqs. (4) and (5) could in principle be calculable, but here they will be regarded as free parameters. Except for the gauge couplings these are the only free parameters of the theories considered here. Specifying these vacuum expectation values specifies simultaneously the masses of the gauge bosons which mediate rare processes and the masses of those bosons which contribute to fermion masses.

In what follows it will be useful to introduce the following variables,

$$z_{ij} = \sum_{a=1}^{m} \phi_i^{a*}\phi_j^{a} , \qquad j,i = 1...m \tag{6}$$

$$x_i^2 = z_{ii} + |a_i|^2 \tag{7}$$

and the variables $x_i^{2}{}'$ and z'_{ij} with ϕ_i^a replaced by $\phi_i^{\alpha}{}'$. Note that the following inequality exists:

$$z_{ij}^2 \leq z_{ii}\, z_{jj} \tag{8}$$

Quark Masses

The $(\mathrm{mass})^2$ matrix for the ETC gauge bosons which couple quarks to techniquarks can be written as follows

$$V_i^a A_{ab} V_b^i + S_i^\alpha B_{\alpha\beta} S_\beta^i + V_i^a C_{a\beta} S_\beta^i + S_i^\alpha C_{\alpha b}^* V_b^i \tag{9}$$

where $V_i^a (V_b^i)$ and $S_i^\alpha (S_\beta^i)$ stand for the gauge bosons corresponding to lowering (raising) operators for the two unitary groups respectively. Here $i,j=1,\ldots N$ and $a,\alpha=1,\ldots m$. The matrices A, B and C have the following properties

$$A = A^+ \qquad B = B^+ \qquad C = C^T. \tag{10}$$

and in terms of the vacuum expectation values X_i^2, Z_{ij}, $X_i^{2\prime}$, Z_{ij}' and v they are given as follows

$$A_{ii}^2 = g_{ETC}^2 (X_i^2 + v^2) \quad ; \quad A_{ij} = g_{ETC}^2 Z_{ij} (i \neq j) \tag{11}$$

and

$$C_{ii} = -2g_{ETC}^2 a_i v^* \quad ; \quad C_{ij} = 0 \ (i \neq j) \tag{12}$$

where $i,j=1\ldots m$. The matrix B is obtained from the matrix A by making the replacement $X_i^2 \to X_i^{2\prime}$ and $Z_{ij} \to Z_{ij}'$. We have assumed here that the coupling constants g_{ETC} are equal for the two SU(N+m) groups. Even if they are not equal they cancel completely in the quark mass matrix.

The four-fermion interaction mediated by the gauge bosons V_i^a and S_i^β is then,

$$\mathcal{L}_{4F} = \frac{1}{2} g^2_{ETC} (J^a_{i\mu} P_{ab} J^{i\mu}_b + J^a_{i\mu} R_{a\beta} J^{i\mu}_\beta$$
$$+ J^\alpha_{i\mu} R^*_{\alpha b} J^{i\mu}_b) + \text{h.c.}, \tag{13}$$

where $J^a_{i\mu}$ and $J^\beta_{i\mu}$ denote the currents to which the gauge bosons V^a_i and S^β_i couple respectively. (We drop the term $J^\alpha_{i\mu} J^{i\mu}_\beta$ as it does not contribute to fermion masses.) The matrices P and R are given as follows:

$$P = [A - CB^{-1}C^*]^{-1} \tag{14}$$

and

$$R = [C^* - BC^{-1}A]^{-1}. \tag{15}$$

Finally, in accordance with the fermion representations (3), we have the currents,

$$J^a_{i\mu} = \bar{u}_a \gamma_\mu U_i + \bar{d}_{La} \gamma_\mu D_{Li}, \tag{16}$$

and

$$J^\alpha_{i\mu} = \bar{d}_{R\alpha} \gamma_\mu D_{Ri}.$$

In order to find the quark mass matrices we insert these currents into Eq. (13), perform a Fierz transformation and use the relation

$$\langle \bar{U}_{iL} U_{iR} \rangle = \langle \bar{D}_{iL} D_{iR} \rangle = \frac{1}{2} \mu^3_{TC} . \tag{17}$$

We find

$$M_d = g_{ETC}^2 R \mu_{TC}^3 \qquad M_u = g_{ETC}^2 P \mu_{TC}^3. \tag{18}$$

where M_d and M_u are defined to be the coefficients of $\bar{d}_R d_L$ and $\bar{u}_R u_L$ respectively. It follows then from Eqs. (10), (14) and (15) that M_u is hermitian but M_d is generally neither hermitian nor symmetric.[7] However if

$$C = C^* \quad \text{and} \quad A = B \tag{19}$$

then M_d is <u>hermitian.</u> We shall discuss the implications of these properties at the end of the paper.

