Fermi National Accelerator Laboratory

FERMILAB-Conf-82/42-THY
June, 1982

QCD and the Space-Time Evolution of High Energy e^+e^-, $p\bar{p}$, and Heavy Ion Collisions

J.D. BJORKEN
Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

*Presented at the 2nd International Conference on: Physics in Collision: High Energy ee/ep/pp Interaction, Stockholm, Sweden, 2-4 June, 1982.

I. INTRODUCTION

With QCD the generally uncontested theory of the strong interactions, it is natural that descriptions of high energy collisions nowadays tend to use the QCD language of quarks and gluons. Nevertheless, we usually don't observe the quarks and gluons — we see hadrons. This creates problems — problems that lead directly to the outstanding issue facing QCD, that of quark and gluon confinement. Some processes, such as e^+e^- annihilation into $q\bar{q}$ or $q\bar{q}g$ appear to permit a relatively easy description in terms of the quark and gluon language. Take the man in the street to a typical PEP or PETRA experiment and show him the on-line displays of two- and three-jet events, and he may well get the idea. He needn't be a theorist or even experimentalist to be able to see the quarks and gluons. In fact, he would do almost as well as the professionals in deciding which of the three jets is the gluon.

In other processes, such as low-p_T (or even high-p_T) particle production by hadrons, it is hard to see obvious evidence for existence of the quarks and gluons. Nevertheless the QCD ideas have been applied with some success to these more complex collisions. No one is willing to say that existing data is inconsistent with QCD, but there is a real problem in weighing the significance of the claimed successes of QCD for high energy collision processes. Most of the successes, I believe, do not test the theory in a fundamental way. By a fundamental QCD test I mean the following: if the outcome of the experimental test were to sharply <u>disagree</u> with the QCD prediction, one would be forced to abandon QCD. Such fundamental tests do exist. They include measurements of e^+e^- total cross sections, observation of 3-jet final states in e^+e^-

annihilation, and observation of at least approximate scaling (i.e., no gross power-law deviations) in deep inelastic lepton-hadron processes. Examples of measurements which I believe fail this test are energy dependence of total multiplicity and approximate scaling (or non-scaling) of final state hadron distributions in any process, including e^+e^- annihilation.

Indeed most measurements, if viewed as fundamental QCD tests, are deficient in some way or another. The deficiencies usually can be traced back to two basic problems. The first problem is understanding the structure of a hadron in terms of quark-gluon degrees of freedom. For many applications we need to know the distribution of quarks and gluons within a hadron (in the parton-model sense), or even the wave function of a hadron in terms of quark-gluon constituents. The quark-gluon distribution functions have a reasonably precise definition in terms of the moments of deep inelastic structure functions and of the Altarelli-Parisi equations that control their QCD behavior. Less precise is the applicability of this concept to Drell-Yan dilepton production and/or high-p_T jet production in hadron-hadron collisions. Tests of QCD in elastic or nearly elastic scattering processes depend upon a Fock-space description of the hadron. For example, in fixed-angle elastic $\pi\pi$ scattering at high energy, the QCD predictions[1] rest upon the assumption that there exists, with finite probability, a bare $q\bar{q}$ component of the pion wave function which can be calculated perturbatively (when q and $\bar{q}$ are close together). Is this a clear consequence of QCD? Are even the concepts of wave function and Fock-space description, very difficult concepts in any relativistic quantum field theory, admissible? Even so, would this picture be compatible with, e.g., the bag model description of hadrons? I have no definite arguments to offer one way or the other, but if experiments which depend upon these ideas were to disagree sharply with the QCD predictions, I would not give up QCD.

The second problem in identifying most measurements in terms of fundamental QCD tests lies in the question of "hadronization" of quarks and gluons. Even in the simplest case of e^+e^- annihilation, the quarks "seen" by the man on the street are manifested as jets of hadrons. As we shall review in more detail later, these jets evolve over large distance and time scales, and again the applicability of QCD perturbation theory may have serious limitations.

The above problems highlight what to me is a central question: to what extent is the diagrammatic, perturbative QCD approach viable at all? Perturbative QCD is applicable at short distances — distances less than the confinement scale and hence the size of ordinary hadrons. It is not a priori clear to me what Feynman diagrams with quarks and gluons as interior lines and with hadrons on exterior lines really mean. How does one derive the Feynman rules for such amplitudes? At short distances the appropriate Hilbert space for describing the dynamics is most likely built from quarks and gluons. At large distances the Hilbert space of asymptotic hadron states, as carefully constructed by axiomatic field theorists long ago, most likely is what is appropriate. What is the transformation function from one to the other? Does it make sense to write down amplitudes which mix together the descriptions? That is, can one use both Hilbert spaces at the same time? Perhaps these questions are answerable by the experts, but I for one remain puzzled.

These dour remarks are not meant to belittle all the recent, beautiful work employing QCD methods. They are especially inappropriate, given that I have not been an active participant in this difficult and demanding field. Perturbative QCD is our best tool for probing the structure of high-energy collisions, but I do feel that there is still a need for a solid foundation under the calculational superstructure.

None of these big problems will be solved by the contents of this talk. I will instead concentrate on the space-time evolution of hadronic final states in various processes. It has been known for a long time[2] that large distances are important at high energies, and that we therefore should be able to at least map out the basic space-time geography of the collision process. This has been a favorite topic of mine for a long time. I feel it may help to sharpen the distinction between non-perturbative and perturbative phenomena. It must be admitted that so far, the space-time pictures have not led to very much in the way of practical (computational) insights, but given the present QCD ideology, it may be useful to look at the subject again. We shall begin in Section II with a discussion of e^+e^- annihilation into hadrons, a process blessed with well-known elements of simplicity. In Section III we consider the opposite extreme of highly relativistic nucleus-nucleus collisions. Here a space-time description has its own elements of simplicity, elements which might conceivably be applicable in hadron-hadron collisions. In Section IV we address the more immediate issues of how these ideas relate to present-day observations, especially high-energy hadron-hadron collisions. Section V is devoted to concluding remarks.

II. SPACE-TIME DESCRIPTION OF e^+e^- ANNIHILATION

Perturbative QCD is designed for short-distance applications, where the QCD force is manifestly weak. One sure way to eliminate large-distance effects in QCD is to eliminate large distances. For example, for very high energy (E_{cms}>>1TeV) e^+e^- collisions, we can envisage a very small collision hall ($\sim 10^{-12}$cm.) into which we put a piece of detection apparatus of size[3] $\lesssim 10^{-13}$cm. with good spatial and angular resolution. In theorists' language, our quantization volume is chosen so small that perturbation theory is manifestly valid. The "asymptotic" scattering states are indeed quarks and gluons, and the very concept of hadron cannot exist because a hadron doesn't fit into the box. We may imagine preparing incident quark and/or gluon beams, and in detecting quarks and gluons in the tiny detectors.

Over what region of space-time, then, may we safely use perturbation theory? Evidently, as shown in Fig. 1a, we may cover the interior of the light-cone up to a time (and distance) $\lesssim$1f, but we may do more. We may view the process in different reference frames. Assume the secondary $q\bar{q}$ pair is collinear with the e^+ beams and then increase the e^- beam energy by a factor 3, while decreasing the e^+ beam energy by the same factor. In that frame we may again cover the interior of the light-cone up to a distance $\sim$1f. In the original cms frame this is a tilted region of space-time as shown in Fig. 1b. By repeating this argument in many reference frames, we may define the perturbative region shown in Fig. 1c. The outer limit is defined by the frame in which the secondary quark (or antiquark) no longer has momentum >>1 GeV, and is thus proportional to the initial cms energy. For PEP/PETRA conditions, this "formation length" (in the cms frame[f1]) is conservatively $\sim$15f.

Within this region we should be able to safely use perturbation theory. Well beyond it, we must expect to see the produced hadrons. For example, in a typical event there are no more than $\sim$3 charged particles ($\lesssim$5 in toto) produced per unit of rapidity. Then within $\sim$2 units of rapidity (i.e., $40^\circ<\Theta<140^\circ$), $\lesssim$10 hadrons are typically emitted. If we say a hadron covers an area of $2f^2$, then at a distance of $\sim$5f these 10 hadrons typically cover only $\lesssim$10% of the detection area. It is reasonable, therefore, to say that for t>5f, the centrally produced, large-angle hadrons are already created and, to good approximation, asymptotic particles. This is not yet true, of course, for the system moving in the directions of the produced quark and antiquark.

Thus we have defined a region of space-time (Fig. 2a) in which the asymptotic state of free, outward-moving pions is certainly realized. Again this argument can be repeated in a boosted reference frame (Fig. 2b). After doing this many times, we obtain the region of space-time where the system is "asymptotic" as well as the region within which the dynamics is QCD-perturbative. These are shown in Fig. 3a. It is also important to keep in mind that we have so far suppressed the transverse motion. The system in the transverse coordinates at a time t_0, say $\sim$6f, in Fig. 3a is shown in Fig. 3b. For the two-jet evolution which we have described, the QCD perturbative region, as well as the transition region separating it from the asymptotic region, is confined to transverse distances $\lesssim$1f.

---

f1) If we choose an extreme reference frame by boosting by a factor 15 (300GeV e^- on 1 GeV $e^+ \rightarrow$ 300 GeV q + 1 GeV $\bar{q}$), the formation length is $\sim$200f.

Some things have been left out. We have essentially discussed only the case of a two-jet final state. In the small collision hall, we might have found a "hard", high-momentum jet at large angles (again, large means $40° < \Theta < 140°$). The perturbative probability of this happening is of order unity (actually $\sqrt{8(\alpha_s/3\pi)}$ log E_{max}/E_{min}). This hard-gluon jet should evolve over a large transverse distance, in a way not dissimilar to the quark jets. Thus the hyperbolic surface defining the boundary of the asymptotic region should be "spiky" (Fig. 4). At really high energies these spikes will themselves grow more spikes, etc., and the surface will have a fractal structure.

Thus far, the space-time map depends very little on dynamical assumptions, other than what is needed to account for gross properties of the data. We may now begin to pose the main question. The perturbative QCD region of space-time evidently does not join contiguously onto the asymptotic region. The intermediate transition region is what is most interesting: How thick is it? What goes on inside? Can perturbative QCD concepts be used in at least most of this region?

