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GAUGE HIERARCHY AND DECOUPLING

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ABSTRACT

This is the first of two papers in which we deal with the problems of a large gauge hierarchy and decoupling in theories with spontaneously broken symmetry. They are relevant to the current developments of grand unification, in which several vacuum expectation values of scalar fields are introduced, with one of them (V) much larger than all the others. We shall show to all orders in the loop expansion that: (1) Once we make a proper identification of the light particles and of the heavy particles at the tree level, then such a division will be maintained order by order in the loop expansion without the necessity of fine tuning. The correction to the light masses in each order is only logarithmic in V. (2) To $\mathcal{O}(1)$ accuracy, there is a local renormalizable effective Lagrangian, composed of light fields only, which can be used to reproduce all the one light particle irreducible Green's functions for external momenta $<< V$.

In this paper, we shall give a pedagogical discussion of how the general results are obtained. We shall use a gauge model with O(3) symmetry to describe, in particular, how we organize the perturbation series to incorporate the renormalized vacuum expectation values as a part of the input parameters. The one loop calculation is then performed, including a complete treatment of the scalar sector, to illustrate our claim and to extract out all the effective parameters as functions of the original parameters. A complete, all order treatment will be presented in the second paper. Finally, a set of renormalization group equations are written down, wherein one stays in the low energy region to corrrelate these two sets of parameters. A leading logarithmic sum for the gauge coupling is performed to demonstrate the calculational and conceptual simplicity.

I. Introduction

GENERALITY

There are two problems in gauge theories with spontaneously broken symmetry to which definitive solutions are lacking thus far. One has to do with the validity of the decoupling theorem[1] and the other is the gauge hierarchy problem.[2] We shall explain these problems and then give solutions, with emphasis on the first problem.

First, let us state the decoupling theorem.[3] Consider a theory in which there are both light mass and heavy mass fields. We shall require that both the full theory and the reduced or light theory (which is obtained after the heavy fields are deleted) are renormalizable. The decoupling theorem applies to processes with only light external particles in the limit that the heavy mass M becomes very large relative to both the light mass m and the external momenta $|\vec{p}_i|$. If, in this limit, light particle physical quantities calculated to $\mathcal{O}(1)$ from the full theory can also be calculated from the light theory after suitable redefinition of masses, couplings and wave function renormalization of the light theory have been made, then decoupling is said to occur. The utility of the decoupling theorem is that physical quantities can often be calculated more simply from the light theory than from the full theory. The validity of this theorem has already been established in many theories without spontaneous symmetry breaking, for example QED and QCD.[1]

The gist of the argument in these theories is that, to the order of accuracy stated, the contribution of the heavy particles is limited to only the ultraviolet region in the integrals that arise and that the heavy particles matter only for diagrams or subdiagrams with at most four light legs. That is so because the heavy particle mass M always appears in the propagator $(p^2+M^2)^{-1}$. Only for divergent integrals do the M dependent effects become manifest. However, these finite M dependent effects are absorbable by the usual renormalization constants and thus the theorem follows.

In theories with spontaneously broken symmetry, the situation is different. In particular, there are two new aspects which merit discussion:

(1) How does M become big?

The masses of the particles are generally proportional to gv, where g is a generic coupling constant and v is a generic vacuum expectation value of a scalar field. We can make certain masses big by making either the appropriate g or v large. In the former case, we are dealing with a strong coupling theory. An example of this is the SU(2) x U(1) electroweak model with a very heavy Higgs particle (mass M_H). In the limit $M_H \to \infty$ the resulting theory is apparently non-renormalizable and the decoupling theorem is not valid. This was pointed out by Veltman and others and was recently formulated in the language of an effective Lagrangian by Appelquist and Bernard.[4] We shall not dwell on this case any further.

What we shall be concerned with is the case where one or more vacuum expectation value becomes much larger than the other vacuum expectation values. We shall denote the large vacuum expectation value by V. In so doing, we shall find that some of the scalars, vectors and fermions become very massive simultaneously. This is the limit taken in theories of the grand unification category.[5] We shall show that the decoupling theorem applies here.

(2) Where do the heavy mass effects contribute in an integration?

A distinctive feature for a theory with spontaneously broken symmetry is that some of the coupling terms are proportional to M. Thus, one naturally gets contributions to light field processes like the convergent integral

$$M^2 \int d^4k \, \frac{1}{k^2+M^2} \, \frac{1}{(k^2+m^2)^2} \, , \qquad \text{(I.1)}$$

in which m is a light mass. Because of the M dependence in the vertices of theories with spontaneous symmetry breakdown, this integral gives a ℓn (M/m) + constant] factor which is not suppressed by a $1/M^2$ factor, as would be the case in theories without spontaneous symmetry breakdown. The dominant region of the integrand here does not come entirely from the ultraviolet.

It comes both from the ultraviolet ($k \sim M$) and infrared ($k << M$) regions. To control such regions is an added complication in an analysis. Ultimately, we shall see that even though the entire region contributes, the renormalization group technique can still be used to sum up the heavy mass effects.

Now we turn to the gauge hierarchy issue. As is well-known in grand unification schemes, a big local gauge symmetry G is initially assumed in a Lagrangian, which is then badly broken by a huge vacuum expectation value V down to a reduced local gauge symmetry G'. This is further broken by another vacuum expectation value v (<< V) into the final symmetry, presumably $SU(3)_{color} \otimes (SU(2) \otimes U(1))_{electroweak}$. There are two questions which should be posed along the way. On the one hand one may ponder how such drastically different scales can be naturally induced in a theory. To this, we have no answer, and perhaps there is really no answer without further dynamical input. A more serious and pragmatic problem concerns the consistency of dividing particles into light and heavy species. The point is that if we proceed to calculate radiative corrections in a loop expansion, then light and heavy particles get mixed. How to separate 'heavy' and 'light' naturally is a crucial aspect of the gauge hierarchy problem.

What we shall show is that while there is at all stages mixing between light and heavy particles in the loop expansion, we shall see that once we make an assignment at the tree level so that some of the particles are light with mass m_{tree}, then at any loop order the mass of the light particles is given by

$$m = m_{tree} \left(1 + \sum_{i=loop} A_i (\ell n \frac{M}{\mu})^{n_i}\right) \tag{I.2}$$

where A_i is a function of the coupling constants, n_i is some integer, and μ is a subtraction constant. This is accomplished by way of our organization of the perturbation series. Disregarding the technical details for the moment, it is then clear that we do not need any fine tuning to maintain a heavy-light separation in the spectrum.[6] It should also be evident to the reader that this is a necessary and essential step in order to push through the proof of the decoupling theorem. Had it not been possible to make a clean division of particles into light and heavy sectors, the meaning of the decoupling theorem would have been very obscure.

