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Hyperfine Splittings in Heavy Quark Systems

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ABSTRACT

The hyperfine splittings in heavy quark systems are calculated to 4th order in the strong coupling constant. The hypothesis, that spin-spin interactions are dominated by short distances, is shown to be self-consistent. The $\Psi - \eta_c$ mass difference is compatible with the range of the QCD scale parameter Λ expected on the basis of QCD-like potential models. From the $\Upsilon - \eta_b$ mass difference we expect a determination of Λ with an uncertainty of about $\pm$ 100 MeV.

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Non-relativistic bound state spectroscopies (such as hydrogen atom, positronium etc.) have provided invaluable tools to guide us in our understanding of quantum mechanics and quantum electrodynamics. They also allow us to determine accurately the fine structure constant and the electron mass. Non-relativistic bound systems are even more important in quantum chromodynamics (QCD) since there are no free quarks or free gluons upon which direct measurements can be performed. Although our present understanding of the large distance behavior of QCD is still far from complete, quarkonia (i.e. heavy quark-antiquark bound systems) can provide an accurate determination of the scale parameter Λ and heavy quark masses in QCD.[1-3] This is possible only after care is taken to separate the short distance effects from the large distance effects. We argue that this can be achieved for the hyperfine splitting. Here we present the result of the complete one-loop calculation for the hyperfine splitting of heavy quark systems. The result is then applied to the Ψ and Υ spectroscopies.

To lowest order in perturbative QCD, the hyperfine splitting[4] is given by

$$H_{HFS} = \frac{8\pi}{3} \frac{\alpha_s}{m^2} \vec{s}_1 \cdot \vec{s}_2 \; C_2(R) \; |\phi(0)|^2 \qquad , \tag{1}$$

where $\alpha_s = g^2/(4\pi)$ and m is the quark mass; $\vec{s}_1$ and $\vec{s}_2$ are the spins of the quark and the antiquark. The group factor $C_2(R) = 4/3$. The splitting is proportional to the bound state wavefunction at the origin. Thus the spin force, which is responsible for the hyperfine splitting, is short-ranged. As we shall see, this force remains short-ranged even after the one-loop corrections have been incorporated. For the wave function at short distances we need the quarkonium potential which, however, can be accurately determined via a phenomenological approach.[2,5] Hence we believe that perturbative QCD is applicable to calculate the hyperfine splitting.

This is in contrast to the fine structure (e.g. the splitting of the P states), where the responsible spin-dependent force extends to large distances. Consequently, the fine structure is more sensitive to the nature of the confining force and the result derived from perturbative QCD may be unreliable in this case.

The calculation is carried out in two steps. We first calculate, in momentum space, the effective Hamiltonian, ΔH, which governs the spin-spin interaction of a quark-antiquark pair with on-shell mass m. In the second step we obtain the hyperfine splittings ΔE by evaluating expectation values of the Fourier transform of ΔH with the bound state wavefunctions given in Ref. 2. The Feynman diagrams which contribute to ΔH are shown in Fig. 1. This set of graphs is necessarily gauge invariant as can be easily checked. The results for the individual diagrams given below refer to the Feynman gauge. We have used dimensional regularization for ultraviolet divergences, and a small gluon mass λ to regularize infrared divergences. The result reads

$$\Delta H = \frac{8\pi}{3} \frac{\alpha_s^{(0)}}{m^2} C_2(R)\, \vec{s}_1 \cdot \vec{s}_2 \left\{ 1 + \frac{\alpha_s^{(0)}}{\pi} K^{(0)} \right\} \quad , \tag{2}$$

where $\alpha_s^{(0)}$ is the unrenormalized strong fine structure constant. The first term arises from Fig. 1a, and the various one-loop contributions are given by

$$\begin{aligned} K^{(0)}(1b) = {} & \frac{1}{4} C_2(G) \left\{ \frac{5}{3} \left[\frac{2}{\varepsilon} + \ln(4\pi) - \gamma_E - \ln\left(\frac{Q^2}{\mu^2}\right) \right] + \frac{31}{9} \right\} \\ & - \frac{1}{4} T(R) N_f \left\{ \frac{4}{3} \left[\frac{2}{\varepsilon} + \ln(4\pi) - \gamma_E - \ln\left(\frac{Q^2}{\mu^2}\right) \right] + \frac{20}{9} \right\} \quad , \end{aligned} \tag{3}$$

