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HYPERCHARGE GENERATORS IN SU(7) GRAND UNIFICATION MODELS

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ABSTRACT

We present a thorough analysis of U(1) generators in all of the nine possible symmetry breaking patterns that reduce SU(7) down to $SU_c(3) \times SU(2) \times U(1)$. There is one allowed representation that contains three generations of leptons and at least two generations of quarks, and satisfy anomaly freedom, reality under $SU_c(3)$ and $U_{em}(1)$, and complexity under $SU_c(3) \times SU(2) \times U(1)$. In addition, three other representations, two having only one and two of the three lepton-quark generations complex with respect to $SU_c(3) \times SU(2) \times U(1)$, and the other having fractionally charged color singlets in pair embedded in the fundamental representation [7], are also analyzed for completeness.

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I. INTRODUCTION

Recently there has been considerable interest in grand unified scheme based on SU(7).[1,2] This is partly motivated by the persistently elusive nature of t-quark in experiments[3] and partly by the possibility that the recent observation[4] of fractional charge + (1/3)e might be the evidence of fractionally charged color singlet particles.[5] Also one may need to require additional intermediate natural mass scale as it has been discussed in connection with the possible neutrino oscillations.[6]

In this paper, we consider all classes of SU(7) which are anomaly-free and complex[7] with respect to $SU_c(3) \times SU(2) \times U(1)$ and yet allowing three generations of leptons and at least two generations of quarks. We restrict ourselves to those representations containing color 1, 3 and 3^* only[8] but allow repetitions of the same irreducible representation as long as the combination does not involve any common factors in the coefficients.[9] Also, we require the representation to be real under $SU_c(3)$ and $U_{em}(1)$. Complex representations[10] can be found in gauge groups SU(N), SO(4N + 2), and E_6. Among these, SO(4N + 2) and E_6 are anomaly free. Spinor representation of $SO(4N+2) = D_{2N+1}$ with dimensionally of 2^{2N} is complex. But the charge assignment in the spinorial representation of $SU(2N+1) = A_{2N}$, i.e., the anomaly-free combination which becomes the spinor representation of D_{2N+1} into which A_{2N} can be naturally embedded, encompasses the charge assignment of D_{2N+1}. On the other hand, the fundamental representation [27] of E_6 is complex and contains no color exotics.[11] But it can not accommodate more than two generations of quark and lepton. Thus we prefer to use the anomaly-free combination of totally antisymmetric A_{2N} representations.

Despite the simplicity, the minimal SU(5) model[12] suffers from the inability of accommodating more than one quark-lepton generation in the spinor representation. SU(6) is no improvement over SU(5) as, for example, the anomaly-free combination $2[6]^* + [15]$ can entertain only one generation as in SU(5). One may hope to use a single irreducible representation. Indeed the anomaly-free and complex irreducible representation[13] occurs in SU(6) but with dimension D = 374,556 corresponding to the highest weight (0,5,0,0,4) in the Dynkin basis. The next lowest dimensional representation which is complex and anomaly-free is in SU(5) with D = 1,357,824 or (0,7,3,3). Thorough search[14] has been carried out for the range of dimension up to $D = 4 \times 10^9$ for SU(N) where N is less than 17. There are 28 anomaly-free complex representations. Such representations are only of mathematical curiosity and do not have any practical use due to awesome dimensionality. In addition, they usually contain color exotics. Even then, closer examinations reveal that the representation (0,5,0,0,4) can accommodate only one generation of quarks and leptons, but with many exotic particles. So we go to the next simplest case, i.e., the complex and anomaly-free combination of the irreducible representations in $SU(7) = A_6$.

The problem of flavor unification has been discussed before but with the requirement of the three generations usually. Such scheme assumes that the third quark doublet will be eventually discovered, a fact at the moment far from the reality. It may be that the t-quark is not what we have sought after and has totally unexpected properties and mass value. In fact, certain theoretical arguments based on the evolution of the coupling constants via renormalization equations tend to imply either the t quark is as heavy as 240 GeV or it is accompanied by still another 4th quark doublet. In short, there is no persuasive reasoning to insist on the doublet of (t,b) even theoretically. Thus we choose to require the SU(7) representation to accommodate three doublets of leptons and at least two doublets of quark with the hypercharges as given in the standard flavor gauge model.

In this paper, we give special emphasis on the various possible patterns of symmetry-breaking, all of which will eventually reduce SU(7) down to $SU_c(3) \times SU(2) \times U(1)$. But there are actually three U(1) generators and a different symmetry-breaking mode will need different combinations of the three U(1) diagonal generators for the hypercharge. Tracking down those U(1) generators and their eigenvalues is by no means a trivial task. The method we use here is to utilize the projection operators. It turns out that there are nine non-trivial routes to break SU(7) into $SU_c(3) \times SU(2) \times U(1)$ including the one going through SU(5) at the intermediate stage. We give a complete analysis of U(1) generators in all of these cases and give the structure of corresponding charge and hypercharge generators.[15] Such thorough U(1) analysis has, as far as we know, never been made before. The projection operator method is not only very economical but also enormously powerful and it can easily be extended to any group. In this paper, we take the case of spinor representation to apply the projection operator method explicitly. For other representations we will give the final results only, which can be obtained similarly.

Another non-trivial result of this paper is that there is exactly one complex and anomaly-free representation with $D \leq 110$ of SU(7) that satisfies the required number of quarks and leptons. That is: $[21] + [35^*] + [7^*]$. But it will be shown that the charge generators for this representation discussed in the literature[1] are corresponding to some particular modes of symmetry-breaking out of the possible nine. Without complete control of the U(1) eigenvalues, it will be impossible to see how general they are.