Scope of our Work

To be concrete, we consider an O(3) gauge model where there are two scalar triplets. After the Higgs mechanism is invoked twice, we shall find that there exist in the light sector a vector boson together with its would-be Goldstone boson partner and associated Fadeev-Popov ghost and a physical Higgs scalar.

Of particular interest to us in the full theory are the renormalized Green's functions $\Gamma^{(n)}$, which have n light external legs and are one light particle irreducible (by one light particle irreducible we mean that $\Gamma^{(n)}$ may have diagrams which are one heavy particle reducible.)[7] Besides being pertinent to the physics under investigation, these Green's functions will properly and automatically take into account mixing between heavy and light particles.

The renormalized Green's functions in the light theory $\Gamma^{*(n)}$ are of course just one particle irreducible. They can be constructed according to the rules dictated by the light Lagrangian, which is obtained by striking out all terms involving heavy fields in the full Lagrangian and by redefining the light masses and coupling constants and performing finite wave function renormalizations.

The results we have obtained to all orders in the loop expansion are as follows:

(1) Decoupling theorem: We have succeeded in showing that for the n light particle Green's function, if the external momenta satisfy $|\vec{p}_i| << M$,

$$\Gamma^{(n)}\ (g;m,M,\mu;\ \vec{p}_i)$$
$$= Z^{n/2}\Gamma^{*(n)}\ (g^*;\ m^*,\mu;\ \vec{p}_i) + \mathcal{O}(1/M^2) \qquad \text{(I.3)}$$

in which g is a generic coupling constant in the full theory and μ (<< M) is a subtraction scale. For definiteness one may use the minimal subtraction scheme to renormalize these Green's functions. g^* and m^* are effective parameters in the light theory and Z is a generic finite wave function renormalization constant.

(2) Natural separation of the light particles from the heavy particles (gauge hierarchy): To all orders, we can show that

$$m^* = m\,(1 + \sum_{i=\text{loop}} b_i(g)\ (\ln \frac{M}{\mu})^{n_i}) \qquad (\text{I.4})$$

In words, it says that up to logarithmic correction, light particles are naturally light in the effective light theory. There is no need for fine tuning to arrive at this conclusion.

Throughout our approach, we stay in the low energy region. All the $\ln M/\mu$ effects are just radiative corrections and are contained in the effective parameters. There are natural renormalization group equations which can be deployed to sum up such contributions. In our approach to incorporate correctly the heavy fields, we do not have to deal with either high energy boundary conditions, or thresholds in the β-functions of the renormalization group.[8]

Let us briefly sketch the basic technique used to obtain these results. Due to the conciseness of a Lagrangian description we can construct the integrand for any diagram. Conversely, if we know how to construct any arbitrary integrand, we know the Lagrangian. Hence it is evident that the proof of a decoupling theorem rests on how one can make a systematic and correct approximation on integrands of the full theory so that the resulting integrals are accurate to the order in heavy mass we set out to achieve. When these resulting approximate integrands are precisely reproducible by the effective Lagrangian we have the theorem.

Now, this combinatoric problem is in fact solved by an algebraic identity, which rearranges the integrands in the way we have just described. In order to assure that we know how to make the correct approximation, all we need is some power counting rules, which have been developed by us.

There are of necessity some technical issues along the way. Thus, we have divided the material into two articles, of which this is the first. The more general, all order aspects will be given in a sequel.[9] Here, we shall be pedagogical and shall substantiate our approach with explicit one loop calculations. We hope that the reader will see the logic behind our manipulation, as guided by concrete examples. In this connection, another contribution of this article is that we shall give a complete and detailed account of the scalar sector, which is where most of the complication lies for a theory with spontaneously broken symmetry.

The plan of this paper is as follows: in the next section, we shall write down the Lagrangian for the full theory and display the spectrum after symmetry breaking. The large mass limit will be taken at the tree level to deduce the operator structure of the effective light

Lagrangian. In Section III, we shall describe how we shall develop the perturbation series. Here, we shall state our approach to the relationship between the mass parameters and the vacuum expectation values. The general approach to extract large mass effects and how decoupling is accomplished will be outlined in Section IV. This is followed by Section V, where explicit calculations at the one loop order are given. An appendix has been prepared to record contributions from various diagrams. We shall digress momentarily in Section VI, to focus on the decoupling aspects and the gauge hierarchy issue based on these one loop results. Furthermore armed with these one loop results, we shall use the renormalization group technique in Section VII to improve them in the spirit of performing a leading logarithmic sum. It should be noted that while the renormalization group equations governing the effective parameters can be trivially derived, our approach is nevertheless novel. A short conclusion to recapitulate some of the results will make up Section VIII.

II. Model Lagrangian

The model Lagrangian we shall consider possesses an O(3) local gauge symmetry. The gauge fields are $A_\mu^{a=1,2,3} \equiv \vec{A}_\mu$. To induce a gauge hierarchy, we introduce two real scalar triplets $\phi_{1,a}$ $(= \vec{\phi}_1)$ and $\phi_{2,a}$ $(= \vec{\phi}_2)$ with a=1,2,3. The classical Lagrangian is

$$\begin{aligned}\mathcal{L} = &- \frac{1}{4} (\partial_\mu \vec{A}_\nu - \partial_\nu \vec{A}_\mu - e\, \vec{A}_\mu \times \vec{A}_\nu)^2 \\ &- \frac{1}{2} (\partial_\mu \vec{\phi}_1 - e\, \vec{A}_\mu \times \vec{\phi}_1)^2 - \frac{1}{2} (\partial_\mu \vec{\phi}_2 - e\, \vec{A}_\mu \times \vec{\phi}_2)^2 \\ &- \frac{1}{2} m_1^2\, \vec{\phi}_1^2 - \frac{1}{2} m_2^2\, \vec{\phi}_2^2 - \frac{1}{4} \lambda_1\, (\vec{\phi}_1^2)^2 \\ &- \frac{1}{4} \lambda_2\, (\vec{\phi}_2^2)^2 - \frac{1}{2} \lambda_3\, \vec{\phi}_1^2\, \vec{\phi}_2^2 - \frac{1}{2} \lambda_4\, (\vec{\phi}_1 \cdot \vec{\phi}_2)^2 \qquad \text{(II.1)}\end{aligned}$$

where the metric is (-1,1,1,1).

