Pub-81-051

ILL-(TH)-81-18
March, 1981

On the Modified Jet Calculus of Bassetto, Ciafaloni and Marchesini

by

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ABSTRACT

We reexamine some of the equations derived by Bassetto, Ciafaloni and Marchesini for the various functions in their extended jet calculus, and derive an alternative set of equations based on the same concepts.

PACS Index: 13.90.+i
13.65.+i
14.80.Dq

Typed by Kandy Meredith

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I. INTRODUCTION

We were very interested in the extended jet calculus of Bassetto, Ciafaloni and Marchesini (BCM),[1,2,3] which allows one to compute the production of colorless (quark-antiquark-multigluon) clusters in jets. In preparation for some applications of their formalism, we rederived the basic equations for the modified propagators; we present our findings in this short note.

In our derivation we attempt to keep all planar graphs, and only planar graphs. This results in a somewhat different set of equations for the modified propagators. The solutions of these equations do, however, obey the same sum rules which BCM found for their case; hence they are as appealing physically.

The colorless clusters produced by extension of the BCM arguments to our propagators are more general than in the BCM case. Instead of consisting solely of one quark-antiquark pair and multiple gluons, they may contain additional quark-antiquark pairs. This does not detract from their usefulness, however.

Solutions of our equations are damped in Q^2 in a manner similar to those of BCM, although the exact behavior is a little different. The most striking result of their investigation, finite mass for the color singlets as $Q^2 \to \infty$, is preserved.

II. EQUATIONS FOR THE GENERATING FUNCTIONS

As pointed out in Eq. (3.5) of Ref. 1, the ordinary jet calculus of Konishi, Ukawa and Veneziano[4)] can be summarized by the equation

$$
\begin{aligned}
k^2 \frac{d}{dk^2} F_a\left(k^2, Q_o^2; \{U_d(x)\}\right)
&= \frac{\alpha(k^2)}{2\pi} \sum_c \int dz\, P_a^c(z) F_c\left(\lambda(z)k^2, Q_0^2; \{U_d(xz)\}\right) \\
&\quad + \frac{\alpha(k^2)}{2(2\pi)} \sum_{c_1 c_2} \int dz\, \hat{P}_a^{c_1 c_2}(z)\, F_{c_1}\left(\lambda(z)k^2, Q_0^2; \{U_d(xz)\}\right) \\
&\quad * F_{c_2}\left(\lambda(1-z)k^2, Q_0^2; \{U_d(x(1-z))\}\right)
\end{aligned}
\qquad (2.1)
$$

where

$$
\begin{aligned}
F_a\left(Q^2, Q_0^2; \{U_d(x)\}\right) &= \sum_{\{c_i\}} \left(n_q!\, n_{\bar{q}}!\, n_g!\right)^{-1} \int dx_1 \,.\,.\,.\, dx_n \\
&\quad * U_{c_1}(x_1) \,.\,.\,.\, U_{c_n}(x_n)\, D_a^{c_1 \ldots c_n}\left(Q^2, Q_0^2, x_1 \ldots x_n\right)
\end{aligned}
\qquad (2.2)
$$

is a generating functional for the normalized inclusive distributions of n partons of off-shell mass Q_o in the a parton jet of mass up to Q.

This can then be used to obtain an equation for the exclusive quantities if one writes

$$
G_a\left(Q^2, Q_0^2; \{\xi_d(x)\}\right) = 1 + F_a\left(Q^2, Q_0^2; \{-1 + \xi_d(x)\}\right) \qquad (2.3)
$$

and expands in ξ_d to obtain the exclusive probabilities. The new generating functional G obeys the related equation $\left(\varepsilon = Q_0^2/k^2\right)$

$$k^2 \frac{d}{dk^2} G_a\left(k^2,\ Q_0^2;\ \{\xi_d\}\right) = -G_a \frac{\alpha(k^2)}{2\pi} \int_{\varepsilon(k)}^{1-\varepsilon(k)} dz\ P_a^v(z)$$

$$+ \frac{1}{2} \frac{\alpha(k^2)}{2\pi} \sum_{c_1c_2} \int dz\ \hat{P}_a^{c_1c_2}(z)\ G_{c_1}\left(\lambda(z)k^2,\ Q_o^2; \{\xi_d(xz)\}\right)$$

$$* \quad G_{c_2}\left(\lambda(1-z)k^2,\ Q_0^2;\ \{\xi_d(x(1-z))\}\right) \tag{2.4}$$

It is especially important to note that if all the functions ξ are set equal to 1, each G must be 1 and the terms on the right hand side cancel due to

$$\frac{1}{2} \sum_{c_1c_2} \int_{\varepsilon}^{1-\varepsilon} dz\ \hat{P}_a^{c_1c_2}(z) = \int_{\varepsilon}^{1-\varepsilon} dz\ P_a^v(z) \tag{2.5}$$

III. PLANAR GRAPHS

We now wish to apply this formalism to an idealized world in which only planar graphs are included. When the number of colors is infinite all QCD-like theories go over to this situation. Assuming that three colors are already basically infinite, we wish to only include the planar graphs from the start. Following Witten[5)] and BCM,[1)] we draw the quarks as oriented lines and the gluons as quark-antiquark pairs - i.e., we are labelling the color indices.

In this regard, we notice that there are two possible ways of drawing a gluon - either one can have the quark line on top as the gluon proceeds from left to right across the page, or vice versa (see Fig. 1). We will call the first possibility g_u and the second possibility g_ℓ. As far as can be determined, the gluon of field theory can be represented either by g_u or by g_ℓ, but we will specify which is being used each time since it then is much easier to describe the planar graphs exactly.

All of the vertices in Eqs. (2.4) above have one parton splitting into two others with momentum fractions z and 1-z. We will use the convention that momentum fraction z is carried by the parton which goes off toward the top of the page, and fraction 1-z is carried by the parton which goes toward the bottom of the page. Once this is done, we see that the planar graphs lack some of the possible branchings used previously. In particular, when we have an incident gluon g_u, only the graph of Fig. 2 is allowed; we cannot have the vertex involving $P_g^{\bar{q}q}(z)$.

This means that when G is restricted to the probabilities coming from planar graphs (which should equal the probabilities coming from all graphs in the $N_c \to \infty$ limit), the basic evolution equations must be modified. We must remove the non-planar branching $g_u \to \bar{q}(z)+q(1-z)$. In order to have the desired probability conservation we must therefore also alter the virtual potential for the gluon, so that

$$P^{v}_{g_u}(z) = \frac{1}{2}\,\hat{P}^{gg}_{g_u}(z) + \frac{1}{2}\,\hat{P}^{q\bar{q}}_{g_u}(z) \qquad (3.1)$$

rather than

$$P^{v}_{g}(z) = \frac{1}{2}\,\hat{P}^{g}_{g_u}(z) + \hat{P}^{q}_{g_u}(z)$$

as in Eq. (2.3) of Ref. 1. No such modification is necessary for the quarks; there is a planar interpretation of all the possible vertices.

