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An Upper Bound on the Top Quark Mass from Rare Processes

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ABSTRACT

We study the $K_L - K_S$ mass difference and the $K_L \to \mu^+\mu^-$ decay rate in the Kobayashi-Maskawa model. We show that if the matrix element $\langle \bar{K}^o | [\bar{s}\gamma_\mu(1-\gamma_5)d]^2 | K^o \rangle$ is smaller by at least a factor of two than its vacuum insertion estimate, then a rather stringent upper bound on the top quark mass (m_t) can be obtained. In particular using the MIT bag model estimate of the matrix element in question and taking into account the lower experimental limit on m_t we find $m_t = 33 \pm 14$ GeV if the free quark model estimate of the relevant short distance functions is used. Inclusion of the perturbative QCD effects leads to $m_t = 26 \pm 7$ GeV. We comment on how these values might be affected by long distance effects not included in the analysis.

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Exactly seven years ago Gaillard and Lee[1] made a quantitative prediction for the mass of the (at that time conjectured) charmed quark by studying the K_L - K_S mass difference in the four quark version[2] of the standard model.[3] Soon after their prediction was confirmed by experiment.[4] In this letter we make a quantitative prediction for the mass of the (as yet undiscovered) top quark by studying the $K_L - K_S$ mass difference and the $K_L \to \mu^+\mu^-$ decay rate in the six quark version of the standard model due to Kobayashi and Maskawa (KM).[5] For a certain range of values for the matrix element $M \equiv \langle \bar{K}^0 | [\bar{s}\gamma_\mu(1 - \gamma_5)d]^2 | K^0 \rangle$, which enters the formula for $\Delta m = m_{K_L} - m_{K_S}$, a rather stringent <u>upper</u> bound on m_t can be obtained. In particular we shall compute the upper bound on m_t corresponding to M as calculated[6] in the MIT bag model.

Our estimates of the <u>short</u> distance contributions to the relevant amplitudes are done both in the free quark model and in QCD. Since the analysis has various uncertainties we shall calculate the dependence of $(m_t)_{max}$ on the values of M and m_c, the charm quark mass. Finally we shall comment that the inclusion of long distance effects not contained in M may likely decrease our bound.

Our letter is organized as follows. We shall first discuss the formulae for $K_L - K_S$ mass difference and $K_L \to \mu^+\mu^-$ decay rate as obtained in the free quark model and in QCD. Subsequently we shall combine these formulae to obtain an upper bound on m_t. Finally we shall briefly discuss how the upper bound in question might be affected by long distance effects not included in our analysis.

<u>$K_L - K_S$ Mass Difference</u>

Quite generally we can write

$$\Delta m = 2\,\mathrm{Re}\, C \langle \bar{K}^0 | [\bar{s}\gamma_\mu(1 - \gamma_5)d]^2 | K^0 \rangle + LD \tag{1}$$

where the first term on the r.h.s. of (1) represents the short distance contribution coming from the box diagrams of Fig. 1 and LD stands for all long distance contributions which cannot be absorbed into the matrix element which appears in the first term. Whereas the matrix element $<\bar{K}^o|\ldots|K^o>$ cannot be evaluated by perturbative techniques, the short distance function C (coefficient function of the four-fermi operator $O = [\bar{s}\gamma_\mu(1-\gamma_5)d]^2$) can be calculated for instance in the free quark model or in the perturbative QCD. The evaluation of the coefficient C in the free quark model involves the box diagrams of Fig. 1 where the blob represents the $W^\pm$ gauge bosons and the unphysical scalar $\phi^\pm$ exchanges. In the 't Hooft-Feynman gauge the box diagrams with $W^\pm$ exchanges dominate[1] if the internal fermion masses $m_u, m_c, m_t << M_W$. For the case discussed here $m_t/M_W > 0.25$ and the inclusion of unphysical scalar contributions[7] as well as an exact[8] evaluation of the diagrams of Fig. 1 is necessary. In the literature only Inami and Lim[9] have done such an exact calculation. We confirm their result.

