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Constraints on Charge 2/3 Quark Masses

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ABSTRACT

We have investigated the constraints on the masses of the top quark and the corresponding member of a conjectured fourth generation, which are imposed by the K_L-K_S mass difference and the $K_L \to \mu^+\mu^-$ decay rate, in the context of the standard model. It was found that the top quark mass is bounded above, as previously shown by Buras for the case of three generations, but by an increasing function of the mass of the fourth charge 2/3 quark. Viewed differently, the mass of any additional charge 2/3 quark is bounded below by a function of the top quark mass. The constraints were found to be quite sensitive to the value of the K^0-$\bar{K}^0$ matrix element of the $\Delta S=2$ four-quark operator, as in the three-generation case, and numerical results are presented for a range of values for this model-dependent matrix element, including the MIT Bag Model prediction.

Recently Buras[1] has shown that the K_L-K_S mass difference and the $K_L \to \mu^+\mu^-$ decay rate can be used to obtain an upper bound on the top quark mass. The bound depends on the value for the matrix element, $<\bar{K}^0|[\bar{s}\gamma_\mu(1-\gamma_5)d]^2|K^0>$, and for the MIT Bag Model value of this matrix element, Buras found $m_t < 47$ GeV/c^2 in the free quark model approximation, and a more stringent result when QCD radiative corrections were included. Though one can perhaps question the quantitative validity of some of the assumptions made in the analysis, nonetheless, the bound provides interesting information concerning very massive charged 2/3 quarks.

In view of the growing interest in the possibility that there is yet a fourth generation,[2] we have investigated the implications of another charge 2/3 quark, t', for the upper bound on m_t obtained by Buras.[1] We have found that the K_L-K_S mass difference and the $K_L \to \mu^+\mu^-$ decay rate still constrain the t and t' quark masses, in spite of the great freedom afforded by the large number of (unknown) mixing parameters in the case of four generations. Specifically, m_t remains bounded above, but by an increasing function of $m_{t'}$; alternatively, $m_{t'}$ is bounded below by an increasing function of m_t. As in the case of three generation, the quantitative results remain sensitive to the value of the $\Delta S=2$ matrix element $<\bar{K}^0|[\bar{s}\gamma_\mu(1-\gamma_5)d]^2|K^0>$.

In the following we summarize several arguments suggesting a fourth generation, review the assumptions underlying the upper bound on m_t obtained by Buras and extend the analysis to four generations. Numerical computations are then presented illustrating the resulting constraints on the masses m_t and $m_{t'}$ for an interesting range of values of the model-dependent $\Delta S=2$ matrix element. We conclude with a critique of the underlying assumptions, discussing their validity and indicating how the constraints on m_t and $m_{t'}$ are affected by possible modifications of these assumptions.

The possibility that there may be more than the usual three generations of quarks and leptons has been advanced to explain a number of astrophysical observations. Slowly moving massive neutrinos, possibly belonging to a fourth generation of quarks and leptons, has been suggested by de Rujula and Glashow[3] to account for the invisible mass in galactic halos and the missing mass of the universe. And it has been suggested by Sciama and Melott[4] that a heavy fourth neutrino decaying radiatively into a light neutrino would give a significant ultraviolet background which could be responsible for the ionization of the interglactic medium. Bachall, et al.[5] have argued that more than three neutrinos are probably necessary if neutrino oscillations are to explain the discrepancy between the predicted and observed solar-neutrino fluxes. And Segre and Turner[6] have shown that a fourth generation of fermions is needed to obtain the observed baryon asymmetry within a minimal SU(5) grand unified theory. While, no one of these arguments is compelling, taken together, they suggest an interesting possibility worth investigating.

On the theoretical side, work[7] on the renormalization group properties of Higgs-Yukawa coupling constants in grand unified theories has suggested a natural scale of 200 to 250 GeV/c^2 for fermion masses, well above Buras upper bound, pointing to more than the usual three generations of quarks and leptons. With such a mass scale arising naturally there is the attractive possibility that the light quark masses are entirely radiative in origin and such models have been discussed.[8] It should be noted that all other upper bounds[9] on quark masses do admit objects as heavy as 250 GeV/c^2. Clearly, it is of interest to understand the implications for such heavy fermions of the very precise, low energy measurements of properties of the neutral kaons.