$$K^{(0)}(1c + 1d) = -\frac{3}{2} C_2(R) - \frac{3}{4} C_2(G) \left\{ \frac{1}{3} - \frac{1}{2} \ln\left(\frac{Q^2}{m^2}\right) + \frac{2}{3} \ln\left(\frac{Q^2}{\lambda^2}\right) \right\} \quad , \tag{4}$$

$$K^{(0)}(1e + 1f) = C_2(R) - \frac{1}{4} C_2(G) \left\{ \frac{2}{\varepsilon} + \ln(4\pi) - \gamma_E - \ln\left(\frac{m^2}{\mu^2}\right) + 6 + 2\ln\left(\frac{\lambda^2}{m^2}\right) \right\}, \quad (5)$$

$$K^{(0)}(1g) = \frac{1}{4} C_2(G) \left\{ 3\left[\frac{2}{\varepsilon} + \ln(4\pi) - \gamma_E - \ln\left(\frac{m^2}{\mu^2}\right) + \frac{4}{3} \right] + 2\ln\left(\frac{Q^2}{m^2}\right) + 2 \right\}, \quad (6)$$

$$K^{(0)}(1h + 1i) = \frac{9}{16} C_2(R)[1 \ \ \ln 2] \quad , \quad (7)$$

where we have dropped terms of order Q^2/m^2. $\varepsilon = 4 - D$ for D space-time dimensions, μ is the renormalization scale, Q the modulus of the space-like momentum transfer, $\gamma = 0.5772...$ the Euler constant, N_f the number of massless quark flavors, and in QCD the group factors read $T(R) = ½$, $C_2(G) = 3$. In Eq. (4) the "1/v-singularity," which arises from Fig. 1c has been dropped in the standard manner. Summing Eqs. (2) - (7) we obtain

$$\Delta H = \frac{8\pi}{3} \frac{\alpha_{\overline{MS}}(\mu)}{m^2} C_2(R) \vec{s}_1 \cdot \vec{s}_2 \left\{ 1 + \frac{\alpha_{\overline{MS}}(\mu)}{\pi} K \right\} \quad , \quad (8a)$$

with

$$K = \frac{1}{16}(1 - 9\ln 2) C_2(R) + \frac{11}{18} C_2(G) - \frac{5}{9} T(R) N_f \quad , \quad (8b)$$

$$- \frac{1}{12}[11 C_2(G) - 4T(R) N_f] \ln\left(\frac{Q^2}{\mu^2}\right) + \frac{7}{8} C_2(G) \ln\left(\frac{Q^2}{m^2}\right)$$

where we have used the $\overline{MS}$-scheme[7] which absorbs the pole term as well as the constant $(\ln(4\pi) - \gamma_E)$ into the definition of the renormalized, scale dependent coupling constant $\alpha_{\overline{MS}}(\mu)$. As expected, the infrared divergences which appear in Eqs. (4) and (5) have cancelled in the sum Eq. (8b). The existence of the $\ln(Q^2/m^2)$ contribution in Eq. (8b) was first pointed out by Dine.[8] However, results quoted in literature[8-10] for the coefficient of this term, disagree mutually with one another and with Eq. (8). We note that a complete evaluation of the 4th order hyperfine splitting is necessary in order to determine the scale parameter Λ.

The effective Hamiltonian Eq. (8) does not include possible non-perturbative effects (except for those which can be absorbed into the potential and thereby influence the wave function at short distances). In particular we assume that the energy shift of the pseudoscalar state due to the U(1) anomaly[11] is negligible. This assumption can eventually be tested in a quarkonium system where quark and antiquark carry different flavor (e.g. $(b\bar{c})$, $(t\bar{b})$ etc). In this case, the effective Hamiltonian is given by (m_1 and m_2 are quark mass and antiquark mass respectively)

$$\Delta\tilde{H} = \frac{8\pi}{3}\frac{\alpha_{\overline{MS}}(\mu)}{m_1 m_2} C_2(R)\,\vec{s}_1\cdot\vec{s}_2 \left\{ 1 + \frac{\alpha_{\overline{MS}}(\mu)}{\pi}\tilde{K} \right\} , \tag{9a}$$

with

$$\tilde{K} = \left(-3\frac{m_1 m_2}{m_1{}^2 - m_2{}^2} \ln\left(\frac{m_1}{m_2}\right) + 1 \right) C_2(R) + \left(\frac{3}{8}\frac{m_1 + m_2}{m_1 - m_2} \ln\left(\frac{m_1}{m_2}\right) - \frac{5}{36} \right) C_2(G) - \frac{5}{9} T(R) N_f$$
$$- \frac{1}{12} [11\, C_2(G) \;\; 4T(R) N_f] \ln\left(\frac{Q^2}{\mu^2}\right) + \frac{7}{8} C_2(G) \ln\left(\frac{Q^2}{m_1 m_2}\right) . \tag{9b}$$

Note that the annihilation diagrams (Figs. (1h), (1i)) do not contribute in Eq. (9). Equations (8) and (9) represent our final results.