The free quark model estimate of the coefficient C is modified by the QCD corrections. For completeness we have included these effects in our analysis by following the work of Gilman and Wise.[10] These corrections are not negligible.

Neglecting for the moment the LD contributions, the ratio of Δm to the kaon mass, which is measured[11] to be 0.71×10^{-14}, is given by

$$\frac{\Delta m}{m_K} = \left[\frac{0.42}{R}\right] \frac{G_F}{\sqrt{2}} \frac{2}{3} f_K{}^2 \frac{\alpha}{4\pi} \frac{1}{\sin^2\theta_W} F(x_i, \theta_j)$$

$$= 1.6 \times 10^{-10} \left[\frac{1}{R}\right] F(x_i, \theta_j) = 0.71 \times 10^{-14} \tag{2}$$

where

$$F(x_i, \theta_j) = \left[(\mathrm{Re}A_c)^2 - (\mathrm{Im}A_c)^2\right] B(x_c, x_c)\eta_1$$

$$+ \left[(\mathrm{Re}A_t)^2 - (\mathrm{Im}A_t)^2\right] B(x_t, x_t)\eta_2$$

$$+ 2\left[\mathrm{Re}A_c\mathrm{Re}A_t - \mathrm{Im}A_c\mathrm{Im}A_t\right] \tilde{B}(x_t, x_c)\eta_3 \quad , \qquad (3)$$

and

$$\mathrm{Re}A_c = -s_1c_2\left[c_1c_2c_3 - s_2s_3\cos\delta\right]$$

$$\mathrm{Im}A_c = -\mathrm{Im}A_t = s_1s_2s_3c_2\sin\delta$$

$$\mathrm{Re}A_t = -s_1s_2\left[c_1s_2c_3 + c_2s_3\cos\delta\right] \qquad (4)$$

where $c_i = \cos\theta_i$, $s_i = \sin\theta_i$, and δ are the standard KM parameters.[5] Furthermore $G_F = 1.1785 \times 10^{-5}\ \mathrm{GeV}^{-2}$, $f_K = 1.23m_\pi$ is the kaon decay constant and θ_W is the Weinberg angle ($\sin^2\theta_W \approx 0.23$). The parameter R[12] depends on the estimate of the matrix element $\langle\bar{K}^o|\ldots|K^o\rangle$ in eq. (1). In the MIT bag model R = 1,[6] whereas in the vacuum insertion approximation[1] R = 0.42. In obtaining Eq. (3) we have exploited the generalized GIM[2] condition $A_u = -A_c - A_t$, where $A_u = c_1s_1c_3$.

The functions $B(x_i, x_i)$ and $\tilde{B}(x_i, x_j)$ are given by[9]

$$B(x_i, x_i) = x_i\left[\frac{1}{4} + \frac{9}{4}\frac{1}{(1-x_i)} - \frac{3}{2}\frac{1}{(1-x_i)^2}\right] + \frac{3}{2}\left[\frac{x_i}{x_i - 1}\right]^3 \ln x_i \qquad (5)$$

$$\tilde{B}(x_i, x_j) = x_ix_j\left\{\frac{1}{x_j - x_i}\left[\frac{1}{4} + \frac{3}{2}\frac{1}{(1-x_j)} - \frac{3}{4}\frac{1}{(1-x_j)^2}\right]\ln x_j\right.$$

$$\left. + (x_j \leftrightarrow x_i) - \frac{3}{4}\frac{1}{(1-x_j)(1-x_i)}\right\} \qquad (6)$$

where

$$x_i = \frac{m_i^2}{m_W^2} \quad . \tag{7}$$

In obtaining (5) and (6) x_u has been set to zero. $B(x_c, x_c) \approx x_c$. It is interesting to observe that for $m_t \leq m_W$ the functions $B(x_t, x_t)$ and $\tilde{B}(x_c, x_t)$ can be represented with 10% accuracy by

$$B(x_t, x_t) \approx x_t \quad , \quad \tilde{B}(x_c, x_t) \approx x_c \ln \frac{m_t^2}{m_c^2} \quad . \tag{8}$$

Thus the functional forms of the low x_t approximations[1,13] for $B(x_t, x_t)$ and $\tilde{B}(x_c, x_t)$ work well up to $x_t \approx 1$. This is to a large extent coincidental, and due to the unphysical scalars contributions which compensate for a slower than x_t dependence of $W^{\pm}$ contributions (as calculated in 't Hooft-Feynman gauge). For $x_t > 1$ this compensation is incomplete and the functions $B(x_t, x_t)$ and $\tilde{B}(x_c, x_t)$ as given in (5) and (6) increase substantially more slowly than the estimates of Eq. (8).