There does exist a school of thought[4] which argues that perturbative QCD can account for almost all of this boundary region — that by pushing down on the infrared cutoffs, enough gluons and quark-antiquark pairs are (perturbatively) created to account for the observed hadrons. Furthermore, it is argued that the planar structure of the leading Feynman graphs allows, even within perturbation theory, color rearrangement into color-singlet low-mass quark-antiquark systems locally in phase-space, so that very little in the way of non-perturbative effects need take place. This is the phenomenon of "preconfinement". Even within our qualitative and descriptive space-time picture, we may see the plausibility of this view. Within the large-angle region, we need to account for ~ 4 GeV ($\approx 10\times 0.4$ GeV) of produced energy. Can this be the spoor of the perturbatively emitted gluons? We argued that the number of such gluons was $\sim \alpha_s \log E_{max}/E_{min}$. The mean energy of these gluons is $\sim E_{max}/(\log E_{max}/E_{min})$, leading to an amount $\sim \alpha_s E_{max}$ of perturbative energy emitted into the large angles. Now for $E_{max} > 3$ GeV, the gluon is identifiable as a distinct extra jet, and we double-count. Hence we should take $E_{max} \lesssim 3$ GeV, implying $\alpha_s \gtrsim 1.3$ to get the energy budget satisfied. This lies on the boundary of perturbative calculation ($\alpha_s/\pi \ll 1$??), and might be admissible. Furthermore, the above estimate is evidently very crude.

On the other hand, this estimate can be regarded by a skeptic that the perturbative mechanisms fall short of producing enough energy. I find myself among the skeptics, although uncomfortably so, inasmuch as I have not put pen to paper and done any real calculations myself. Nevertheless I shall submit other arguments favoring skepticism.

The first is based on calculations of Bassetto, et al.,[5] who compute the inclusive spectrum of soft gluons and hence total multiplicity in, say, the process $\eta_t \to gg \to$ gluons (Quark-pair creation, for simplicity, is neglected.) This calculation is done in leading-log approximation, with terms of order $\alpha_s \log^2$ kept. In this limit they find that the dominant amplitudes have a tree structure (as in an electromagnetic cascade in matter). If one searches inclusively for a soft gluon, the leading contribution comes from traveling from the main trunk of the tree down the lesser branches[f2] until the specified gluon is reached. Bassetto, et al. find that the main contribution does come from a limited region of phase-space where not only are the gluon energies down the branches strongly ordered, but also the angles; in Fig. 5 one has

$$E_1 \gg E_2 \gg \ldots E_n \qquad \theta_1 \gg \theta_2 \gg \ldots \theta_{n-1} \tag{2.1}$$

The inclusive spectrum is calculated to be

$$x\frac{dN}{dx} = \sum_{n=1}^{\infty}\left(\frac{3\alpha_s}{\pi}\right)^n \frac{[\log(Q^2x^2/Q_0^2)]^n[\log(1/x)]^{n-1}}{n!\,(n-1)!}$$

$$= \frac{1}{[\log(1/x)]}\,\xi I_1(2\xi) \tag{2.2}$$

with $x = E_{gluon}/E_{jet}$ the usual longitudinal fraction, and with I_1 a Bessel-function

$$I_1(2\xi) = \xi + \frac{\xi^3}{1!2!} + \frac{\xi^5}{2!3!} + \ldots \tag{2.3}$$

and with

$$\xi = \sqrt{\frac{6\alpha_s}{\pi}\left(\log\frac{Q}{Q_0} - \log\frac{1}{x}\right)\log\frac{1}{x}} \tag{2.4}$$

---

f2) At a branching, the ratio of gluon momentum in the major branch to that in the minor branch is typically of order $(\log)^{-1}$.

Notice that the maximal value of ξ occurs not at the minimal $x \sim Q_0/Q$, but rather at $x \sim \sqrt{Q_0/Q}$. This implies at asymptotic energies that a two-fireball structure should emerge, with peaking at cms momentum halfway (on a multiplicative scale) between the minimum and the maximum allowed momentum. Very few large-angle gluons are emitted, and this cannot be beneficial to the preconfinement picture; the two "fireballs" must communicate to produce color-screening. However a small-angle approximation has been made, and the minimum value of the rapidity distribution can be expected to be of order one per unit rapidity.

Integration of the inclusive spectrum (ignoring the running[f3] of α_s) gives the total jet multiplicity with the traditional form emergent from perturbative QCD:

$$\bar{n}_{jet} \sim \cosh\left(\sqrt{\frac{6\alpha_s}{\pi}} \log \frac{Q}{Q_0}\right) - 1 \quad . \tag{2.5}$$

At PEP/PETRA energies, with $Q_0 \sim 1$ GeV, $Q \sim 40$ GeV, and $\alpha_s \lesssim 0.2$, one gets

$$\bar{n}_{jet} \sim 4/\text{jet} \tag{2.6}$$

---

f3) This <u>overestimates</u> the yield.

leaving quite a bit of multiplicity to be accounted for.[f4] The calculation is also for pure gluon couplings, for which branchings are most frequent. In $e^+e^- \rightarrow q\bar{q} \rightarrow$ quarks and gluons, we should expect a smaller number.

Thus we see again that the perturbative calculation tends to fall short of accounting for the full multiplicity. But there is another feature which makes one suspicious that perturbative gluons are really the mechanism which accounts for bulk hadronization at existing energies. This is the aforementioned strong ordering of emission angles. The soft gluons appear at small angles relative to the natural jet angle. If one looks down the (quark) beam direction, this will imply a highly coplanar structure for the soft emission (Fig. 6). This indicates to me that this extra multiplicity is to be associated with additional hard jets, and will not[f5] easily account for the

---

f4) Note that the mean number of orders of perturbation theory being used to compute this multiplicity is, because of the double factorial, quite small

$$\langle N\rangle = \frac{\alpha_s}{\bar{n}}\frac{\partial\bar{n}}{\partial\alpha_s} = \sqrt{\frac{3\alpha_s}{2\pi}}\left(\log\frac{Q}{Q_0}\right)\cdot\coth\left(\sqrt{\frac{3\alpha_s}{2\pi}}\log\frac{Q}{Q_0}\right) \sim 1.4 \quad ,$$

and grows with energy very slowly.

---

f5) The whole picture of event structure would be much less bizarre and more conventional if this structure represented the single quark jet which is seen in a frame in which the $\bar{q}$ has momentum $\sim$1 GeV. (cf. footnote f1). Is something left out of the calculation?

azimuthally symmetric multiplicity.

Another argument for additional nonconfining effects is simply that in the perturbative framework there exists a finite probability per event that no additional gluons are emitted and that a single two-jet final state emerges. This should be, at PEP/PETRA energies, at least of order a few percent. In those cases one must invoke nonperturbative mechanisms. Then why are they negligible in other events?

Also Gupta and Quinn[6] have argued that in a QCD world where only heavy quarks exist, the non-perturbative effects become manifest. The typical final state in, say, $e^+e^-\to t\bar{t}$ will (in a world without light quarks) be a highly excited state of the $t\bar{t}$ system in a (linear?)

---

f6) Because of the angular collimation, one might suspect that these "soft" gluons, which are emitted "after" the hard gluons will be emitted at too late a time to allow the conjectured preconfinement mechanism to operate. However, this is not a problem; the emission time is no longer than what the basic space-time geometry which we have discussed would imply. This happens because the time scale of the early stages of the cascade is so short that the relatively long time scale of the later stages is not a problem. Specifically, the emission time for the k-th virtual gluon in the ladder is

$$t_k \sim \frac{E_k}{q_k^2} \quad ,$$

where q_k^2 is the squared virtual mass of the emitting gluon. Using

$$q_k^2 \sim E_k^2 \theta_{k+1}^2 \quad ,$$

this implies that

$$t_k >> t_{k-1}$$

and thus $t \sim t_n$ is the "natural" time scale $\sim E_n/Q_0^2$.

potential, with size proportional to energy and with lifetime probably large[f7] in comparison with the period of oscillatory motion. Thus there must be a mechanism[7] for producing the string (or some alternative long-range confining field). Can the perturbative QCD of gluons do that?

Finally, there may be some evidence essentially within perturbative QCD for conversion of collision energy into "nonperturbative" structures, be they strings or something else. This comes about from looking at QCD canonically quantized (in temporal, $A_0=0$, gauge) in a small volume ($\ll 10^{-39}\text{cm}^3$). In such a small volume an infrared cutoff is provided by the box size, and QCD behaves very much like perturbative QED. There is a small distinction that goes beyond the interactions of transverse gluons with themselves and with the instantaneous Coulomb field. Upon expanding the gauge potential $A(\vec{x},t)$ in a Fourier series (using periodic boundary conditions)

$$A(\vec{x},t) = \bar{A}(t) + \sum_{k\neq 0} A_k(t)e^{i\vec{k},\vec{x}}$$

the space-averaged mode of the gauge potential $\bar{A}$ possesses interesting dynamics. This is not the case in QED, where a constant A field is a

---

f7) I once thought[3] that this would not be the case, owing to soft gluonium emission by the t and $\bar{t}$ as they moved in the constant force field. However, this is wrong; the emission amplitude is exponentially small ($\sim e^{-\sqrt{m_t}\,t}$ for large top-quark mass m_t) owing to the nonzero gluonium mass and a bad overlap integral for the toponium wave functions. This observation is due to S. Gupta, whom I thank for very enlightening discussions.

gauge-artifact.[f8] However, in QCD $\vec{G} = \vec{A} \otimes \vec{A}$ can be nonvanishing. Ignoring all transverse gluons, the dynamics of the $\vec{A}$ mode is that of a rotor,[3] with a rich spectrum of excited states. In the presence of the produced quark pair, there is an extra interaction energy $e_s \vec{j} \cdot A$, where $\vec{j}$ is the (color-octet) space-averaged dipole current of the quark pair. This coupling should excite the rotor. It will tend to align the A field along the quark color in internal space and along the direction of motion of the quarks in ordinary space. This is perhaps the first step (and all that can be seen in such a small box) in preparing the line integral P exp $ie\int A \cdot ds$ (bare string?) that should connect a widely separated $q\bar{q}$ pair. The excitation energy of the rotor is independent of the quark-pair energy and inversely proportional to the size of the quantization volume. We may conjecture that this excitation energy, which is crudely seen at short distances, will find its way into the isotropically produced pions.