In addition, we present in this paper the U(1) analysis of the representations $[21] + [35] + 5[7^*]$ and $2[25] + [21^*] + [7^*]$ as well as of the spinor representation but with fractionally-charged color singlets embedded in pair. The former can have only two and one respectively out of three generations of quarks and leptons complex. But in view of the smallness of the electron and muon mass compared to that of tau, such models may have some potential use. The spinor representation with fractionally-charged color singlets in pair can embrace only two generations of leptons and quarks. Our complete U(1) analysis reveals again that the SU(7) model of this latter type as discussed by others[5] corresponds to only one particular symmetry-breaking.

We give a simple formula for $\sin^2\theta_W$ at the grand unification mass and point out that different symmetry-breaking patterns will contribute differently to the renormalization corrections. Finally, we compare the characteristic differences of the fermion content in different types of representations.

The paper is organized as follows: In Section II, we define a set of guielines that must be satisfied by the model and by the allowed representations of SU(7). Section III contains the nine possible symmetry-breaking patterns that make $SU(7) \to SU_c(3) \times SU(2) \times U(1)$. In particular, the intermediate breaking stage may not necessarily go through SU(5). In Section IV, we give the projection operators for the nine modes of symmetry-breaking and show how they can be used, in particular, to obtain U(1) eigenvalues to construct the hypercharge generators. The most general charge assignment in SU(7) turns out to be rather restricted: there is only one charge assignment possible when the irreducible representation is allowed to repeat. In this case, two SU(5) singlets in the fundamental representation [7] of SU(7) are electrically neutral. But with charged particle embedding to the SU(5) singlets, the spinor representation is the only one allowed whether or not these singlets have fractional charges. Finally in Section V, the difference of the fermion content between the different SU(7) representations is briefly sketched.

II. SU(7) REPRESENTATIONS

We impose the following guidelines for SU(7).

(1) The representation should be anomaly free,[16,17] for the renormalizability.

(2) The representation of left-handed fermions should be real with respect to $SU_c(3)$.

(3) The representation should contain 1_c, 3_c, and 3_c^* only of $SU_c(3)$.[8]

(4) The representation should be complex[7] with respect to $SU_c(3) \times SU(2) \times U(1)$.

(5) The representation should accommodate at least two generations of quarks and all three generations of leptons.

In the case of SU(5), the first two conditions are equivalent but in general they are not. The representation of the charged fermions is real with respect to $U_{em}(1)$ in any case, as there are no known massless and charged fermions in nature. We do not require the overall asymptotic freedom but it turns out this condition[18] is satisfied automatically in our case. Nor do we insist on having all three generations of quarks. This will imply that the lepton-quark symmetry holds for the first two generations only.

SU(7) models can be classified depending on whether or not the same irreducible representation is repeated in the anomaly-free combinations:

Type I. No irreducible representations appear more than once.

Type II. The same irreducible representation can be repeated in such a way that no common coefficients are to exist in the anomaly-free combinations.

We considered all the possible anomaly-free combinations having D = 42 - 110. Of all possible combinations, there is exactly one representation that satisfies the five conditions listed above. That is:

$$f_L = [21] + [35^*] + [7^*] \tag{1}$$

If we relax conditions and require at least two of the three quark-lepton generations to be complex with respect to $SU_c(3) \times SU(2) \times U(1)$, then we obtain the representation,

$$f_L = [21] + [35] + 5[7^*] \tag{2}$$

If we relax conditions further to allow at least one of the three quark-lepton generations to be complex with respect to $SU_c(3) \times SU(2) \times U(1)$, then the representation

$$f_L = 2[35] + [21^*] + [7^*] \tag{3}$$

is also possible. In representation (3), the first generation would have considerably smaller mass than the other two generations. In view of the smallness of the electron mass compared to those of muon and tau, some may like to entertain such a scenario. For this reason, we will give the hypercharge operators corresponding to this case along with the other two cases.

Generally there is only one anomaly-free and complex representation of Type I in SU(2N + 1), and that is constructed out of 2^{2N} spinorial representation of SO(4N + 2).

III. SYMMETRY-BREAKING PATTERNS

Although there is still a great deal of freedom in defining the hypercharge operators with the constraints imposed above, the structure of hypercharge operators falls into one of the following nine categories of symmetry-breaking pattern, all of which reduce SU(7) to $SU_c(3) \times SU(2) \times U(1)$.

Case (A):

$$SU(7) \rightarrow SU(2) \times SU_{EWS}(5) \times U^a(1) \tag{4}$$

followed by

$$SU(2) \rightarrow U^b(1)$$

and

$$SU_{EWS}(5) \rightarrow SU(2) \times SU_c(3) \times U^c(1) \quad .$$

This case takes SU(5) as an intermediate step from which $SU_c(3) \times SU(2)$ comes. The symmetry breakings into $U^b(1)$ and $U^c(1)$ do not necessarily occur at the same stage. But the charge operator is independent of the order in the second stage of symmetry breakings. In general, there are three U(1) generators.

Case (B):

$$SU(7) \rightarrow SU(2) \times SU_S(5) \times U^a(1) \tag{5}$$

followed by

$$SU_S(5) \rightarrow SU(2) \times SU_c(3) \times U^c(1)$$

and then

$$SU(2) \rightarrow U^b(1) \quad .$$

Note that weak SU(2) emerges at the first stage of symmetry-breaking and remains that way.

Case (C):

$$SU(7) \rightarrow SU(2) \times SU_S(5) \times U^a(1) \tag{6}$$

and

$$\begin{aligned} SU_S(5) &\rightarrow SU_S(4) \times U^b(1) \\ &\rightarrow SU_c(3) \times U^c(1) \times U^b(1) \quad . \end{aligned}$$

The first stage of symmetry-breaking is the same as in Case (B), but is followed by

$$SU_S(5) \rightarrow SU_S(4) \rightarrow SU_c(3)$$

successively.