The parameters η_i in Eq. (3) represent QCD corrections in the leading logarithmic approximation, which depend very weakly on m_t. Choosing the QCD scale parameter Λ to be 0.3 GeV and setting $m_c = 1.5$ GeV (constituent quark mass) one finds[10]

$$\eta_1 \approx 0.90 \quad , \quad \eta_2 \approx 0.62 \quad , \quad \eta_3 \approx 0.33 \tag{9}$$

for all values of m_t considered in this paper. For $m_c = 1.2$ GeV (current quark mass) we find $\eta_1 \approx 1.0$ with η_2 and η_3 unchanged. The free quark model estimate corresponds to $\eta_1 = \eta_2 = \eta_3 = 1$. The parameters η_i depend very weakly on Λ for $0.2 \leq \Lambda \leq 0.5$ GeV.

$\underline{K_L \to \mu^+\mu^-}$

The evaluation of the short distance contribution to the decay $K_L \to \mu^+\mu^-$ involves in the free quark model the diagrams of Fig. 2 where the blob represents the induced $Z\bar{s}d$ coupling. The full list of diagrams contributing to this induced coupling can be found in refs. 1, 9 and 14. In the approximation of neglecting the muon mass the box diagrams with $\phi^{\pm}$ exchanges do not contribute. In the literature only in refs. 9 and 14 exact calculations of the diagrams of Fig. 2 have been done. We confirm the results of these papers. For completeness we have also included QCD corrections in our analysis by making a straightforward generalization of the results of ref. 15 to the 6 quark model.

Combining our calculations with the upper bound on the short distance contribution to $K_L \to \mu^+\mu^-$, as extracted by various authors[16, 17] from the data, we are led[9,16] to the following inequality ($M_W = 80.5$ GeV)

$$|\,\mathrm{Re}A_t\,|G(x_t)\eta \leq |\,s_1 c_3\,|\,0.85 \times 10^{-2} \quad . \tag{10}$$

Here[9,14]

$$G(x_t) = \frac{3}{4}\left[\frac{x_t}{x_t - 1}\right]^2 \ln x_t + \frac{x_t}{4} + \frac{3}{4}\frac{x_t}{1 - x_t} \tag{11}$$

and η, which represents QCD corrections is equal to 0.9 for $\Lambda = 0.3$ GeV. The free quark model estimate corresponds to $\eta = 1$. The charm contribution to $K_L \to \mu^+\mu^-$ is smaller by two orders of magnitude than the upper bound of Eq. (10) and has been neglected. On the other hand, as noticed first by Shrock and Voloshin[16] the bound of Eq. (10) is very useful for finding bounds on KM angles if the mass of the top quark is larger than 20 GeV. The function $G(x_t)$ as given in (11) increases slightly slower than its low x_t approximation[16,18] $G(x_t) \approx x_t$. For instance $G(1) = 0.62$.

Formulae like (2) and (10) (without the account for QCD effects and in the small x_t approximation) have been already used by various authors[9,12,13,16,19] with the aim to find bounds on the mixing angles θ_i and δ. Here we shall use them to find an upper bound for m_t as a function of the parameter R which enters Eq. (2). It is probably useful to get a feeling why a bound on m_t can be at all obtained from the formulae (2) and (10) alone. After all these equations contain three unknown parameters θ_2, θ_3 and δ (θ_1 is known[20]) and it would appear that by making a suitable choice for their values an arbitrary large top quark mass would be allowed. In order to see that this is not the case let us first make a very crude approximation (justified for large R) and neglect in Eq. (3) all terms but $\eta_2[\mathrm{Re}A_t]^2 B(x_t, x_t)$. Equations (2) and (10) lead then to the following inequality

$$\frac{G^2(x_t)}{B(x_t, x_t)} \leq \frac{1}{R}\,\frac{\eta_2}{\eta^2}\,[s_1 c_3]^2 \times 1.63 \tag{12}$$