Given this, the basic equations for the generating functionals are

$$\frac{4\pi k^2}{\alpha(k^2)}\,\frac{d}{dk^2}\,G_{q_i}\left(k^2,Q_0^2;\{\xi_d(x)\}\right) = G_{q_i}(k^2,Q_0^2;\{\xi_d(x)\}\;3C_F +$$

$$C_F \int_{\varepsilon(k)}^{1-\varepsilon(k)} \{(-z-1)\;G_{q_i}\left(k^2,Q_0^2;\{\xi_d(xz)\}\right)G_g\left(k^2,Q_0^2;\{\xi_d(x(1-z))\}\right)$$

$$+\,(z-2)\;G_g\left(k^2,\;Q_0^2;\{\xi_d(xz)\}\right)\;G_{q_i}\left(k^2,Q_0^2;\{\xi_d(x(1-z))\}\right)\}\;dz$$

$$+\int_{\varepsilon(k)}^{1-\varepsilon(k)} \frac{2C_F dz}{1-z}\,\Big[-G_{q_i}\left(k^2,Q_0^2;\{\xi_d(x)\}\right)$$

$$+\,G_{q_i}\left(k^2,Q_0^2;\{\xi_d(xz)\}\right)G_g\left((1-z)k^2,Q_0^2;\{\xi_d(x(1-z))\}\right)\Big]$$

$$+\int_{\varepsilon(k)}^{1-\varepsilon(k)} \frac{2C_F dz}{z}\Big[-G_{q_i}\left(k^2,Q_0^2;\{\xi_d(x)\}\right)+G_g\left(zk^2,Q_0^2;\{\xi_d(xz)\}\right)G_{q_i}\left(k^2,Q_0^2;\{\xi_d(x(1-z))\}\right)\Big] \qquad (3.2)$$

and

$$\frac{4\pi k^2}{\alpha(k^2)} \frac{d}{dk^2} G_g(k^2,Q_0^2;\{\xi_d(x)\}) = G_g(k^2,Q_0^2;\{\xi_d(x)\})[+\frac{11}{3} N_c - \frac{N_f}{3}]$$

$$+ \int_\varepsilon^{1-\varepsilon} 2C_A dz[-2+z-z^2]G_g(k^2,Q_0^2;\{\xi_d(xz)\})G_g(k^2,Q_0^2;\{\xi_d(x(1-z))\})$$

$$+ \int_\varepsilon^{1-\varepsilon} dz \frac{[2z^2-2z+1]}{2} \sum_{q_i} G_{q_i}(k^2,Q_0^2;\{\xi_d(xz)\})G_{\bar{q}_i}(k^2,Q_0^2;\{\xi_d(x(1-z))\})$$

$$+ \int_\varepsilon^{1-\varepsilon} \frac{2C_A dz}{z}[-G_g(k^2,Q_0^2;\{\xi_d(x)\})+G_g(zk^2,Q_0^2;\{\xi_d(xz)\})G_g(k^2,Q_0^2; \{\xi_d(x(1-z))\})]$$

$$+ \int_\varepsilon^{1-\varepsilon} \frac{2C_A}{1-z} dz \ [-G_g(k^2,Q_0^2;\{\xi_d(x)\})+G_g(k^2,Q_0^2;\{\xi_d(xz)\})$$

$$* \ G_g((1-z)k^2,Q_0^2;\{\xi_d(x(1-z))\})] \qquad (3.3)$$

If we now ask for the probability that gluons go only to gluons, this is the same as inserting $\xi = 1$ for all the gluons in these equations, and $\xi = 0$ for all the quarks. This then gives the equation for $\sigma_g(k^2,Q_0^2)$, the probability that gluons go only to gluons:

$$\frac{\tau\partial\sigma}{\partial\tau}(\tau,\tau_0) = \sigma_g(\tau,\tau_0) \ [-\frac{C_A}{\pi b}\int_{\tau_o}^{\tau}[1-\sigma(\tau')]d\tau' + (1-\sigma_g)] - \delta\sigma^2 + \frac{N_f\sigma}{12\pi b} \quad (3.4)$$

with $\tau = \ln(k^2/\Lambda^2)$; $\delta = \frac{N_f}{6\pi b}$ $b = \frac{11N_c-2N_f}{12\pi}$; $C_A = N_c$; and Λ is defined by $\alpha_s = \frac{1}{b\tau}$.

This differs somewhat from Eq. (3.11) of Ref. 1 but they agree in the $N_c \to \infty$ limit.

We now wish to compute the color connected propagators $\Gamma^q(x)$. We define Γ^q by saying that we have a tree (see Fig. 3) and we count clockwise around the branches of the tree, starting with the trunk (particle incident from the left) until we come to the first non-gluon. This is the object whose x is labelled in Fig. 3; the kind of object is the upper label q or $\bar{q}$.

Offhand, therefore, one might think one could have $\Gamma_g^{q_i}$, $\Gamma_{q_i}^{q_j}$, $\Gamma_{q_i}^{\bar{q}_j}$, or $\Gamma_g^{\bar{q}_i}$ (the i,j indices label flavor of the quarks). However there are no $\Gamma_{q_i}^{\bar{q}_i}$ or $\Gamma_{g_u}^{\bar{q}_i}$. This can be seen by drawing a typical planar diagram such as Fig. 3. We see that the requirement of planarity will force the first non-gluon coming clockwise off a quark to be a quark - not an antiquark. Furthermore, the first non-gluon coming off an "upper" gluon is a quark, whereas the first non-gluon coming off a "lower" gluon is an antiquark.

We now write the generating functional for the planar graphs in the form

$$G_a(Q^2, Q_0^2; \{U_d(x)\}) = \sum_{\{c_i\}} \int dx_1 \ldots \int dx_n U_{c_1}(x_1) \ldots U_{c_n}(x_n) \\ * E_a^{c_1 \ldots c_n}(Q^2, Q_0^2; x_1 \ldots x_n) \tag{3.5}$$

where $E_a^{c_1 \ldots c_n}$ is the <u>exclusive</u> probability to see c_1 at x_1, c_2 at x_2, and c_n at x_n. Note that because we are insisting on planar graphs, the order $c_1 \ldots c_n$ matters. Define δ^q to be a variation which searches out the first U which is a quark U

$$\left. \frac{\delta^q G_a(Q^2, Q_0^2; \{U\})}{\delta U(x)} \right|_{\substack{\text{all other} \\ U\text{'s} = 1}} = \Gamma_a^q \left(Q^2, Q_0^2; x\right) \tag{3.6}$$

Acting with this variation on Eqs. (3.2) and (3.3) will either pick the "first" quark out of the "upper" leg, or out of the "lower" leg after the bifurcation. Using the graphical technique of BCM, we therefore expect the equations to take the form depicted in Fig. 4.

Note that BCM make a special equation for ϕ, which is for the case where the flavor carrying quark coming out is the same one as came in. This is not necessary; $\Gamma_i^i \neq \Gamma_i^j$ due to different starting conditions at $Q^2 = Q_0^2$. We will use Γ_i^i if we need the same flavor out as in.

Noting that

$$\frac{\delta^q}{\delta u(x)} G_a\big(\ldots U(xz)\big) = \frac{1}{z}\,\Gamma_a\big(\tfrac{x}{z}\big)\ , \tag{3.7}$$

we obtain the equations for general x:

$$\begin{aligned}\frac{4\pi k^2}{\alpha(k^2)}\frac{d}{dk^2}\Gamma_i^j(k^2,x) &= 3C_F\Gamma_i^j(k^2,x)+C_F\int_\varepsilon^{1-\varepsilon}\Big\{\frac{(-z-1)}{z}\,\Gamma_i^j\big(k^2,\tfrac{x}{z}\big)+\big(\tfrac{z-2}{z}\big)\Gamma_g^j\big(k^2,\tfrac{x}{z}\big)\\ &+\big(\tfrac{z-2}{1-z}\big)\sigma(k^2)\Gamma_i^j\big(k^2,\tfrac{x}{1-z}\big)\Big\} + 2C_f\int_\varepsilon^{1-\varepsilon}\frac{dz}{1-z}\Big[-\Gamma_i^j(k^2,x)+\frac{1}{z}\,\Gamma_i^j\big(k^2,\tfrac{x}{z}\big)\Big]\\ &+2C_F\int_\varepsilon^{1-\varepsilon}\frac{dz}{z}\Big[-\Gamma_i^j(k^2,x)+\frac{1}{z}\,\Gamma_g^j\big(zk^2,\ \tfrac{x}{z}\big)+\frac{\sigma(zk^2)}{1-z}\,\Gamma_i^j\big(k^2,\ \tfrac{x}{1-z}\big)\Big]\end{aligned} \tag{3.8}$$