We shall assume, following Gaillard and Lee,[10] that the K_L-K_S mass difference is given by the short-distance contributions of the box diagrams of Fig. 1. One finds[1]

$$\frac{\Delta m_K}{m_K} = \frac{\alpha}{8\pi}\frac{G}{\sqrt{2}}\,(m_K\sin^2\theta_W)^{-1} <\bar{K}^0|[\bar{s}\gamma_\mu(1-\gamma_5)d]^2|K^0> \mathrm{Re}\; F(x_i,\theta_i). \tag{1}$$

Here, $F(x_i,\theta_i)$ depends upon quark masses, $x_i=m_i^2/m_W^2$, and the generalized Kobayashi-Maskawa[11] mixing parameters, θ_i. Allowing for QCD corrections by including factors η_{ij}, which are unity in the free quark model, we have

$$F(x_i,\theta_i) = \sum_{j,k} A_jA_kB(x_j,x_k)\eta_{jk} , \tag{2}$$

where the double sum extends over all charge 2/3 quarks ($j,k=u,c,t,t'$) and for[12] $i=j$

$$B(x_i,x_i) = x_i\left[\frac{1}{4} + \frac{9}{4}(1-x_i)^{-1} - \frac{3}{2}(1-x_i)^{-2}\right] - \frac{3}{2}[x_i/(1-x_i)]^3 \ln x_i , \tag{3}$$

while for $i \neq j$

$$B(x_i,x_j) = x_ix_j\left\{(x_j-x_i)^{-1}\left[\frac{1}{4} + \frac{3}{2}(1-x_j)^{-1} - \frac{3}{4}(1-x_j)^{-2}\right]\ln x_j + (x_i \leftrightarrow x_j) - \frac{3}{4}[(1-x_i)(1-x_j)]^{-1}\right\}. \tag{4}$$

For all practical purposes, the u quark contribution in Eq. (2) can be neglected, since m_u is small.

The factors A_i in Eq. (2) are functions of the generalized K-M parameters, being products of elements of the weak charged current mixing matrix[11] V:

$$A_i = V_{di}V_{si}^* \qquad (i=u,c,t,t'). \tag{5}$$

In the four generation case, V depends on six angles θ_i and three phases δ_i, and we have adopted the following convenient generalization[13] of the standard[14] 3×3 K-M matrix:

$$V = \begin{bmatrix} c_1 & s_1c_3 & s_1s_3c_5 & s_1s_3s_5 \\ -s_1c_2 & \begin{matrix} c_1c_2c_3 \\ +s_2s_3c_6e^{i\delta_1} \end{matrix} & \begin{matrix} c_1c_2s_3c_5 \\ -s_2c_3c_5c_6e^{i\delta_1} \\ +s_2s_5s_6e^{i(\delta_1+\delta_3)} \end{matrix} & \begin{matrix} c_1c_2s_3s_5 \\ -s_2c_3s_5c_6e^{i\delta_1} \\ -s_2c_5s_6e^{i(\delta_1+\delta_3)} \end{matrix} \\ -s_1s_2c_4 & \begin{matrix} c_1s_2c_3c_4 \\ -c_2s_3c_4c_6e^{i\delta_1} \\ -s_3s_4s_6e^{i\delta_2} \end{matrix} & \begin{matrix} c_1s_2s_3c_4c_5 \\ +c_2c_3c_4c_5c_6e^{i\delta_1} \\ -c_2c_4s_5s_6e^{i(\delta_1+\delta_3)} \\ +c_3s_4c_5s_6e^{i\delta_2} \\ +s_4s_5c_6e^{i(\delta_2+\delta_3)} \end{matrix} & \begin{matrix} c_1s_2s_3c_4s_5 \\ +c_2c_3c_4s_5c_6e^{i\delta_1} \\ +c_2c_4c_5s_6e^{i(\delta_1+\delta_3)} \\ +c_3s_4s_5s_6e^{i\delta_2} \\ -s_4c_5c_6e^{i(\delta_2+\delta_3)} \end{matrix} \\ -s_1s_2s_4 & \begin{matrix} c_1s_2c_3s_4 \\ -c_2s_3s_4c_6e^{i\delta_1} \\ +s_3c_4s_6e^{i\delta_2} \end{matrix} & \begin{matrix} c_1s_2s_3s_4c_5 \\ +c_2c_3s_4c_5c_6e^{i\delta_1} \\ -c_2s_4s_5s_6e^{i(\delta_1+\delta_3)} \\ -c_3c_4c_5s_6e^{i\delta_2} \\ -c_4s_5c_6e^{i(\delta_2+\delta_3)} \end{matrix} & \begin{matrix} c_1s_2s_3s_4s_5 \\ +c_2c_3s_4s_5c_6e^{i\delta_1} \\ +c_2s_4c_5s_6e^{i(\delta_1+\delta_3)} \\ -c_3c_4s_5s_6e^{i\delta_2} \\ +c_4c_5c_6e^{i(\delta_2+\delta_3)} \end{matrix} \end{bmatrix} \quad (6)$$