Let us now summarize the space-time picture which is suggested. The basic space-time geography must, on kinematic and common-sense grounds alone, be as shown in Figs. 3 and 4. There is a perturbative region $\lesssim$1f proper distance from the light cone, and an asymptotic region $\gtrsim$3-5f proper distance from the light cone. In between there is a transition region of uncertain thickness, within which the dynamics is complicated and hadronization takes place. The surfaces of these regions are not smooth, but are punctured by occasional jets of high transverse momentum. The number of these per unit rapidity is no more than order unity. Some fraction of the energy which flows out through

---

f8) Except for Bohm-Aharanov topological effects.

the bulk of the transition surface (not including the jettish portions) may originate in perturbative production of soft gluons (3 GeV>p_T>300 MeV?), but it is not at all clear (and I doubt it is the case) that this is the dominant part.

This picture is certainly consistent with the conventional one (Feynman-Field, Lund, etc., supplemented with QCD jets) used to simulate e^+e^- reactions at PEP and PETRA.

However, the dynamics in the transition region of space-time remains somewhat out of control. It may in fact be very complicated and require a statistical treatment.[8] That this may be necessary may be indicated by the universality of gross particle production properties in hadron-hadron, lepton-hadron, and lepton-lepton collisions, and the near indistinguishability of quark and gluon jets. Sometimes I think that a hydrodynamic approach[9] might be appropriate,[10] with initial conditions applied at the inner surface and output calculated at the outer boundary of the transition region. The initial energy density by our earlier estimates should be at least a few GeV/f^3, while that of a free, asymptotic pion gas should be considerably less than 100 MeV/f^3. To do hydrodynamics requires an assumption of equilibrium of a fluid of quarks and gluons (nonperturbatively produced?), and of existence of a conserved energy-momentum tensor, along with an equation of state relating energy density and pressure. However, there are plenty of problems with this approach, at least in e^+e^- annihilation. If perturbative QCD cannot make all those quanta in the fluid, when were they produced? Also, the overall time-scale for the hydrodynamic evolution, by our own arguments, is quite short ($\lesssim$3-5f). Is this really long compared to the formation and equilibration times of the purported fluid?

While considering these questions, it seemed prudent to consider a hydrodynamic description using similar space-time boundary conditions for a more macroscopic system. A nice opportunity for doing this is given by the present interest in extreme-relativistic nucleus-nucleus collisions,[11] and I will digress to describe how that system might behave and whether it might teach us about more elementary collisions.

III. NUCLEUS-NUCLEUS COLLISIONS

Consider a central collision of two extreme-relativistic nuclei (energy >>100 GeV/nucleon) in the center-of-mass frame. A time ct=1f or so after the collision we may expect a hot hadronic system is formed between the outward-moving, highly Lorentz-contracted nuclear pancakes (Fig. 7). This system is produced by the independent collisions of small transverse elements of the pancakes of area $\sim d_0^2$; each must act independently of the others because of causality. We assume that the collision of these basic elements produces energy in the same way as two nucleons, or a nucleon and nucleus would. This assumption is bolstered somewhat by the similarity of particle production in nucleon-nucleus and nucleon-nucleon collisions, once one gets away from the fragmentation region of the nucleus. The energy which in the elementary collision moves outward at $\sim 90°$ to the beam direction will get trapped in the region around the collision plane of the projectiles. An elementary estimate for the energy density ε_0 in this midplane at time t_0 after the collision gives[12]

$$E_0 \simeq \frac{d\langle E\rangle}{dy} \cdot \frac{1}{2d_0^2\, t_0} \tag{3.1}$$

where $d\langle E\rangle/dy$ is the energy production per unit rapidity in a hadron-hadron collision. This argument, as before, can be repeated in other reference frames - in particular all "central" frames for which we still have highly Lorentz-contracted pancakes colliding with each other. Since it is a good approximation that nucleon-nucleon interactions produce a "central plateau" of produced hadrons, the energy density deposited in the collision plane should be, in the new frame, about the same as in the old. This implies a symmetry property of the initial conditions which we have imposed. And if indeed the initial conditions at a given proper time are independent of Lorentz boost angle, the subsequent motion will respect the symmetry. Therefore the longitudinal motion everywhere between the pancakes is determined: at time t after the collision the fluid a distance z from the collision plane moves with velocity z/t; that is, the expansion is homogeneous.

In general, the hydrodynamic flow is determined[9] from energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0 \tag{3.2}$$

$$T^{\mu\nu} = (\varepsilon+p)u^\mu u^\nu - g^{\mu\nu}p \tag{3.3}$$

and an equation of state relating the energy density ε and pressure p. With our boost-invariant boundary conditions, ε and p are functions only of the (on-axis) proper time $\tau=\sqrt{t^2-z^2}$. Very simple calculations[12] lead to equivalent formulae which determine their time evolution

$$\frac{d\varepsilon}{d\tau} = \frac{-(\varepsilon+p)}{\tau} \quad \text{or} \quad \frac{dT}{d\tau} = -v_s^2 \frac{T}{\tau} \tag{3.4}$$

with v_s^2=dp/dE the local sound velocity of the medium and T the temperature. With an "ideal" equation of state, ε=3p (as for a photon gas), one gets

$$\varepsilon \sim \tau^{-4/3} \qquad T \sim \tau^{-1/3} \tag{3.5}$$

The entropy is conserved in this ideal hydrodynamic expansion, and it follows that the entropy content per unit of rapidity is a constant of the motion, independent of the equation of state. Because one identifies the final entropy at low temperature with the number of produced pions,[9] this has the important implication that the predicted pion multiplicity does not depend upon the details of the expansion but only on the initial conditions imposed for the expansion.[f9]

The initial conditions (3.1) to be imposed on Eqn. (3.2) or (3.4) imply[13] an initial energy density of a few GeV/f^3 (the same order of magnitude encountered in the e^+e^- annihilation example.) To go further requires the equation of state. At high temperature, one should expect[14] an ideal quark-gluon plasma. At low temperature one must have an ideal, dilute pion gas. Thus an appropriate way of writing the equation of state is

---

f9) Here we neglect during the expansion entropy production associated with heat conductivity and viscosity. The criterion justifying such neglect is that the mean free path of the fluid quanta be small compared to the characteristic scale of the problem. Rough estimates[12] give some cause for optimism.

$$p = \frac{\pi^2}{90}\, n(T)T^4 \tag{3.6}$$

where n(T), the effective number of Bose degrees of freedom in the equivalent ideal fluid, is in the limiting cases[f10]

$$n(T) \to \begin{cases} 42\pm 6 & T >> 200\text{MeV} \\ 3 & T << 200\text{MeV} \end{cases} \tag{3.7}$$

n(T) is continuous and, if $T^{\mu}_{\mu} \geq 0$, it is a monotone increasing function of T.

The transition is generally estimated[15] to occur at a temperature $T \sim 200 \pm 50$ MeV (where the mean energy per quantum is $\sim 2T = 400$ MeV), and the evidence from lattice Monte-Carlo calculations of the thermodynamic functions is that the transition is quite abrupt. The growth of n(T) across the transition is sizeable. This is important; for our (quite uncertain) estimate of initial energy density the system finds itself initially in the quark-gluon phase, but only at a temperature $\gtrsim 200$ MeV. During the longitudinal-expansion stage of the evolution (which for central uranium-uranium collisions might last a time $c t \sim 5$-10f.), the system goes through the transitions and ends up as very dense pionic matter. Thereafter the expansion is three-dimensional. The fluid should rapidly cool and become the cloud of asymptotic produced pions (cf. Fig. 8).

---

f10) The uncertainty for large T has to do with whether to include strange quarks in the fluid.

The transverse motion of the fluid is interesting.[16] As shown in Fig. 9, a rarefaction front moves inward from the boundary at the local sound velocity (typically $\lesssim 3^{-1/2}c$. Within the central on-axis region there is pure longitudinal homogenous expansion. At transverse distances beyond the rarefaction front the fluid expands, cools more rapidly, and soon moves outward at the speed of light. (As pointed out to me by W. Czyz, the initial condition for the transverse motion of the fluid at the midplane is essentially the same as imposed by Landau[9] in the original hydrodynamic model.) The sound velocity can be expected to vary as one goes through the transition region from quark- gluon plasma to pionic matter, so that there may be shock waves in this region as well.[17]

With our estimate of initial energy density and equation of state, we can infer roughly the number of pions which are produced. For a central U-U collision, it is $\sim 10^3$ per unit of rapidity, with a large uncertainty (at least a factor 3) coming from our uncertainty in the appropriate transverse size-scale d_0 to be used.

In summary, the main lesson we have learned, beyond becoming familiarized with the geometry of the hydrodynamic evolution, is that we may naturally (and conservatively) expect that quark-gluon plasma is initially formed in a central ion-ion collision in the central rapidity region, but that is not so hot (200-300 MeV?). Much higher energy densities are needed to significantly raise the temperature. This may happen occasionally owing to the large (KNO) fluctuations in energy deposition seen in nucleon-nucleon collisions. And one must keep in mind that the boost-invariant boundary conditions might be wrong. In nucleon-nucleon collisions, the Landau boundary conditions of total equilibration of the incident projectiles' energy and momentum at the moment of impact does lead to a final distribution of hadrons similar to what is seen. However, given that leading baryons seem to exist even for central nucleon-nucleus collisions, this argues that in all cases, including nucleus-nucleus collisions, the baryon-number is retained in the outgoing nuclear pancakes. Thus at least the valence quarks are not equilibrated at impact.[18] But what about the valence gluons? For them, the case is less clear.[19]

IV. HADRON-HADRON COLLISIONS

Can these space-time descriptions say much about the nature of hadron-hadron collisions? There are at least a few remarks which can be made:

1. Hadron-hadron collisions are probably not the best arena for <u>fundamental</u> QCD tests. It is true that perturbative QCD coupled with parton-model concepts <u>has</u> done extremely well in accounting for a great deal of data on massive dilepton production in hadron-hadron collisions.[20] Nevertheless the remaining open issues regarding higher order effects and the relevance of initial-state interactions would allow considerable tolerance in the comparison of experiment with theory before calling into question the issue of QCD as the correct underlying theory of strong interactions.
2. The space-time geography of hadron-hadron collisions must be basically similar to what we have in e^+e^- collisions. For normal events, the distance scales associated with "transition" and

"asymptotic region" should be similar to e^+e^- annihilation. But the region near the light cone is more problematic; it is not clear-given the size and complexity of incident projectiles - that perturbative QCD can be applied. It is likewise unclear that hydrodynamic ideas can be applied. If they could, the evolution from $t\sim 0.1-1f$ would parallel the nucleus-nucleus evolution from $t\sim 1-10f$.

