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Comment on Tumbling Gauge Theories

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ABSTRACT

We find that for any SU(N) gauge theory, there are no fermion representations which have (1) asymptotic freedom, (2) anomaly cancellation, and (3) a two fermion channel more attractive than the most attractive two gluon channel. The attractive force between fermions is compared to that between gluons in the MIT Bag Model. Therefore mass hierarchies due to tumbling are unlikely to occur in SU(N) gauge theories.

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Dynamical models of the electroweak symmetry breakdown[1,2] require quark and lepton mass generation in a hierarchy starting at an energy scale of 10^4-10^6 GeV[3,4], much below the GUT scale of $\sim 10^{15}$ GeV. At a sufficiently high energy, in the standard scenario, the theory exhibits an unbroken gauge interaction G_s, called the sideways group, which consists of massless fermions and massless gauge bosons ("gluons"). There are also in general additional global symmetries of the fermions, denoted by $\mathscr{G}_f$. If G_s undergoes a series of spontaneous breakdowns at well separated scales into a succession of smaller and smaller subgroups, the hierarchy of quark and lepton masses may be understood as arising from the energy scales of the dynamical symmetry breakdowns.[5,6] Such a process is called "tumbling"[5] and the smallest masses are generated by the highest energy scale.

The two main obstacles to implementing this tumbling scheme are how to generate a set of scales from a single gauge interaction G_s and how to determine the actual pattern of the spontaneous breakdown of the global symmetries $\mathscr{G}_f$. One proposed[5,7] method of determining the pattern of $\mathscr{G}_f$ symmetry breakdown is the maximally attractive fermion channel (MAC) hypothesis. To understand MAC, one considers the interaction energy between two fermions, one in a representation R_1 of G_s and the other in a representation R_2 of G_s, at an energy scale Λ sufficiently large that the gauge field coupling constant is small ($\alpha_G(\Lambda)<<1$). In this region the interaction energy, ΔE_{12}, is calculable in perturbative theory. In the Lorentz scalar channel [1] which transforms under G_s according to the representation R_3 contained in $R_1 \otimes R_2$ ΔE_{12} has the form:

$$\frac{\Delta E_{12}}{\Lambda} = \frac{[C(3)-C(1)-C(2)]}{2} \alpha_G(\Lambda) d_f \qquad (1)$$

where C(i) is the value of quadratic Casmir operator on the representation R_i and d_f is a dynamical factor of order one independent of the group representation and $\alpha_G(\Lambda)$. For $C(3)<C(1)+C(2)$ the force is attractive, and when Λ is such that

$$-\frac{C(3)-C(1)-C(2)}{2} \alpha_G(\Lambda) \simeq 1 \qquad (2.a)$$

and $$\alpha_G(\Lambda)<<1 \ , \qquad (2.b)$$

the strength of the interaction is comparable to the total energy of the two fermion state. The MAC hypothesis states that the channel with the maximum value of $[C(1)+C(2)-C(3)] \equiv C^3_{12}(\max)$ will become strong first (i.e. at the smallest value of $\alpha_G(\Lambda)$-the largest $\Lambda \equiv \Lambda_1$). The resulting bound state spectrum may be treated as a set of scalar fields with the same representation content under the global and gauge symmetries as the bound states. The analysis of symmetry breaking then proceeds using techniques familiar for elementary Higgs potentials. Therefore at the scale Λ_1 symmetry breaking may occur by the "condensation" of the MAC:

$$\mathcal{G}_f \rightarrow \mathcal{G}_f' \text{ and possibly [2] } G_s \rightarrow G_s'$$

Now the process can be repeated for the remaining gauge group G_s' and massless fermions protected by the chiral symmetries in $\mathcal{G}_f'$.

Thus a Λ_2 is sought such that

$$C'^{3}_{12}\,\alpha_{G'}(\Lambda_2) \simeq 1 \tag{4.a}$$

$$\alpha_{G'}(\Lambda_2) << 1 \tag{4.b}$$

with C'^{3}_{12} at maximum value.