In the equal-mass case Eq. (8b) yields

$$K = 0.563 + 2.25 \ln\left(\frac{\mu^2}{m^2}\right) + 0.375 \ln\left(\frac{Q^2}{m^2}\right) , \text{ for } N_f = 3$$

and

$$K = 0.286 + 2.08 \ln\left(\frac{\mu^2}{m^2}\right) + 0.542 \ln\left(\frac{Q^2}{m^2}\right) \quad , \text{ for } N_f = 4 \quad . \tag{10}$$

From Eqs. (8) and (10) one obtains hyperfine splittings ΔE by evaluating the appropriate expectation values,[12]

$$\Delta E = \frac{32\pi}{9} \frac{\alpha_{\overline{MS}}(\mu)}{m^2} < 1 > \left[1 + \frac{\alpha_{\overline{MS}}(\mu)}{\pi} \epsilon(\mu, \xi)\right] \quad , \tag{11a}$$

with

$$<1> \equiv |\phi(0)|^2 \quad , \qquad \xi \equiv \frac{< \ln\left(\frac{Q^2}{m^2}\right) >}{<1>} \quad , \tag{11b}$$

$$\epsilon(\mu,\xi) = 0.563 + 2.25 \ln\left(\frac{\mu^2}{m^2}\right) + 0.375\,\xi \ , \ N_f = 3 \quad ,$$

$$\epsilon(\mu,\xi) = 0.286 + 2.08 \ln\left(\frac{\mu^2}{m^2}\right) + 0.542\,\xi \ , \ N_f = 4 \quad ; \tag{11c}$$

$\phi(0)$ is the wave function in coordinate space at $\vec{x} = \vec{0}$. The quantities $|\phi(0)|^2$ and ξ have to be evaluated in a specific potential model. From Ref. 2 we obtain[13] for J/Ψ and Υ: $\xi(\Psi) = 0.56$, $\xi(\Upsilon) = -0.26$. These values are so small that their contribution to the hyperfine splitting given in Eq. (11) can be neglected phenomenologically. The smallness of ξ for Ψ and Υ shows that momentum transfers of order m dominate the hyperfine splittings in both cases. Thus the appropriate number of flavors is $N_f = 3$ for Ψ and $N_f = 4$ for Υ.

From the hyperfine splitting, the leptonic widths[14] of the vector state, and the hadronic width[14] of the pseudoscalar state we can form the ratios r_1 and r_2 which are independent of the wave function at short distances, and hence, any particular potential model that is used,

$$r_1 \equiv \frac{9}{8} e^2 \alpha_{EM}^2 \frac{\Delta E}{\Gamma_{ee}^V} = \begin{cases} \alpha_{\overline{MS}}(m)\left[1 + 5.9\dfrac{\alpha_{\overline{MS}}(m)}{\pi}\right] & , N_f = 3 \\ \alpha_{\overline{MS}}(m)\left[1 + 5.6\dfrac{\alpha_{\overline{MS}}(m)}{\pi}\right] & , N_f = 4 \end{cases} \quad (12)$$

$$r_2 \equiv \frac{4}{3} \frac{\Gamma_{had}^{PS}}{\Delta E} = \begin{cases} \alpha_{\overline{MS}}(m)\left[1 + 4.7\dfrac{\alpha_{\overline{MS}}(m)}{\pi}\right] & , N_f = 3 \\ \alpha_{\overline{MS}}(m)\left[1 + 4.6\dfrac{\alpha_{\overline{MS}}(m)}{\pi}\right] & , N_f = 4 \end{cases} \quad (13)$$

where e and α_{EM} are the quark charge and the electromagnetic fine structure constant respectively. Experimentally,[15] the hadronic width of the η_c is known only with large errors. Using $\Gamma_{ee}^V(\Psi) = 4.8 \pm 0.6$ keV and $\Delta E(\Psi - \eta_c) = 119 \pm 9$ MeV, we obtain from the ratio r_1 ($N_f = 3$)

$$\Lambda_{\overline{MS}} = 0.41 \pm 0.07 \text{ GeV} \quad ;$$

using the wave functions of Ref. 2[16] and Eq. (11) one obtains ($N_f = 3$)

$$\Lambda_{\overline{MS}} = 0.38 \pm 0.03 \text{ GeV} \quad .$$

In both cases only the experimental uncertainties are given.