3. Another basic geographical fact is the existence of non-central collisions. In nucleus-nucleus collisions it is evident that the multiplicity of produced particles is proportional to the overlap of the colliding nuclear pancakes. Thus the natural distribution of impact parameters, along with the strong correlation of multiplicity with impact parameter, leads to a dispersion in multiplicity proportional to the mean and to (at least approximate) KNO scaling.[21,22] This also leads to strong long-range correlations of the type seen[23] at the SPPS: if the multiplicity of left-movers is high, so also will be the multiplicity of right-movers. Since KNO-type behavior is so obvious from simple geometry, why should there be any "surprise" that it works so well in the high energy $p\bar{p}$ collisions? The reason is that in the heyday of multiperipheral, short-range correlation, and/or "naive" perton models[24] the impact-parameter dependence of particle-production was expected to be very weak, because the "wee" components of the projectiles responsible for particle production were linked to the valence components via a long multiperipheral chain or ladder. While some fraction of the particle production - in particular what is seen at the lower energies - may behave in this way, the SPPS data

demonstrates the need to go further. Use of the old naivete may now be an attractive option.

On the other hand, there are other rationales for the observed KNO behavior. One[25] comes from Reggeon-calculus theory of the Pomeranchuk singularity. In this kind of model, which is consistent with a large amount of diffractive phenomena,[26] the multiplicity depends (linearly) upon the number of pomerons in the forward elastic amplitude which are "cut" in the process of obtaining the inelastic production cross-section. The mean number of cut Pomerons is small, but greater than one. Therefore event-to-event fluctuations in the number of cut Pomerons leads to broad fluctuations in the number of produced hadrons.

A similar argument can be made with regard to the QCD jets. As we saw in the discussion of e^+e^- annihilation, there the mean number of such jets is small and each one carries with it a sizeable hadronic multiplicity. Thus fluctuations in the number of such jets may also give broad fluctuations in multiplicity. This mechanism may be especially important in e^+e^- annihilation, where KNO behavior also seems to be seen, but where the previous mechanisms are not applicable. In this case one might <u>not</u> expect such strong left mover- right mover multiplicity correlations as seen in SPPS data. This in fact seems to be the experimental trend.[27] In the hadron-hadron case this QCD jet mechanism can be expected to be different, because all quarks <u>and</u> gluons in the projectiles are available to fragment and/or radiate. At present attempts are being made[28] to interpret the large KNO multiplicity-energy fluctuations in terms of this mechanism, but I have not seen the details.

4. The origin of KNO scaling and large E_T fluctuations as seen by the NA5 collaboration and others, and especially at the SPPS collider, is as yet unclear. It is remarkable that, at the 0.1mb cross-section level, there exist[29] hadron final states with ~ 6 times the nominal mean transverse energy of ~ 5-6 GeV (into the central detection system; $40° < \Theta < 140°; \Delta y \simeq 2$). This energy presumably radiates outward in straight lines from an initial collision volume $< 1F^3$, implying an initial energy density >30-40 GeV/f^3 and an asymptotic time for the final pion gas to form of $\gtrsim 5$-10f. At least this class of fluctuating events may well have to be described in hydrodynamic (or at least statistical) terms.[30] Without understanding the underlying origin of the large E_T, it may still be reasonable and useful to evolve the 3-dimensional, spherically symmetric expansion from a starting time ~ 1-2f. after the impact. It may be expected that the tendency will be to convert the thermal energy into the kinetic energy of the outward hydrodynamic flow.[31] This may lead, in the extreme limit, to a distorted P_T distribution, with enhancements at high $p_\perp$.

5. The issue of high-P_T jet production in hadron-hadron collisions is a rather murky one, especially with regard to fixed-target observations[32] at CERN and FNAL. Because jet searches based upon a transverse-energy trigger are burdened with the very large background of high-multiplicity, low-p_T events, it is a matter of some controversy whether a true jet signal, identifiably distinct from the tail of the background distribution, does exist. I sense some anxiety in the air with regard to the future of jet studies at hadron colliders, especially given the KNO multiplicity fluctuations

seen at the SP$\bar{P}$S. Such anxiety is to me unfounded for two reasons. First of all, were skepticism somehow to win out and QCD jets don't exist (I doubt that will happen), then it will be easier to find W and Z. It only takes CVC plus the e^+e^- data plus an extrapolation in energy-scale of a factor two to conclude that W and Z do decay into jets. But the second reason is simply that good evidence for jet production at or above the QCD level already exists from the ISR data.[33] The recent UA1 correlation studies are likewise supportive. Furthermore the observation[29] by UA1 of single-particle inclusive spectra out to a p_T of 8 GeV implies such a big high-p_T cross-section that the jets cannot be far behind. A big problem with the fixed-target data is that the $p_T \lesssim 5$GeV of the systems studied is at the threshold of observability;[34] at the ISR "jets" of $p_T \lesssim 10$Gev are markedly cleaner both in morphology and in the way their production scales with energy.

6. There are very interesting issues having to do with high-p_T exclusive processes, best studied at low energy. Elastic scattering at, say, 90° cms angle is viewed by perturbative-QCD theorists[35] as occurring when the projectiles are in a very simple configuration, namely with only valence quarks and no gluons or other bag-gage, and with the spatial separation of the quarks very small. In other words, the participating hadrons are essentially pointlike during the scattering process. To me these seem to be strong assumptions, not obviously justified from QCD first-principles. Much more interesting than my opinion is an experimental test proposed by Mueller:[36] if one does large-angle quasi-elastic scattering in a <u>nucleus</u>, neither the projectile nor the secondary nucleons should be

significantly absorbed in the nuclear matter, and the cross-section should be proportional to A. Mueller has looked into the experimental opportunities at AGS or PS energies and believes the measurement is feasable.

V. CONCLUDING REMARKS

Nowadays no description of collision phenomena can ignore the QCD implications: is the phenomenology (at worst) consistent with general QCD expectations and (at best) in quantitative agreement with sharp QCD predictions? Independent theoretical approaches which do not make any contact with QCD risk being widely ignored.

A challenge to students of hadron-initiated processes is to find any sharp QCD test. Massive dilepton production may be the best and works quite well. High $-P_T$ jet production is not in very good repute, but we may anticipate that before long it will come into its own at the SPPS. The single-particle yields are already encouraging.

There is the intrinsic problem of defining jets in these processes which makes quantitative comparisons difficult. A good meeting ground between theory and experiment might be in precise measurement of inclusive low-mass, high P_T, pairs or triples of hadrons. After all, in an unbiased jet, the fastest three particles should carry most of the jet momentum. (This can be determined from e^+e^- data.) Massive dihadron production[37] is another area where measurements are precisely defined and where theory has a chance of making contact.

The soft collisions may remain more phenomenological for some time to come. The possible connection of KNO scaling to impact-parameter dependence should be explored, as well as alternative views such as multiple production of marginally perturbative QCD jets, or of multipomeron effects. Comparisons with the corresponding phenomena in e^+e^- annihilation will be important.

We must also admit the possibility that, after all, the QCD picture is wrong or perhaps incomplete. The ideas which exist[38] on slightly broken QCD invite searches for massive fractionally charged objects which might accrete nuclear matter. It need hardly be said that every effort should be made to look for such objects.

In this talk we concentrated on the space-time development of high energy collisions. In all cases, the evolution is near the light-cone, implying that low-momentum, large angle secondaries emerge early, and that energetic, leading secondaries emerge late, on a time scale proportional to their laboratory energy.

There is always a region of space time (which we denoted "transition") where "hadronization" occurs, with initial energy densities (in the local rest frame) of at least a few GeV/fermi^3. Understanding this region largely in terms of perturbative QCD probably does not work.

It is possible to experimentally study the transition region. For example, the hadronization region in Fig. 4b is a propagating system containing a leading quark. Its existence and size is probably most directly studied in energetic muon-nucleus scattering (or, even better, by colliding stored electrons with stored heavy ions). Does the struck quark and its associated "hadronization cloud" interact like a hadron? The bare quark would not be expected to do so.

The evolution of the transition, or hadronization region might be so complicated that a statistical or even hydrodynamical approach is appropriate. In this context relativistic nucleus-nucleus collisions provide an arena for exploring such a picture of the collision process; the conditions for a hydrodynamic description are most justifiable there. In the picture we discussed, quark-gluon plasma is probably formed in such extreme-relativistic collisions, but with relatively low temperature. The hydrodynamic evolution might last for a time ct of order 5-10f., with the predicted number of produced hadrons determined by the initial conditions imposed at the starting time ($\sim$1f?,$\ll$1f?) for hydrodynamic flow. The multiplicity is, for heavy nuclei, of course very large.

The major problems in exploring relativistic nucleus-nucleus collisions - other than getting the relativistic ion beams themselves - are in experimental signatures. One would like to know that quark-gluon plasma was really formed and would like to learn something about its equation of state. Several signatures have been discussed,[39] such as enhanced γ and dilepton yield, enhanced K/π ratio, "patchy" multiplicity distributions indicating hydrodynamic instabilities ("flares" or "volcanoes"), and pion or photon intensity-correlation measurements to determine the size of the radiating system. In addition there is the off-chance that new metastable structures of high density might be produced - although that is very speculative[40] and hardly possible to evaluate seriously. Other than the metastable structures, none of the above signatures is especially direct and unambiguous. What would turn out to be the most interesting phenomenon is probably "none of the above."

The relevance of quark-gluon plasma production in pp or $p\bar{p}$ collisions is unclear. However, despite our lack of basic understanding, the observed phenomenon of large multiplicity and transverse-energy fluctuations makes some kind of statistical and/or hydrodynamic treatment almost mandatory, at least for the later stages of the evolution, where one pictures the system as a hot, dense, not-too-thick spherical shell expanding outward at the speed of light. But there is such poor understanding of what is going on that there follows only one safe conclusion: the most important regions of space-time are the collision halls at the SPPS, the e^+e^- machines and the fixed target experiments. Even the most dedicated adherent of QCD must admit that there is still much to understand about high energy collisions, and that the guidance of experiments will be essential in attaining that understanding.

VI. ACKNOWLEDGMENTS

The author thanks his colleagues at Fermilab and, in particular, G. Baym, A. Buras, W. Czyz, E. Friedlander, S. Gupta, T. D. Lee, L. McLerran, A. Mueller, E. Weiner, and A. White for enlightenment, stimulation and criticism.