$$\begin{aligned}\frac{4\pi k^2}{\alpha(k^2)}\frac{d}{dk^2}\Gamma_g^j(k^2,x) &= \Gamma_g^j(k^2,x)\Big[\frac{11N_c}{3}-\frac{N_f}{3}\Big]+2C_A\int_\varepsilon^{1-\varepsilon}dz\Big\{\frac{1}{z}\,\Gamma_g^j\big(k^2,\tfrac{x}{z}\big)+\\ &+\frac{\sigma(k^2)}{1-z}\,\Gamma_g^j\big(k^2,\ \tfrac{x}{1-z}\big)\Big\}\big[-2+z-z^2\big]+\int_\varepsilon^{1-\varepsilon}dz\,\frac{[2z^2-2z+1]}{z}\sum_i\frac{\Gamma_i^j(k^2,\ \tfrac{x}{z})}{z}\\ &+\int_\varepsilon^{1-\varepsilon}2C_A\frac{dz}{z}\Big[-\Gamma_g^j(k^2,x)+\frac{1}{z}\,\Gamma_g^j\big(zk^2,\ \tfrac{x}{z}\big)+\frac{\sigma(zk^2)}{1-z}\,\Gamma_g^j\big(k^2,\ \tfrac{x}{1-z}\big)\Big]\\ &+\int_\varepsilon^{1-\varepsilon}\frac{2C_A dz}{1-z}\Big[-\Gamma_g^j(k^2,x)+\frac{1}{z}\,\Gamma_g^j\big(k^2,\ \tfrac{x}{z}\big)+\frac{\sigma(k^2)}{1-z}\,\Gamma_g^j\big((1-z)k^2,\ \tfrac{x}{1-z}\big)\Big]\end{aligned} \tag{3.9}$$

or alternately,

$$\frac{4\pi k^2}{\alpha(k^2)} \frac{d}{dk^2} \Gamma_i^j(k^2,x) = \frac{3}{2} C_F \Gamma_i^j(k^2,x) - 2C_F\left(\int^{\tau} d\tau'\right)\Gamma_i^j(k^2,x)$$

$$+ \int \frac{dz}{z} \Gamma_i^j\left(\lambda(z)k^2, \frac{x}{z}\right) P_q^{qg}(z) + \int \frac{dz}{z} P_q^{qg}(z)\ \Gamma_g^j\left(\lambda(z)k^2, \frac{x}{z}\right)$$

$$\int dz \left[\frac{1+z^2}{1-z} \right] C_F \frac{\sigma_g(\lambda(1-z)k^2)}{z} \Gamma_i^j\left(\lambda(z)k^2, \frac{x}{z}\right) \tag{3.10}$$

$$\frac{4\pi k^2}{\alpha} \frac{d}{dk^2} \Gamma_g^j(k^2,x) = - \Gamma_g^j(k^2, x)\left[- \frac{11}{6} C_A + 2C_A \int^{\tau} d\tau'\right]$$

$$+ \int \frac{dz}{z} \Gamma_g^j\left(\lambda(z)k^2, \frac{x}{z}\right) P_g^{gg}(z) + \int dz\ \hat{P}_g^{gg}(z)\sigma_g(\lambda(z)k^2)\ \frac{\Gamma_g^j\left(\lambda(1-z)k^2, \frac{x}{1-z}\right)}{1-z}$$

$$+ \frac{1}{N_f} \sum_i \int \frac{dz}{z} \Gamma_i^j\left(\lambda(z)k^2, \frac{x}{z}\right) P_g^{q\bar{q}}(z) \tag{3.11}$$

Here we have used the same convention for $\lambda(z)$ as do BCM.

IV. MOMENT EQUATIONS

We now consider the zero moments of Eqs. (3.8) and (3.9). These give [defining $S_Q(k^2) = \sum_j \int \Gamma_i^j(k^2,x)dx$, $S_G(k^2) = \sum_j \int \Gamma_g^j(k^2,x)dx$]

$$\frac{4\pi k^2}{\alpha}\frac{d}{dk^2} S_Q(k^2) = -\frac{3}{2} C_F\{S_G(k^2) - S_Q(k^2)[1-\sigma(k^2)]\}$$

$$+ 2\, C_F \int_\varepsilon^{1-\varepsilon} \frac{dz}{z} \{S_G(zk^2)-S_Q(k^2)[1-\sigma(zk^2)]\} \tag{4.1}$$

$$\frac{4\pi k^2}{\alpha}\frac{d}{dk^2} S_G(k^2) = S_G(k^2)\left[\frac{11N_C}{3} - \frac{N_f}{3}\right] - \frac{11}{3} C_A S_G(k^2)[1+\sigma(k^2)] + \frac{1}{3} S_Q N_f$$

$$+ 2\, C_A \int_\varepsilon^{1-\varepsilon} \frac{dz}{z}\left[-S_G(k^2) + S_G(zk^2) + \sigma(zk^2)\, S_G(k^2) + \sigma(k^2)\, S_G(zk^2)\right] \tag{4.2}$$

It is tempting to guess that

$$\int \sum_j \Gamma_i^j(x)dx = S_Q(k^2) = S_Q = \text{const}\ [= 1, \text{ by evaluation at } k^2 = Q_0^2] \tag{4.3}$$

since every quark coming in has some "first" quark coupled to it. If we then have

$$S_G(k^2) = S_Q[1 - \sigma(k^2)] \tag{4.4}$$

the first equation, (4.1), will be identically satisfied.

Substituting (4.3) and (4.4) into the second equation, we find exactly the equation for $\sigma(k^2)$, (3.4). The sum rules of BCM, Eq. (4.2) of Ref. 1, therefore, do hold for proper summation over the final index.

We now take higher moments of Eqs. (3.10) and (3.11) and find the moment equations

$$\tau \frac{\partial}{\partial\tau} \Gamma_i^j(\tau, n) = -\frac{C_F}{2\pi b} \Gamma_i^j(\tau, n) \int^{\tau} d\tau' \left[1-\sigma_g(\tau')\right]$$

$$+ \frac{3}{8\pi b} C_F \Gamma_i^j(\tau, n)(1-\sigma_g(\tau))$$

$$+ \Gamma_i^j(\tau, n) \frac{A_q^{qg}(n)\left[1+\sigma_g(\tau)\right]}{4\pi b} + \frac{A_q^{qg}(n)\Gamma_g^j(\tau, n)}{4\pi b} \tag{4.5}$$

$$\tau \frac{\partial}{\partial\tau} \Gamma_g^j(\tau, n) = -\frac{C_A}{2\pi b} \Gamma_g^j \int^{\tau} d\tau' \left[1-\sigma_g(\tau')\right]$$

$$+ \frac{1}{2} \Gamma_g^j (\tau, n) \left[1-\sigma_g(\tau) + \frac{(1+\sigma_g(\tau))A_g^{gg}(n)}{2\pi b}\right] + \frac{1}{4\pi b N_f} A_g^{q\bar{q}}(n) \sum_i \Gamma_i^j(\tau, n)$$

$$+ \frac{N_f}{12\pi b} \Gamma_g^j (\tau, n) \tag{4.6}$$

These also differ somewhat from Eqs. (4.3) of BCM.

V. FORMATION OF COLOR SINGLETS

There are a number of minor differences between our equations and those of BCM; however only two of these are likely to have major consequences.

First, the dominant terms as $Q^2 \to \infty$ in the right hand side of Eq. (4.5) are smaller by a factor of two than those in Eq. (4.3) of Ref. 1. We don't really understand this, since we believe they should agree. Since an overall factor on the right hand side of Eq. (4.1) will not affect the sum rule derivation, we have been unable to find an independent way of testing this.

The second major consequence arises from the inclusion of the second term on the right hand side of the Γ_i^j equation as shown in Fig. 4. This shows up as the term

$$\Gamma_i^j(\tau, n)\ A_q^{qg}(n)/4\pi b$$

in Eq. (4.5). This will create graphs in the Γ_i^j propagator like those in Fig. 5a; to obtain a "color connected" propagator all the particles inside the dotted line must be included. Some of these are emitted toward the bottom of the page. These may include additional $Q\bar{Q}$ pairs whose x is not "measured". This is in contrast to the BCM method, which includes only graphs like those shown in Fig. 5b.

While the BCM method is simpler, we think that it does not include all the possible planar graphs for SU(n). Inclusion of the extra term makes life less beautiful; however, as we show below it does not ruin the major result.

We now form mesonic color singlets following BCM. Samples of the types of graph to be included are displayed in Fig. 6. As in their case, only the $q \to qg$, $g \to gg$, and $q \to gq$ vertices are allowed; the $g \to q\bar{q}$ vertex will not give a color singlet with their construction.