Case (D):

$$SU(7) \rightarrow SU_{EW}(3) \times SU_S(4) \times U^a(1) \tag{7}$$

followed by

$$SU_{EW}(3) \rightarrow SU(2) \times U^b(1)$$

and

$$SU_S(4) \rightarrow SU_c(3) \times U^c(1)$$

We note that the symmetry-breaking mode $SU_{EW}(3) \rightarrow SU(2)$ is also possible, i.e., without $U^b(1)$ contributions. But since we need only doublets and singlets of weak SU(2), this symmetry-breaking is excluded. This can be seen by observing that the projection operator for $A_2 \rightarrow A_1$ is given by

$$P\left\{A_2 \rightarrow A_1 \times U(1)\right\} = \begin{pmatrix} 2 & 2 \\ 0 & 0 \end{pmatrix} \tag{9}$$

so that no doublets can occur. However,

$$P\left\{A_2 \rightarrow A_1 \times U(1)\right\} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{10}$$

which corresponds to Case (D) is allowed. Note that $A_2 \rightarrow A_1$ can take two different projections.

Case (E):

$$SU(7) \rightarrow SU_c(3) \times SU_{EW}(4) \times U^a(1)$$

followed by

$$SU_{EW}(4) \rightarrow SU_{EW}(3) \times U^b(1)$$

$$\rightarrow SU(2) \times U^c(1) \times U^b(1)$$

Note that the color $SU_c(3)$ appears from the first stage.

Mathematically, the symmetry-breaking pattern

$$SU_{EW}(4) \to Sp(4) \times U^b(1)$$

followed by either

$$Sp(4) \to SU(2) \times SU(2) \to SU(2) \times U^c(1)$$

or directly

$$Sp(4) \to SU(2) \times U^c(1)$$

is also possible. In these cases, the eigenvalue of $U^b(1)$, Y^b, is zero necessarily and this makes too restrictive to allow the right combination for charge operators. In addition, $Sp(4) \to SU(2) \times U(1)$ contains higher multiplets for weak SU(2) than just singlets and doublets.

Case (F):

$$SU(7) \to SU_c(3) \times SU_{EW}(4) \times U^a(1) \tag{12}$$

followed by

$$SU_{EW}(4) \to SU(2) \times SU(2) \times U^b(1) \to SU(2) \times U^b(1) \times U^c(1)$$

This case[19] is distinguished from Case (E) in that the second stage is

$$SU_{EW}(4) \to SU(2) \times SU(2) \times U^b(1)$$

followed by any one of the two SU(2)'s going into $U^c(1)$. The symmetry-breaking pattern suggested by Cox, Frampton, and Yildiz[1] for generating the fermion masses is in fact of this type.

The pattern of $SU_{EW}(4) \to SU(2) \times SU(2)$ without $U^b(1)$ is ruled out, as the final weak SU(2) contains triplets necessarily in addition to singlets and doublets.[19] Note that $A_3 \to A_1 \times A_1$ can take two different projections also.

Case (G):

$$SU(7) \to SU_{EWS}(6) \times U^a(1) \tag{13}$$

followed by

$$SU_{EWS}(6) \to SU_{EWS}(5) \times U^b(1)$$

and

$$SU_{EWS}(5) \to SU(2) \times SU_c(3) \times U^c(1) \quad .$$

The first and second stages of symmetry-breaking go through SU(6) and SU(5). But $U^a(1)$ and $U^b(1)$ may play a significant role to give the correct charge generators.

Case (H):

$$SU(7) \to SU_{EWS}(6) \times U^a(1) \tag{14}$$

followed by

$$SU_{EWS}(6) \to SU(2) \times SU_S(4) \times U^b(1)$$

and

$$SU_S(4) \to SU_c(3) \times U^c(1) \; .$$

This case has different symmetry-breaking in the second stage from Case (G).

Case (I):

$$SU(7) \to SU_{EWS}(6) \times U^a(1) \tag{15}$$

followed by

$$SU_{EWS}(6) \to SU(3) \times SU_c(3) \times U^b(1)$$

and

$$SU(3) \to SU(2) \times U^c(1) \quad .$$

Mathematically, SU(6) can branch into SU(2) × SU(3), SU(4), SU(3) and Sp(6) also, but these modes can be shown to fail to give the correct charge generator.

These exhaust all symmetry-breaking patterns[20] which are non-trivial and more importantly relevant to the physics of SU(7) model. Different symmetry-breaking patterns give rise to different physics in detail. Firstly, they entail different combinations of U(1) generators to appear in the charge operator, and secondly, they provide a different evolution of $\sin^2\theta_W$ from its value at the grand unification mass to low energies. We make thorough investigation of the U(1) generators in each and every case of the symmetry-breaking patterns in the next section. We will introduce projection operators[21] in order to obtain the U(1) eigenvalues and their contribution to the hypercharge operator. We show the potential powerfulness of the projection method which can be used to any Lie group.

IV. PROJECTION OPERATORS AND HYPERCHARGE GENERATORS

The projection operators for the nine symmetry-breaking patterns are given in Table I. Here the first and second rows project A_6 into A_2 and the third row projects A_6 into A_1. The remaining three rows represent the three successive $U^a(1)$, $U^b(1)$, and $U^c(1)$ projections.

The charge operator is a generator of SU(7) and is given by

$$Q = T_3 + aY^a + bY^b + cY^c \ , \tag{16}$$

where T_3 is the third generator of SU(2) and Y^a, Y^b, Y^c are the diagonal operators corresponding to the respective U(1) rotations.

Different symmetry-breaking patterns give rise to the different combinations of Y in the hypercharge operator. Here we will work out explicitly for the hypercharge in the Case (D), and simply give the final results for the other cases which can be obtained similarly. The projection operator of Case (D) carries the representations [7], [21], [35] into $SU_c(3) \times SU(2) \times U^a(1) \times U^c(1)$ as in Table II. As expected, [7] decomposes into (3,1) + (1,2) + 2(1,1) in ($SU_c(3)$, SU(2)) multiplets.