We should also remark that the real parts of the mass matrices in Eq. (18) are the same as in Ref. 5. The imaginary parts are however completely different which has important consequences for the CP violation. Also the FCN currents in our model turn out to be different from those in the model of Ref. 5.

We have compared the formulae (18) with the "experimentally" known quark masses for the case of N=2 and two (m=2) generations. (Here we have taken the matrix A to be equal to the matrix B and have put all phases to zero.) A reasonable fit is obtained with

$$\begin{aligned} a_1 &= 12.4 \text{ TeV} & D_1 &= 43.8 \text{ TeV} \\ a_2 &= -1.7 \text{ TeV} & D_2 &= 14.9 \text{ TeV} \\ v &= 5.9 \text{ TeV} & Z_{12} &= D_1 \cdot D_2 \end{aligned} \tag{20}$$

where $D_i \equiv \sqrt{Z_{ii}}$. These parameters give

$$m_c = 1.25 \text{ GeV} \qquad m_u = 28 \text{ MeV} \qquad m_s = -122 \text{ MeV}$$
$$m_d = 7 \text{ MeV} \qquad \sin\theta_c = 0.21 \quad . \tag{21}$$

The corresponding fit for three (m=3) generations is obtained with the parameters

$$\begin{aligned} a_1 &= 3.0 \text{ TeV} & D_1 &= 9.85 \text{ TeV} \\ a_2 &= 18.4 \text{ TeV} & D_2 &= 67.2 \text{ TeV} \\ a_3 &= -1.65 \text{ TeV} & D_3 &= 20.3 \text{ TeV} \\ v &= -4.5 \text{ TeV} & Z_{ij} &= D_i \cdot D_j \end{aligned} \tag{22}$$

These parameters give

$$\begin{aligned} m_t &= 7.2 \text{ GeV} & m_c &= 1.3 \text{ GeV} & m_u &= 12 \text{ MeV} \\ m_b &= -6.0 \text{ GeV} & m_s &= 83 \text{ MeV} & m_d &= -1.8 \text{ MeV} \\ \sin\theta_1 &= 0.13 \ . \end{aligned} \tag{23}$$

where θ_1 is the mixing angle which replaces θ_{Cabibbo} in the KM matrix.[8]

To obtain the values in Eqs. (21) and (23) μ_{TC}=400 GeV has been used. For G_{TC}=SU(2), this value corresponds to M_W=80 GeV. The sign's of the masses will be discussed later.

The quark mass matrices, M_u and M_d, are singular when det A=0. When $Z_{ij}=D_iD_j$, as required by our best fit,[9] the determinant of A is proportional to the small parameters a_j and hence the mass matrices are near this singularity. Hence it is possible to obtain acceptable light quark masses with relatively large values for the parameters D_i.[5]

We note that the models are not able to reproduce all of the masses exactly. In particular, the mass of the top quark is too small. We have found that requiring $m_t > 20$ GeV worsens the fit for the masses of the lighter quarks and the value of the Cabbibo angle.

The mass spectra in Eqs. (21) and (23) are however close enough to the experimental values that an analysis of $\bar{K}^0K^0$ mixing and $\bar{D}^0D^0$ mixing makes sense. Moreover, radiative corrections will alter these masses.

Flavor-Changing Neutral Currents

We write an effective Lagrangian for FCN currents in the class of models we consider as,

$$\mathcal{L}_{|\Delta S|=2} = C_1(\theta_d)(\bar{s}\gamma_\mu d)^2 + C_2(\theta_d)(\bar{s}\gamma_\mu\gamma_5 d)^2 + \text{h.c.} \tag{24}$$

$$\mathcal{L}_{|\Delta C|=2} = C_3(\theta_u)(\bar{u}\gamma_\mu c)^2 + \text{h.c.} \tag{25a}$$

$$C_3(\theta_u) = C_1(\theta_u) + C_2(\theta_u) \tag{25b}$$

where $\theta_d(\theta_u)$ are the sets of angles describing the unitary matrices which diagonalize the down (up) quark mass matrices. Notice that the two processes are described by the same functions C_1 and C_2 but different angles θ_u and θ_d. In the two generation case $\theta_{\text{Cabbibo}} = \theta_d - \theta_u$.