VI. NUMERICAL RESULTS

We restrict ourselves here to presentation of a few illustrative results; the applications of the formalism which were our original goal will be discussed in another paper.

In Figure 7 we show the solution $\sigma_g(\tau)$ to equation (3.4), the probability that gluons go only to gluons. Note the rapid damping as $Q^2 \to \infty$.

Similar damping is seen for the propagators Γ^q. In Figs. 8, 9, and 10 we compare various propagators Γ^q with the corresponding "ordinary" propagators D^q computed using the Altarelli-Parisi-Owens equations. Figure 8 is perhaps the most illustrative of the damping created by the semi-inclusive definition of the Γ propagators. We see that Γ_i^i and D_i^i have the same values for small τ (in fact all moments start at 1 at $\tau = \tau_o$), but that all the moments of Γ_i^i drop rapidly at large Q^2. Similar effects can be seen in Figs. 9 and 10; of course since these Γ and D propagators start at 0 at $\tau = \tau_o$, they are not required to have the same values at other small Q^2.

It is this damping at large Q which restricts the mass of produced colorless clusters to a finite value. Followed BCM, we write the cross sections for production of $q(x_2)$ and $\overline{q}(x_1)$ in colorless clusters such as those in Fig 6 as

$$\left.\frac{k^2 d\sigma}{\sigma dk^2 dx_1 dx_2}\right|_{c.s.} = \frac{\alpha(k^2)}{2\pi} \sum_c{}' \sum_{c_1 c_2} \int \frac{dx}{x^2} D_q^c(Q^2, k^2 x)$$
$$\int \frac{dz}{z(1-z)} \hat{P}_c^{c_1 c_2}(z) \Gamma_{c_1}^{\overline{q}} \left(\lambda(z)k^2, Q_0^2; \frac{x_1}{xz}\right) \Gamma_{c_2}^{q} \left(\lambda(1-z)k^2, Q_0^2; \frac{x_2}{x(1-z)}\right) \quad (6.1)$$

An approximation for the mass of the colorless clusters M^2 can be obtained as follows:

First we note that if a partion of mass k^2 decays to a colorless cluster of momentum fraction W and some other system of p'^2 with momentum fraction 1-W, with the two systems having perpendicular momentum $p_\perp$ relative to the direction of k, then in the infinite momentum frame

$$k^2 > \frac{M^2+p^2_\perp}{W} + \frac{p'^2+p^2_\perp}{1-W}$$

Now in Eq. (6.1) the parton of momentum k carries momentum fraction x of the initial large momentum in the jet, so that $W = X/x$, where X is the momentum fraction of the jet carried by the colorless cluster. It is also true that $X > x_1 + x_2$, so the relation

$$k^2 > \frac{M^2 X}{x_1+x_2}$$

is reasonable.

To estimate the color singlet mass spectrum, BCM then substitute the boundaries $k^2 = \frac{M^2 X}{x_1+x_2}$ and $X = x_1 + x_2$ into Eq. (6.1) to obtain

$$\frac{1}{\sigma} M^2 \frac{d\sigma}{dM^2 dX} = \frac{M^2}{\sigma} \int \int dx_1 dx_2 \; \delta(x_1 + x_2 - X) \frac{d\sigma}{dM^2 dx_1 dx_2}$$

The next problem is then to estimate the integral over the Γ propagators,

Again following the lead of BCM, we return to our equations (3.10) and (3.11). We define the functions

$$x\Gamma(k^2, Q_0^2, x) = M(k^2 Q_0^2, x)$$

and obtain equations for them by multiplying (3.10) and (3.11) through by x.

With the hypothesis that $M(k^2,x) = \Delta(k^2x)$, the equations simplify considerably. In fact if we take the limit $k^2 \to \infty$ with kx fixed, the functions $\Delta(k^2x)$ then become the solutions of extremely simple equations

$$
\begin{aligned}
y \frac{d}{dy} \Delta_i^j(y) &= - \Delta_i^j(y) \frac{C_F}{2\pi b} \\
y \frac{d}{dy} \Delta_g^i(y) &= - \Delta_g^i (y) \frac{C_A}{2\pi b}
\end{aligned}
\tag{6.2}
$$

with the solutions

$$
\begin{aligned}
\Gamma_i^j &\sim \frac{1}{x} \left(\frac{1}{k^2x}\right)^{C_F/2\pi b} \\
\Gamma_g^i &\sim \frac{1}{x} \left(\frac{1}{k^2x}\right)^{C_A/2\pi b}
\end{aligned}
\tag{6.3}
$$

This is similar to the behavior found by BCM, except that the exponent $\frac{C_F}{2\pi b}$ for the quark propagator Γ_i^j is smaller than theirs by a factor of 2. This does not affect their conclusion about the damping of the mass clusters: Eq. (6.1) becomes

$$
\begin{aligned}
\frac{M^2 d\sigma}{\sigma dM^2 dX} &= \frac{\alpha(M^2)}{2\pi X} \sum_c{}' \int_X^1 dx\, D_q^c \left(Q^2, \frac{M^2x}{X}, x\right) \\
&\sum_{c_1c_2} \int_\varepsilon^{1-\varepsilon} \frac{dW}{W(1-W)} \Delta_{c_1}^{\bar{q}} \left(M^2W, Q_0^2\right) \Delta_{c_2}^q \left(M^2(1-W), Q_0^2\right) \\
&\int_{XW/x}^{1-X(1-W)/x} dz\, \hat{P}_c^{c_1c_2} (z)
\end{aligned}
\tag{6.4}
$$

and the large M^2 behavior of Δ damps the result as before.

ACKNOWLEDGMENTS

This work was supported in part by NSF PHY79-00272. K. E. Lassila would like to thank Fermilab for their hospitality.

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FIGURE CAPTIONS

1) Representation of basic particles in our graphs. The particles are proceeding from left to right across the page.
2) The only planar graph for the splitting of an "upper" gluon into a quark-antiquark pair.
3) Sample graphs for the color connected propagators.
4) Graphical depiction of equations (3.9) and (3.8). The box depicts the virtual potential; solid circles depict all possible QCD happenings; half circles with lines on both sides stand for the color connected propagators Γ; and the open circle stands for the probability, σ_g, that gluons go only to gluons.

5) Sample graphs for the propagators Γ. BCM include only graphs like 5b); our equations also have graphs like 5a.

6) Sample graphs contained in the sum in Eq. (6.1). The "bubble" encloses a color singlet.

7) The probability that gluons go only to gluons

8) Comparison of propagators for quark $\rightarrow$ quark (same flavor) for moments $n = 1$, 6 and 21.

9) Comparison of propagators for quark $\rightarrow$ quark (different flavor) for moments $n = 1$,6 and 21.

10) Comparison of propagators for gluon $\rightarrow$ quark for moments $n = 1$, 6 and 21.

