Fermi National Accelerator Laboratory

FERMILAB-Pub-82/35-THY
May, 1982

ZEROS IN AMPLITUDES: GAUGE THEORY AND RADIATION INTERFERENCE

Stanley J. Brodsky[(a)]

Institute for Advanced Study, Princeton, New Jersey 08540

and

Robert W. Brown[(b)]

Fermi National Accelerator Laboratory, Batavia, Illinois 60510

We find that any tree amplitude for single-photon emission can be written in a general canonical form which vanishes identically in certain kinematical zones provided that the electromagnetic couplings and any other derivative couplings are as prescribed by gauge theory. The location of these null zones depends only on the charge and momentum of the external legs. We give a classical picture for their occurrence based on radiation interference that is a generalization of the well-known absence of dipole radiation for nonrelativistic collisions involving particles with the same charge-to-mass ratio and g-factor.

(a) On leave from Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94305

(b) On leave from Case Western Reserve University, Cleveland, OH 44106

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

The most striking feature[1,2] of the differential cross section predicted by gauge theory for the annihilation of quarks into a weak boson plus a photon,

$$u + \overline{d} \rightarrow W^{+} + \gamma, \tag{1}$$

is the fact that the lowest-order amplitude[3,4] vanishes identically and spin-independently at a specific scattering angle. For massless quarks, the angle is given by[2] $\cos\theta_{cm}^{\gamma,u} = -1 - 2Q_d = -1/3$, independent of energy. Although the experiments are difficult,[5] the possibility of the empirical confirmation of a sharp minimum at this angle is important since it provides a check on the magnetic moment of the W and quarks as predicted by renormalizable gauge theories as well as a direct measure of fractional quark charge by real photons.

In this letter we will discuss the physical origin of such "gauge zeros" and the general conditions necessary for the occurrence of "radiation null zones" in photon production processes, as well as a new canonical form for the amplitude. The vanishing of the Born amplitude for (1) is of course not due to any conventional selection rule or conservation law. In essence the zero is due to the complete destructive interference of the classical radiation patterns of the incoming and outgoing charged lines. The null zone conditions are the generalization of the well-known nonrelativistic result where electric dipole radiation vanishes in collisions of particles with the same charge-to-mass ratio and where magnetic dipole radiation vanishes for such collisions if in addition their g-factors are identical.

We have found that the electromagnetic couplings of particles specified by gauge theory guarantee that null zones exist for a very general class of tree graphs and are completely prescribed by the classical currents of the external lines, independent of spin. More fundamentally, this result is due to the close connection between the electromagnetic gauge couplings and a Lorentz transformation of the particles' spin. The same connection is responsible for $g = 2$, i.e. the identity of the orbital and spin precession frequencies of a charged particle in a uniform field if its couplings are given by gauge theory tree graphs.[6] In each case, the null zones and the value $g = 2$ are destroyed by quantum corrections from loop graphs.

The above result can be stated as a theorem. Consider single photon emission (four-momentum q) by an n-external-particle system (momenta p_i, charges Q_i, masses m_i, and spin ≤ 1) with the internal particles unspecified. The photon couplings to the particles are to be those of gauge theory. The interactions of the particles among themselves can involve any number of fields with constant or single derivative couplings, and the derivative couplings must be of gauge theory form. (This includes all of the renormalizable theories of interest as well as an infinite class of nonrenormalizable theories.) <u>Then any set of tree graphs, defined by photon emission in all possible ways, vanishes if the quantities</u> $Q_i/p_i \cdot q$ <u>are the same for all particles.</u> We may write this condition as n-2 equations

$$\frac{Q_i}{p_i \cdot q} = \frac{Q_1}{p_1 \cdot q} , \quad i = 2, 3, \ldots, n-1 \tag{2}$$

arbitrarily choosing $i = 1$ as a standard and $i = n$ as the ratio that is determined by the rest through charge and momentum conservation (or by gauge choice). The system may be termed "on-the-charge-cone" if Eq. (2) is satisfied, since the denominators are essentially light-cone energies.[7] The kinematic null zone is determined by Eq. (2) and momentum conservation.

It is straightforward to check[8] that Eq. (2) leads to the cancellation of the radiation due to the convection or classical currents[9] of the relativistic external lines. What is remarkable is that the same cancellation goes through when the spin and contact (seagull) currents are added and when internal line radiation is included. We shall return to an outline of the proof of the theorem after a few remarks.

The null zone defined by Eq. (2) will always lie in the physical domain for a given scattering or decay transition if all of the incoming and outgoing charges are of the same sign[10] and if $n-1$ masses are neglected. If we consider "turning on" the masses to some arbitrary set of sufficiently large values, the zone may move out of the physical region. The constraint for $n = 2$ is precisely the gauge zero discussed previously for scattering[1-4] and decay[11] of weak bosons. Through the generality of the theorem we discover previously unnoticed zeros in ancient radiative processes such as

$$e^- + e^- \to e^- + e^- + \gamma \qquad (3)$$

where the zero occurs for the photon at right angles to the beams in the c.m. and for the final electrons at equal energies. It is a two-dimensional zone consisting of the common electron energy and a final electron azimuth relative to the photon axis.