Thus, the most general charge assignment is

$$Q = \mathrm{diag}(q,q,q,a,a-1,b,1-3q-2a-b). \tag{17}$$

If we identify the charge of (3,1) to be -1/3 and those of (1,2) as (1,0), then

$$Q = \mathrm{diag}(-1/3,\ -1/3,\ -1/3,\ 1,0,q,-q)\ . \tag{18}$$

Using the charges of the five components of [7], (3,1) and (1,2), as input, one can determine the contribution of each U(1) to Q from Eq. (16) and Table IIA.

One can easily verify that there are actually two solutions for Case (D): either

$$a = -1/6, \qquad b = -1/6, \qquad c = 1/6, \tag{19}$$

or

$$a = 0, \qquad b = 1/2, \qquad c = -\ 1/3\ . \tag{20}$$

Having determined U(1) contributions to Q, we can then obtain the charges of the sixth and seventh elements, i.e., q = -1 for Eq. (20) and q = +1 for Eq. (19), as well as those of [21] and [35]. The hypercharges of [21] and [35] given in Table IIB and IIC are corresponding to Eq. (20), but it is obvious that the combination (19) would amount only to the interchange of 6 $\leftrightarrow$ 7 in ψ_α, $\psi_{\alpha\beta}$ and $\psi_{\alpha\beta\gamma}$. On the other hand, the two different U(1) combinations can result in different evolutions of the coupling constants from the renormalization group equations. And in particular $U^a(1)$ contributions to the evolution of $\sin^2\theta_W$ will be different depending on whether or not a = 0. Also different monopoles may be produced for the two different solutions, Eqs. (19) and (20). However, it is not clear if estimate of the monopole contributions[22] to the energy density of the universe is reliable enough to prefer one solution to the other.

In any case, one may say that from the five conditions imposed, only allowed solutions are q = ±1 for Eq. (1), i.e., for Type I representation. However, it turns out that for Type II representations q = 0 is the only one allowed. This is because, for example, Eq. (2) contains many more, presumably heavy, particles which are SU(5) real besides those in

$$4(5^*,1) + (10,2) + (5\ ,2) + 5(1,2) + (1,1)$$

of (SU(5), SU(2)) decomposition. In this case, the charge assignment is decided by the content of the light particles in the mixed combinations of SU(5) × SU(2). This is to be contrasted with Type I representation for which the fundamental representation dictates the charge assignment.

We summarize in Table III the contributions of the various U(1) generators for the Type I (Eq. (1)) and for the Type II (Eqs. (2) and (3)). Then the charge operator can readily be read off from Table III and Eq. (16). We note that the SU(7) model constructed by Kim[1] has the hypercharge generator corresponding to the second solutions in Case (D) and Case (C) for Type I, and the SU(7) model constructed by Cox, Frampton, and Yildiz[1] corresponds to Case (A) for Type II.

The anomaly-free combination of Eq. (3) contains at least one of three generations that is complex with respect to $SU_c(3) \times SU(2) \times U(1)$. As mentioned above, this representation may be of some interest. The charge operator in each mode of symmetry-breaking is precisely the same as that of Eq. (2), which may be due to the fact that the charge assignment of the fundamental representation [7] is identical to that in Eq. (2), i.e., $q = 0$ in Eq. (18). This does not, however, mean the same content of fermions. For example, in Eq. (2) while there can be no light t quark from the usual wisdom of survival hypothesis[7] (i.e., real with respect to SU(5)), b quark is real with respect to $SU_c(3) \times SU(2) \times U(1)$. However, in the representation (3), a doublet of (t,b) can exist but its representation is real with respect to $SU_c(3) \times SU(2) \times U(1)$.

Fractional charge assignment such as $q = \pm 1/3$, $\pm 2/3$, and $\pm 4/3$ are also possible for Type I but in this case representation (1) can accommodate only two generations of quarks and leptons. SU(7) model with $q = \pm 1/3$ has been discussed recently by some authors.[5] For this reason, we give the complete U(1) analysis as well as the charge operators in Table IV. We stress that such fractional charge assignments are not possible for Type II (Eqs. (2) and (3)). This type of model with fractional q may provide another exciting usefulness of SU(7) if the recent report[4] of the particle with the charge + 1/3 implies, indeed, the existence of the fractionally-charged color singlets. The structure of the charge operator considered by Li and Wilczek[5] corresponds to Case (A) for Type I while they suggest the symmetry-breaking mode Case (D) for generating fermion masses. This type of model necessarily contains, amongst others, integrally-charged $(3^*,2)$, and fractionally-charged $(1,2)$ multiplets of $(SU_c(3), SU(2))$. Then such integrally-charged quarks can form bound states with the usual color-triplet quarks, which will then materialize as fractionally-charged hadrons.[23]

Having determined U(1) eigenvalues, we proceed to find the usual weak hypercharge Y that is given by a linear combination of Y^a, Y^b, and Y^c with coeffients a, b, and c respectively. With the convention

$$\mathrm{Tr}\ T_3^2 = \mathrm{Tr}\ Y_1^2 = 1/2\ ,$$

we obtain from Eqs. (16) and (18) that

$$Q = T_3 + Y = T_3 + C\ Y_1 = T_3 + (5/3 + 4q^2)^{1/2}\ Y_1 \qquad (21)$$

This gives the SU(2) × U(1) mixing angle at the grand unification mass M_G by

$$\sin^2\theta_W\ (M_G) = \mathrm{Tr}\ T_3^2/\mathrm{Tr}Q^2 = (1 + C^2)^{-1} = 3/(8 + 12q^2) \qquad (22)$$

which implies

$$\sin^2\theta_W\ (M_G) = 0.15,\ 0.375,\ \text{and}\ 0.32$$

for $q = \pm 1$, 0, and $\pm 1/3$ respectively. Obviously one needs to increase (decrease) $\sin^2\theta_W$ in the case $q = \pm 1$ ($q = 0$, and $+ 1/3$), when it is extrapolated to the low-energy region via the renormalization equation. This may choose to prefer particular modes of symmetry-breaking in each case. Such renormalization corrections can indeed be made as has been done by Kim.[1] They usually contain two or three parameters coming from intermediate mass scales and we leave this problem as an exercise to the interested readers.