Fig. 1

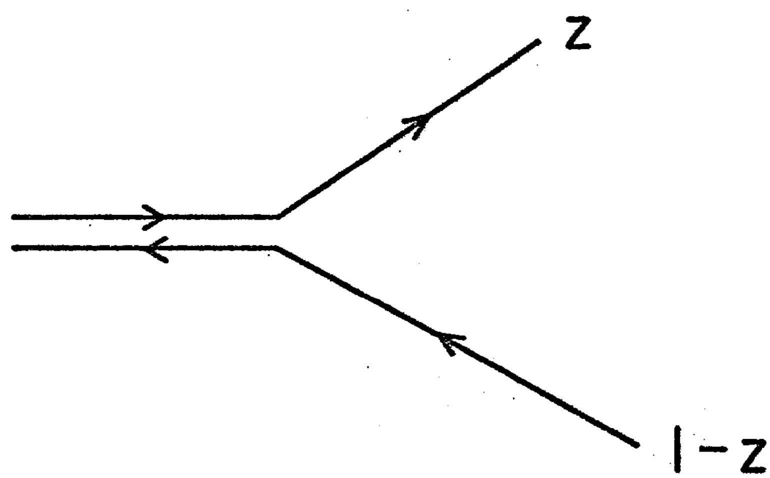


Fig. 2

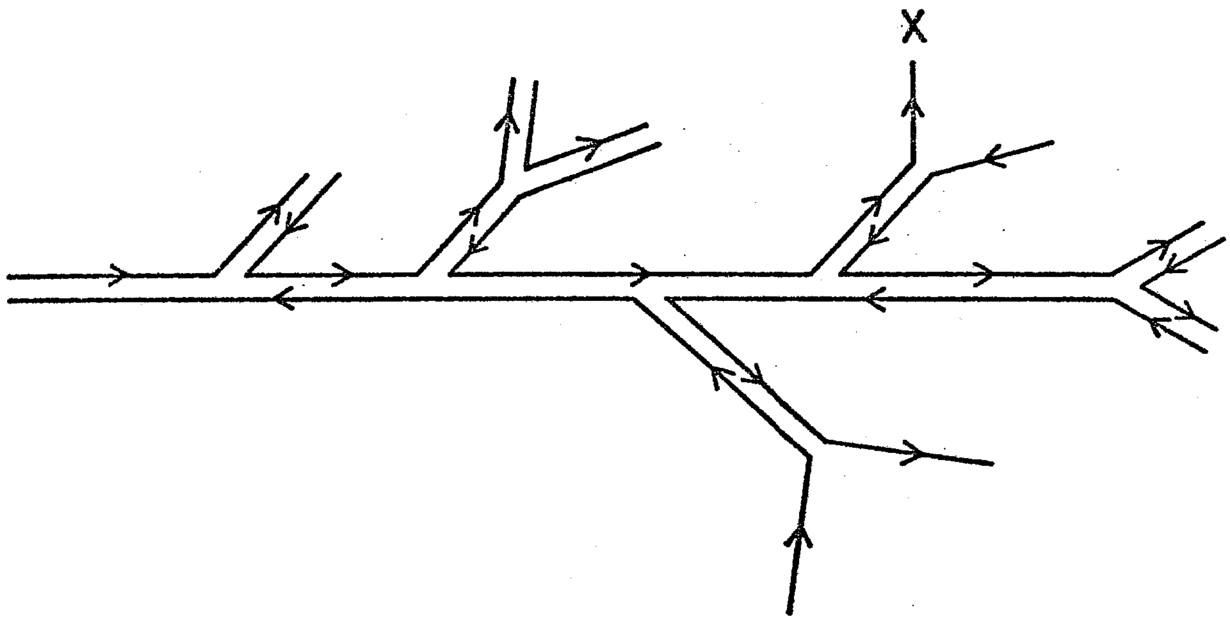


Fig. 3a

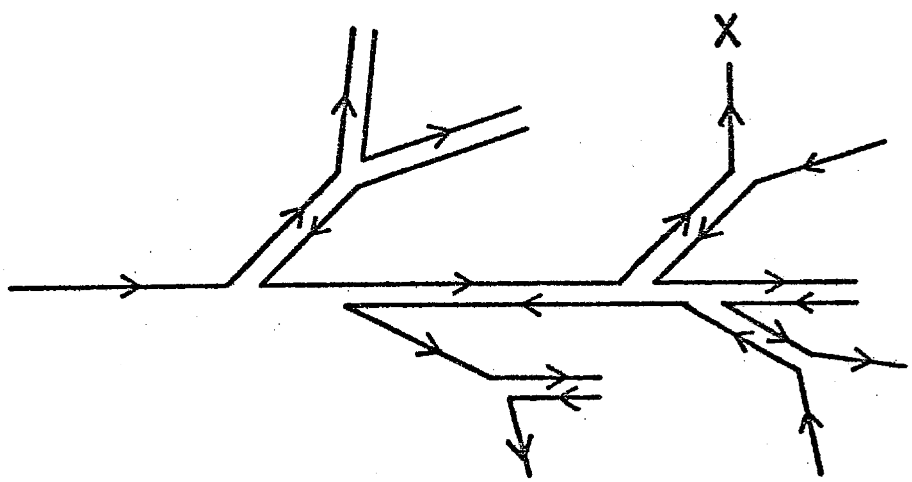


Fig. 3b