Using the vacuum expectation values of Eqs. (20) and (22), we find the following results for the coefficient functions C_i, (in units of GeV^{-2}):

$$C_1(\theta_d)=3\times10^{-10} \quad C_2(\theta_d)=1\times10^{-10} \quad C_3(\theta_u)=2\times10^{-13} \tag{26}$$

for the two generation model and

$$C_1(\theta_d)=8\times10^{-11} \quad C_2(\theta_d)=3\times10^{-11} \quad C_3(\theta_u)=7\times10^{-14} \tag{27}$$

for the three generation model.

The coefficients $C_1(\theta_d)$ and $C_2(\theta_d)$ in our models are somewhat smaller than the ones found in the model of Ellis and Sikivie, yet our models still fail to account for the smallness of the $\overline{K}^0K^0$ mixing by <u>two</u> orders of magnitude. We find however that the contribution of the ETC gauge boson exchanges to the $D^0\overline{D}^0$ mixing is two orders of magnitude smaller than the one in the model of Ref. 5 and consequently it is well below the experimental limits.

This suppression of the ETC contributions to the $D^0\overline{D}^0$ mixing can be easily understood in the following way. In the two generation model, the functions C_1 and C_2, defined in Eqs. (24) and (25), have a simple dependence on the mixing angles θ_d and θ_u,

$$C_i(\theta) = \alpha_i\cos 4\theta + \beta_i\sin 4\theta + \gamma_i \ . \tag{28}$$

The coefficients α_i, β_i, γ_i depend in a complicated, (but calculable), way on the Higgs vacuum expectation values. An estimate of the order of magnitude for FCN currents gives $C_i \approx 10^{-8}\ \mathrm{GeV}^2$. If θ_d and θ_u were not related to the coefficients α_i, β_i and γ_i any suppression of FCN currents would be purely accidental. However, these angles are determined by the same vacuum expectation values and therefore it is possible that θ_d or θ_u minimizes the coefficients C_1 and C_2. That is exactly what happens for the $\Delta C=2$ processes in the limit where $a_i \equiv \lambda r_i$ as λ goes to zero with r_i, Z_{ij}, and v held fixed. In this limit we find that θ_u approaches the angles θ_1 and θ_2 which minimize the functions $C_1(\theta)$ and $C_2(\theta)$ respectively:

$$\tan(2\theta_1) \simeq \tan(2\theta_2) \simeq \tan(2\theta_u) = \frac{2Z_{12}}{Z_{11}-Z_{22}} . \tag{29}$$

The angle θ_d however is different,

$$\tan(2\theta_d) \simeq \frac{2Z_{12}(a_2Z_{11}+a_1Z_{22})}{a_2Z_{11}^2-a_1Z_{22}^2+(a_1-a_2)Z_{12}^2} . \tag{30}$$

In the same limit,[10] the coefficient functions approach their minima:

$$C_1(\theta_u) = \frac{Z_{12}^2(a_1-a_2)^2}{2[(Z_{11}^2-Z_{22}^2)^2+4Z_{12}^2(Z_{11}+Z_{22})^2]} \simeq -C_2(\theta_u). \tag{31}$$

We conclude that in this limit $C_1(\theta_u)$ and $C_2(\theta_u)$ are suppressed by two powers of λ, whereas the Cabbibo angle, $(\theta_u - \theta_d)$, goes to a non-zero value. This is the same suppression as was found in Ref. 5. In our model, there is however a second surprise: the two vector coefficients C_1 and C_2 of the $\Delta C=2$ neutral currents cancel to leading order in λ. Expanding to higher order, we find that their sum, the $\Delta C=2$ neutral current, vanishes as λ^4.

The suppression can be understood as follows. Consider first the extreme limit $a_i=0$. In that limit the matrices m_u and Z (see Eq. 6) can be diagonalized simultaneously. The horizontal gauge boson $(\text{mass})^2$-matrix is given by:

$$(M^2)^{ab} = \text{Tr}\ \lambda^a \lambda^b Z \qquad (32)$$

where λ^a are the generators of the (broken) horizontal symmetries. A convenient basis for our purpose is to choose the off-diagonal generators as raising or lowering operators working on the generations of quarks. Because Z is diagonal all matrix elements of M^2 vanish, except when both λ^a and λ^b are diagonal, or when λ^b is a raising operator and λ^a the corresponding lowering operator. This property is preserved when the matrix is inverted. To get a $\Delta C=2$ term a raising-raising matrix element is needed in the inverse of M^2, and since it is absent there are no $\Delta C=2$ processes in this limit.