V. FERMION CONTENTS

Representations (1) and (2) agree for the first two generations of leptons and quarks. However, as for t quark, they predict completely different physics. Whereas there is no light t quark in Eq. (2), there are two more doublets of quarks, in Type I with $q = \pm 1$. But b and t have different doublet partners: b is forming a doublet with an exotic partner x with charge -4/3 while t has another exotic partner y whose charge is 5/3. The semi-leptonic b decay in Type I is discussed in Ref. 1. As for the leptons, there is an additional doublet of charge +2 and +1 besides the usual tau doublet in Type I. In fact, Type II does not contain any exotic particles, while exotic quarks and leptons are naturally occurring in Type I:

$$(x^c, b^c)_L , \qquad (t^c, y^c)_L , \qquad (L^{++}, L^{+})_L \tag{23}$$

where the two quark doublets correspond to two $(3^*, 2)$, $\psi^*_{6\alpha i}$ and $\psi^*_{7\alpha i}$, with $Y = 5/6$ and $Y = -7/6$ respectively, and the lepton doublet to $(1,2)$, $\psi_{7\alpha}$. Here α and i denote 4, 5, and 1, 2, 3 respectively. Also $x_L = \psi_{6i}$ with $Y = -4/3$, $y_L = \psi^*_{6ij}$ with $Y = 5/3$, and $L^{--}_L = \psi^*_{457}$ with $Y = -2$. Thus the representation (1) contains four doublets of quarks and leptons.

We note that there are two b-type quarks in Eq. (2) which are real with respect to the $SU_c(3) \times SU(2) \times U(1)$. Also there is a fourth lepton doublet with the ordinary charge, but both third and fourth lepton doublets are real with respect to $SU_c(3) \times SU(2) \times U(1)$. Representation (2) of Type II may not give any substantial hadronic or semi-leptonic b decay.[24] Since the two b-type quarks and two τ-like lepton doublets are real with respect to $SU_c(3) \times SU(2) \times U(1)$, the mass of these particles are expected to be heavier than those of the first two generations according to the usual survival hypothesis.[7] In any case, the τ neutrinos must be massive.

As mentioned before, the representation of Type I can allow the fractional charges such as $\pm 1/3$, $\pm 2/3$, $\pm 4/3$ in the fundamental representation [7], if we require at least two generations of quark and lepton to exist with the usual quantum numbers as given in the standard electroweak theory.

Exotic quarks and leptons are characteristic of this type of SU(7) models. For example, Eq. (1) with $q = \pm 1/3$ contains two additional doublets of "lepton" with the fractional charge,

$$(L^{+2/3},\ L^{-1/3})_L\ , \qquad (L^{+4/3},\ L^{1/3})_L \tag{24}$$

having $Y = 1/6$ and $5/6$ respectively, and two additional doublets of "antiquark", i.e., two $(3^*,2)$'s with unconventional charges,

$$(\bar{Q}^{2/3},\ \bar{Q}^{-1/3})_L\ , \qquad (\bar{Q}^{0},\ \bar{Q}^{-1})_L \tag{25}$$

with $Y = 1/6$ and $Y = -1/2$ respectively as well as their right-handed singlet partners in addition to the two neutral lepton singlets. Since the representation is complex with respect to $SU_c(3) \times SU(2) \times U(1)$, most of these exotic particles would not be so heavy. It is interesting to note that, in addition to the fractionally-charged leptons of Eq. (24), fractionally-charged hadrons are predicted in this model from the color singlet bound state of Eq. (25) and the usual quark.

The most general Yukawa interaction for Type I is also discussed by Kim[1]. We will suffice to state that fermion masses can be generated by giving non-zero vacuum expectation values to the appropriate $SU_c(3) \times U_{em}(1)$ neutral components of Higgs scalars, i.e., to (0,0,0,0,-1,1) of $[7]_H$; (0,0,-1,0,0,1) of $[35]_H$; (0,0,1,0,-2,1) and (0,0,0,0,1,-1) of $[140]_H$; (-1,1,0,1,0,-1), (0,0,0,0,1,-1), (0,0,1,0,-2,1), (1,-1,0,-1,2,-1), (-1,1,-2,1,-1,2) and (1,-1,1,-1,-1,1) of $[588]_H$ in case (D) of symmetry-breaking.

VI. CONCLUDING REMARKS

We have given thorough analysis of U(1) eigenvalues and their contributions to the charge and hypercharge generators in SU(7) grand unification models. There are nine distinct and physically interesting modes of symmetry-breaking for

$$SU(7) \to SU_c(3) \times SU(2) \times U(1) \ .$$

In general, the final hypercharge operator U(1) is given by a linear combination of the three U(1) generators emerging at the various stages of each symmetry-breaking. Such U(1) analysis is carried out by making use of the projection operators of branching modes of SU(n) group. The projection operator method is most convenient to use and to find the U(1) eigenvalues and can easily be applied to any Lie group.

By imposing a set of five guidelines, we narrowed down the allowed SU(7) representations to be exactly one in the case where the same irreducible representation is not permitted to repeat itself. This case is referred to as Type I in the text.

In Type II where the same irreducible representation is allowed to repeat as long as there are no common factors in the coefficients, we found two more representations with $D \leq 110$ under somewhat relaxed requirements. The representation (2) contains two generations of quarks and leptons whose representations are complex with respect to $SU_c(3) \times SU(2) \times U(1)$. Representation (3) allows only one family of the quark-lepton to have complex representation with respect to $SU_c(3) \times SU(2) \times U(1)$.

Both representations (1) and (2) have been the subjects of recent discussions,[1,5] but usually with a particular mode of symmetry-breaking pattern.

In Type II, one or more generations may have heavier mass than the first generation.