Irrespective of whether or not the charge cone condition (2) can be satisfied in the physical region for a given set of charges and masses, the theorem implies that there is an (n-2)-dimensional first-order zero. The tree amplitudes described can be shown[8] to reduce to a new canonical (factored) form. For no internal-line emission it is

$$M = \sum_{i=2}^{n-1} p_i \cdot q \left(\frac{Q_1}{p_1 \cdot q} - \frac{Q_i}{p_i \cdot q} \right) J_i \cdot \varepsilon ,$$

$$J_i = \frac{j_n}{p_n \cdot q} - \frac{j_i}{p_i \cdot q} \ . \tag{4}$$

The quantity j_i is the product of the current for the emission of the photon by the i^{th} leg, discussed below, and the rest of the matrix element. Internal currents modify J_i in a well-defined manner.[8] At n = 3, the four-body factorization discussed in Ref. 3 is reproduced, but with an interesting variant in form. The generalized factorization displayed applies to <u>any</u> complete set of photon emission helicity amplitudes, including seagulls, if necessary, and thus can simplify calculations.

The proof of the theorem rests on the relationship between a Lorentz transformation and the external and internal photon couplings. Let us first define the currents for tree diagrams. For photon emission by an external leg with charge Q flowing along momentum p, the electromagnetic current factors $Qj\cdot\varepsilon/p\cdot q$ in the general Feynman amplitude are

$$\text{outgoing} \quad \frac{Q}{p\cdot q}\,\tilde{\chi}\,[p\cdot\varepsilon + \text{spin current} + \text{contact current}]\ldots \tag{5a}$$

$$\text{incoming} \quad \ldots[-\,p\cdot\varepsilon - \text{spin current} - \text{contact current}]\,\chi\,\frac{Q}{p\cdot q} \tag{5b}$$

for particle wave function χ and photon polarization ε. The "contact" current arises from both the momentum change (due to photon emission) in and the gauging (seagull) of any derivative coupling for the external leg. As examples, $\chi = \{1;\ u(p);\ \varepsilon_\alpha(p)\}$, $\tilde{\chi} = \{1;\ \bar{u}(p);\ \varepsilon_\alpha^+(p)\}$, and the spin current is $\{0;\ \frac{i}{4}\,\sigma^{\beta\alpha}\omega_{\beta\alpha};\ -\,\omega_{\beta\alpha}\}$ for a {scalar; Dirac; vector} particle, respectively, where

$$\omega_{\mu\nu} = q_\mu\varepsilon_\nu - \varepsilon_\mu q_\nu. \tag{6}$$

If there is a derivative coupling ∂^α, the contact current is always $-\,\omega_{\beta\alpha}$. Any internal line emission factor involving propagators D can be rewritten quite generally as

$$\frac{Q}{p\cdot q}\,[D(p-q)(p\cdot\varepsilon + \text{spin} + \text{contact}) - (p\cdot\varepsilon + \text{spin} + \text{contact})D(p)] \tag{7}$$

where the indices have again been suppressed. A simple (scalar) example of this is

$$\frac{1}{(p-q)^2-m^2}\,Q(2p-q)\cdot\varepsilon\,\frac{1}{p^2-m^2} = \frac{Q}{p\cdot q}\,[\frac{1}{(p-q)^2-m^2}\,p\cdot\varepsilon - p\cdot\varepsilon\,\frac{1}{p^2-m^2}]. \tag{8}$$

If the external legs are on-the-charge-cone [Eq. (2)], then so too are the internal legs ($Q/p\cdot q = Q_1/p_1\cdot q$). The theorem then follows if the currents [square brackets of Eqs. (5) and (7)] cancel throughout the graph. (Closed loop internal legs do not follow the external legs on-cone.) The convection currents clearly cancel by momentum conservation plus transversality ($q\cdot\varepsilon = 0$). An elegant way to see the remaining currents vanish is through Lorentz invariance: The spin terms are the first-order wave function corrections for the Lorentz transformation $\Lambda_{\mu\nu} = g_{\mu\nu} + \omega_{\mu\nu}$. The contact terms are also the same first-order Lorentz transformation of any derivative coupling in momentum space. Therefore Lorentz invariant tree graphs for photon emission with the prescribed couplings vanish on-the-charge-cone.