One may wonder why these arguments are restricted to the up quarks. The reason is that if $a_i=0$ the down-quark masses are all zero, and therefore strangeness and $\Delta S=2$

processes are not defined. For any infinitisimal value of the a_i's, strangeness will correspond to some direction which is not related to the matrix Z. This explains why on the one hand the Cabbibo-angle is a free parameter, but on the other hand as discussed below the $\Delta S=2$ processes are not suppressed unless $\theta_c=0$.

When a_i is unequal to zero, the $\Delta C=2$ current appears with some power of a_i. To determine this power we treat these vev's as a small perturbation. Since we have diagonalized the matrix Z we can no longer assume that the matrix G (Eq. 5) is diagonal. In general the $m \times m$ block of this matrix in generation space will be some non-diagonal matrix which we will call a. The two SU(N+m) groups operate on this matrix in the following way:

$$\begin{array}{lll} a \rightarrow U\, a & \text{first SU(N+m)} & \\ a \rightarrow a\, U^* & \text{second SU(N+m)} & (33) \end{array}$$

where U is a unitary matrix. Because of this transformation property, the contribution of the matrix a to the horizontal gauge boson masses is :

$$\Delta(M^2)^{ab}_{vv} = \Delta(M^2)^{ab}_{ss} = \mathrm{Tr}\ \lambda^a \lambda^b a a^+ \tag{34}$$

$$\Delta(M\ )^{ab}_{vs} = \Delta(M^2)^{ba}_{sv} = \mathrm{Tr}\ \lambda^a a (-\lambda^{b*}) a^+ \tag{35}$$

where M^2_{vv} and M^2_{ss} are the mass-matrices for the gauge bosons of the first and second ETC group respectively and M^2_{vs} is the mixing term. The lowest order contribution to $\Delta C=2$ processes is shown in Fig. 2. This diagram can only be non-vanishing if the perturbation can couple two identical raising operators. However, $\Delta(M^2)^{ab}_{vv}$ vanishes if λ^a and λ^b are identical raising operators, because in the fundamental representation the product of two such operators is zero. The mixing term (Eq. 35) does have raising-raising couplings, but since the gauge boson S does not couple directly to up quarks, the lowest order diagram which can contribute is the one in Fig. 3. Since $\Delta(M)^2_{vs}$ is of order λ^2, this diagram is of order λ^4. Because the matrix a is not diagonal $\Delta(M)^2_{vv}$ has matrix elements between diagonal and off-diagonal generators, but they too can only contribute in second order, through diagrams like Fig. 3.

The choice of the representations used to break the ETC group is very important for this suppression mechanism. For example, the square of a raising operator vanishes only in the fundamental representation. Since this fact is crucial to make the diagram of Fig. 2 vanish, we expect the suppression to be only by two powers of λ if the representation $(N+m, \overline{N+m})$ is changed. The same is true for the breaking of SU(N+m) to SU(N). For example, if this breaking is due to symmetric tensors instead of vectors, the suppression disappears.

Although in our fits of Eqs. 20 and 22 the values of a_i are not extremely small they are substantially smaller than the values of D_i and hence our suppression mechanism is effective. In the case of $\Delta S=2$ processes, the suppression mechanism discussed above cannot work because if θ_u already minimizes the coefficient functions C_i, θ_d cannot minimize these same coefficient functions if the Cabibbo angle is to be non-zero. Furthermore, there is no cancellation between $C_1(\theta_d)$ and $C_2(\theta_d)$ since they multiply different operators. Hence in our model, the $\Delta S=2$ processes are suppressed only by two powers of θ_c, which is not sufficient to agree with the experimental results.

Since our models are so successful in suppressing the $D^0\bar{D}^0$ mixing for which the experimental bounds are less stringent than for $K^0\bar{K}^0$ mixing the possibility appears that by interchanging the assignment of right-handed up and down quarks in Eq. (3) we could suppress sufficiently the $K^0\bar{K}^0$ mixing while still be consistent with the $D^0\bar{D}^0$ mixing.