Here $s_i=\sin\theta_i$, $c_i=\cos\theta_i$ and, by convention, $0 \leq \theta_i \leq \pi/2$ and $-\pi \leq \delta_i \leq \pi$. Finally, combining the known values of the constants in Eq. (1), the relation among quark masses and generalized K-M parameters resulting from the K_L-K_S mass difference is given by

$$\mathrm{Re}\, F(x_i,\theta_i) = 4.4 \times 10^{-5}\, R\ , \tag{7}$$

where R^{-1} is the physical $\Delta S=2$ matrix element, normalized, for convenience, to its value calculated in the MIT Bag Model:[15]

$$R^{-1} = \langle\bar{K}^0|[\bar{s}\gamma_\mu(1-\gamma_5)d]^2|K^0\rangle/\langle\bar{K}^0|\ldots|K^0\rangle_{BAG} \tag{8}$$

Turning to the $K_L \to \mu^+\mu^-$ decay rate, the short-distance contributions, in the unitary gauge, come from three diagrams: the annihilation box diagram like Fig. 1 with one of the quark lines replaced by a muon-lepton line, and the two neutral current annihilation diagrams in which the effective flavor-changing $Z\bar{s}d$ coupling is induced by charged current interactions. The short-distance contributions to $K_L \to \mu^+\mu^-$ coming from these three diagrams have been shown[16] to be bounded by the dispersive part of the amplitude, which in turn can be bounded in terms of the two-photon contribution to the absorptive part of the amplitude and the measured $K_L \to \mu^+\mu^-$ and $K_L \to \gamma\gamma$ decay rates. Generalizing these results to the case of four generations one obtains

$$\left| \sum_i \mathrm{ReA}_i \; G(x_i)\eta_i \right| \leq 1.9\times10^{-3} \tag{9}$$

where the summation extends over all charge 2/3 quarks (i=u,c,t,t'). The dependence on the quark masses, for arbitrary x_i, is given by[17]

$$G(x_i) = \frac{3}{4}\left(\frac{x_i}{1-x_i}\right)^2 \ln x_i + \frac{x_i}{4} + \frac{3}{4}\,\frac{x_i}{1-x_i}\ . \tag{10}$$

The parameters η_i in Eq. (9) represent small QCD corrections to the free quark model, for which all η_i=1. The u and c quark contributions to the LHS of Eq. (9) are, in fact, negligible compared to the RHS and will be ignored in the following.

Clearly, the above analysis can trivially be further extended to any number of generations in the standard model.

Before proceeding to use Eqs. (7) and (9) to obtain constraints on m_t and $m_{t'}$, we observe that, since CP nonconservation is very small ($\sim10^{-3}$), the CP nonconserving contributions to the kaon mass matrix coming from the box diagrams (Fig. 1) must be small compared to the CP conserving contributions; i.e., $|\mathrm{Im}\, F(x_i\ \theta_i)/\mathrm{Re}F(x_i,\ \theta_i)|<<1$. For simplicity, we shall therefore neglect CP nonconservation in the following numerical computations and restrict the phases δ_i in the generalized K-M matrix [Eq. (6)] to the values 0 and π.

To illustrate the constraints imposed on m_t and $m_{t'}$ by the K_L-K_S mass difference [Eq. (7)] and the $K_L \to \mu^+\mu^-$ decay rate [Eq. (9)], we have used the free quark model ($\eta_{ij}=1$, $\eta_i=1$) and have allowed the unknown K-M parameters in the factors A_i to vary over their entire allowed ranges. The numerical computations were organized as follows: With the measured value[14] $A_u = 0.213 \pm 0.012$ fixed, unitarity of the K-M matrix was used to determine $A_c=-(A_u+A_t+A_{t'})$, [c.f., Eq. (5)], while A_t and $A_{t'}$ were varied. The region of possible values of A_t and $A_{t'}$ was then determined, allowing all the K-M parameters to vary over their full range, subject only to the empirical constraints[18] $|\cos\theta_1| = 0.9737 \pm 0.0025$ and $|\sin\theta_1 \cos\theta_3| = 0.219 \pm 0.011$. Finally, for each of the admissable values of A_t and $A_{t'}$, constraints on m_t and $m_{t'}$, imposed by Eqs. (7) and (9), were found numerically.

Figure 2 shows the results of the computations for the free quark model for R=1, which corresponds to the MIT Bag Model value of the $\Delta S=2$ matrix element; and for comparison, a smaller value, R=3/4, and a larger value, R=2. As Buras found[1] for three generations, the constraints on the quark masses are quite sensitive to the value of R[19].