Since the function $G^2(x_t)/B(x_t, x_t)$ increases monotonically with increasing x_t an upper bound on m_t can be obtained. Furthermore since $\eta_2/\eta^2 \simeq 0.77$ the upper bound in question is reduced by QCD corrections relative to its free quark model value. The bound on m_t also decreases with increasing R. All these qualitative features remain valid when all the terms in Eq. (3) are retained. It should be emphasized that it is crucial for obtaining the bound that Δm and $K_L \to \mu^+\mu^-$ are considered simultaneously, and that R is larger than 0.42, the value used by Gaillard and Lee.[1] Parenthetically we would like to remark that the choice $R \approx 1$ would not totally destroy the successful prediction of ref. 1 for m_c. With $R = 1$, $m_c \approx 2.2$ GeV, which is not too bad.

In a numerical analysis of Eqs. (2) and (10) we have used

$$c_1 = 0.97 \quad , \quad |s_3| \leq 0.5 \tag{13}$$

as obtained in ref. 20 from the data on nuclear β decay and hyperon decays respectively. The upper bound on m_t can then be found for fixed values of R and m_c by varying s_2, s_3 and $\sin\delta$ in the full range $|s_2| \leq 1$, $|\sin\delta| \leq 1$ and $|s_3| \leq 0.5$. The result is shown in Fig. 3. We observe that QCD corrections substantially lower the bound. The bound is also lowered when the current quark mass $m_c = 1.2$ GeV is used instead of the constituent quark mass $m_c = 1.5$ GeV. We also observe that the strong dependence of $(m_t)_{max}$ on the parameter R for $R < 1$ is somewhat weakened for $R > 1$. For $R = 0.42$ (not shown in Fig. 3), which corresponds to vacuum insertion estimate of ref. 1 no useful upper bound on m_t can be obtained. Even for $m_c = 1.2$ GeV and after the inclusion of QCD effects the upper bound on m_t corresponding to $R = 0.42$ is higher than m_W. Much more stringent bounds are obtained in the MIT bag model ($R = 1$). For the four cases considered in the Fig. 3 the upper bounds on m_t in the MIT model are: 47 GeV, 38 GeV, 33 GeV and 29 GeV.

Combining these results with the experimental lower bound on m_t[21] ($m_t \geq 19$ GeV), we conclude that if the matrix element $<\bar{K}^0|[\quad]|K^0>$ in Eq. (1) is evaluated in the MIT Bag model then our analysis leads to

$$m_t = \begin{cases} 33 \pm 14 \text{ GeV} & \text{Free Quark Model} \qquad (14a) \\ 26 \pm 7 \text{ GeV} & \text{QCD} \qquad . \qquad (14b) \end{cases}$$

It is amusing to observe that the result (14b) is very close to Bjorken's[22] estimate $m_t = 27$ GeV, obtained from empirical considerations. It should also be noticed that the experimental lower bound on m_t leads to an upper bound on R. This bound is roughly $R \approx 3$ and $R \approx 2$ for the Free Quark Model and QCD estimate respectively.

So far in our analysis we have neglected the long distance (LD) term which enters Eq. (2). As pointed out by Wolfenstein[12] and recently by Hill[23] the contribution of low mass intermediate states (e.g. π, η), which are not accounted for by the box diagrams (first term in (2)) may give a sizeable contribution to Δm.[24] Following Wolfenstein and Hill we write (z is a parameter)

$$LD = -z\,\Delta m \tag{15}$$

which together with the box contribution gives Eq. (2) with

$$R \rightarrow R(1 + z) \quad . \tag{16}$$

Thus for $z > 0$ the upper bound on m_t, which we found above is lowered, but it is increased if $z < 0$. In this respect the PCAC estimate of the LD term by Hill,[23,25] who finds $z > 0$, is very interesting. On the other hand Wolfenstein[12] attaches greater unreliability to the estimate of z and considers also negative values of z, which would increase our bound. However as pointed out by Hill,[23] independently of the PCAC estimate, a positive sign of z is preferred if the "penguin" diagram contributions to the CP violation ratio ε'/ε are as large as claimed by Gilman and Wise.[26] Negative z together with the results of ref. 26 would lead[23] to the violation of the experimental bound on ε'/ε. Thus in the end it may well be that z is indeed positive.