The two scales Λ_1 and Λ_2 are related by the expression for $\alpha_G(\Lambda)$ valid when $\alpha_G(\Lambda)<<1$: [3]

$$\alpha_G(\Lambda) = \frac{1}{(b_G/2\pi)\ln(\Lambda/\Lambda_0)} \tag{5}$$

which implies

$$\frac{\Lambda_1}{\Lambda_2} \sim \exp\{C'^{3}_{12}(\max) - C'^{3}_{12}(\max)\ \} \tag{6}$$

Therefore a large ratio of scales may arise from factors

$C^3_{12}(\max)$ and $C'^3_{12}(\max)$ of the same order of magnitude and a "tumbling" pattern could arise such that:

$$\Lambda_1 >> \Lambda_2 >> \Lambda_3 \cdots \text{ and}$$

$$\mathcal{G}_f \xrightarrow[\Lambda_1]{} \mathcal{G}'_f \xrightarrow[\Lambda_2]{} \cdots$$

A more thorough examination of MAC, however, reveals that it may be deceptively simple. H. Georgi, L. Hall, and M. Wise[8] have noticed that there are examples in which $C'^3_{12}(\max) > C^3_{12}(\max)$ and therefore $\Lambda_2 > \Lambda_1$ a contradiction. Furthermore they observe that in the string picture a large separation between Λ_1 and Λ_2 cannot be simply obtained. In this note we argue that even within the confines of the MAC hypothesis, the basic assumption for tumbling, that the gauge self-interactions are weak at the energy scale determined by the condition of Eq. 2, is never satisfied for any set of fermion representations which satisfy both the constraints of asymptotic freedom and anomaly cancellation. The most interesting cases arise for SU(N) gauge groups; the analysis is similar for other simple groups and the same conclusion is reached[4].

Qualitatively it is easy to understand why tumbling may fail in asymptotically free gauge theories. The requirement that $C^3_{12}(\max)$ be large tends to require large dimensional fermion representations (e.g. in SU(N) the p^{th} rank completely symmetric tensor representation has a value for the quadratic Casmir operator $C_2=p(N+p-1-\frac{p}{N})/2$) in order to satisfy Eq.2 with Λ in the weak gauge coupling regime, whereas asymptotic freedom severely limits the dimensionality of the representations. Therefore, a more precise determination of the strength of the gauge self interactions is needed. In the original tumbling schemes[5] this condition was taken to be $\alpha_G(\Lambda)<1$, but this criterion must certainly be refined as can be seen in the large N limit in SU(N). For large N the effective coupling governing the strength of the gauge self interaction is $N\alpha_G(\Lambda)$ and the existence of tumbling scenario requires

$$C_{12}(\max)\alpha_G(\Lambda)\simeq 1 \text{ for } C_2(G)\alpha_G(\Lambda)<<1 \tag{7}$$

instead of Eq.2[5]. Moreover, for large N ($N>17$) the only fermion representations which are asymptotically free and satisfy the gauge anomaly constraint are (in Young Tableaux form)

$$m_a(\square\square \oplus (N+4)\overline{\square}) \oplus m_b\left(\begin{smallmatrix}\square\\ \square\end{smallmatrix} \oplus (N-4)\overline{\square}\right) \oplus m_c\left(N-1 \begin{array}{ll}\square & \square\\ \vdots & \\ \square & \end{array}\right)$$

$$\oplus m_d(\square \oplus \overline{\square}) \oplus m_e(\square\square \oplus \overline{\square\square}) \oplus m_f\left(\begin{smallmatrix}\square\\ \square\end{smallmatrix} \oplus \overline{\begin{smallmatrix}\square\\ \square\end{smallmatrix}}\right) \tag{8.a}$$

where m_a, m_b, m_c, m_d, m_e, and m_f are non-negative integers satisfying the constraint imposed by asymptotic freedom:

$$(N+2)(m_a+2m_e)+(N-2)(m_b+2m_f)+2Nm_c$$

$$+(N+4)m_a+(N-4)m_b+2m_d\leq 11N \qquad (8.b)$$

Eq.(8) implies $C^3_{12}(\max)\leq(N+1-\frac{2}{N})$ but $C_2(G)=N$ so that the condition for tumbling (Eq.7) is never satisfied.

To investigate finite N all asymptotically free and anomaly free fermion representations are characterized in Table 1. In terms of eight types of representations C_i i=1,...,8 any other solution can be represented as the linear combination

$$\sum_{i=1}^{\infty} m_i C_i \oplus \sum_i n_i (c_i \oplus c_i^*) \oplus \sum_i p_i (R_i) \qquad (9.a)$$

where c_i is a complex fermion representation and r_i is a real or pseudoreal representation satisfying the constraint of asymptotic freedom. (See Table 2). The $\{m_i\}$, $\{n_i\}$, and $\{p_i\}$ are sets of non-negative integers chosen so that asymptotic freedom is guaranteed:[11]

$$\sum_{i=1}^{8} T_2(C_i)m_i+\sum_j T_2(c_j)2n_j+\sum_k T_2(r_k)p_k\leq 11N/_2 \qquad (9.b)$$

In simple groups other than SU(N) there are no solutions of the C_i type since the anomaly constraint is trivial[11]. The complex representations (c_j) which allow asymptotic freedom are the lowest dimensional spinor representation (dimension 2^{2N}, $T_2=2^{2N-3}$) in O(4N+2) for $2\leq N\leq 4$ ($C_2(G)=22N$) and two higher dimensional spinor representations (dimensions 126 and 144; T_2 = 35 and 34 respectively) in O(10) as well as the 27 dimensional representation (T_2=1) in $E_6(C_2(G)=22)$.