We conclude with the following remarks:

(i) The hypothesis, that the hyperfine splittings of heavy quark systems are dominated by short distance effects, is self-consistent. The smallness of the parameter ξ indicates that in the Ψ and Υ families the relevant distances are of the order of the quark compton wavelength.

(ii) Contrary to the radiative corrections to the van Royen-Weisskopf formula the one-loop contribution to the hyperfine splitting is small! This result has been anticipated[2,4] on the basis of QCD-like potential models and provides further evidence for a short distance behavior of the quark-antiquark potential as predicted by perturbative QCD.

(iii) The $\Psi - \eta_c$ mass difference gives a scale parameter $\Lambda_{\overline{MS}} \backsim 0.4$ GeV, compatible with the range of scale parameters $\Lambda_{\overline{MS}} = 0.2 - 0.5$ GeV, which has been considered in connection with the Ψ and Υ spectroscopies.[2] However, a more precise determination of Λ from the hyperfine splittings in the Ψ system is not possible due to theoretical uncertainties such as relativistic and higher order radiative corrections, scheme dependence, etc. All these corrections and ambiguities will be much less important in the Υ spectroscopy, and we expect a determination of the scale parameter Λ within ± 100 MeV. The predictions for the $\Upsilon - \eta_b$ mass difference and the ratios r_1 and r_2 are shown in Fig. 2 as functions of $\Lambda_{\overline{MS}}$.

(iv) Perturbative QCD predicts various quantities of the Υ spectroscopy, such as hyperfine splittings, electromagnetic and hadronic decay widths, etc. Thus the experimental measurement of the parameters of the Υ family will provide stringent tests of the fundamental theory of strong interactions. The detailed comparison of the various quantities will also shed light on the theoretical approximations and uncertainties, and in particular elucidate the influence of nonperturbative effects.

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[12] The expectation value $<\ln(Q^2/m^2)>$ reads in terms of coordinate space wave functions

$$<\ln\left(\frac{Q^2}{m^2}\right)> \quad = \frac{1}{2\pi}\int d^3x\,\frac{1}{r}(\ln(mr)+\gamma_E)\Delta(\phi^*(\vec{x})\phi(\vec{x})) \qquad ,$$

where Δ is the Laplace operator.

[13] Here we have used the potential of Ref. 2 with the scale parameter of $\Lambda_{\overline{MS}} = 0.5$ GeV. For the potential with $\Lambda_{\overline{MS}} = 0.2$ GeV one obtains $\xi(\Psi) = 0.42$ and $\xi(\Upsilon) = -0.53$. The values for $|\phi(0)|^2$ read: a) potential 1 ($\Lambda_{\overline{MS}} = 0.5$ GeV): $|\phi(0)|^2_\Psi = 0.0628\ \mathrm{GeV}^3$, $|\phi(0)|^2_\Upsilon = 0.488\ \mathrm{GeV}^3$; b) potential 2 ($\Lambda_{\overline{MS}} = 0.2$ GeV): $|\phi(0)|^2_\Psi = 0.0584\ \mathrm{GeV}^3$, $|\phi(0)|^2_\Upsilon = 0.390\ \mathrm{GeV}^3$. The quark masses are given by $m_c = 1.48$ GeV, $m_b = 4.88$ GeV respectively.

[14]The leptonic width of the vector state reads $\Gamma^V_{ee} = 4\pi e^2 \alpha^2_{EM}/m^2 <1>[1 - 5.33\ \alpha_s/\pi]$ (for a discussion of the radiative corrections see Ref. 2, Appendix B). The hadronic width of the pseudoscalar state is given by (cf. Ref. 1)

$$\Gamma^{PS}_{had} = \frac{8\pi}{3} \frac{\alpha^2_{\overline{MS}}(m)}{m^2} <1> \times \begin{cases} 1 + 5.27\dfrac{\alpha_{\overline{MS}}(m)}{\pi} , & N_f = 3 \\ 1 + 4.85\dfrac{\alpha_{\overline{MS}}(m)}{\pi} , & N_f = 4 \end{cases} .$$