In summary we may conclude that if the matrix element of Eq. (1) is not larger than its MIT bag model estimate, and if $z > 0$ as suggested by Hill's paper[23] then our analysis (within the six quark version of the standard model[27]) indicates that the top quark should weigh <u>not more than 30-40 GeV</u>. Experimentalists will tell us in a not too distant future whether this is indeed the case.

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FOOTNOTES AND REFERENCES

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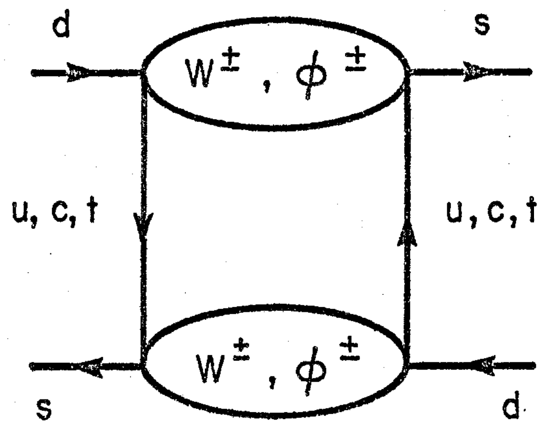
FIGURE CAPTIONS

Fig. 1: Box diagram contributions to the coefficient C of Eq. (1). $W^{\pm}$ are the $SU(2)_L$ gauge bosons and $\phi^{\pm}$ are the corresponding unphysical scalars. The crossed diagram is not shown.

Fig. 2: Diagrams contributing to the left-hand side of the bound in Eq. (10); (a) box diagram (b) induced Z^o contribution.

Fig. 3: The upper bound on m_t for various cases considered in the text as function of the parameter R. FQM stands for the free quark model. The horizontal line shows the approximate experimental lower bound on m_t.


d
s
$W^{\pm}$, $\phi^{\pm}$
u, c, t
u, c, t
$W^{\pm}$, $\phi^{\pm}$
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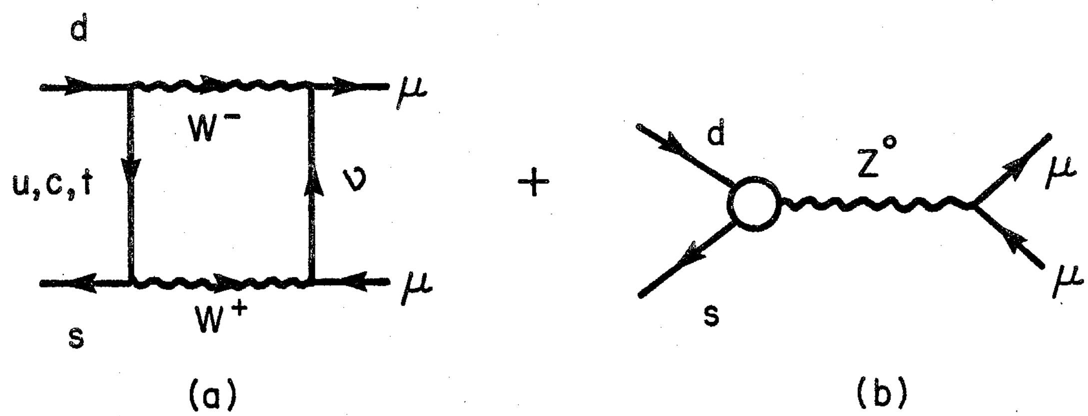