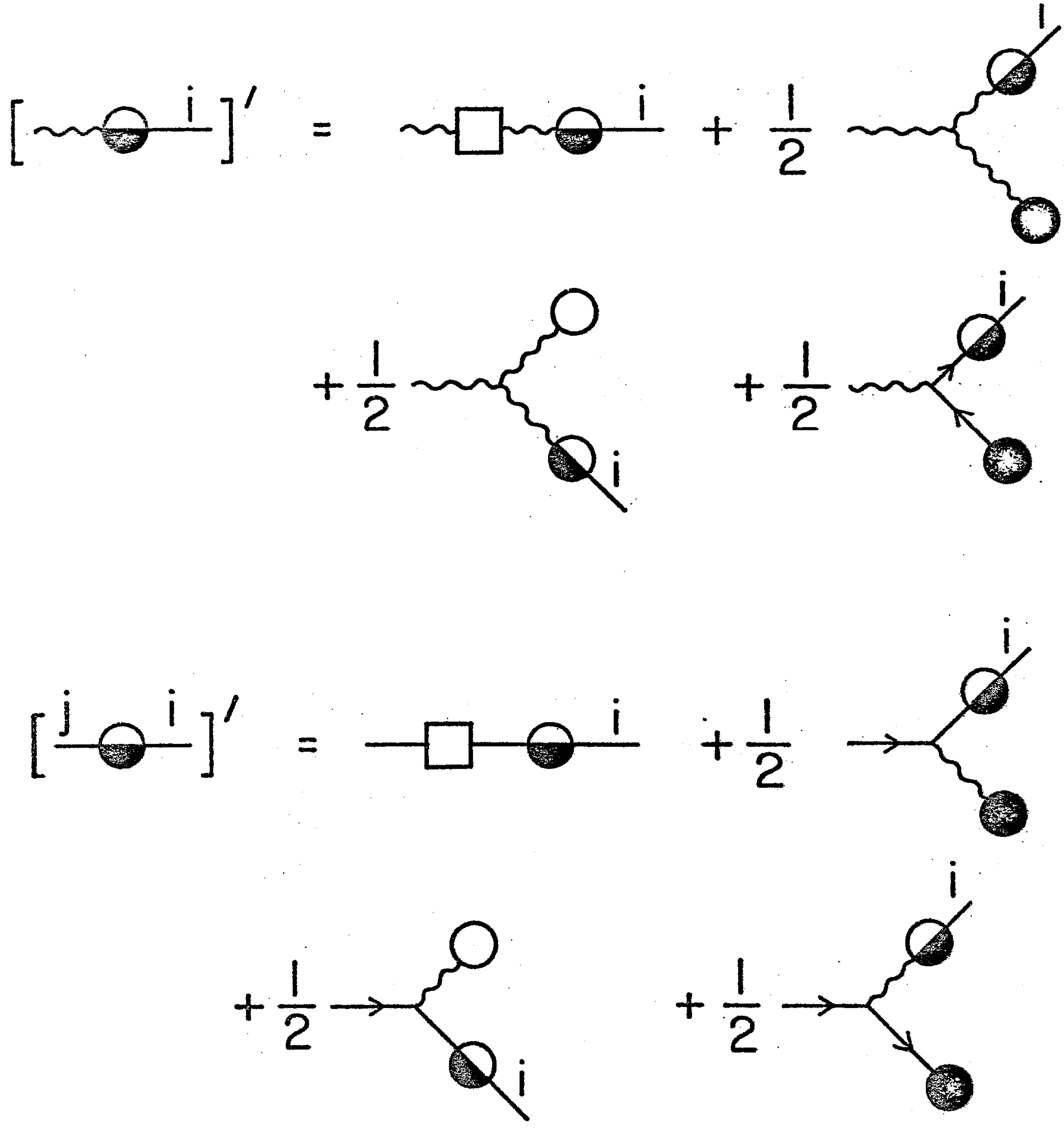


Fig. 4

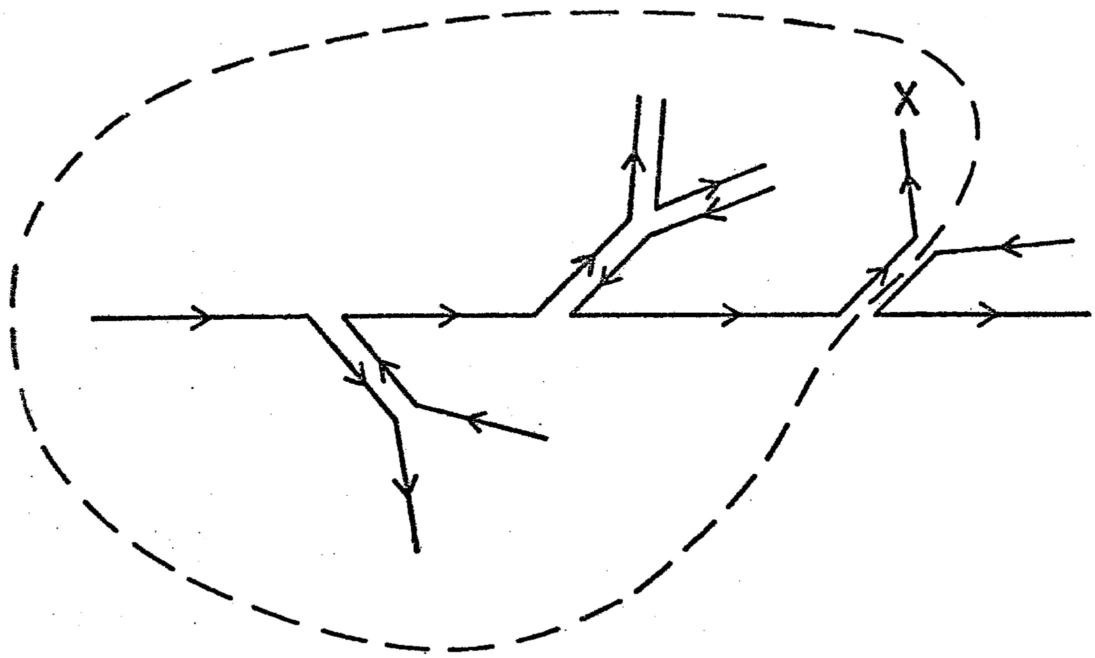


Fig. 5a

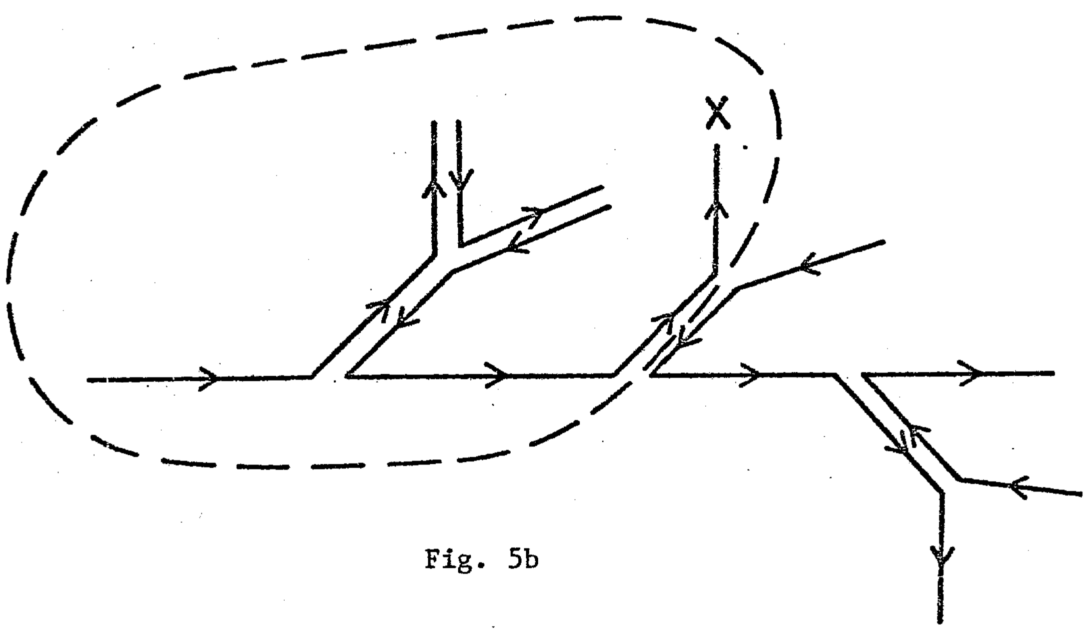


Fig. 5b