This is unfortunately impossible because of the following bound, which can be derived from Eqs. (11), (12), (14), and (15):

$$\frac{\det M_u}{\det M_d} = - \frac{\det B}{\det C^*}$$

$$|\det B| \geq |\det C^*| \tag{36}$$

and therefore $|\det M_u| \geq |\det M_d|$, as observed experimentally. This bound clearly does not allow the opposite assignments of the up and down sectors.

We should also remark that in addition to the heavy ETC gauge bosons there exist also in ETC models light neutral pseudo-Goldstone bosons which mediate FCN processes. However if all quarks of the same charge get their masses from the same technifermion condensates (monophagy)[11] the flavor changing couplings of these pseudo-scalars to the quarks are only $O(g^2\, m_f^2/M_W^2)$[12] instead of $O(g\, m_f/M_W)$ and the relevant transitions can be suppressed below the experimental limits.

CP Violation

Finally we would like to discuss the question of strong and weak CP violation in our models.

The source of CP-violation is the Higgs-system that breaks G_{ETC}. All CP-violating θ-parameters of G_{ETC} and QCD can be set to zero by axial U(1)-rotations. We will assume that the TC and QCD vacua do not break CP. Then all phases in the quark masses are due to phases in the Higgs-Lagrangian. We will not try to explain the origin of these phases nor their magnitude, but we will only investigate whether it is possible to make them appear in the weak interactions, but not in the strong interactions.

The criterion for the absence of a contribution to the θ-parameters of QCD is

$$\arg[\det(M_u M_d)] = 0 . \tag{37}$$

Because M_u is hermitian, $\arg\det(M_u)$ is 0 or π. Using

(Eq. 15) we derive

$$\arg(\det M_d) = \arg\det C \pmod{\pi} . \qquad (38)$$

Therefore, to satisfy Eq. (38), det C must be real, and have the right sign to make $\det(M_u M_d)>0$.

Notice that the phases of the matrices A and B of Eq. (9) do not affect the strong interactions. Therefore, by making C real and A, B complex we may have a chance to achieve our goal.

The phase of C is proportional to the phase of v. There are now two possibilities:

(i) The phase of v is not determined by the Higgs potential. This means that all vacua with different phases of v are equivalent; the Higgs-Lagrangian must then have had a U(1)-symmetry which was spontaneously broken. Consequently there must be a (pseudo) Goldstone boson.

(ii) The Higgs potential determines the phase of v. Then we have to make sure that C is real by imposing a symmetry on the Higgs-system. The matrix C may still have the wrong sign; if the number of generations is odd this can be cured by changing the signs of some parameters so that v changes sign. If the number of generations is even this does not work. Indeed in our two generation model, we find $\mathrm{sgn}(\det M_d) = -1$. This relative minus sign cannot be removed by changing the signs of the parameters. In the three generation case, however, we can define the parameters such that sgn(det M_d) = 1. A discrete symmetry which seems suitable is interchange of H_a and H'_α and replacement of G by its hermitian conjugate. This symmetry implies that Eq. (19) is satisfied, if it is a symmetry of the vacuum. Then the down quark mass matrix is hermitian.

Now we have to investigate if higher order corrections respect this hermiticity. It can be shown easily that if there is no mixing between gauge bosons with vector couplings to all quarks and gauge bosons with axial vector couplings to all quarks, only hermitian mass matrices can be generated. In our model, the bosons $V_i^a+S_i^a$ have vector couplings, whereas the bosons $V_i^a-S_i^a$ have a vector coupling to up-quarks and an axial vector coupling to down quarks. If Eq. (19) is satisfied these bosons do not mix. Therefore m_d is hermitian in the absence of up quarks, but hermiticity is broken by diagrams with up-quark loops. These diagrams are suppressed by at least two powers of μ_{TC}/M_{ETC}.

In addition to this, the discrete symmetry is broken because H_a and H'_α couple to different gauge bosons (V and S respectively) which in turn couple to different quarks. Even if the coupling constants for S and V gauge bosons are equal at the scale $\Lambda^{T'C}$ at which G_{ETC} is broken, symmetry breaking terms of order $g^4_{ETC}(\Lambda^{T'C})$ are generated, which will induce a phase in the mass-matrix. Because $\Lambda^{T'C}$ is a few orders of magnitude larger than the technicolor scale, at which g_{ETC} is large, such diagrams are suppressed. But unless there is an unexpected cancellation this effect appears to be too large to be acceptable. Nevertheless the induced θ-parameter is small in this model, and further improvement may be possible.