The two types of representations have rather different fermionic contents. Whereas there are no exotic particles with unusual charges in Type II, exotic particles are natural commodities of Type I. This is because representations of Type II permit only trivial embedding of SU(5) in the fundamental representation of SU(7), i.e., the two additional SU(5) singlets are electrically neutral under due conditions. On the other hand, in Type I, the SU(5) singlets in [7] have non-trivial charges. Not only can they allow $q = \pm 1$ but also fractional charges like $q = \pm 1/3$, $\pm 2/3$ or $\pm 4/3$.

The most crucial constraints that force only trivial embedding of SU(5) in Type II representations, i.e., $q = 0$, are the reality conditions under $SU_c(3)$. We think this may be of the general characteristics of the Type II representations in any SU(n) grand unification models where $n \geq 7$. Therefore, non-trivial SU(5) embedding appears to be possible only for Type I representations.

The representation (1) contains four doublets of quarks and leptons, some of which are made of particles with unconventional charges. For $q = \pm 1$, they are given in Eq. (23) in addition to the usual three lepton doublets and two quark doublets. For $q = \pm 1/3$, they are the four doublets of Eqs. (24) and (25) besides the usual two families of (u, d, ν_e, e) and (c, s, ν_μ, μ). The representation (2) contains two topless b-quarks, i.e., without their t-type partners, and two additional τ-type lepton doublets along with the usual two families of particles. These additional particles have representations which are real with respect to $SU_c(3) \times SU(2) \times U(1)$, so that their masses are expected to be heavier than those of the usual families.

The representation (3) has the same fermionic content as in Eq. (2) except for the fact that all particles besides the first family (u, d, ν_e, e) have real representations in $SU_c(3) \times SU(2) \times U(1)$.

Further experimental confirmation of significant semileptonic and hadronic b decay may not favor the representations of Type II. On the other hand, if fractionally-charged leptons as well as hadrons are found, the representations of Type I with the fractional charge such as $q = \pm 1/3$ either in SU(7) or SU(9) may be of some interest.

We have constructed projection operators for all of the symmetry-breaking modes of interest in Table I. With these, we carried out projections of SU(7) into $SU_c(3) \times SU(2) \times U^a(1) \times U^b(1) \times U^c(1)$ for the two types of representations. We have explicitly given the projections of representation (1) for Case (D) in Table II to indicate how the projection operator method results in various eigenvalues. As mentioned before, the U(1) combinations obtained by others are only particular cases of many possible solutions. We then showed how these U(1) eigenvalues are combined to give the right hypercharge operator for representations considered in Table III and Table IV. Different symmetry-breaking patterns generally have different U(1) combinations for hypercharge. For given charge assignment of [7], the weak mixing angle at the grand unification mass is given by the same value Eq. (18), but different symmetry-breaking patterns will give rise to different evolutions of the running coupling constants and in particular $\sin^2\theta_W$. Such estimates can be made and have been done at least for two cases of symmetry-breaking. But they will usually involve two or three parameters corresponding to the threshold values of the intermediate stages of symmetry breaking.

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TABLE I. Projection operators for nine symmetry-breaking patterns of SU(7) into $SU_c(3) \times SU(2) \times U^a(1) \times U^b(1) \times U^c(1)$.

Case (A)

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 2 & 4 & -1 & 1 & -4 & -2 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ -2 & 1 & 1 & -1 & -1 & 2 \end{pmatrix}$$

Case (B)

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 2 & 4 & -1 & 1 & -4 & -2 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ -2 & 1 & 1 & -1 & -1 & 2 \end{pmatrix}$$

Case (C)

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 2 & 4 & -1 & 1 & -4 & -2 \\ 1 & 2 & 2 & -2 & -2 & -1 \\ 1 & 2 & 2 & 2 & 2 & -1 \end{pmatrix}$$

Case (D)

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 3 & -1 & 2 & -2 & 1 & -3 \\ 0 & 1 & 1 & -1 & -1 & 0 \\ 1 & 1 & 2 & 2 & -1 & -1 \end{pmatrix}$$

Case (E)

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 3 & -1 & 2 & -2 & 1 & -3 \\ 1 & 1 & 2 & 2 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 \end{pmatrix}$$

Case (F)

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 3 & -1 & 2 & -2 & 1 & -3 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Table I - cont'd.

Case (G)

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 2 & 3 & -3 & -2 & -1 \\ 1 & 2 & 3 & 3 & -2 & -1 \\ -2 & 1 & -1 & -1 & -1 & 2 \end{pmatrix}$$

Case (H)

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 2 & 3 & -3 & -2 & -1 \\ 1 & 2 & 0 & 0 & -2 & -1 \\ 1 & 2 & 2 & 2 & 2 & -1 \end{pmatrix}$$

Case (I)

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 2 & 3 & -3 & -2 & -1 \\ 1 & 0 & 1 & 1 & 0 & -1 \\ 0 & 1 & 1 & 1 & -1 & 0 \end{pmatrix}$$

TABLE II. Complete weight system of [7], [21], and [35] with $SU_c(3) \times SU(2) \times U^a(1) \times U^b(1) \times U^c(1)$ properties. Here $_m(SU_c(3),\ SU(2))_n$ denotes the m^{th} and n^{th} element of $SU_c(3)$ and $SU(2)$ representations and A: $[7] = \psi_\alpha$; B: $[21] = \psi_{\alpha\beta}$; C" $[35] = \psi_{\alpha\beta\gamma}$, where $\alpha,\beta,\gamma = 1, 2, 3,\cdots,7$.

A: $[7] = \psi_\alpha$

Weight						A_2		A_1	Y^a	Y^b	Y^c	Y	$(SU_c(3),SU(2))$	α
1	0	0	0	0	0	1	0	0	3	0	1	-1/3	$_1(3,1)$	1
-1	1	0	0	0	0	0	0	1	-4	1	0	1/2	$(1,2)_1$	4
0	-1	1	0	0	0	-1	1	0	3	0	1	-1/3	$_2(3,1)$	2
0	0	-1	1	0	0	0	0	0	-4	-2	0	-1	$(1,1)$	6
0	0	0	-1	1	0	0	0	0	3	0	-3	+1	$(1,1)$	7
0	0	0	0	-1	1	0	0	-1	-4	1	0	1/2	$(1,2)_2$	5
0	0	0	0	0	-1	0	-1	0	3	0	1	-1/3	$_3(3,1)$	3

Table II. cont'd.