FIGURE CAPTIONS

1. Region of space-time in which QCD pertubation theory is certainly valid: (a) cms frame, (b) boosted frame, (c) composite of all boosted frames.

2. Region of space-time in which asymptotic hadrons may be detected: (a) cms frame, (b) boosted frame.

3. Summary of important space-time regions in 2-jet e^+e^- annihilation; (a) longitudinal evolution, (c) fixed-time picture.

4. Summary of important space-time regions in 3-jet e^+e^- annihilation: (a) longitudinal evolution, (b) fixed-time picture.

5. Dominant Feynman graphs in the calculation of Bassetto et. al. for the inclusive soft gluon spectrum.

6. "Target diagram" of multijet structure in e^+e^- annihilation, as predicted by the perturbative QCD calculation of Bassetto et. al. The view is along the axis of the "spectator" quark.

7. Geometry of a nucleus-nucleus central collision a short time after the moment of impact.

8. Equation of state for the produced plasma, showing estimates of the initial and final conditions for the one-dimensional flow.

9. Details of the transverse motion at impact parameters of order the nuclear radius.

REFERENCES

[1]For reviews, cf. A. Mueller, Physics Repts. 73, 237 (1981); G. Altarelli, Physics Repts. 81, 1 (1982).

[2]E. Feinberg and I. Ya. Pomeranchuk, Nuovo Cimento Suppl. 3, Ser. 10, 652 (1956).

[3]See J. D. Bjorken, Proceedings of the 1979 SLAC Summer Institute on Particle Physics, ed. A. Mosher, for some details of the apparatus. There may be some problems, not considered there, with regard to spontaneous pair creation by the strong electromagnetic fields. I thank J. Hamilton for some stimulating conversations on practicalities.

[4]K. Konishi, A. Ukawa, and G. Veneziano, Nucl. Phys. B157, 45 (1979); Phys. Letts. 78B, 243 (1978); Phys. Letts. 80B, 259 (1979).

[5]A. Bassetto, M. Ciafaloni, G. Marchesini, and A. Mueller, University of Florence preprint 82/11.

[6]S. Gupta and H. Quinn, Phys. Rev. D25, 838 (1982).

[7]For example, the Lund model; cf. B. Andersson, these proceedings; also B. Andersson, G. Gustafson, and T. Sjöstrand, Nucl. Phys. B197, 45 (1982).

[8]A more optimistic viewpoint prevails for the Lund model, cf. ref. 7.

[9]"Collected papers of L. D. Landau," ed. D. Ter Haar, Gordon and Breach (N.Y.), 1965.

[10]P. Carruthers and Minh Duong Van, Phys. Letters 4B, 597 (1972); Phys. Rev. D8, 7 (1973). A rather extensive treatment is given by F. Cooper, G. Frye, and E. Schonberg, Phys. Rev. D11, 192 (1974).

[11]See, for example, E. Shuryak, Phys. Reports 61, 71 (1979). Also, Proceedings of the Bielefeld Workshop on Quark Matter Formation in Heavy Ion Collisions, May 1982, World Scientific Publishing Co. (Singapore), to be published.

[12]J. Bjorken, in preparation.

[13]This conclusion has been found by others, in particular cf. A. Mueller, Proceedings of the 1981 Isabelle Summer Workshop, ed. H. Gordon, BNL 51443, p. 636.

[14]J. Collins and M. Perry, Phys. Rev. Letters 34, 1353 (1975).

[15]For example, L. McLerran and B. Svetitsky, Physics Letters 98B, 195 (1981); Phys. Rev. D24, 450 (1981); J. Engels, F. Karsch, I. Montvay, and H. Satz, Phys. Letters 101B, 89 (1981); K. Kajantie, C. Montonen, and H. Pietarinen, Zeit. Phys. C9, 253 (1981); J. Kuti, J. Polonyi, and K. Szlachanyi, Phys. Letters 98B, 199 (1981); I. Montvay and H. Pietarinen, University of Helsinki preprint HU-TFT-82-8; DESY 81-077 (1981); D. Boal, J. Schachter, and R. Woloshyn, TRIUMF preprint TRI-PP- 82-14.

[16]This has been investigated by G. Baym and W. Czyz (private communication).

[17]G. Baym (private communication).

[18]R. Anishetty, P. Koehler, and L. McLerran, Phys. Rev. D22, 2793 (1980).

[19]The importance of distinguishing quark and gluon interactions in nuclear matter has been emphasized by A. Krzywicki, J. Engels, B. Petersson, and U. Sukhatme, Phys. Lett. 85B, 407 (1979).

[20]See for example, A. Buras, Proceedings of the 1981 International Symposium on Lepton and Photon Interactions at High Energies, ed. W. Pfeil. (Univ. Bonn, 1981).

[21]Z. Koba, H. Nielsen, and P. Olesen, Nucl. Phys. B40, 317 (1972).

[22]Z. Koba and A. Buras, Lett. Nuovo Cimento 6, 629 (1973); A. Buras, J. Dethlefsen, and Z. Koba, Acta Physica Polonica B5, 473 (1974) and also T. T. Chou and C. N. Yang, SUNY (Stony Brook) preprint (1982).

[23]UA5 Collaboration: K. Alpgård et.al., Phys. Letters 107B, 310, 315 (1981). See also N. Yamadagni, these proceedings.

[24]See for example: J. Kogut and L. Susskind, Phys. Repts 8, 75 (1973).

[25]L. Caneschi and R. Jengo, Nucl. Phys. B89, 19 (1975).

[26]For a summary, see A. White, Fermilab preprint FERMILAB-CONF-82/16/THY.

[27]TASSO Collaboration, R. Brandelik et.al., Phys. Lett. 100B, 357 (1981). I do not know of an e^+e^- analysis parallel to the UA5 analysis. This would be nice to see.

[28]G. Fox and R. Kelly, Caltech preprint CALT68-890 (1982).

[29]M. Pimiä, these proceedings.

[30]An initial attempt in this direction is given by J. Kapusta, CERN preprint, to be published in the Proceedings of the 17th Rencontre de Moriond, Les Arcs, March 1982, ed. Tran Thanh Van.

[31]This has been worked out for a conjectured hydrodynamic description of e^+e^- annihilation. See in particular F. Cooper et.al., Ref. 10, for the formalism, and Ref. 30 for a short discussion.

[32]C. DeMarzo et.al.,, Physics Letters 112B, 173 (1982).

[33]P. Darriulat, Ann. Rev. Nucl. Sci. 30, 159 (1980).

[34]J. D. Bjorken, Phys. Rev. D8, 4098 (1973).

[35]G. Farrar and S. Jackson, Phys. Rev. Letts. 43, 246 (1979); S. Brodsky and G. Lepage, Phys. Rev. Letters 43, 545 (1979); Phys. Letters 87B, 359 (1979); A. Duncan and A. Mueller, Phys. Rev. D21, 1636 (1980).

[36]A. Mueller, Columbia University preprint CU-TP-232, to be published in the proceedings of the 17th Moriond Conference (op.cit.; reference 15).

[37]J. H. Cobb et.al., Phys. Rev. Letts. 40, 1420 (1978); A. Angelis et.al., Nucl. Physics B (to be published); Fermilab proposal P-711 (D. Levinthal, spokesperson).

[38]A. DeRujula, R. Giles, and R. Jaffe, Phys. Rev. D17, 285 (1978).

[39]See references 11 and 12; also, T. D. Lee, Columbia University preprint CU-TP-226.

[40]For an example, cf. J. Bjorken and L. McLerran, Phys. Rev. D20, 2353 (1979).