It is well known that gauge theory couplings can be derived[12] by assuming a constraint on the high-energy behavior of tree graph amplitudes consistent with unitarity. By turning our argument on its head, electromagnetic gauge theory couplings can be derived by the constraint that the canonical form (4) is maintained in tree graph approximation for nonzero spin. Indeed, $g \neq 2$ destroys the spin current cancellation by adding terms that are no longer a universal Lorentz transformation of the field. Although we have not investigated systems with spin > 1 it is possible to build a spin-J system out of a composite of 2J spin - 1/2 collinear fermions.[13] It is interesting to note that in the tree graph approximation gauge couplings and identical $Q_i/p_i\cdot q$ for collinear constituents translate into an effective gauge coupling for the spin-J composite which preserves the null zone.

What experimental tests can be proposed? The original[1] reaction (1) is part of the plans[5,14] for future $p\bar{p} \to W\gamma X$ experiments at CERN and Fermilab. Although the actual external legs are integrally charged hadrons with anomalous moments, the high transverse momentum recoil photon couples in leading twist only to the u, d, and W^+; diagrams involving radiation from spectators, etc., are suppressed by powers of m^2/M_W^2 where m is the hadronic mass scale. In addition there are quantum corrections from QCD loop diagrams that are of order $\alpha_s(M_W^2)/\pi$ by the standard renormalization group analysis. To this accuracy, the quark and boson charges and magnetic moments can be probed. With the discovery of more general null zones, reactions such as hard quark scattering, $q+q \to q+q+\gamma$, or τ radiative decays may give a measure of heavy quark and lepton magnetic moments.

Other massless gauge bosons are known[3,4] to give rise to four-body gauge zeros and the theorem presented here may be adapted for the determination of canonical form and possible null zones in the general gauge group case. The "charges" involve the representations of the particles and the amplitude needs to be invariant under the transformations of the internal symmetry group. Unfortunately, color and the zones are washed away by the necessary averaging/summing that goes on when the quark and gluon reactions are converted from and to hadron singlets. An interesting and open question is the extension of the canonical form (4) to graviton radiation and even to supersymmetry.

Local gauge theory exposes an intimate relationship between internal symmetry and space-time. Null zones, including the ancient dipole result show another face of this relationship, leading to more equations involving the internal variables (e.g. charge) and space-time (e.g. masses and angles). Many more details of these equations are to be reported elsewhere.[8]

RWB is grateful for the hospitality of the theory group at Fermilab and for the support of the National Science Foundation.

References and Footnotes

1. R. W. Brown, D. Sahdev, and K. O. Mikaelian, Phys. Rev. D 20, 1164 (1979).

2. K. O. Mikaelian, M. A. Samuel, and D. Sahdev, Phys. Rev. Lett. 43, 746 (1979).

3. C. J. Goebel, F. Halzen, and J. P. Leveille, Phys. Rev. D 23, 2682 (1981).

4. Z. Dongpei, Phys. Rev. D 22, 2266 (1980). See also K. O. Mikaelian, Phys. Rev. D 25, 66(1982).

5. R. W. Brown, Proc. Topical Workshop II (eds. V. Barger, D. Cline, and F. Halzen, Wisconsin, 1982). C. L. Bilchak, R. W. Brown, J. D. Stroughair, and R. Kinnunen (to be published).

6. V. Bargmann, L. Michel, and V. L. Telegdi, Phys. Rev. Lett. 2, 435 (1959); S. J. Brodsky and J. R. Primack, Ann. Phys. 52, 315 (1969).

7. In a frame such that $\vec{q}$ is along the positive z axis, $q^+ = q^0 + q^z = 2\omega$ and $q^- = q^0 - q^z = 0$. In terms of light-cone coordinates, $p_i^+ = x_i q^+$ and $p_i^- = \left((m^2 + k_\perp^2)/p^+\right)_i$, the null zone condition Eq. (2) requires $Q_i/(m_\perp^2/x)_i$ to be identical for all external lines.

8. A detailed discussion will be presented elsewhere.

9. D. R. Yennie, Lectures on Strong and Electromagnetic Interactions (Brandeis, Massachusetts, 1963). J. D. Jackson, Classical Electrodynamics (Wiley, New York, 1975), Chs. 14 and 15.

10. A physical null zone can exist for a subset of neutral particles provided that they are massless, can propagate along the photon's direction, and their spin terms vanish in that configuration. Massless Dirac and scalar particles can satisfy these criteria while external vectors cannot, so $\bar{\nu}_e + e^- \rightarrow W^- + \gamma$ has a gauge zero (Ref. 1) while Compton scattering $\gamma + e^- \rightarrow \gamma + e^-$ does not.

11. T. R. Grose and K. O. Mikaelian, Phys. Rev. D 23, 123 (1981).

12. C. H. Llewellyn-Smith, Phys. Lett. 46B, 233 (1973); J. M. Cornwall, D. N. Levin, and G. Tiktopoulos, Phys. Rev. Lett. 30, 1268; 31, 572(E) (1973).

13. J. Schwinger, Particles, Sources, and Fields (Addison-Wesley, Massachusetts, 1970).

14. D. Cline and C. Rubbia, private communication.