For finite N, dynamics also must be included so that Eq.(7) is modified to be

$$C_{12}(\max)\alpha_G(\Lambda)d_f \simeq 1 \text{ for } C_2(G)\alpha_G(\Lambda)d_G \ll 1 \tag{10}$$

where d_f contains the dynamics of the two fermion channel while d_G is associated with the dymnamics of the gauge bosons self interactions. The more naive analysis of tumbling of Eq.7 is valid if $d_f \simeq d_g$. To evaluate these dynamical factors, the MIT Bag Model is used to calculate the energy splittings of the two "gluon" state and the two fermion state. The maximally attractive two gluon channel always has vacuum quantum numbers. It has been argued by K. Johnson[13] that the ratio of the energy shift of this channel $\Delta E_G(R)$ in a spherical bag of radius R (here the scale R provides an infrared cutoff) to the ground state energy of the two gluon in the absence of interactions, $2E_G(R)$, determines the confinement scale R_c; more precisely he argued that the condition for gluon condensation (confinement) is $\{(2E_G(R_c)-|\Delta E_G(R_c)|)^2 - 2E^2{}_G(R_c)\} \simeq 0$ or $1.5\ |\Delta E_G(R_c)|/E_G \simeq 1$ where the center of mass motion has been removed. Approximately then, MAC applied to gauge bosons determines the scale at which gluon self interactions become strong. For the two gluon MAC

$$1.5 \frac{|\Delta E_G|}{E_G} = C_2(G)\alpha_G(R)d_G \tag{11.a}$$

with $1/R > B^{1/4}$ and for the two fermion MAC

$$1.5 \frac{|\Delta E_f|}{E_f} = C_{12}(\max)\alpha_G(R)d_f \tag{11.b}$$

A detailed calculation[14] gives

$$d_f = .41 \qquad \text{and} \qquad d_G = .62 \tag{12}$$

so that $d_f \simeq d_G$. Using the values from Eq.12 in Eq.10 we find no solutions for Eq.9.a and Eq.9.b.

It is therefore reasonable to conclude that tumbling gauge theories realized by the MAC hypothesis do not exist for simple gauge groups if the theory is asymptotically free and anomaly free. This result further suggests that since the gauge boson channels are the most attractive these theories do not realize the Higgs phase but are universally confining.

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FOOTNOTES

[1]. As is usual in these discussions only the Lorentz scalar channels are allowed as possible condensates. It is not expected that Lorentz invariance will be spontaneously broken.

[2]. If the MAC is not a singlet under the gauge group, the local gauge symmetry may be broken as well as global symmetries.

[3]. This behaviour is dictated by the renormalization group equations. The constant $b_G = \frac{11}{3}C_2(G) - \sum_{R_i} \frac{2}{3}T_2(R_i)$, where R_i is the sum over fermion representation.

[4]. The restriction to simple gauge groups is natural, since each separate simple component of a semisimple group has a coupling which evolves at its own rate (barring the imposition of some discrete symmetry to relate coupling constants). In a given energy range only one coupling can become strong, the weaker couplings can, to good approximation, be ignored.

[5]. All the examples of tumbling suggested in Ref.5 fail to meet the conditions of Eq.7. Also the suggestion of Marciano[10] that chiral symmetry breaking for quarks in exotic representations may provide the weak interaction symmetry breaking scale is untenable under the conditions of Eq.7.

TABLE CAPTIONS

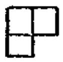
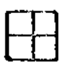
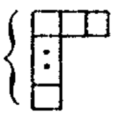
Table 1: The eight independent anomaly free and asymptotically free complex fermion representations in SU(N) are listed in the first column by their Young diagrams. Numbers preceding an irreducible representation indicate its multiplicity in the representation. The second column gives the values of N for which the resulting theory is asymptotically free (upper bound) and independent of the other solutions (lower bound). The lowest order coefficient in the Beta function is $b_G=(\frac{11}{3}C_2(SU(N))-\frac{1}{3}B)$. The fermion contribution B is given in the last column. The maximum B consistent with asymptotic freedom is 11N.