There are two attitudes one can take regarding the parameters in systems where spontaneous symmetry breakdown occurs, such as this. Gildener and other people[2] require that λ_1 and λ_2 are positive, m_1^2 and m_2^2 are negative and that

$$-\sqrt{\lambda_1 \lambda_2} \le \lambda_3 \le \min\, [\sqrt{\lambda_1 \lambda_2}, \lambda_2 m_1^2/m_2^2, \lambda_1 m_2^2/m_1^2] \qquad \text{(II.2)}$$

The vacuum is then determined by locating the absolute minimum of the potential. When the above conditions on λ_1, λ_2, λ_3, m_1^2 and m_2^2 are satisfied and when $\lambda_4>0$, the absolute minimum occurs for both scalar triplets having non-zero vacuum expectation values and for the triplets having non-zero vacuum expectation values and for the relative orientation of the vacuum expectation values of the scalar triplets being orthogonal to each other. One then writes

$$\vec{\phi}_1 = (v_1 + \sigma, \pi_2, \pi_3) \quad \text{(II.3)}$$

and

$$\vec{\phi}_2 = (\psi_1, v_2 + \phi, \psi_3) \quad \text{(II.4)}$$

The local gauge symmetry is spontaneously broken $0(3) \xrightarrow{v_1} 0(2) \xrightarrow{v_2}$ no local gauge symmetry. In this approach, $v_1 \equiv V$ and v_2 ($<< v_1$) are functions of e, the λ's, and the m's. To achieve a large gauge hierarchy $v_1 >> v_2$, one has to fine tune the λ's, and the m's. For example, at the tree level

$$v_1^2 = \frac{\lambda_3 m_2^2 - \lambda_2 m_1^2}{\lambda_1 \lambda_2 - \lambda_3^2} \quad \text{(II.5)}$$

$$v_2^2 = \frac{\lambda_3 m_1^2 - \lambda_1 m_2^2}{\lambda_1 \lambda_2 - \lambda_3^2} \quad \text{(II.6)}$$

and for $v_1^2 >> v_2^2$ we must fine tune parameters so that

$$\lambda_3 m_1^2 - \lambda_1 m_2^2 \approx 0 . \quad \text{(II.7)}$$

This is one aspect of the gauge hierarchy problem.

Another way to look at the parameters is that the symmetric vacuum is assumed to be unstable. The vacuum expectation values v_1 and v_2 are taken to be free parameters which one puts in by hand and which satisfy $v_1 >> v_2$. Now m_1^2 and m_2^2 are determined, as functions of the λ's, v's and e by the minimum conditions. Furthermore to ensure we are at the absolute minimum of the potential, at least in the tree approximation, we require that λ_1, λ_2 and λ_4 are positive and that

$$\max \left[-\sqrt{\lambda_1 \lambda_2} , - \frac{\lambda_1 v_1^2}{v_2^2} , - \frac{\lambda_2 v_2^2}{v_1^2}\right] \leq \lambda_3 \leq \sqrt{\lambda_1 \lambda_2} \quad \text{(II.8)}$$

This is the approach we shall follow. This completely skirts the question, which is perhaps philosophical, as to whether one can naturally induce a very large gauge hierarchy. The issue we address is whether this approach is self-consistent in the context of perturbation theory. At the tree level, the minimum conditions are

$$-m_1^2 = \lambda_1 v_1^2 + \lambda_3 v_2^2 \tag{II.9}$$

$$-m_2^2 = \lambda_2 v_2^2 + \lambda_3 v_1^2 \tag{II.10}$$

When we add radiative corrections, the right hand sides of Eqs. (II.9) and (II.10) will have extra terms. This will be discussed further in the next few sections.

Now, it is easy to display the spectrum in this model. We have

(1) $$m_{A_1}^2 = (e\, v_2)^2, \tag{II.11}$$

$$m_{A_2}^2 = (e\, v_1)^2, \tag{II.12}$$

$$m_{A_3}^2 = e^2\, (v_1^2 + v_2^2). \tag{II.13}$$

(2) We define

$$\xi \equiv (v_1\pi_2 - v_2\psi_1)/\sqrt{v_1^2 + v_2^2}, \tag{II.14}$$

and

$$\eta \equiv (v_2\pi_2 + v_1\psi_1)/\sqrt{v_1^2 + v_2^2}. \tag{II.15}$$

It can readily be seen that ξ is the Goldstone partner to A_3. In the 't Hooft-Feynman gauge, which will be used throughout for our calculation,

$$m_{\xi}^2 = m_{A_3}^2 \qquad (II.16)$$

whereas

$$m_{\eta}^2 = \lambda_4(v_1^2 + v_2^2) \qquad (II.17)$$

(3) π_3 is the Goldstone partner for A_2 and ψ_3 is that for A_1, where their masses are respectively

$$m_{\pi_3}^2 = m_{A_2}^2 \qquad (II.18)$$

and

$$m_{\psi_3}^2 = m_{A_1}^2 \qquad (II.19)$$

(4) σ and ϕ mix. The mass eigenstates are

$$H = \sigma \cos\theta - \phi \sin\theta, \qquad (II.20)$$

$$h = \sigma \sin\theta + \phi \cos\theta, \qquad (II.21)$$

with

$$\sin\theta \cong -\left(\frac{\lambda_3 v_2}{\lambda_1 v_1} + \frac{\lambda_3(\lambda_2 - \frac{3}{2}\frac{\lambda_3^2}{\lambda_1})}{\lambda_1^2}\left(\frac{v_2}{v_1}\right)^3\right), \qquad (II.22)$$

where higher order terms in v_2/v_1 have been dropped. Their masses are, respectively,

$$m_H^2 = \lambda_1 v_1^2 + \lambda_2 v_2^2 + \sqrt{(\lambda_1 v_1^2 - \lambda_2 v_2^2)^2 + 4(\lambda_3 v_1 v_2)^2}$$

$$\cong 2\left(\lambda_1 v_1^2 + \frac{\lambda_3^2}{\lambda_1} v_2^2\right) \qquad (II.23)$$