The constraints on m_t and $m_{t'}$ can be viewed either as an <u>upper</u> bound on m_t , which is an increasing function of $m_{t'}$; or, alternatively, as a <u>lower</u> bound on $m_{t'}$ which increases with m_t. Of course, they are only interesting if Buras bound, which corresponds to $m_t=m_{t'}$ for each R in

Fig. 2, is found to be violated when the t quark is discovered. Even then, the numerical results for the free quark model illustration are not very restrictive unless R is at least as large as in the MIT Bag Model. It is, nevertheless, noteworthy that the K_L-K_S mass difference and the $K_L \rightarrow \mu^+\mu^-$ decay rate do indeed constraint m_t and $m_{t'}$, in spite of the great freedom afforded by the numerous, presently unknown, K-M parameters in the case of four generations.

In this free quark model calculation, all QCD corrections were neglected and these are expected to strengthen the constraints significantly. However, the QCD correction factors, η_{ij} in Eq. (2) and η_i in Eq. (9), have been estimated only for three generations and then assuming[19] $m_{W,Z} >> m_t >> m_b >> m_c >> m_{u,d,s}$. Using these results, viz., $\eta_{cc}=0.90$, $\eta_{tt}=0.62$, $\eta_{ct}=0.33$ and $\eta_t=0.90$ Buras found[1] that the upper bound on m_t was substantially strengthened, changing from $m_t<47$ GeV/c^2 in the free quark model to $m_t<33$ GeV/c^2 with these QCD corrections for R=1. If we assume these same QCD corrections remain, at least approximately, valid in the regime of heavy quarks, we have found the numerical results for the case of four generations are, indeed, similarly improved. However, the complete calculation of the QCD corrections, valid for arbitrary quark masses, has not been done.[21]

Finally, we note that it has been suggested[22] that contributions to the K_L-K_S mass difference, other than the box diagrams (Fig. 1) considered here, may be significant. The contributions of low-mass intermediate states, or any such long-distance contributions to Δm_K , probably increase the effective value of R, as Buras has argued,[1] thereby strengthening the constraints.

We have also ignored possible Higgs boson or off-diagonal neutral current contributions to the K_L-K_S mass difference and the $K_L \to \mu^+\mu^-$ decay rate. Though such effects are expected to be small in the more conventional models, in principle, multi-Higgs schemes can be invented in which they are large. However, there is no compelling reason to do so.

Barger, et al,[23] have recently argued that the two photon contribution to the dispersive part of the $K_L \to \mu^+\mu^-$ amplitude, neglected in obtaining the bound on the short-distance contribution, is substantially larger than previously thought.[16] If so, this would effectively increase the RHS of Eq. (9), perhaps even as much as a factor of[23] 3; and this would roughly weaken the constraint on m_t and $m_{t'}$, shown in Fig. 2 by about a factor of $\sqrt{3}$, for this worst case.

To summarize, we have shown that the K_L-K_S mass difference and the $K_L \to \mu^+\mu^-$ decay rate do, indeed, constrain the masses of the charge 2/3 quarks, even if a fourth generation should exist. However, the numerical calculations, at least for the free quark model, indicate the bound on m_t is considerably less stringent if a t' quark exists; an interesting possibility if m_t is found to exceed the upper bound found by Buras.

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FIGURE CAPTIONS

Fig. 1: The annihilation box diagram contributing to the K_L-K_S mass difference Δm_K. The scattering box diagram is obtained by crossing.

Fig. 2: Constraints on m_t and $m_{t'}$ in the free quark model assuming m_c=1.5 GeV/c^2 for R=3/4, 1 and 2. The horizontal and vertical lines represent the present experimental lower bounds.