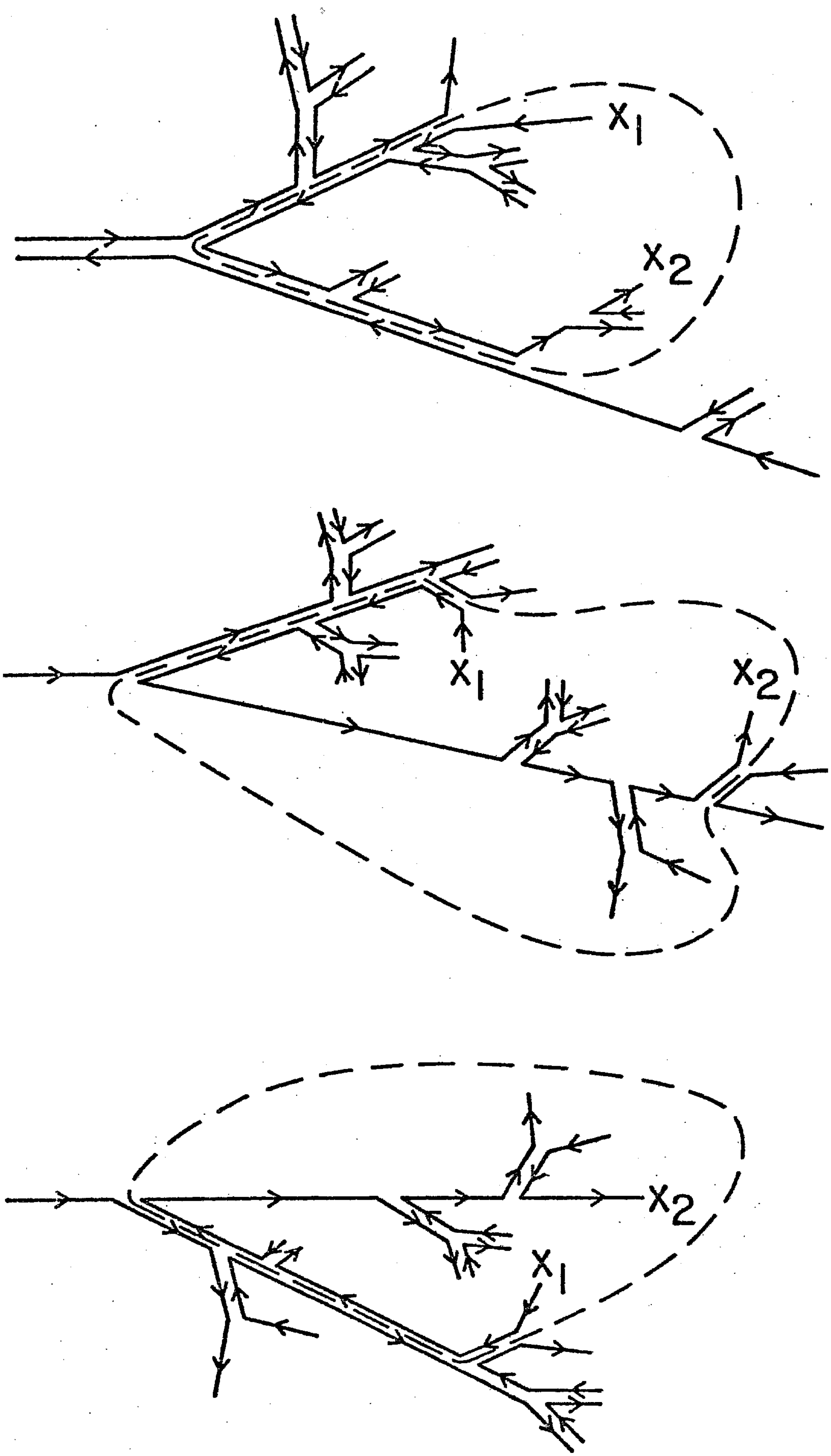


Fig. 6

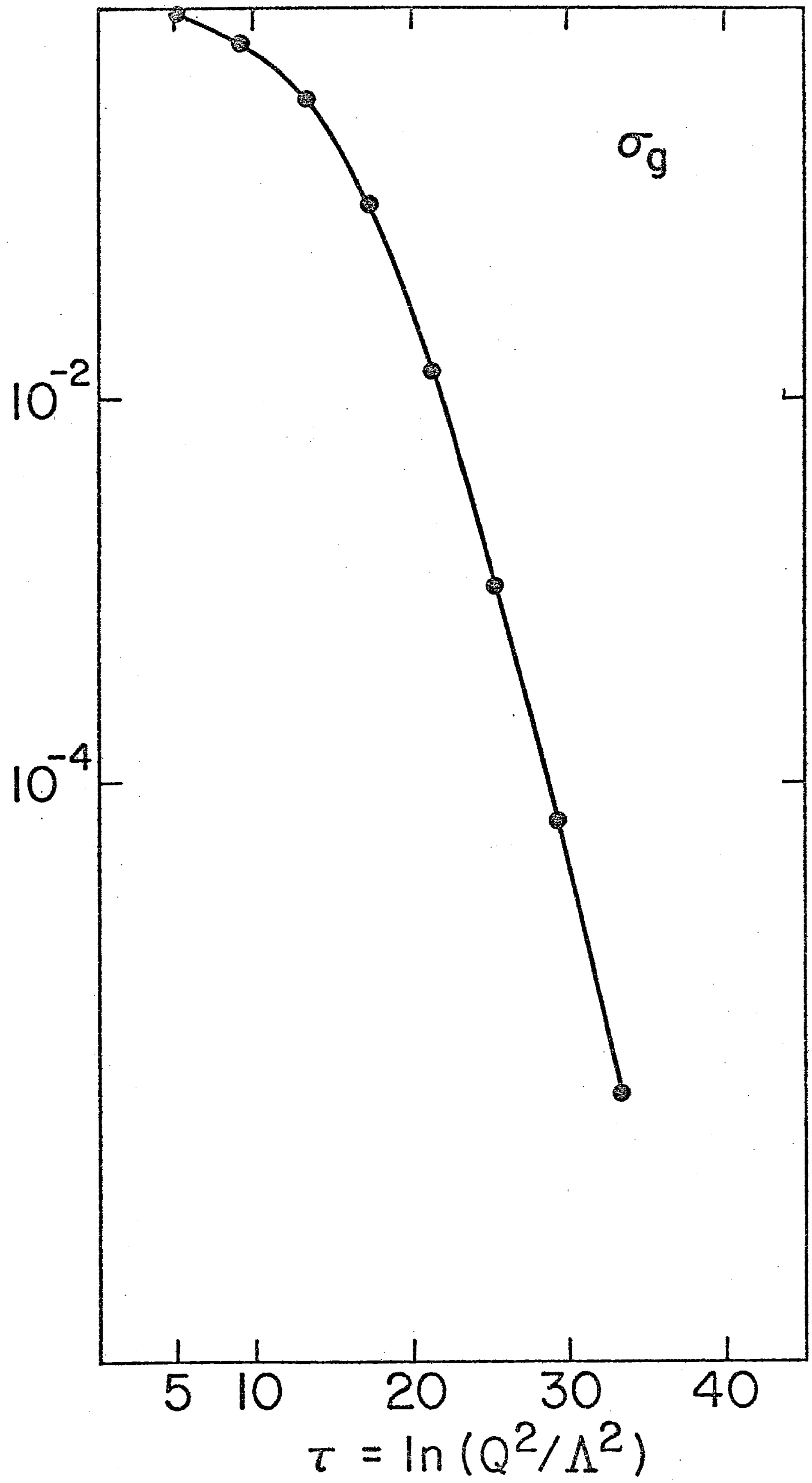


Fig. 7

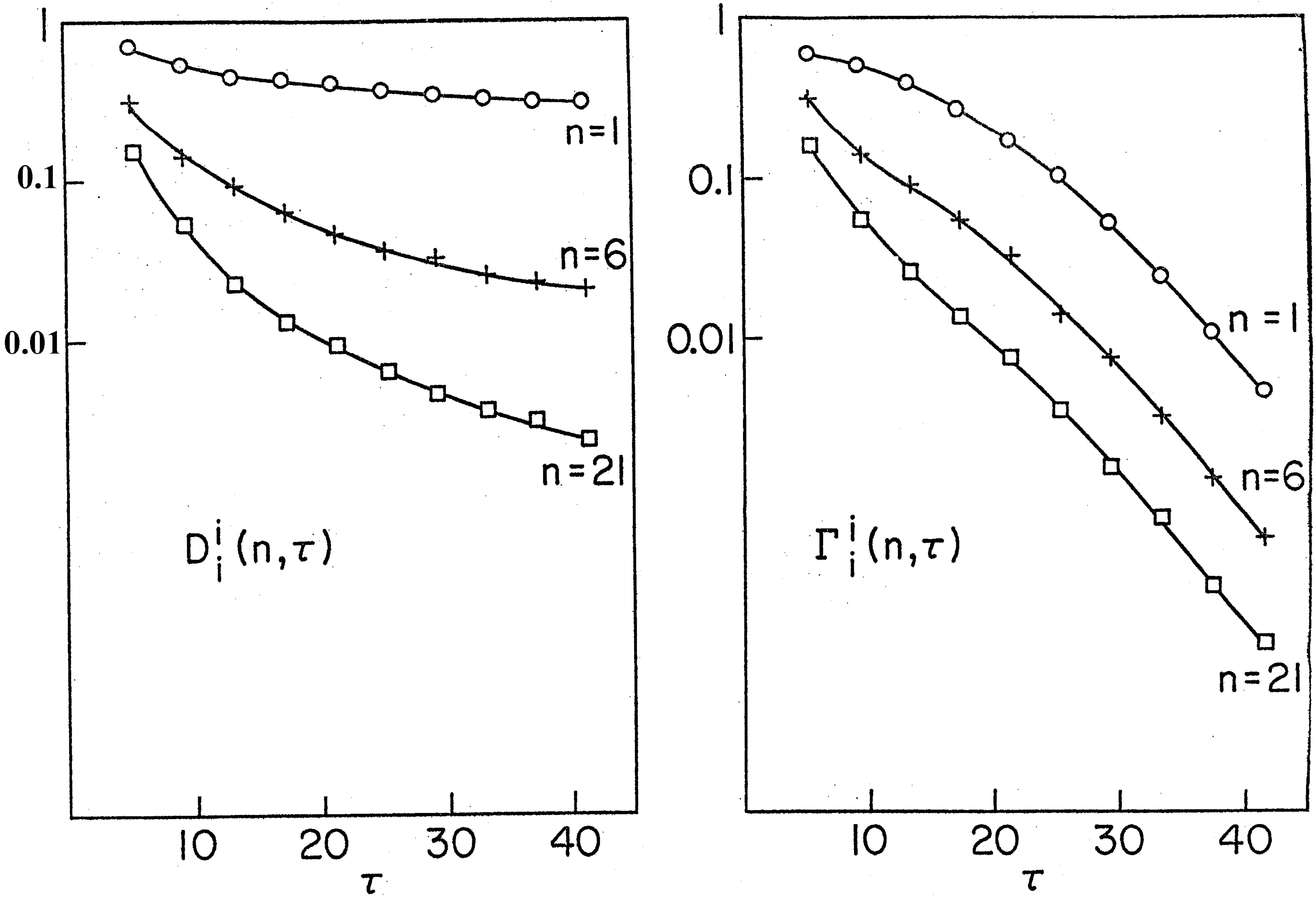
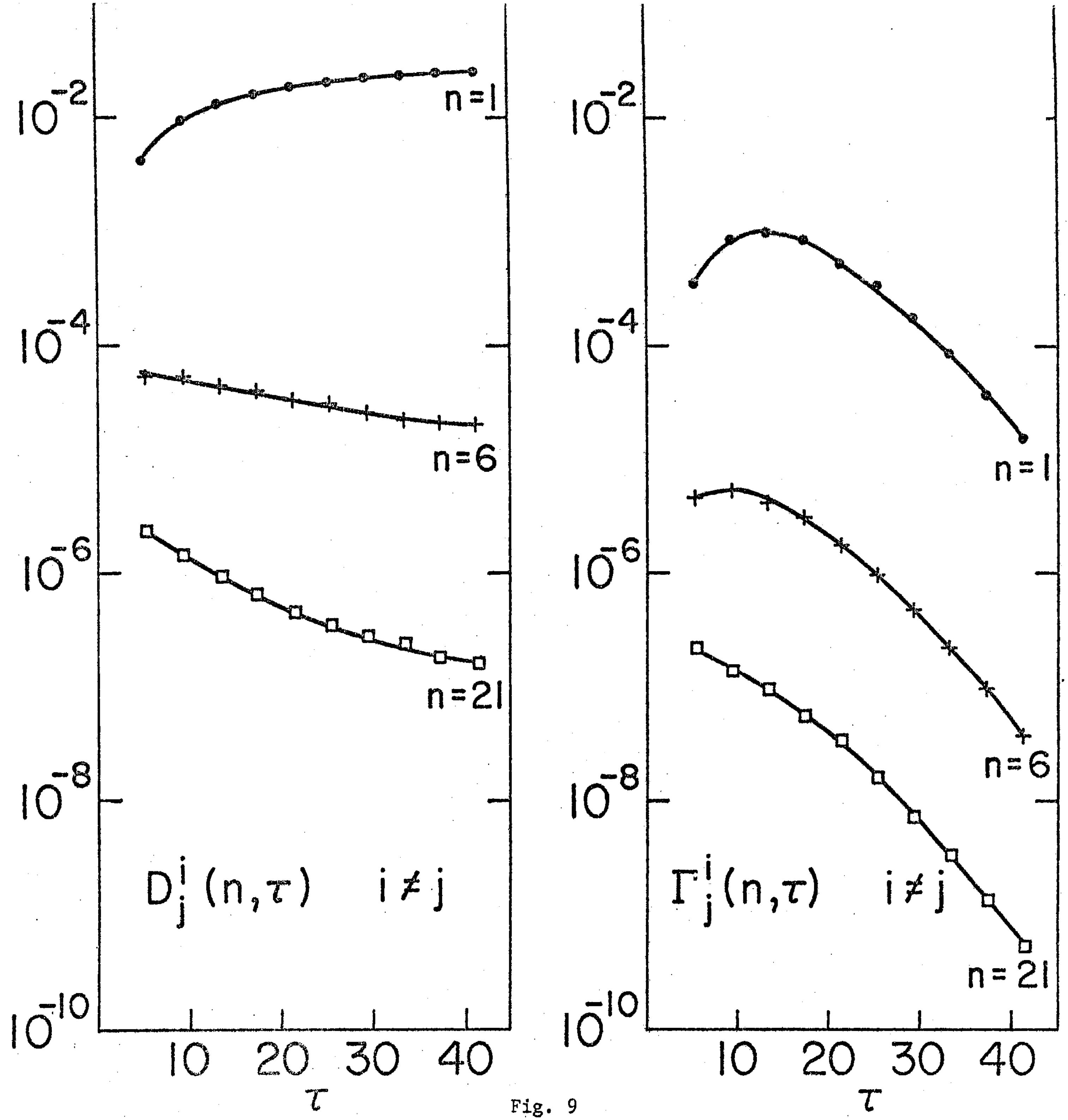