and

$$m_h^2 = \lambda_1 v_1^2 + \lambda_2 v_2^2 - \sqrt{(\lambda_1 v_1^2 - \lambda_2 v_2^2)^2 + 4(\lambda_3 v_1 v_2)^2}$$

$$\cong 2 (\lambda_2 - \lambda_3^2/\lambda_1) v_2^2 \tag{II.24}$$

We can identify A_1, ψ_3, and h as the light fields. For matrix elements at low momenta with only light field external lines, we can show that to $\mathcal{O}(1)$ at the tree level they are reproduced by the following light effective Lagrangian

$$\mathcal{L}_{light} = -\frac{1}{4} (\partial_\mu A_{1\nu} - \partial_\nu A_{1\mu})^2$$

$$- \frac{1}{2} |(\partial_\mu - i e^* A_{1\mu})\Phi|^2$$

$$- \frac{1}{2} m^{*2} |\Phi|^2 - \frac{1}{4} \lambda^* (|\Phi|^2)^2 \tag{II.25}$$

where

$$\Phi = v^* + h + i \psi_3 \tag{II.26}$$

$$v^*_{tree} = v_2 \tag{II.27}$$

$$e^*_{tree} = e \tag{II.28}$$

$$\lambda^*_{tree} = \lambda_2 - \lambda_3^2/\lambda_1 \tag{II.29}$$

and from which we can obtain

$$(m^*_{h\ tree})^2 = 2\lambda^*_{tree}(v^*_{tree})^2 = -2(m^*_{tree})^2 \tag{II.30}$$

and

$$(m^*_{A_1\ tree})^2 = (e^*_{tree})^2 (v^*_{tree})^2 \tag{II.31}$$

That the effective Lagrangian is correct at the tree level is illustrared well by the following example. For h-h scattering in the full theory, we have the four diagrams of Fig. 1, which give

$$-\Gamma^{4h} = 6\lambda_2 - 4\lambda_3^2 v_1^2 \left(\frac{1}{(p_1+p_2)^2+m_H^2} + \frac{1}{(p_1+p_3)^2+m_H^2} + \frac{1}{(p_1+p_4)^2+m_H^2}\right)$$

$$\cong 6\ (\lambda_2 - \lambda_3^2/\lambda_1) \qquad \text{(II.32)}$$

for $m_H >> m_h$ and $|\vec{p}_i|$ $(i = 1,2,3,4)$. We readily identify $\lambda^*_{tree} = \lambda_2 - \lambda_3^2/\lambda_1$. This example also serves to explain why we must consider one light particle irreducible Green's functions.

III. Perturbation Expansion

In this section, we shall discuss how the perturbation series is organized. There are two issues here that merit detailed discussion. They are (1) how we work about the true minimum of the potential which changes order by order and how that is incorporated into the perturbation series and (2) how by including all one light particle irreducible graphs we automatically take care of the mixing of the light and heavy fields that occurs in each order of the perturbation series.

First let us develop our notation and introduce some preliminary details. We shall work in the 't Hooft-Feynman gauge throughout. Then, supplementing the Lagrangian of Eq. (II.1) is the following gauge fixing term and its associated ghost term

$$
\begin{aligned}
\mathcal{L}_{\text{gauge}} = & -\frac{1}{2\alpha}(\partial_\mu A_1^\mu - \alpha a_1 \psi_3)^2 - \frac{1}{2\alpha}(\partial_\mu A_2^\mu + \alpha a_2 \pi_3)^2 \\
& -\frac{1}{2\alpha}(\partial_\mu A_3^\mu - \alpha(a_2\pi_2 - a_1\psi_1))^2 + \bar{C}_1(\partial^2 - \alpha a_1 e v_2)C_1 \\
& +\bar{C}_2(\partial^2 - \alpha a_2 e v_1)C_2 + \bar{C}_3(\partial^2 - \alpha(a_2 e v_1 + a_1 e v_2))C_3 \\
& -e(\vec{\bar{C}} \times ((\partial_\mu \vec{A}^\mu) + \vec{A}^\mu \partial_\mu))\cdot\vec{C} \\
& +\alpha e(-a_1\bar{C}_1\phi C_1 + a_2\bar{C}_2\pi_2 C_1 + a_2\bar{C}_3\pi_3 C_1 + a_1\bar{C}_1\psi_1 C_2 \\
& \qquad -a_2\bar{C}_2\sigma C_2 + a_1\bar{C}_3\psi_3 C_2 - \bar{C}_3(a_2\sigma + a_1\phi)C_3),
\end{aligned}
\tag{III.1}
$$

where $\alpha_{tree} = 1$, $(a_1)_{tree} = ev_2$, and $(a_2)_{tree} = ev_1$. The $\vec{C}$'s and $\vec{\bar{C}}$'s are the ghost and the anti-ghost fields of the corresponding $\vec{A}$'s. There is a discrete symmetry in the total Lagrangian, which helps us discuss the mixing problem and determine which Feynman diagrams are allowable. We assign the following indices

$$0: \quad \text{for } \sigma \text{ and } \phi \tag{III.2}$$

$$1: \quad \text{for } A_1,\ \psi_3 \text{ and } C_1 \tag{III.3}$$

$$2: \quad \text{for } A_2,\ \pi_3 \text{ and } C_2 \tag{III.4}$$

$$3: \quad \text{for } A_3,\ \pi_2,\ \psi_1 \text{ and } C_3 \tag{III.5}$$

It can easily be verified that the indices 1,2,3 (and 0) are cyclically conserved. Details will be given in the companion paper.[9] In particular, this observation tells us that σ will mix only with ϕ and that π_2 will mix only with ψ_1, to all orders.

We shall now address issue (1) above of how the vacuum conditions are incorporated into the perturbation series. Let us consider the σ,ϕ sector of the theory alone and let us examine the relevant linear and quadratic terms from the Lagrangian of Eq. (II.1). These are

$$\begin{aligned}
\mathcal{L}_{(\sigma,\phi)} \equiv & - \frac{1}{2} m_h^2 h^2 - \frac{1}{2} m_H^2 H^2 \\
& + H(-D_1 v_1 \cos\theta + D_2 v_2 \sin\theta) \\
& + h(-D_1 v_1 \sin\theta - D_2 v_2 \cos\theta) \\
& - \frac{1}{2} H^2 (D_1 \cos^2\theta + D_2 \sin^2\theta) \\
& - \frac{1}{2} h^2 (D_1 \sin^2\theta + D_2 \cos^2\theta) \\
& - h\, H \sin\theta \cos\theta\, (D_1 - D_2)
\end{aligned} \tag{III.6}$$

where

$$D_1 = m_1^2 + \lambda_1 v_1^2 + \lambda_3 v_2^2 \tag{III.7}$$

$$D_2 = m_2^2 + \lambda_2 v_2^2 + \lambda_3 v_1^1 \tag{III.8}$$

The quantities m_h and m_H are the tree values of the masses of the h and H fields respectively, which are given by Eqs. (II.23) and (II.24). The angle θ is the rotation angle obtained at the tree level and is given by Eq. (II.22).