B: [21] = $\psi_{\alpha\beta}$

Weight						A_2		A_1	Y^a	Y^b	Y^c	Y	$(SU_c(3),SU(2))$	$\alpha\beta$
0	1	0	0	0	0	1	0	1	-1	1	1	1/6	${}_1(3,2)_1$	41
1	-1	1	0	0	0	0	1	0	6	0	2	-2/3	${}_1(3^*,1)$	21
-1	0	1	0	0	0	-1	1	1	-1	1	1	1/6	${}_2(3,2)_1$	42
1	0	-1	1	0	0	1	0	0	-1	-2	1	-4/3	${}_1(3,1)$	61
-1	1	-1	1	0	0	0	0	1	-8	-1	0	-1/2	$(1,2)_1$	64
1	0	0	-1	1	0	1	0	0	6	0	-2	2/3	${}_1(3,1)$	71
0	-1	0	1	0	0	-1	1	0	-1	-2	1	-4/3	${}_2(3,1)$	62
-1	1	0	-1	1	0	0	0	1	-1	1	-3	3/2	$(1,2)_1$	74
1	0	0	0	-1	1	1	0	-1	-1	1	1	1/6	${}_1(3,2)_2$	51
0	-1	1	-1	1	0	-1	1	0	6	0	-2	2/3	${}_2(3,1)$	72
-1	1	0	0	-1	1	0	0	0	-8	2	0	1	$(1,1)$	45
1	0	0	0	0	-1	1	-1	0	6	0	2	-2/3	${}_2(3^*,1)$	13
0	0	-1	0	1	0	0	0	0	-1	-2	-3	0	$(1,1)$	76
0	-1	1	0	-1	1	-1	1	-1	-1	1	1	1/6	${}_2(3,2)_2$	25
-1	1	0	0	0	-1	0	-1	1	-1	1	1	1/6	${}_3(3,2)_1$	34
0	0	-1	1	-1	1	0	0	-1	-8	-1	1	-1/2	$(1,2)_2$	56
0	-1	1	0	0	-1	-1	0	0	6	0	2	-2/3	${}_3(3^*,1)$	23
0	0	0	-1	0	1	0	0	-1	-1	1	-3	3/2	$(1,2)_2$	57
0	0	-1	1	0	-1	0	-1	0	-1	-2	1	-4/3	${}_3(3,1)$	36
0	0	0	-1	1	-1	0	-1	0	6	0	-2	2/3	${}_3(3,1)$	37
0	0	0	0	-1	0	0	-1	-1	-1	1	1	1/6	${}_3(3,2)_2$	35

Table II. cont'd

C: [35] = $\psi_{\alpha\beta\gamma}$

Weight						A_2		A_1	Y^a	Y^b	Y^c	Y	$(SU_c(3),SU(2))$	$\alpha\beta\gamma$
0	0	1	0	0	0	0	1	1	2	1	2	-1/6	${}_1(3^*,2)_1$	421
0	1	-1	1	0	0	1	0	1	-5	-1	1	-5/6	${}_1(3,2)_1$	641
1	-1	0	1	0	0	0	1	0	2	-2	2	-5/3	${}_1(3^*,1)$	621
0	1	0	-1	1	0	1	0	1	2	1	-2	7/6	${}_1(3,2)_1$	741
-1	0	0	1	0	0	-1	1	1	-5	-1	1	-5/6	${}_2(3,2)_1$	642
1	-1	1	-1	1	0	0	1	0	9	0	-1	1/3	${}_1(3^*,1)$	721
0	1	0	0	-1	1	1	0	0	-5	2	1	2/3	${}_1(3,1)$	541
-1	0	1	-1	1	0	-1	1	1	2	1	-2	7/6	${}_2(3,2)_1$	742
1	0	-1	0	1	0	1	0	0	2	-2	-2	-1/3	${}_1(3,1)$	761
1	-1	1	0	-1	1	0	1	-1	2	1	2	-1/6	${}_1(3^*,2)_2$	521
0	1	0	0	0	-1	1	-1	1	2	1	2	-1/6	${}_2(3^*,2)_1$	431
-1	1	-1	0	1	0	0	0	1	-5	-1	-3	1/2	$(1,2)_1$	764
-1	0	1	0	-1	1	-1	1	0	-5	2	1	2/3	${}_2(3,1)$	542
1	0	-1	1	-1	1	1	0	-1	-5	-1	1	-5/6	${}_1(3,2)_2$	651
1	-1	1	0	0	-1	0	0	0	9	0	3	-1	$(1,1)$	321
0	-1	0	0	1	0	-1	1	0	2	-2	-2	-1/3	${}_2(3,1)$	762
-1	1	-1	1	-1	1	0	0	0	-12	0	0	0	$(1,1)$	654
-1	0	1	0	0	-1	-1	0	1	2	1	2	-1/6	${}_3(3^*,2)_1$	432
1	0	0	-1	0	1	1	0	-1	2	1	-2	7/6	${}_1(3,2)_2$	751
1	0	-1	1	0	-1	1	-1	0	2	-2	2	-5/3	${}_2(3^*,1)$	631
0	-1	0	1	-1	1	-1	1	-1	-5	-1	1	-5/6	${}_2(3,2)_2$	652