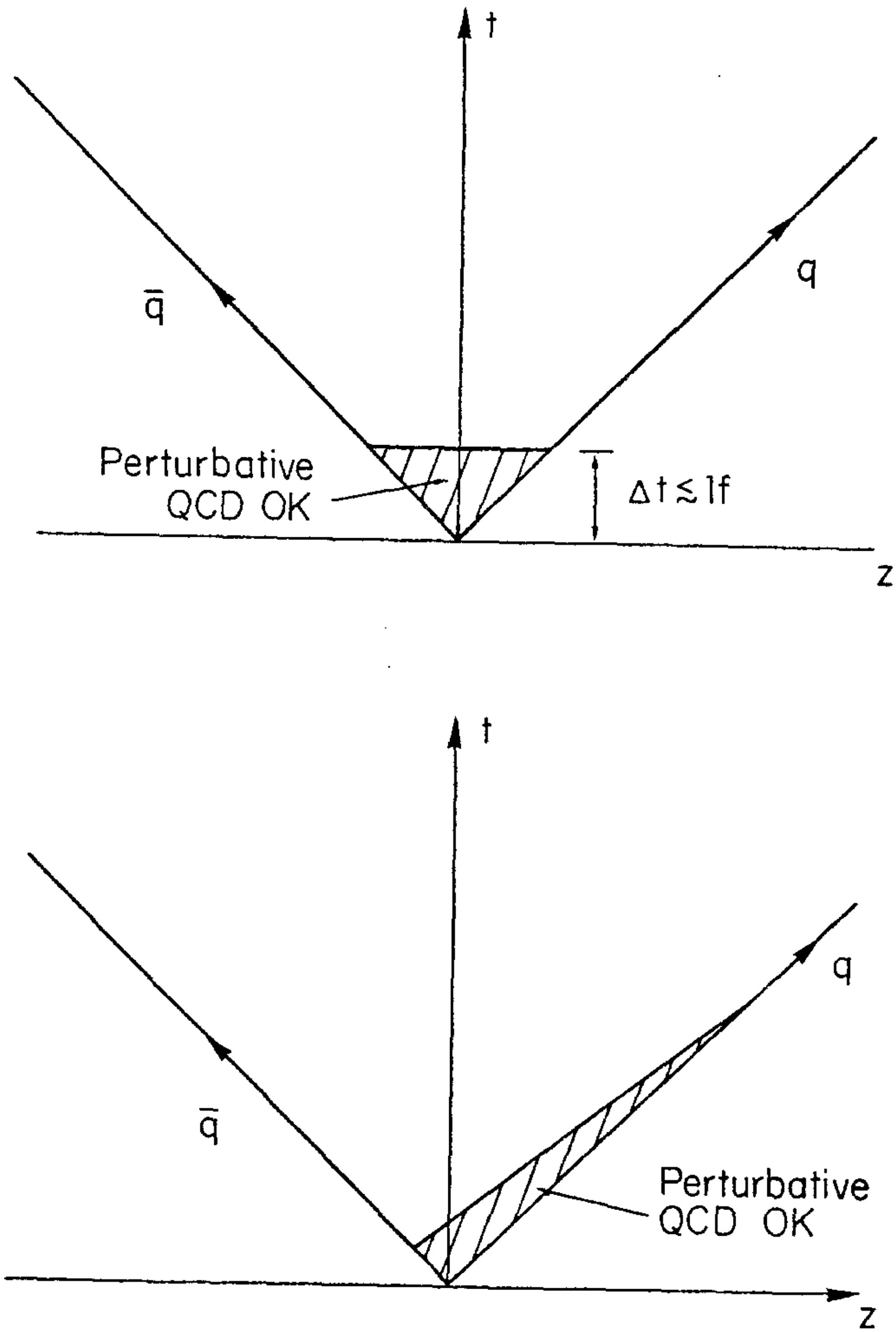


Fig. 1

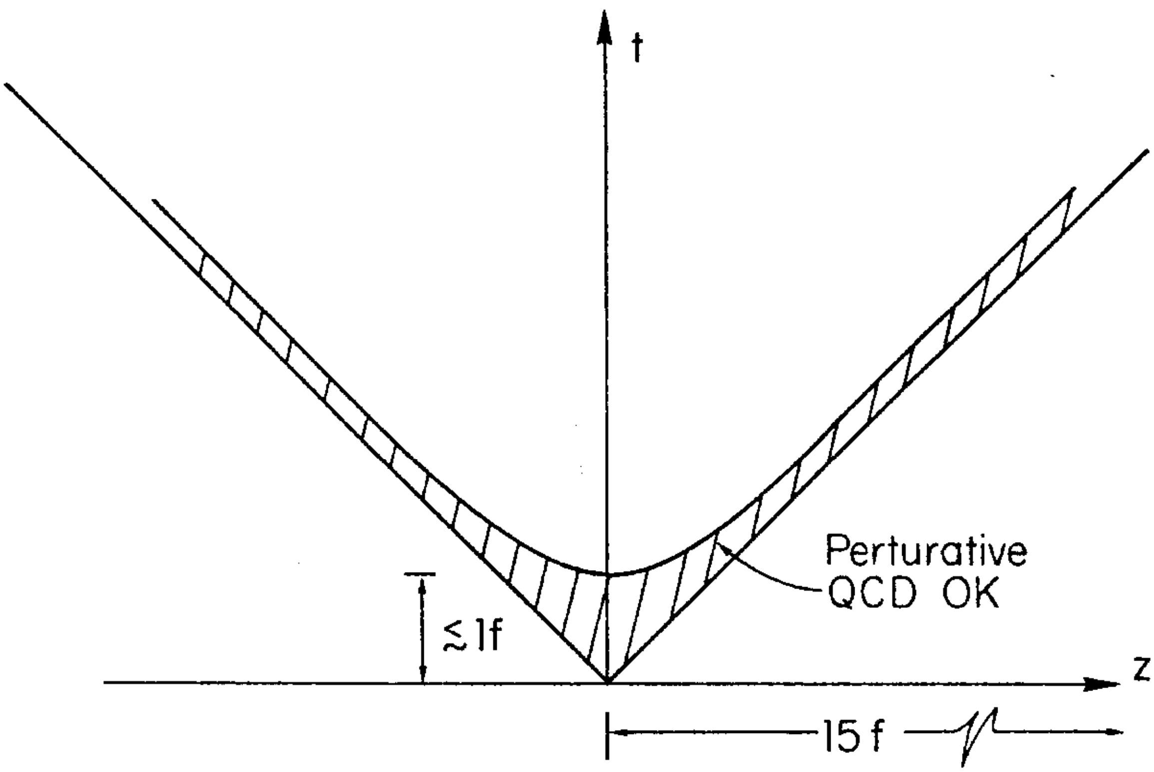


Fig. 2

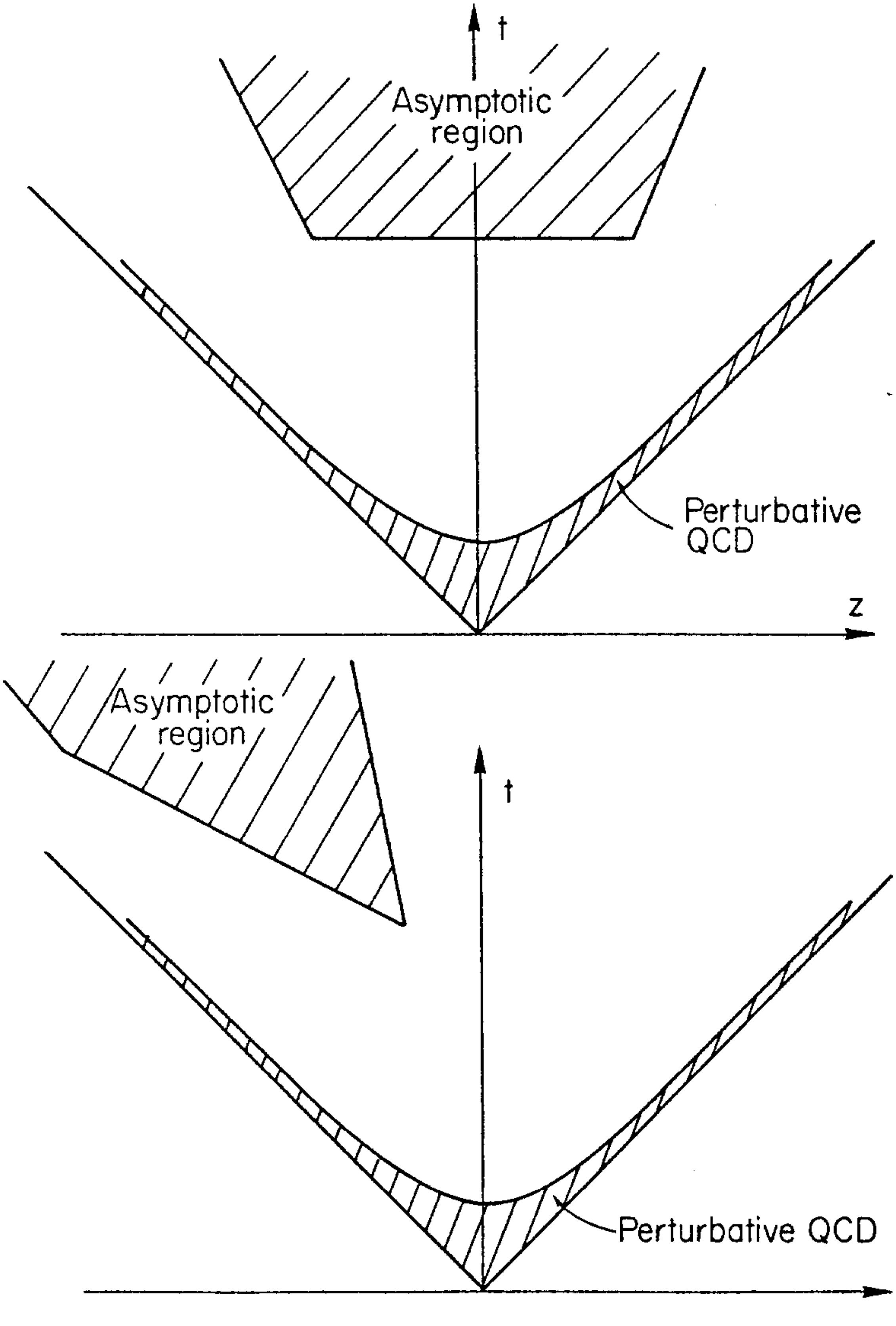

t
Asymptotic region
Perturbative QCD
z
Asymptotic region
t
Perturbative QCD

Fig. 3

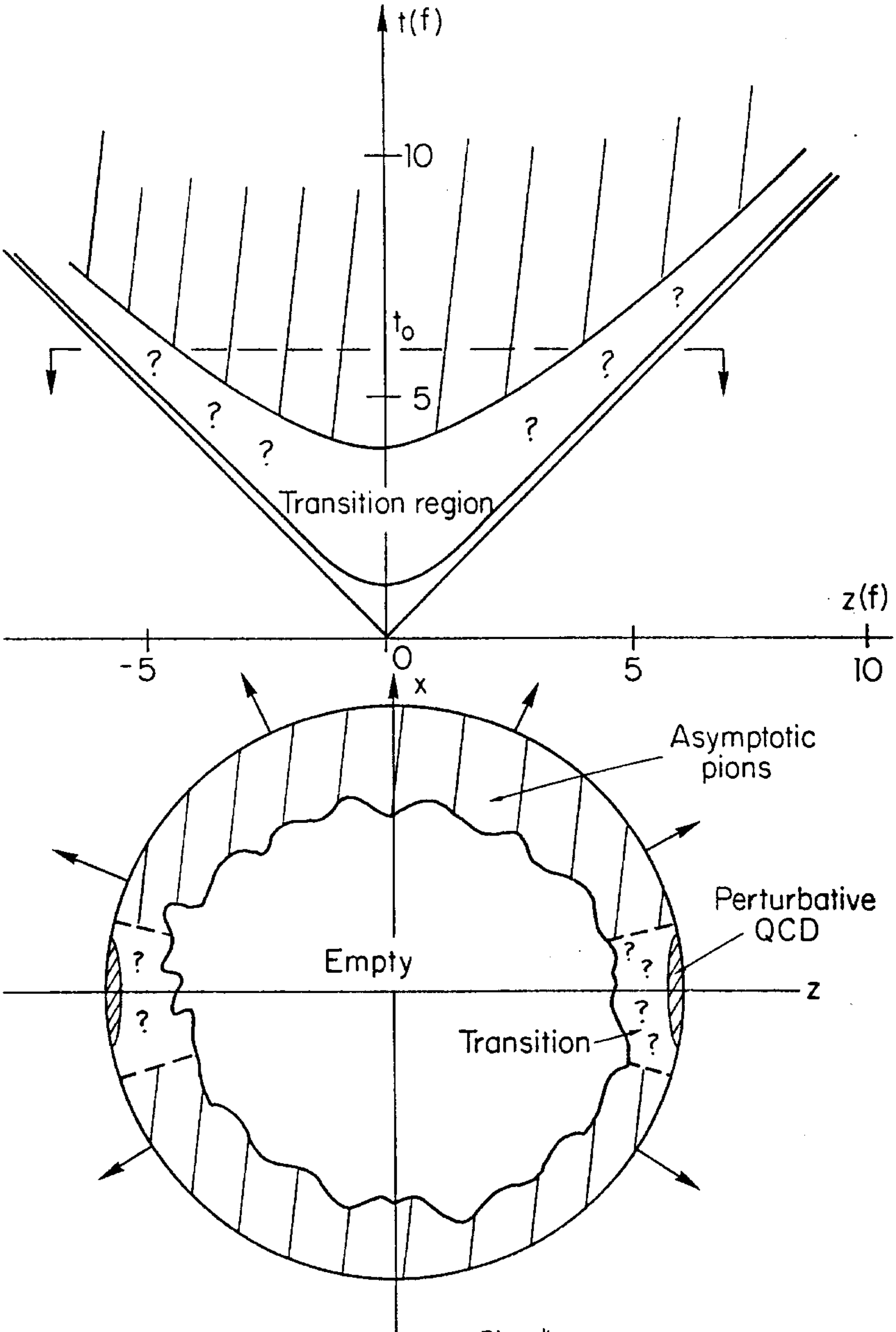