We shall take as our free Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{free}} = &- \tfrac{1}{2}\,\partial_\mu h \partial^\mu h - \tfrac{1}{2}\, m_h^2 h^2 - \tfrac{1}{2}\partial_\mu H \partial^\mu H - \tfrac{1}{2}\, m_H^2\, H^2 \\ &- \tfrac{1}{4}\,(\partial_\mu \vec{A}_\nu - \partial_\nu \vec{A}_\mu)^2 - \tfrac{1}{2} m_{A_1}^2\, A_{1\mu}^2 - \tfrac{1}{2} m_{A_2}^2\, A_{2\mu}^2 - \tfrac{1}{2} m_{A_3}^2\, A_{3\mu}^2 - \tfrac{1}{2}(\partial_\mu \vec{A}^\mu)^2 \\ &- \tfrac{1}{2}\,\partial_\mu \xi\, \partial^\mu \xi - \tfrac{1}{2} m_{A_3}^2 \xi^2 - \tfrac{1}{2}\,\partial_\mu \eta \partial^\mu \eta - \tfrac{1}{2} m_\eta^2 \eta^2 \\ &- \tfrac{1}{2}\,\partial_\mu \pi_3 \partial^\mu \pi_3 - \tfrac{1}{2} m_{A_2}^2 \pi_3^2 - \tfrac{1}{2}\,\partial_\mu \psi_3 \partial^\mu \psi_3 - \tfrac{1}{2} m_{A_1}^2 \psi_3^2 \\ &+ \overline{C}_1\,(\partial^2 - m_{A_1}^2) C_1 + \overline{C}(\partial^2 - m_{A_2}^2) C_2 + \overline{C}_3(\partial^2 - m_{A_3}^2) C_3\end{aligned} \tag{III.9}$$

where the first line of $\mathcal{L}_{\text{free}}$ is the piece relevant for the (σ,ϕ) system. All the quantities in $\mathcal{L}_{\text{free}}$ are renormalized ones. The renormalized values of v_1 and v_2 are input parameters. We do not expand them in loops. We treat the rest of the Lagrangian as a perturbation. The terms D_1 and D_2 are determined by the requirement that we are working at the exact minimum which corresponds to the conditions $\langle h \rangle = \langle H \rangle = 0$ and these in turn give us

$$-D_1 v_1 \cos\theta + D_2 v_2 \sin\theta + R_1 = 0 \tag{III.10}$$

and

$$-D_1 v_1 \sin\theta - D_2 v_2 \cos\theta + R_2 = 0 \tag{III.11}$$

where R_1 and R_2 are, respectively, radiative corrections to H and h tadpoles. Of course, Eqs. (III.10) and (III.11) can also be used to determine m_1 and m_2, but that is unnecessary, because they never appear in any calculation as we can easily show. It should be noted D_1 and D_2 are zero in the tree approximation only and receive contributions starting at the first order in the perturbation expansion. Therefore it is consistent in our perturbation theory to treat the quadratic terms of $\mathcal{L}_{(\sigma,\phi)}$ (excluding of course $m_h^2 h^2$ and $m_H^2 H^2$) as interaction terms. Having now determined D_1 and D_2 we can use them for calculating physical quantities, such as mass shifts. An example is

$$\begin{aligned}\Delta m_h^2 &= D_1 \sin^2\theta + D_2 \cos^2\theta - \Delta\Gamma^{2h}(p=0)\\ &= R_1 \left(\frac{1}{v_1} \cos\theta \sin^2\theta - \frac{1}{v_2} \sin\theta \cos^2\theta\right)\\ &+ R_2 \left(\frac{1}{v_1} \sin^3\theta + \frac{1}{v_2} \cos^3\theta\right) - \Delta\Gamma^{2h}(p=0)\end{aligned} \qquad (III.12)$$

where $\Delta\Gamma^{2h}(p=0)$ is the one light particle irreducible radiative correction to the two point h-function at zero momentum.

Let us now address issue (2) above of how the light-heavy mixing is taken care of. It is clear that h and H will mix in every order of a calculation. Obviously it would be a nuisance to have to re-diagonalize to find the masses and mass eigenstates after each order to yield new propagators and vertices to proceed to calculate the next order. However, as stated above, if we deal with one light particle irreducible graphs then this mixing problem is automatically taken care of to $\mathcal{O}(1)$. Actually, one is assured that for any physical process, as far as internal lines are concerned, mixing has no consequence, because all states are summed over. Therefore we can work with any representation of the fields. However, to construct physical matrix elements which pertain to the physical particles (external lines) under consideration, we need to project out the proper combinations. This is where mixing has to be properly treated. We shall work with the h-H basis and identify the physical light Higgs as the lighter eigenstate of the mass matrix.

$$\mathcal{m}^2 = \begin{pmatrix} m_H^2 + \Sigma_{HH} & \Sigma_{hH} \\ \Sigma_{hH} & m_h^2 + \Sigma_{hh} \end{pmatrix} \tag{III.13}$$

where the Σ's are the self energy operators evaluated at zero momentum. We shall show that Σ_{hh} is of order v_2^2 and Σ_{hH} is of order $v_1 v_2$, at least to one loop order (see Sections V and VI). The all order result will be shown in the sequel. Let the orthogonal matrix which diagonalizes $\mathcal{m}^2$ be

$$A = \begin{pmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{pmatrix} \tag{III.14}$$