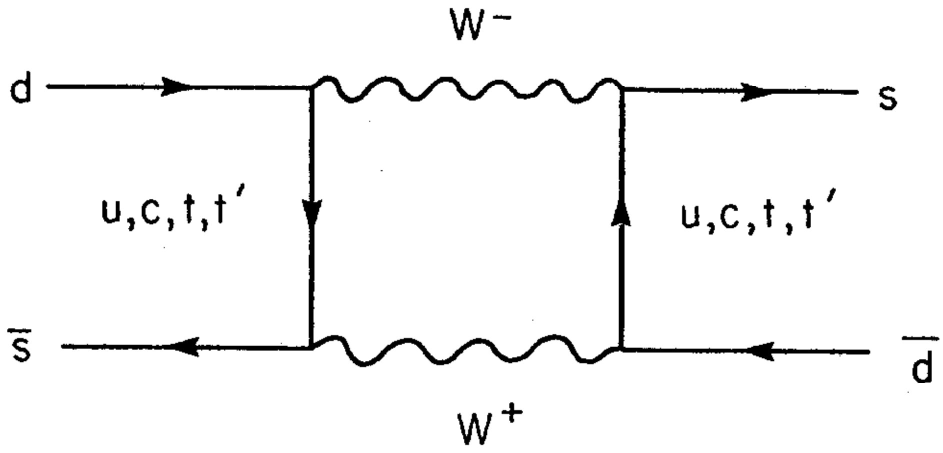

Fig. 1

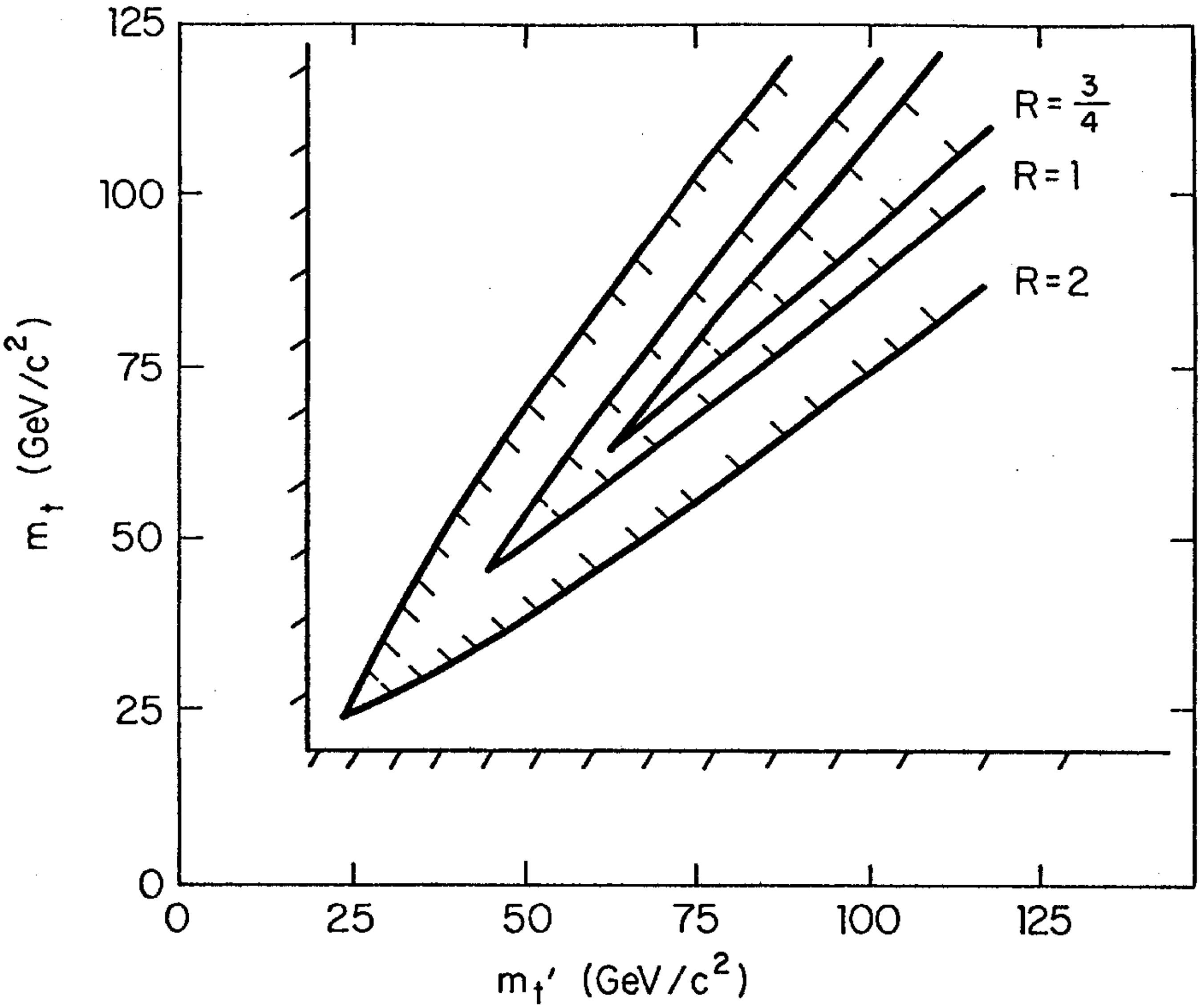

Fig. 2