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Two-loop Corrections to the Evolution of the Higgs-Yukawa Coupling Constant

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Abstract

Second order corrections to the β-function for the coupling $g\bar{\psi}\psi\phi$ are calculated, including the terms of order g_H^5, $g_H^3 g_{QCD}^2$ and $g_H g_{QCD}^4$. It is noted that the two-loop terms are of opposite sign to the one-loop terms.

In the 'standard' model of strong and electroweak interactions, the "mass" of the fermions comes from a Yukawa interaction term, $g_\psi \bar{\psi}_R \psi_L \phi$, with a Higgs scalar of non-zero vacuum expectation value. The evolution of g_ψ with energy, described by β_ψ, is important both in determining low energy mass ratios [1] in Grand Unified Theories, and in its effects on $\sin^2\theta_W$ due to terms in the β-functions for $g_{U(1)}$ and $g_{SU(2)}$ proportional to $g_\psi^2 g_{U(1)}^3$ or $g_\psi^2 g_{SU(2)}^3$ [2]. The terms in β_ψ proportional to g_ψ^3(and g_ψ^5) are also important in discussing what low energy values of g_ψ (and hence fermion mass) are permissible or likely for very heavy fermions in a Grand Unified Theory. [3]

We present here the two-loop β-functions for the coupling of the Higgs to the top quark of a doublet, the bottom quark, and a heavy lepton, ignoring the electroweak effects (these are presented in reference [4]). The important point is that the two-loop terms are of opposite sign to the one-loop terms, which will modify the analysis of fixed points in references [2,3].

The β-function was calculated using an arbitrary gauge-fixing term $\frac{1}{\alpha}(\partial_\mu A)^2$ and in the MS and $\overline{\text{MS}}$ renormalization schemes; of course, the result to the 2-loop level is gauge-and scheme-independent.

For the evolution of the top quark,

$$(16\pi^2)^2 \frac{dg_T}{dt} = g_t\{[\text{one-loop term}] - 12\ g_T^4$$

$$- \frac{11}{4}\ g_T^2 g_B^2 - \frac{9}{4}\ g_T^2 g_E^2 - \frac{27}{4}\ g_T^2 g_c^2 - \frac{27}{4}\ g_T^2 g_s^2$$

$$- \frac{1}{4}\ g_B^4 + \frac{5}{4}\ g_B^2 g_E^2 + \frac{15}{4}\ g_B^2 g_c^2 + \frac{15}{4}\ g_B^2 g_s^2$$

$$- \frac{9}{4}\ g_E^4 - \frac{27}{4}\ g_c^4 + \frac{3}{2}\ g_c^2 g_s^2 - \frac{27}{4}\ g_s^4 \qquad (1)$$

$$+ 36\ g_T^2 g_3^2 + 4\ g_B^2 g_3^2 + 20\ g_c^2 g_3^2$$

$$+ 20\ g_s^2 g_3^2 + (\frac{80}{9}\ N_G - \frac{404}{3})\ g_3^4\}$$

where $t = \ell n\ Q$ and g_3 is the SU(3) coupling constant, N_G is the number of fermion generations, and g_T, g_B, g_c, g_s and g_E, are the couplings of the Higgs scalar to the top, bottom, charmed and strange quarks, and to the lepton. The couplings to the charmed and strange quarks have been included to represent couplings between two different heavy generations. Here, the one-loop term is

$$(16\pi^2)[\frac{9}{2}\ g_T^2 + \frac{3}{2}\ g_B^2 - 8g_3^2$$

$$+ 3g_c^2 + 3g_s^2 + g_E^2] \qquad (2)$$

The result for the evolution of g_B is identical, with T exchanged with B. The result for g_E is

$$(16\pi^2)^2 \frac{dg_E}{dt} = g_E\{[\text{one-loop term}] +$$

$$- 3\ g_E^4 - \frac{27}{4}\ g_E^2 g_T^2 - \frac{27}{4}\ g_E^2 g_B^2 - \frac{9}{4}\ g_E^2 g_u^2$$

$$- \frac{27}{4}\ g_T^4 + \frac{3}{2}\ g_T^2 g_B^2 - \frac{27}{4}\ g_B^4 - \frac{9}{4}\ g_u^2 + 20\ g_3^2 g_T^2 + 20\ g_3^2 g_B^2\} \tag{3}$$

where here the one-loop term is

$$(16\pi^2)[\frac{5}{2}\ g_E^2 + 3\ g_T^2 + 3\ g_B^2 + \quad g_\mu^2] \tag{4}$$

and g_μ refers to a lepton of another generation.

It is worth noting that the calculation of the anomalous dimension of a fermion mass operator in reference [15] is not gauge invariant. The correct method for determining the evolution of fermion masses is to calculate the complete β-function for g_ψ, as we do here. The method of reference [5] does give the correct $g_3^4 g_\psi$ term in β, because the gluon does not interact directly with the Higgs. If one wants the g_ψ^5 terms, or the electroweak corrections (which for energies of $10^2 - 10^{14}$ GeV are comparable to the g_3^2 terms), then one must compute the full 3-point function and the 2-point functions for the Higgs as well as the fermions,

Consider, now, the case of only one very heavy quark. Previously, the $+\frac{9}{2}16\pi^2\ g_T^2$ term in equation (2) put a limit on the possible mass of such a quark (in the context of SU(5)): If $M_T \gtrsim 240$ GeV, then g_T would evolve to infinity at an energy below the unification mass. That is, for any arbitrary (but large) g_T at M_X, M_T at low energy will be close to 240 GeV. [3] In this sense, 240 GeV is a (one loop) "fixed point"; if one knew nothing about $g_T(M_X)$ then the probability distribution for m_T (given a uniform distribution of g_T at M_X) would peak strongly at 240 GeV.

The two loop term, however, is given in equation (1) as $-12\ g_T^5$. Thus when g_T is as large as $\sqrt{6\pi}$ (which corresponds to a mass of ~ 1350 GeV), the two-loop term balances with the one-loop term, and the β-function changes sign. The consequence of this (assuming that the 3-loop term is not important until g_T grows larger still) is that M_{Top} (or M of a fourth generation quark) can be more than 240 GeV. Very heavy quarks can have important consequences for $\sin^2\theta_W$, M_x and the ratio M_b/m_t [2]. Of course, there are indications that the actual third generation top quark is much lighter than that [6], and very heavy masses raise the question of whether perturbation theory remains valid.

The terms in the β-function proportional to $g_{electroweak}$ appear in reference [4]; details of the β function calculation and numerical results for the new fixed points and effects on m_B/m_τ, $\sin^2\theta_W$ and M_X appear in reference [7].

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References

1. A. Buras, J. Ellis, M. Gaillard and D. Nanopoulos, Nucl. Phys. B135, 66 (1978).

2. M. Fischler and C. Hill, Nucl. Phys. B193, 53 (1981).

3. C. Hill, Phys. Rev. D24, 691 (1981).

4. M. Fischler and J. Oliensis, University of Pittsburgh publication PITT-82-3.

5. D. Nanopoulous and D. Ross, Nucl. Phys. B157, 273 (1979).

6. A. Buras, Phys. Rev. Lett. 46, 1354 (1981)

7. M. Fischler and J. Oliensis, University of Pittsburgh publication PITT-82-4.