Then

$$\sin\delta = -\frac{\Sigma_{hH}}{m_H^2 + \Sigma_{HH}} + \mathcal{O}\left(\frac{1}{v_1^3}\right) \tag{III.15}$$

and the light eigenstate is

$$h^{\text{true}} = h\cos\delta + H\sin\delta = h - \frac{\Sigma_{hH}}{m_H^2 + \Sigma_{HH}} H + \mathcal{O}\left(\frac{1}{v_1^2}\right) \tag{III.16}$$

which symbolically gives the amputated Green's function

$$\langle h^{\text{true}} \ldots \rangle_{\text{amp}} = \langle h \ \ldots \rangle_{\text{amp}} - \Sigma_{hH} \frac{1}{m_H^2+\Sigma_{HH}} \langle H \ldots \rangle_{\text{amp}} + \mathcal{O}\left(\frac{1}{v_1^2}\right) \tag{III.17}$$

But this combination of terms is precisely what we mean by one light particle irreducible Green's functions. Furthermore, it is not hard to see that

$$\langle H^{\text{true}} \ H^{\text{true}} \rangle = \langle HH \rangle + \mathcal{O}\left(\frac{1}{v_1^2}\right) \tag{III.18}$$

which affirms that we can just classify the heavy internal lines by H. Thus, as claimed, mixing is automatically taken care of by the use of one light particle irreducible Green's functions.

The theory is iteratively renormalized by minimal subtraction.[10] This is effected by rescaling

$$\vec{A}_\mu = (Z_A)^{1/2} \ (\vec{A}_\mu)_r \tag{III.19}$$

$$\vec{C} = (\tilde{Z})^{1/2} \ (\vec{C})_r \tag{III.20}$$

$$\vec{\phi}_1 = (Z_{\phi_1})^{1/2} \ (\vec{\phi}_1)_r \tag{III.21}$$

$$\vec{\phi}_2 = (Z_{\phi_2})^{1/2} \ (\vec{\phi}_2)_r \tag{III.22}$$

$$v_1 = Z_{v_1} \ (v_1)_r \tag{III.23}$$

$$v_2 = Z_{v_2} \ (v_2)_r \tag{III.24}$$

$$a_1 = Z_A^{-1/2} \ Z_{\phi_2}^{-1/2} \ e_r (v_2)_r \tag{III.25}$$

$$a_2 = Z_A^{-1/2} \ Z_{\phi_1}^{-1/2} \ e_r (v_1)_r \tag{III.26}$$

$$e = Z_e \ e_r \tag{III.27}$$

$$\lambda_i = Z_{\lambda_i} (\lambda_i)_r , \; i = 1,2,3,4 \tag{III.28}$$

and

$$\alpha = Z_A \alpha_r \tag{III.29}$$

If we recall the calculational procedure, we see that since the mass parameters m_1^2 and m_2^2 do not appear in any Green's function, their renormalization is of no interest to us.

For completeness, we remark that the light theory is also quantized in the 't Hooft-Feynman gauge ($\alpha^*_{tree}=1$).

$$\begin{aligned} \mathcal{L}^{light}_{gauge} = &- \frac{1}{2\alpha^*} (\partial_\mu A_1^\mu - \alpha^* a^* \psi_3)^2 \\ &+ \bar{C}_1 (\partial^2 - \alpha^* e^* v^* a^*) C_1 - \alpha^* e^* a^* \bar{C}_1 \; h \; C_1 \end{aligned} \tag{III.30}$$

The free Lagrangian is

$$\begin{aligned} \mathcal{L}^{light}_{free} = &- \frac{1}{4}(\partial_\mu A_{1\nu} - \partial_\nu A_{1\mu})^2 - \frac{1}{2}(\partial_\mu A_1^\mu)^2 \\ &- \frac{1}{2} e^{*2} v^{*2} (A_{1\mu})^2 - \frac{1}{2} \partial_\mu h \, \partial^\mu h \\ &- \frac{1}{2} (2 \lambda^* v^{*2}) h^2 - \frac{1}{2} \partial_\mu \psi_3 \, \partial^\mu \psi_3 \\ &- \frac{1}{2} e^{*2} v^{*2} \psi_3^2 + \bar{C}_1 (\partial^2 - e^{*2} v^{*2}) C_1 \end{aligned} \tag{III.31}$$

where all the quantities are renormalized. The rest is treated as a perturbation in the spirit outlined for the full theory. Renormalization is performed through minimal subtraction, which is in turn carried out iteratively by rescaling

$$A_{1\mu} = (Z^*_{A_1})^{1/2} \, (A_{1\mu})_r \tag{III.32}$$

$$C_1 = (Z^*_{C_1})^{1/2} \, (C_1)_r \tag{III.33}$$

$$h = (Z^*_h)^{1/2} \, (h)_r \tag{III.34}$$

$$\psi_3 = (Z^*_h)^{1/2} \, (\psi_3)_r \tag{III.35}$$

$$v^* = Z^*_v \, (v^*)_r \tag{III.36}$$

$$a^* = (Z^*_{A_1})^{-1/2} \, (Z^*_h)^{-1/2} \, e^*_r \, (v^*)_r \tag{III.37}$$

$$e^* = (Z^*_{A_1})^{-1/2} \, (e^*)_r \tag{III.38}$$

$$\lambda^* = Z^*_\lambda \, (\lambda^*)_r \tag{III.39}$$

and

$$\alpha^* = Z^*_{A_1} \, \alpha^*_R \tag{III.40}$$

IV. Outline of General Approach

In this paper we shall limit ourselves to a pedagogical description of our approach in obtaining the results given by Eqs. (I.3 and I.4). A thorough treatment and other ramifications will be given in a separate publication.[9]

We shall follow two steps in giving a proof. First, we shall establish the existence of a local renormalizable effective field theory for light processes at low energy. This first step is by construction and it is, in principle, possible to work out the combinatorical problem of the induced local vertices and identify the effective Lagrangian. However this task is made much easier by utilizing the next step, which consists of deriving the Becchi-Rouet-Stora (BRS) identities[11] satisfied by the one particle irreducible generating functional of the effective theory. This then uniquely determines the structure of the effective light theory. Let us describe these two steps:

(1) Existence of a Local Renormalizable Effective Field Theory: The aim here is to show that given any diagram we have a procedure to shrink the diagram, or parts of it, systematically into points to exhaust all the $\mathcal{O}(1)$ contributions. Furthermore, we shall establish a set of power counting rules which shows that there are no more than four bosons entering or leaving these induced vertices. The existence of a local effective Lagrangian is then assured.