In summary we have presented in this paper a class of m generation models based on $[SU(N+m) \otimes SU(N+m)]_{ETC}$ extended technicolor groups which are spontaneously broken down to $SU(N)_{TC}$. These models while giving an acceptable quark mass spectrum, acceptable values of weak mixing angles, and having (under certain conditions) hermitian mass matrices in the up and down quark sectors

i) still fail to account for the smallness of the $\bar{K}^0K^0$ mixing by <u>two</u> orders of magnitude.

<u>However</u> in these models

ii) the contribution of the ETC gauge boson exchanges to the $D^0\bar{D}^0$ mixing is found to be two orders of magnitude smaller than the one in the model of Ellis and Sikivie[5] and

iii) the strong CP problem is "almost" solved, because weak CP violation can be introduced, while strong CP violation is naturally suppressed.

We hope that our findings in connection with the suppression of $D^0\bar{D}^0$ mixing and the question of the strong CP problem turn out to be useful in a search for a realistic model with dynamical symmetry breaking.

While completing this paper we have received a preprint by Masiero, Papantonopoulos and Yanagida.[13] These authors in accordance with our results find that breaking the G_{ETC} group by scalars in the fundamental representation implies the absence of tree level FCN currents in purely hadronic and leptonic processes. The models discussed in Ref. 13 are however different from ours.

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[1]S. Weinberg, Phys. Rev. D13, 974 (1976) and references therein.

[2]S. Dimopoulos and L. Susskind, Nucl. Phys. B155, 237 (1979); E. Eichten and K. Lane, Phys. Lett. 90B, 125 (1980).

[3]L. Susskind, Phys. Rev. D20, 2619 (1979); S. Weinberg, Phys. Rev. D19, 1277 (1979).

[4]S. Dimopoulos and J. Ellis, Nucl. Phys. B182, 505 (1981).

[5]J. Ellis and P. Sikivie, Phys. Lett. 104B, 141 (1981).

[6]S. Raby, S. Dimopoulos and L. Susskind, Nucl. Phys. B169, 373 (1980).

[7]W. Konetschny, Univ. of Vienna preprint (1982).

[8]M. Kobayashi and K. Maskawa, Prog. Theor. Phys. 49, 652 (1973).

[9]In the extreme limit, $Z_{ij}=D_iD_j$, G_{ETC} is not broken in the desired fashion. However, we can take $Z_{ij}=P\ D_iD_j$ with P slightly below unity to break the symmetry correctly without materially affecting the quark masses.

[10]In the limit where $Z_{ij}=D_iD_j$, there are corrections to Eq. (31) which tend to spoil the cancellation. These corrections are however small and numerically our best fits (Eq. 20 and 22) still exhibit this cancellation (Eq. 26 and 27).

[11]J. Ellis, M.K. Gaillard, D.V. Nanopoulos and P. Sikivie, Nucl. Phys. B182, 529 (1981).

[12] J. Ellis, D.V. Nanopoulos and P. Sikivie, Phys. Lett. 101B, 387 (1981).

[13] A. Masiero, E. Papantonopoulos and T. Yanagida, MPI-PAE/PTh 25/82.

FIGURE CAPTIONS

Fig. 1. The mechanism for light fermion mass generation through the exchange of a heavy ETC boson which couples a fermion f to a technifermion F.

Fig. 2. The lowest order diagram which can contribute to $\Delta C=2$ processes. The X indicates that the diagram is proportional to $(\Delta M_{VV}^{ab})^{-2}$. As explained in the text, this diagram vanishes.

Fig. 3. The second order contribution to $\Delta C=2$ processes. The X's indicate mixing between the massive ETC V and S bosons.

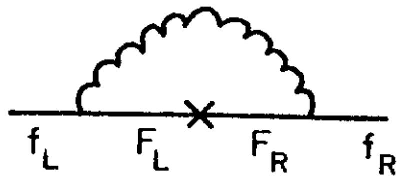

Fig. 1

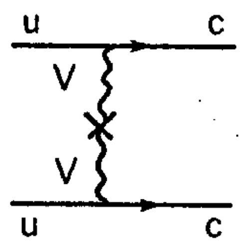

Fig. 2

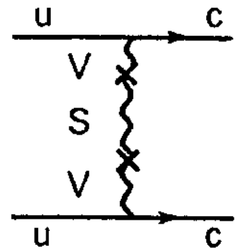

Fig. 3