1
0.1
0.01
n=1
n=6
n=21
$D^{i}_{i}(n,\tau)$
10
20
30
40
τ
1
0.1
0.01
n = 1
n=6
n=21
$\Gamma^{i}_{i}(n,\tau)$
10
20
30
40
τ

Fig. 8



10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10
20
30
40
τ
$n=1$
$n=6$
$n=21$
$D^{i}_{j}(n,\tau) \quad i \neq j$
$\Gamma^{i}_{j}(n,\tau) \quad i \neq j$

Fig. 9

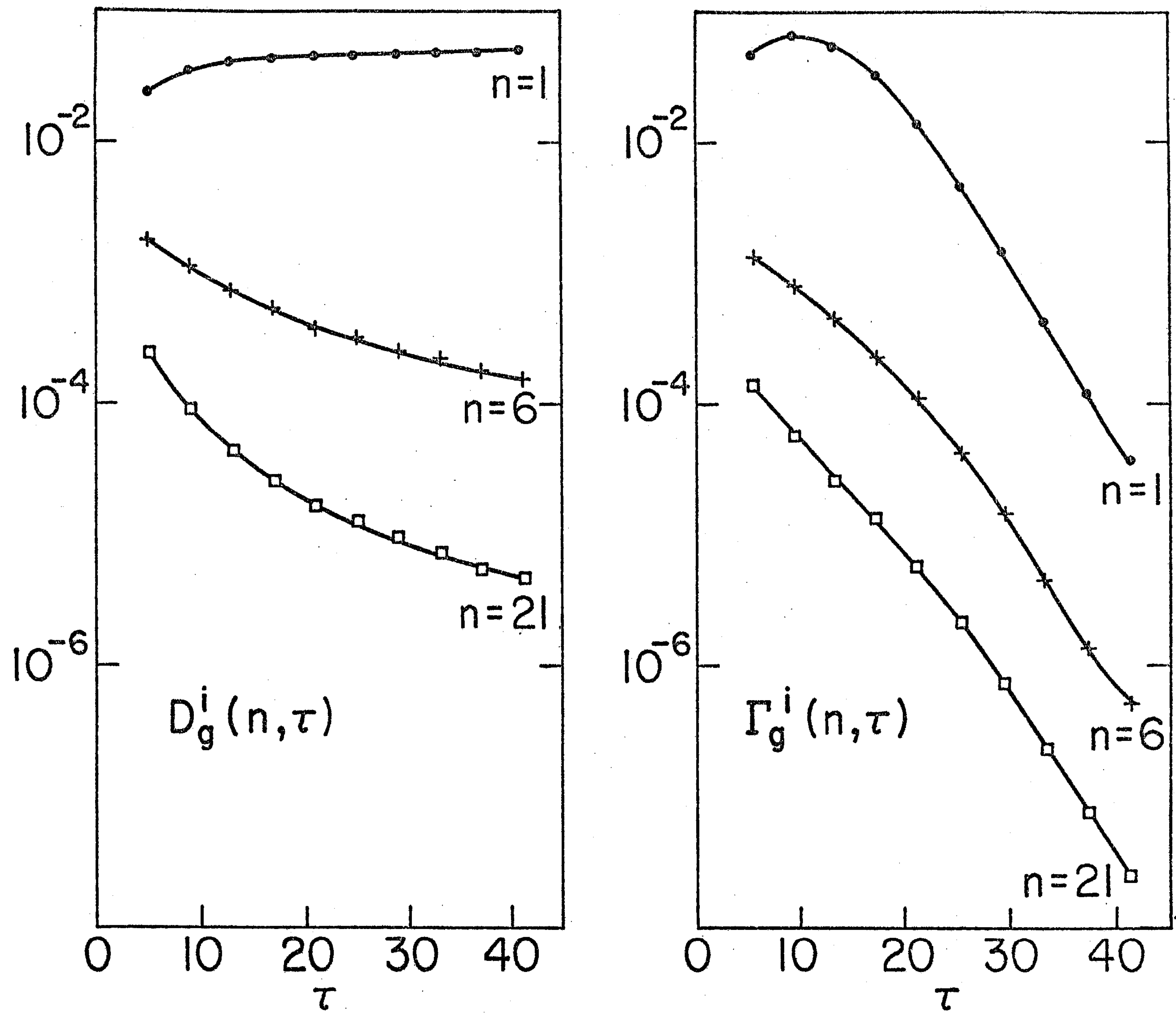


Fig. 10