Table 2: All complex fermion representations satisfying the constraint of asymptotic freedom are listed. If we denote the representation matrices by $T^aQ_a|R_i\rangle=\sum_j T^a_{ij}|R_j\rangle$ then $Tr(T^aT^b)=T_2\delta^{ab}$, $\sum_a T^aT^a=C_2 1$, and $Tr(T^aT^bT^c)_{sym} \equiv A\ d^{abc}$. For explicit expression for these invariants for all simple groups and representations see Ref.9. $SU(N_{max})$ is the largest asymptotically free SU(N) group which can contain the associated fermion representation.

Table 1:

Fermion Content	SU(N)	B
$\mathcal{C}_1 = \ydiagram{3,1} \oplus 2\,\overline{\ydiagram{2}}$	$N=3$	30
$\mathcal{C}_2 = \ydiagram{2,2} \oplus 15\,\overline{\square}$	$N=5$	48
$\mathcal{C}_3 = N-2\left\{\ydiagram{2,1,1,1}\right. \oplus \frac{1}{2}(-N^2+7N+2)\overline{\square}$	$4 \leq N \leq 7$	$N(N+1)$
$\phantom{\mathcal{C}_3 =} N-2\left\{\ydiagram{2,1,1,1}\right. \oplus \frac{1}{2}(N^2-7N-2)\square$	$N=8$	78
$\mathcal{C}_4 = \ydiagram{2,1} \oplus (N^2-9)\,\overline{\square}$	$5 \leq N \leq 6$	$2(N^2-6)$
$\mathcal{C}_5 = \ydiagram{1,1,1,1} \oplus \frac{1}{6}(N-3)(N-4)(N-8)\overline{\square}$	$9 \leq N \leq 11$	$\frac{(N-3)(N-4)(N-5)}{3}$
$\mathcal{C}_6 = \ydiagram{1,1,1} \oplus \frac{(N-3)(N-6)}{2}\,\overline{\square}$	$7 \leq N \leq 17$	$(N-3)(N-4)$
$\mathcal{C}_7 = \ydiagram{2} \oplus (N+4)\,\overline{\square}$	$N \geq 3$	$2(N+3)$
$\mathcal{C}_8 = \ydiagram{1,1} \oplus (N-4)\,\overline{\square}$	$N \geq 5$	$2(N-3)$

Table 2

Representation	Dimension	T_2	C_2	A	N_{MAX}
	N	$\frac{1}{2}$	$\frac{N^2-1}{2N}$	1	NONE
	$\frac{N(N-1)}{2}$	$\frac{N-2}{2}$	$\frac{(N+1)(N-2)}{N}$	$N-4$	NONE
	$\frac{N(N+1)}{2}$	$\frac{N+2}{2}$	$\frac{(N-1)(N+2)}{N}$	$N+4$	NONE
	$\frac{1}{6}N(N-1)(N-2)$	$\frac{(N-2)(N-3)}{4}$	$\frac{3(N-3)(N+1)}{2N}$	$\frac{(N-3)(N-6)}{2}$	26
	$\frac{1}{3}N(N^2-1)$	$\frac{N^2-3}{2}$	$\frac{3}{2}\left(\frac{N^2-3}{N}\right)$	(N^2-9)	11
	$\frac{1}{6}N(N+1)(N+2)$	$\frac{(N+2)(N+3)}{4}$	$\frac{3}{2}\ \frac{(N+3)(N-1)}{N}$	$\frac{(N+3)(N+6)}{2}$	16
	$\frac{N(N-1)(N-2)(N-3)}{24}$	$\frac{(N-2)(N-3)(N-4)}{12}$	$\frac{2}{N}(N^2-N-4)$	$\frac{(N-8)(N-3)(N-4)}{6}$	12
	$\frac{1}{12}N^2(N^2-1)$	$\frac{N(N^2-4)}{6}$	$\frac{2}{N}(N^2-4)$	$\frac{N(N^2-16)}{3}$	6
$N-2$ 	$\frac{1}{2}\ N(N+1)(N-2)$	$\frac{(N-2)(3N+1)}{4}$	$\frac{(3N+1)(N-1)}{2N}$	$\frac{(-N^2+7N+2)}{2}$	9
$N-1$ 	$\frac{1}{2}\ N(N-1)(N+2)$	$\frac{(N+2)(3N-1)}{4}$	$\frac{(3N-1)(N+1)}{2N}$	$\frac{(N^2+7N-2)}{2}$	5