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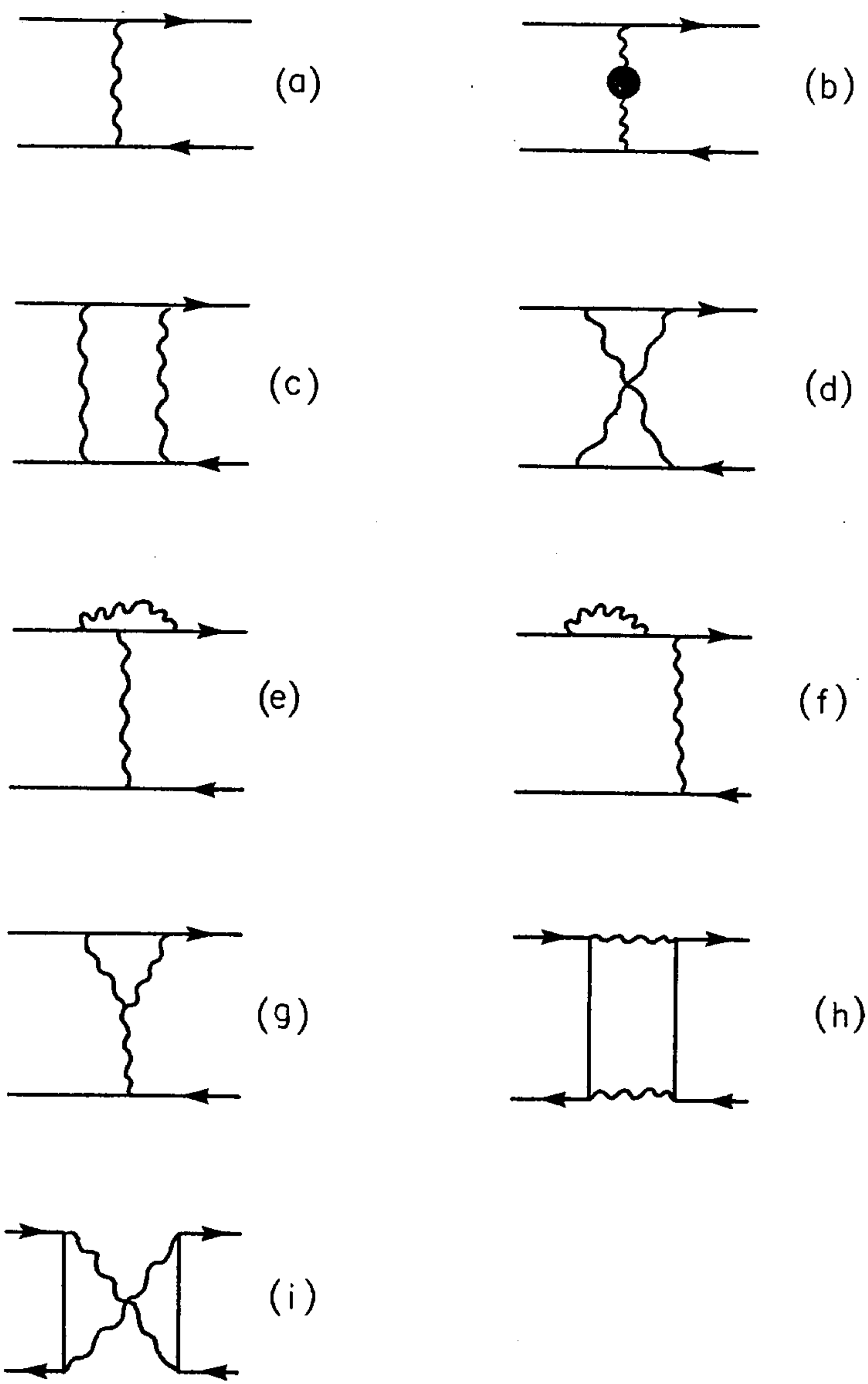
[16]We have estimated the wave function at the origin for arbitrary scale parameters $\Lambda_{\overline{MS}}$ by taking the linear interpolation determined by the two values of $|\phi(0)|^2$ for $\Lambda_{\overline{MS}} = 0.2$ GeV and $\Lambda_{\overline{MS}} = 0.5$ GeV, which are given in Ref. 13.

FIGURE CAPTIONS

Fig. 1: Feynman diagrams contributing to the hyperfine splitting to 4th order in the strong coupling constant.