Fig. 1

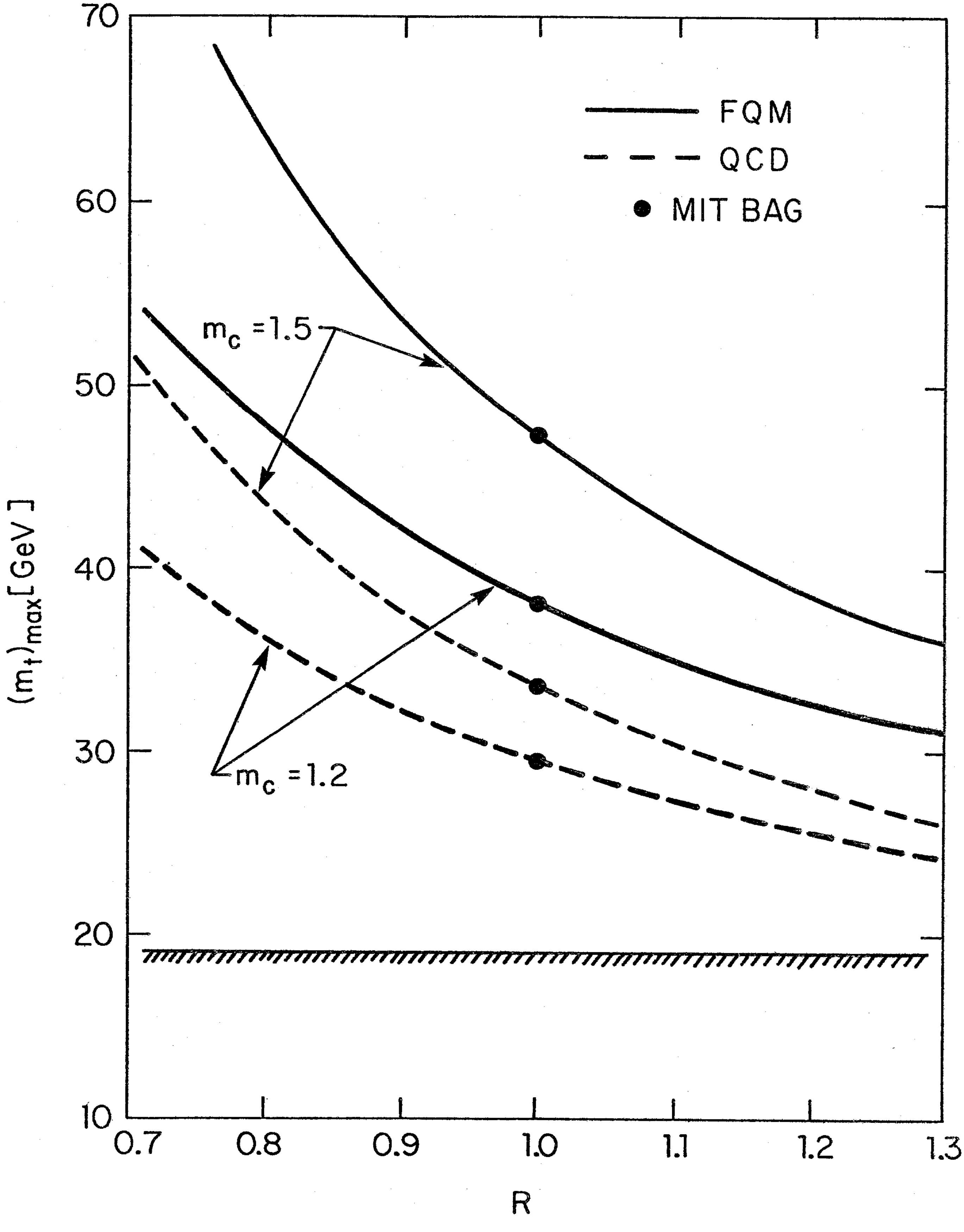
FQM
QCD
MIT BAG
m_c =1.5
m_c =1.2
$(m_t)_{max}$ [GeV]
R
70
60
50
40
30
20
10
0.7
0.8
0.9
1.0
1.1
1.2
1.3

Fig. 2