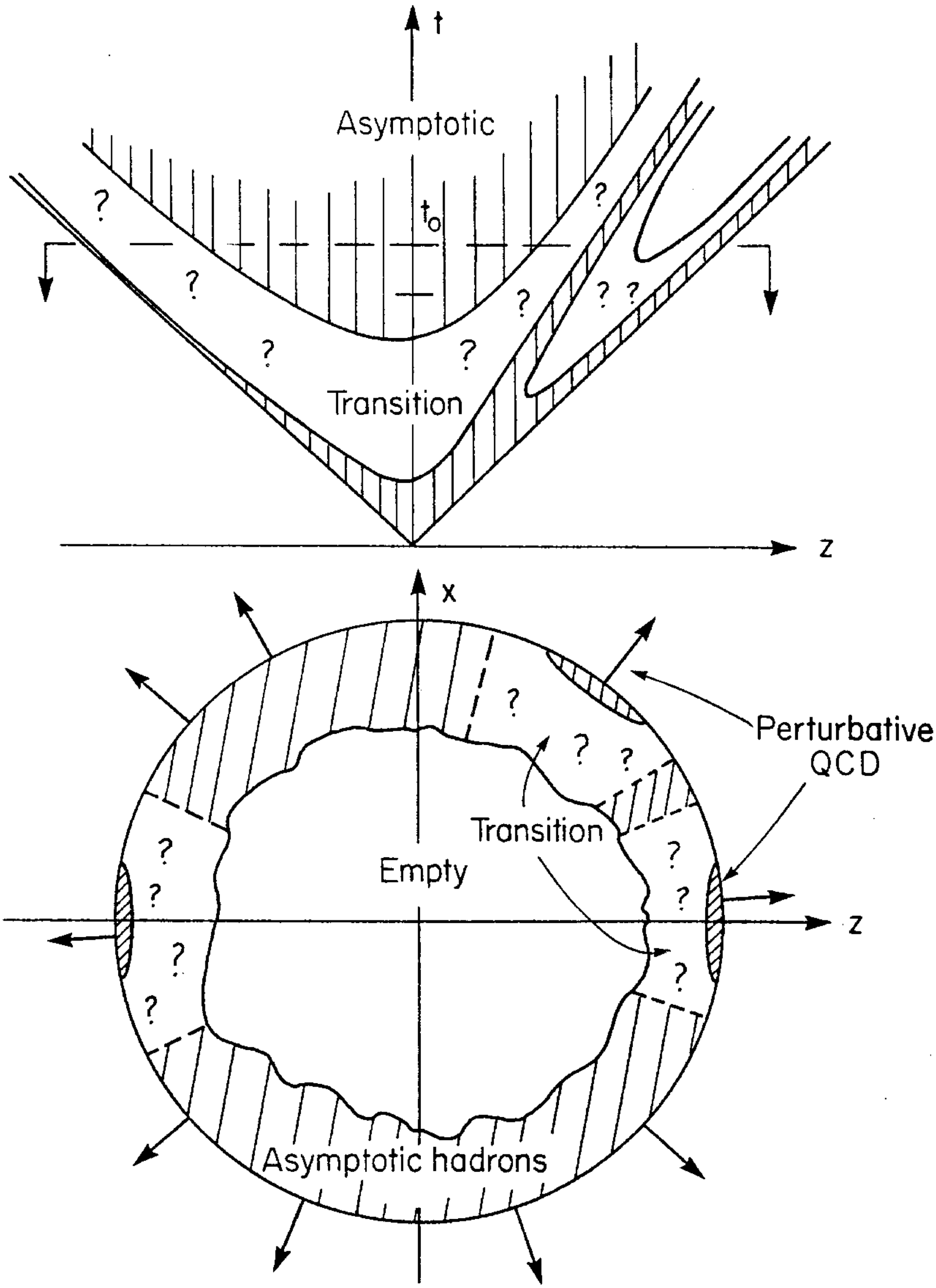

t(f)
10
t_0
5
Transition region
z(f)
-5
0
5
10
x
Asymptotic pions
Perturbative QCD
Empty
z
Transition
?

Fig. 4



t
Asymptotic
?
t_0
?
?
?
?
?
? ?
Transition
z
x
?
?
?
Perturbative
QCD
Transition
Empty
?
?
?
?
?
?
?
?
z
Asymptotic hadrons