Table II. cont'd (C: [35] = $\psi_{\alpha\beta\gamma}$

Weight						A_2		A_1	Y^a	Y^b	Y^c	Y	$(SU_c(3),SU(2))$	$\alpha\beta\gamma$
-1	1	0	-1	0	1	0	0	0	-5	2	-3	2	$(1,1)$	754
-1	1	-1	1	0	-1	0	-1	1	-5	-1	1	-5/6	${}_3(3,2)_1$	643
1	0	0	-1	1	-1	1	-1	0	9	0	-1	1/3	${}_2(3^*,1)$	731
0	-1	1	-1	0	1	-1	1	-1	2	1	-2	7/6	${}_2(3,2)_2$	752
0	-1	0	1	0	-1	-1	0	0	2	-2	2	-5/3	${}_3(3^*,1)$	632
-1	1	0	-1	1	-1	0	-1	1	2	1	-2	7/6	${}_3(3,2)_1$	743
1	0	0	0	-1	0	1	-1	-1	2	-1	2	-1/6	${}_2(3^*,2)_2$	531
0	0	-1	0	0	1	0	0	-1	-5	-1	-3	1/2	$(1,2)_2$	765
0	-1	1	-1	1	-1	-1	0	0	9	0	-1	1/3	${}_3(3^*,1)$	732
-1	1	0	0	-1	0	0	-1	0	-5	2	1	2/3	${}_3(3,1)$	543
0	0	-1	0	1	-1	0	-1	0	2	-2	-2	-1/3	${}_3(3,1)$	763
0	-1	1	0	-1	0	-1	0	-1	2	1	2	-1/6	${}_3(3^*,2)_2$	532
0	0	-1	1	-1	0	0	-1	-1	-5	-1	1	-5/6	${}_3(3,2)_2$	653
0	0	0	-1	0	0	0	-1	-1	2	1	-2	7/6	${}_3(3,2)_2$	753

TABLE III. U(1) contributions to the charge operator Q in nine possible cases of symmetry-breaking patterns. For the representations of Type I and Type II, a, b, and c are determined by $Q = T_3 + aY^a + bY^b + cY^c$.

	Type I			Type II		
	a	b	c	a	b	c
Case A	0	∓ 1	1/6	0	0	1/6
Case B	-1/10	∓ 1	1/15	-1/10	0	1/15
Case C	-1/10	-3/10	1/6	-1/10	-1/20	-1/12
	-1/10	1/5	-1/3			
Case D	-1/6	-1/6	1/6	-1/12	1/6	-1/12
	0	1/2	-1/3	-1/12	1/6	-1/12
Case E	1/12	-1/4	1/2	1/12	1/12	1/6
	1/12	5/12	-1/6			
Case F	1/12	1/4	± 1	1/12	$\pm 1/4$	0
Case G	$\mp 1/6$	$\pm 1/6$	1/6	0	0	1/6
Case H	-1/6	-1/3	-1/6	0	-1/4	-1/12
	1/6	-1/6	-1/3			
Case I	-1/6	1/6	-1/2	0	1/3	1/6
	1/6	1/2	-1/6			

TABLE IV. U(1) contributions to the charge operator Q in nine possible cases of symmetry-breaking patterns for [21] + [35*] + [7*] with fractionally charged color singlet. Charge operator Q is given by $Q = T_3 + aY^a + bY^b + cY^c$.

	21 + 35* + 7*								
	q = ±1/3			q = ±2/3			q = ±4/3		
	a	b	c	a	b	c	a	b	c
Case A	0	±1/3	1/6	0	±2/3	1/6	0	±4/3	1/6
Case B	-1/10	±1/3	1/15	-1/10	±2/3	1/15	-1/10	±4/3	1/15
Case C	-1/10	-2/15	0	-1/10	13/60	1/12	-1/10	-23/60	1/4
	-1/10	1/30	-1/6	-1/10	7/60	-1/4	-1/10	17/60	-5/12
Case D	-1/9	1/18	0	-5/36	-1/18	1/12	-7/36	-5/18	1/4
	-1/18	5/18	-1/6	-1/36	7/18	-1/4	1/36	11/18	-5/12
Case E	1/12	-1/36	5/18	1/12	-5/36	7/18	1/12	-13/36	11/18
	1/12	7/36	1/18	1/12	11/36	-1/18	1/12	19/36	-5/18
Case F	1/12	1/4	±1/3	1/12	1/4	±2/3	1/12	1/4	±4/3
Case G	∓1/18	±1/18	1/6	∓1/9	±1/9	1/6	∓2/9	±2/9	1/6
Case H	-1/18	-5/18	0	-1/9	-11/36	1/12	-2/9	-13/36	1/4
	1/18	-2/9	-1/6	1/9	-7/36	-1/4	2/9	-5/36	-5/12
Case I	-1/18	5/18	5/18	-1/9	2/9	7/18	-2/9	1/9	11/18
	1/18	7/18	1/18	1/9	4/9	-1/18	2/9	5/9	-5/18

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19. We thank Dr. W. Bardeen for pointing out this important possibility to us. The projection operator for $A_3 \rightarrow A_1 \times A_1 \times U(1)$ is $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$. This branching mode is to be distinguished from the mode $A_3 \rightarrow A_1 \times A_1$ without U(1) whose projection is $\begin{pmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 0 \end{pmatrix}$.

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21. Complete summary of projection operators for all Lie algebra can be found in Y. D. Kim, K. Y. Kim, I. G. Koh, Y. J. Park, W. S. l'Yi, Y. S. Kim, C. H. Kim, and H. T. Park, "Tables of All Irreducible Representations for All Classical Group: Complete Weight System, Branching Matrices with U(1) and Anomaly Table", Sogang University Press, Seoul, Korea, 1981.

22. See, for example, J. P. Preskill, Phys. Rev. Lett. 43, 1365 (1979); P. Langacker and S. Y. Pi, Phys. Rev. Lett. 45, 1 (1980); A. B. Guth and S. H. H. Tye, Phys. Rev. Lett. 44, 631 (1980).

23. We thank Dr. C. K. Zachos for bringing the work of Dr. Kabir to our attention.

24. C. Bebek, et al., Phys. Rev. Lett. 46, 84 (1981); K. Chadwick, et al., Phys. Rev. Lett. 46, 88 (1981); J. Burger (Ref. 3).