We shall in fact construct these vertrices explicitly by algebraic rearrangement. An example will now be given to introduce and illustrate the general procedure. Consider Fig. 2 in which there are two light internal lines and a heavy internal line with mass M. The relevant integral for it is of the form:

$$I = M^2 \int d^4k \frac{1}{(k+p_1)^2+m^2} \frac{1}{(k-p_2)^2+m^2} \frac{1}{k^2+M^2} \qquad \text{(IV.1)}$$

The multiplicative factor M^2 comes from the heavy-light vertices. There are two parts in this diagram which contain the heavy line. We denote them as part 1 and part 2. We now define a shrinking operator τ which short circuits all the external momenta entering or leaving the part under consideration. Thus

$$\tau_1 \frac{1}{k^2+M^2} = \frac{1}{M^2} \qquad \text{(VI.2)}$$

and

$$\tau_2 \frac{1}{(k+p_1)^2+m^2} \frac{1}{(k-p_2)^2+m^2} \frac{1}{k^2+M^2} = \frac{1}{(k^2+m^2)^2} \frac{1}{k^2+M^2} \qquad \text{(VI.3)}$$

τ is a Taylor operator which localizes a vertex. Next we use the following algebraic identity

$$1 = (1-\tau_2)(1-\tau_1)+(1-\tau_2)\tau_1 + \tau_2 \qquad \text{(VI.4)}$$

to split the integral into

$$I = I' + I_{(1)} + I_{(2)} \tag{IV.5}$$

where

$$I' \equiv (1-\tau_2)(1-\tau_1)I$$

$$= M^2 \int d^4k \left[\frac{1}{(k+p_1)^2+m^2} \frac{1}{(k=p_2)^2+m^2} - \frac{1}{(k^2+m^2)^2}\right] \left(\frac{1}{k^2+M^2} - \frac{1}{M^2}\right) \tag{IV.6}$$

Note that the original integral has $\mathcal{O}(1)$ and $\mathcal{O}(\ln M^2)$ contribution coming from the $k^2 \lesssim M^2$ region, but because of the $(1-\tau_2)(1-\tau_1)$ operation, the power behavior of the integrand has been 'improved', so the I' is of $\mathcal{O}(\frac{1}{M^2})$ and can be discarded. The second term of Eq. (IV.5) is

$$I_{(1)} \equiv (1-\tau_2)\tau_1 \, I$$

$$= M^2(1-\tau_2) \int d^4k \frac{1}{(k+p_1)^2+m^2} \frac{1}{(k-p_2)^2+m^2} \frac{1}{M^2}$$

$$= \int d^4k \left(\frac{1}{(kp_1)^2+m^2} \frac{1}{(k-p_2)^2+m^2} - \frac{1}{(k^2+m^2)^2}\right) \tag{IV.7}$$

in which we have made two moves: (i) the heavy line has been shrunken into a point vertex, which is graphically represented in Fig. 3i and can be identified as a part of a vertex in the effective light theory and (ii) the $(1-\tau_2)$ operation renders the reduced integral finite. This last feature is too important not to delve into some more. It is because of the built-in mechanism to auto-renormalize the resulting integral that a meaningful large M limit can be taken. This is a distinct advantage of our method over the formal functional method, which integrates over heavy fields but cannot be used judiciously to induce a local theory in the large M limit. $I_{(1)}$ gives $\mathcal{O}(1)$ contribution.

The third term of Eq. (IV.5) is just a part of an effective coupling, because

$$I_{(2)} \equiv \tau_2 I .$$
$$= M^2 \int d^4k \frac{1}{(k^2+m^2)^2} \frac{1}{(k^2+M^2)} \tag{IV.8}$$

is just a number of $\mathcal{O}(\ell n\ M^2)$ with no external momentum dependence. The τ_2 operation induces a vertex shown in Fig. 3ii.

Let us recapitulate the spirit of the procedure employed in the above example. Given a one light particle irreducible graph with light external legs, we first look for (non-trivial) two, three and four light particle sub-graphs, which are the potential diagrams that give rise to local vertices of the effective theory. We call them partition elements. Then, we enumerate all possible ways of reducing these sub-graphs into points. Reduction here is accomplished by applying appropriate Taylor operators τ to produce local vertices. In this way, we obtain various reduced diagrams, which may be specified by a list of reduced partition elements (reduction elements). Some of these diagrams still contain partition elements which are not reduced, i.e. they still have heavy particle propagators inside. We may have to make these diagrams finite by applying extra zero momentum subtractions upon the non-reduced partition elements, aside from the usual minimal subtractions. These 'over-subtractions' improve the large M behavior of the integrands sufficiently to render the integrals vanishing as $M\to\infty$. What are left over are only the 'fully reduced' diagrams in which heavy particle propagators are no longer present. They have the structure of graphs in the light particle theory and only these give contributions of $\mathcal{O}(1)$. This is the essential idea which is applicable to any general diagram. The actual implementation, however, involves rather complicated combinatorics and is best handled in the formalism of Bogolubov-Parasiuk-Hepp-Zimmermann (BPHZ)[12]. In this paper, we shall simply recall the results with brief explanation.

Let I_Γ be the integrand corresponding to a diagram constructed from the full Lagrangian, the renormalized integrand according to BPHZ is

$$R_\Gamma = \sum_{U_0 \in F(\Gamma)} \prod_{\sigma \in U_0} (-t^\sigma)\, I_\Gamma \tag{IV.9}$$

in which U_0 is a forest consisting of a set of non-overlapping renormalization parts σ and F is the set of all forests.

We shall use $\Gamma/\{\pi_1, \pi_2, \ldots \pi_m\}$ to denote the reduced graph corresponding to the set of reduction elements $\{\pi_1, \pi_2, \ldots, \pi_m\}$. We claim that Eq. (IV.9) can be rewritten as:

$$R_\Gamma = \sum_{\{\pi_1,\pi_2,\ldots,\pi_m\}} \sum_{U\epsilon F(\Gamma/\{\pi_1,\pi_2,\ldots,\pi_m\})} \prod_{\gamma\epsilon U} (-T^\gamma) \prod_{i=1}^{m} (\tau^{\pi_i} \sum_{U_i\epsilon F_O(\pi_i)} \prod_{\sigma'\epsilon U_i} (-t^{\sigma'}))I_\Gamma \qquad \text{(IV.10)}$$

in which only the sum ($\sum_{\{\pi_1,\pi_2,\ldots,\pi_m\}}$) corresponding to full reductions contribute to $\mathcal{O}(1)$. Let us explain the content of this formula in words. U_i is a forest of the reduction element π_i before it is shrunken and $F_O(U_i)$is the collection of all such forests. The operation $\sum_{U_i\epsilon F_O(\pi_i)} \prod_{\sigma'\epsilon U_i} (-t^{\sigma'})$, in which σ' is a renormalization part, merely renormalizes the reduction element as is usually done. For a fixed reduced graph, we gather together all the partition elements and renormalization parts and denote them by γ. T is an operation such that if a particular γ is a renormalization part, we apply t. If it is a partition element other than a renormalization part, we apply τ. The rest of Eq. (IV.11) just says that we sum over all the possible reduced graphs and 'renormalize' the integrands as if all the partition elements were divergent. The complete derivation of this identity and other clarification will be given elsewhere.