Fig. 5

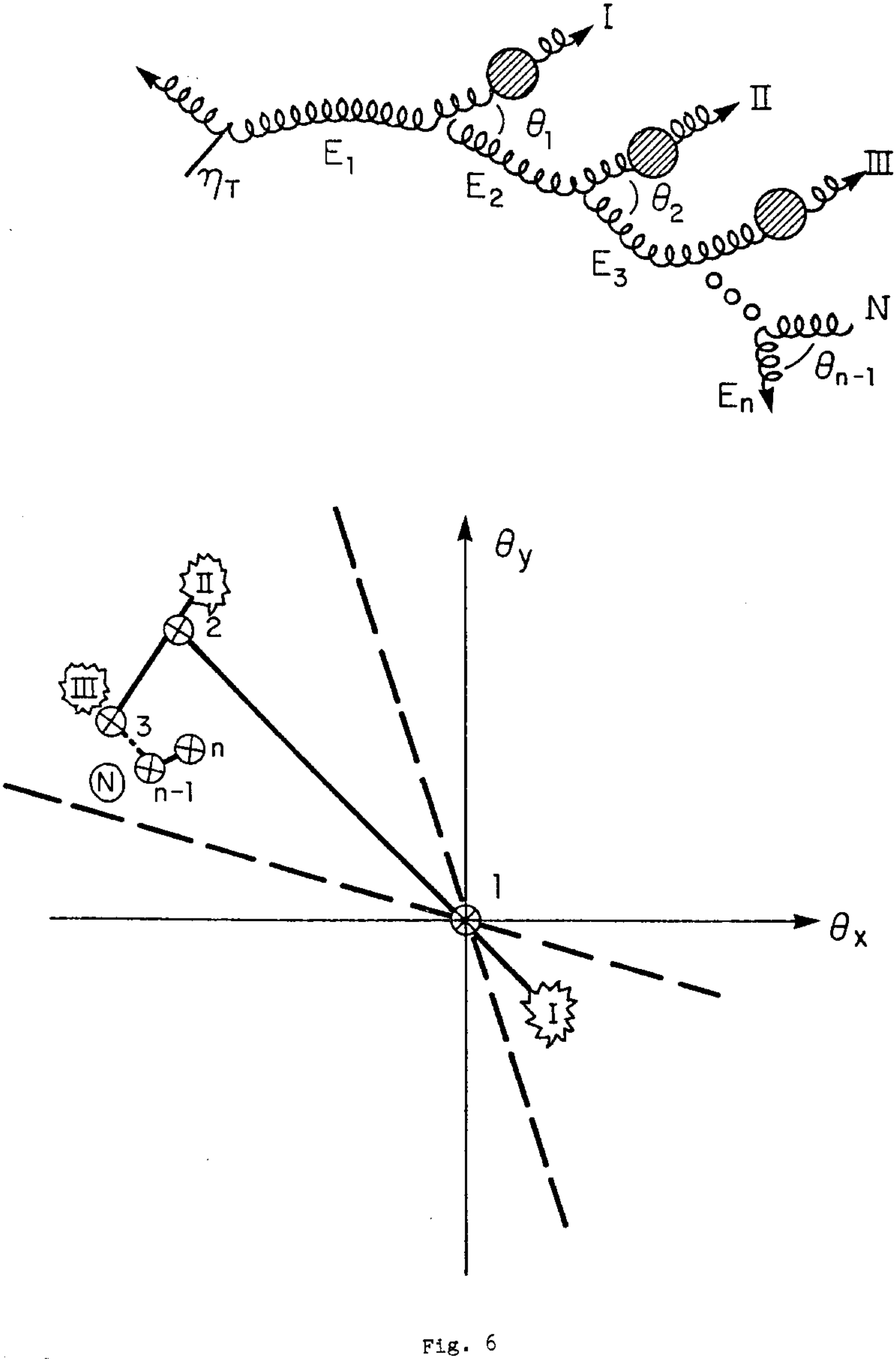

I
II
III
θ_1
θ_2
η_T
E_1
E_2
E_3
N
θ_{n-1}
E_n
θ_y
θ_x
II
2
III
3
n
N
n-1
1
I

Fig. 6

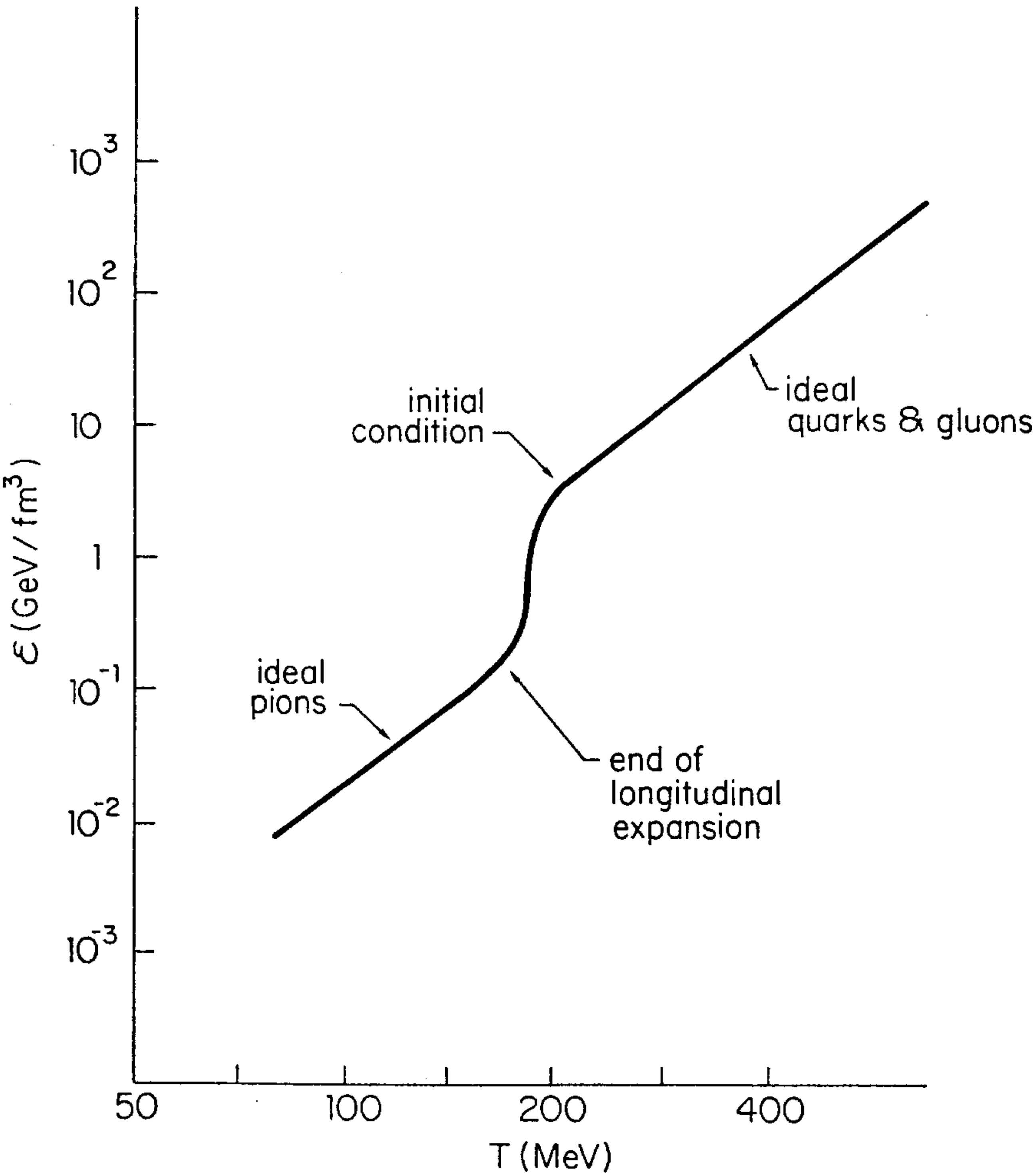

initial
condition
ideal
quarks & gluons
ideal
pions
end of
longitudinal
expansion
10^3
10^2
10
1
10^-1
10^-2
10^-3
ε (GeV/fm^3)
50
100
200
400
T (MeV)

Fig. 7

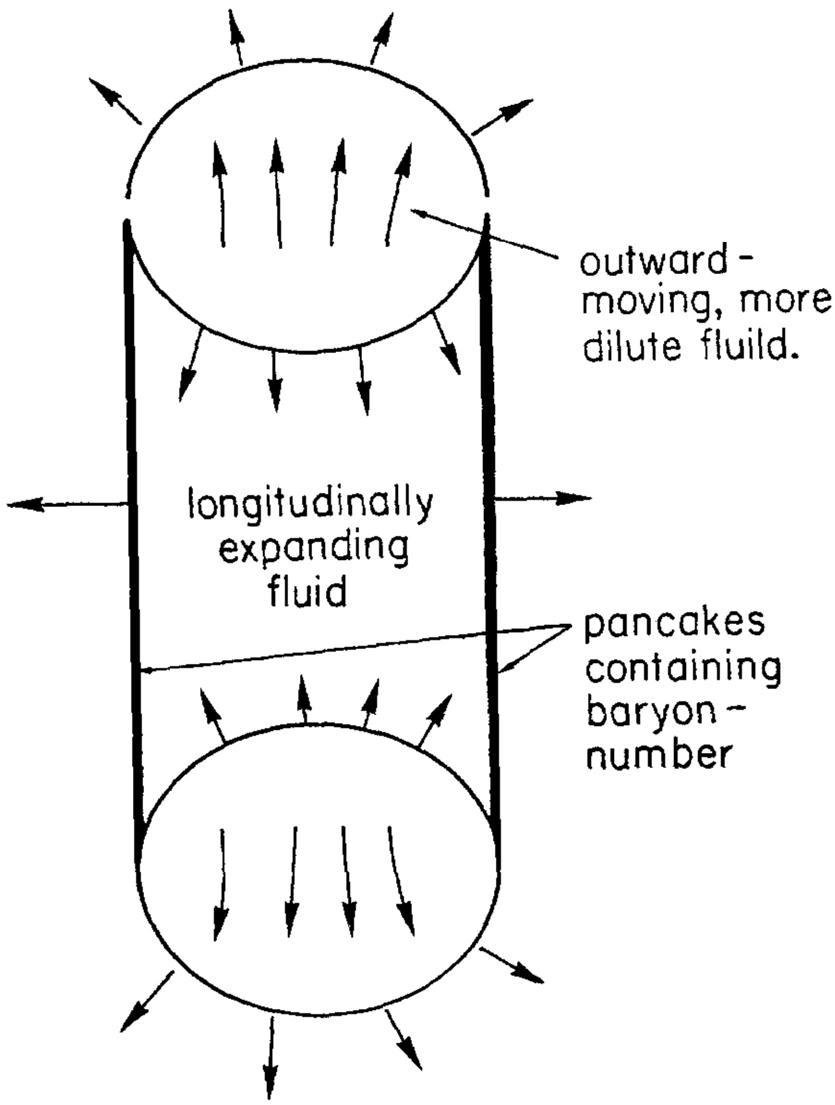

outward-
moving, more
dilute fluild.
longitudinally
expanding
fluid
pancakes
containing
baryon-
number

Fig. 8

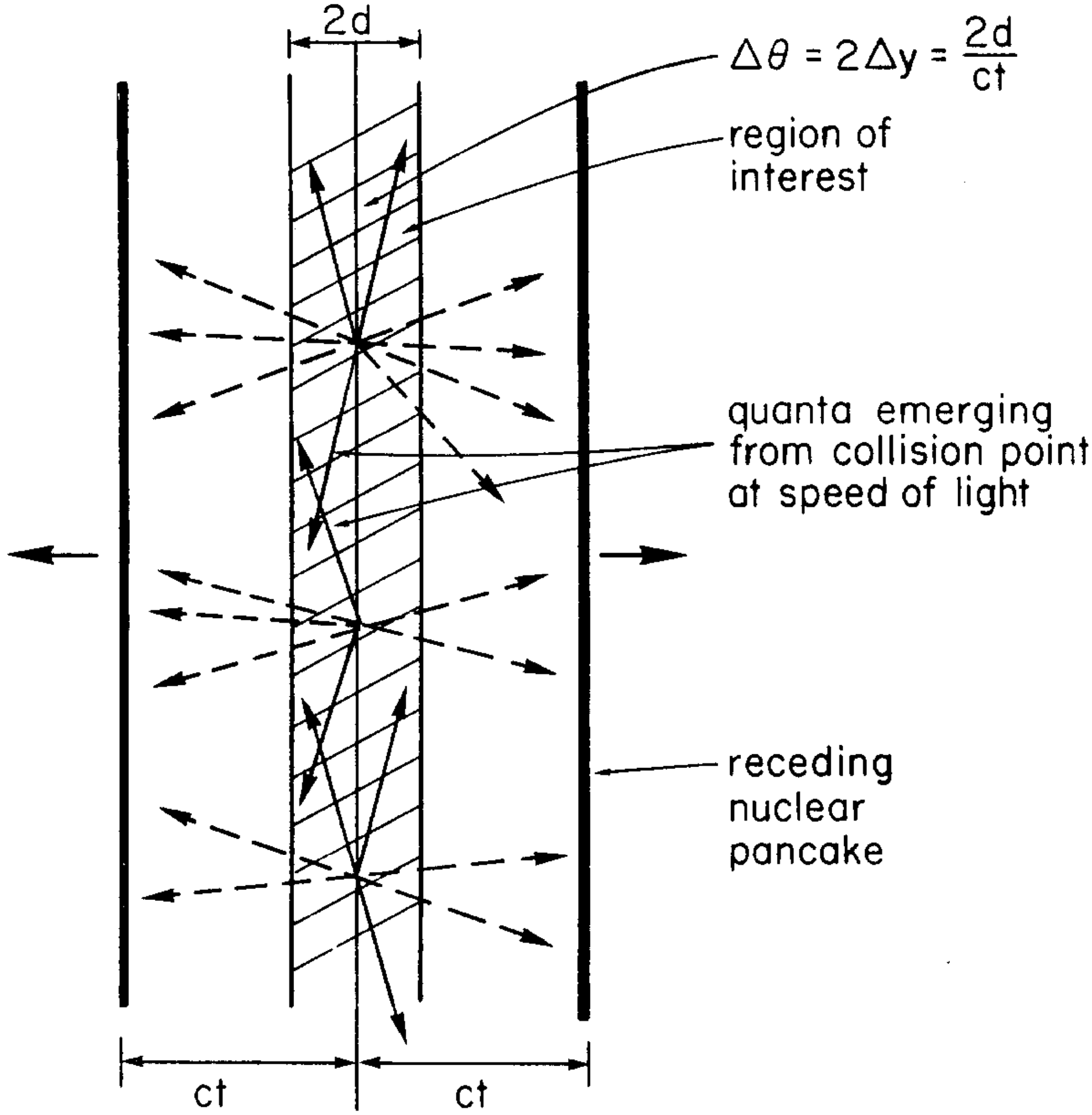

2d
Δθ = 2Δy = 2d/ct
region of interest
quanta emerging from collision point at speed of light
receding nuclear pancake
ct
ct

Fig. 9