Fig. 2: $\Upsilon - \eta_b$ system. The hyperfine splitting $\Delta E(\Upsilon - \eta_b)$ and the ratios $r_1 = \frac{9}{8} e^2 \alpha^2_{EM} \Delta E/\Gamma^V_{ee}$ and $r_2 = \frac{4}{3} \Gamma^{PS}_{had}/\Delta E$ (see text). For the hyperfine splitting the wave functions of Ref. 2[16] have been used.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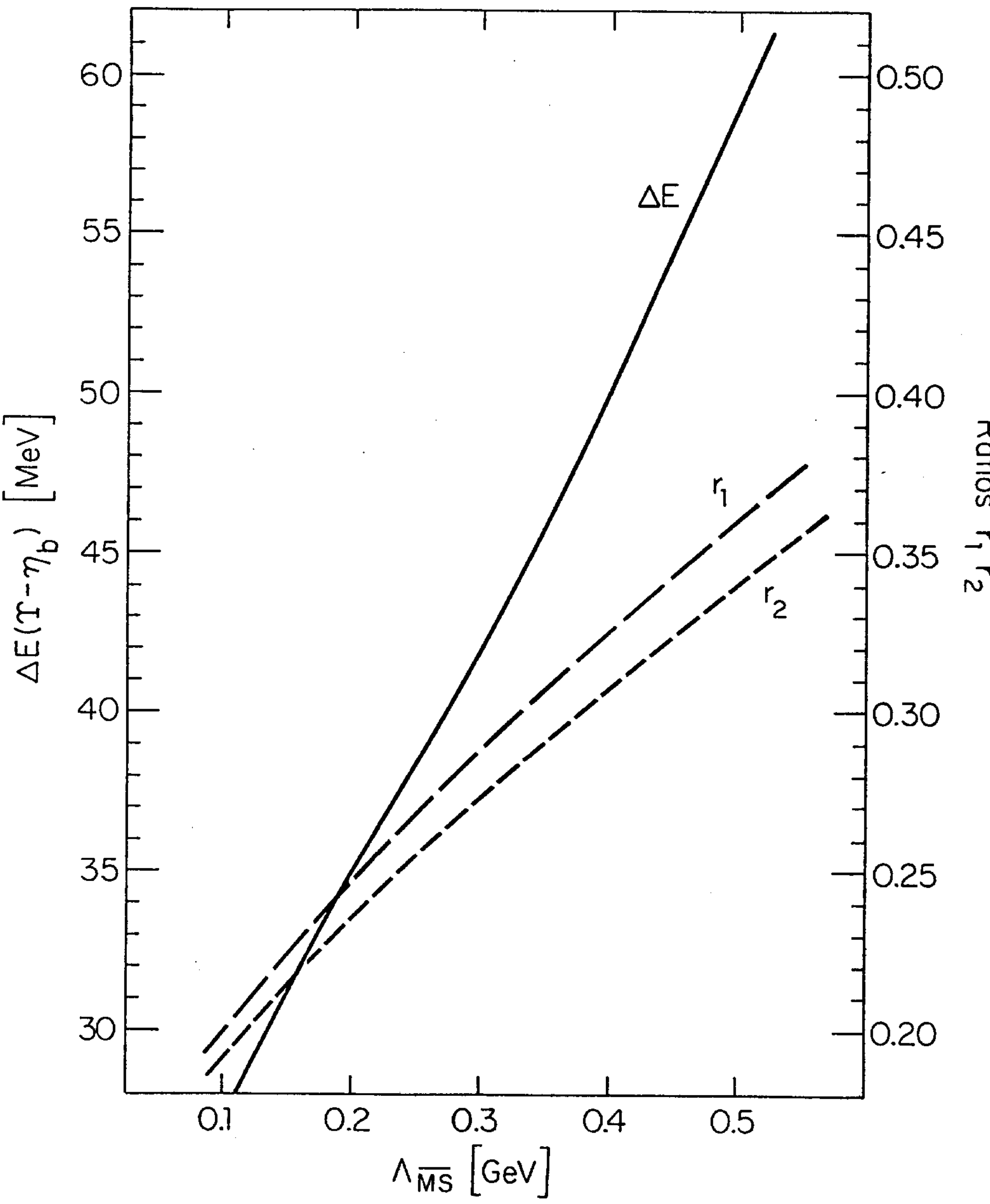

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ΔE
r_1
r_2
$\Delta E(\Upsilon - \eta_b)$ [MeV]
Ratios r_1 r_2
0.1
0.2
0.3
0.4
0.5
$\Lambda_{\overline{MS}}$ [GeV]