The most important point we wish to register here is that the effective vertices are identifiable with the reduction elements. In fact the effective couplings are just the corresponding sub-integrals evaluated at zero external momenta. These we shall explicitly give at the one loop level in the next few sections.

(2) The Exact Form of the Effective Lagrangian: Although it is in principle possible to dig deeply into the structure of the algebraic identities to identify all the fundamental effective vertices, this task is horrendous. On the one hand, these vertices have very complicated coefficients as functions of the orginal parameters, as we shall exhibit even at the one loop level. More importantly, we are dealing with a gauge theory and it is necessary to be assured that gauge invariance is not violated in taking the large v_1 limit. It happens that gauge invariance is actually a blessing in disguise. We may recall that in proving the renormalizability of gauge theories, the BRS identities, together with power counting, actually specify all the counter terms allowable. We can apply a parallel consideration here. We shall start with the BRS identities for the one light particle irreducible functional of the full theory. Upon setting the heavy particle sources to zero, we can show by power counting that the BRS identities go over to O(2) BRS identities for the one particle irreducible functional of the effective light theory. This then uniquely determines the operator structure of the effective Lagrangian. In addition, we shall see that the only vacuum expectation value which is explicitly left behind and enters in the O(2) BRS identities is v_2. This should complete our proof of decoupling and the stability of the mass hierarchy in perturbation expansion to all orders. This program will be carried out in a companion article.

V. One Loop Calculation

We shall determine the effective parameters at the one loop level in this and the next sections. It will be ill advised for us to show how individual diagrams have been evaluated in the way described in the last section, as this will make our presentation rather tedious and vexing. Nevertheless, we have recorded the detailed contributions of various diagrams or sets of diagrams in the Appendix. We confine ourselves here to pointing out the salient features, whenever necessary.

In the light effective theory, there are three parameters, e^*, λ^*, and v^*. However, to go from the full theory to the light theory, even if we restrict oursevles to physical processes with only h and A_1 external lines, there are also two finite wave function renormalization constants which appear in the decoupling equation, Eq. (I.3),

$$\Gamma^{n_1A_1, n_2h} = Z_{A_1}^{n_1/2} Z_h^{n_2/2} \Gamma^{*n_1A_1, n_2h} \tag{V.1}$$

We need, therefore, at least five independent equations of the from (V.1) to extract e^*, λ^* and v^*. As it turns out, we shall have calculated seven such equations so that there are two extra equations to serve as consistency checks.

As already stated we shall use dimensional regularization in n space time dimensions to regulate our integrals and perform minimal subtraction renormalization. Furthermore we shall require that all renormalized parameters have the same dimension, for any n, as the corresponding bare parameters have for n=4. Hence we introduce a dimensionful scale μ via the substitution for the renormalized coupling constants $e^2 \to e^2(\mu^2)^{2-n/2}$ and $\lambda \to \lambda(\mu^2)^{2-n/2}$ As is well-known in such an approach the combination $1/(2-n/2) + \ell n\ \mu^2$ goes together. The $\mathcal{O}(1)$ terms we retain in the following are those that are proportional to $\ell n\ v_1^2$ or $\ell n\ \mu^2$. The constant terms, except those from the tree level, will be neglected.

We now describe the various Green's functions we have calculated:

(1) Γ^{2h}

We have divided the contributions into two parts, as given in Eq. (III.12) and these are summarized in the Appendix. It is worthwhile pointing out that $\Delta\Gamma^{2h}$ and the "tadpole" contributions separately have terms proportional to v_1^2. They cancel out in the sum to maintain naturally the heavy light division. The one light particle, irreducible 2-point function is given by:

$$\begin{aligned}
-\Gamma^{2h} = {} & v_2^2 \Big[\, 2\big(\lambda_2 - \frac{\lambda_3^2}{\lambda_1}\big) + A \cdot \frac{1}{16\pi^2}\, \ell n\ v_1^2 \\
& - \big(A + 6e^4 + 20\ (\lambda_2 - \frac{\lambda_3^2}{\lambda_1})^2\big)\ \frac{1}{16\pi^2}\, \ell n \mu^2 \Big] \\
& + p^2\ \Big[1 + 2e^2\ \frac{1}{16\pi^2}\, \ell n\ v_1^2 - 4e^2\ \frac{1}{16\pi^2}\, \ell n\ \mu^2\Big] \\
& + \mathcal{O}(p^4)
\end{aligned} \qquad \text{(V.2)}$$

where

$$A = 2\lambda_2^2 + 8\lambda_3^2 + 2\lambda_4^2 + 20\,\frac{\lambda_2\lambda_3^2}{\lambda_1} - 16\,\frac{\lambda_3^3}{\lambda_1}$$
$$- 4\,\frac{\lambda_2\lambda_3\lambda_4}{\lambda_1} - 4\,\frac{\lambda_3\lambda_4^2}{\lambda_1} - 14\,\frac{\lambda_3^4}{\lambda_1^2} + 4\,\frac{\lambda_3^3\lambda_4}{\lambda_1^2}$$
$$+ 2\,\frac{\lambda_3^2\lambda_4^2}{\lambda_1^2} + 6\,e^4 - 12\,\frac{e^4\lambda_3}{\lambda_1} + 12\,\frac{e^4\lambda_3^2}{\lambda_1^2} \qquad (V.3)$$

For the reduced theory, we have

$$-\Gamma^{*2h} = 2\lambda^* v^{*2} - v^{*2}\,(20\lambda^{*2} + 6e^{*4})\,\frac{1}{16\pi^2}\,\ell n\,\mu^2$$
$$+ p^2\,(1-2e^{*2}\,\frac{1}{16\pi^2}\,\ell n\,\mu^2) + \mathcal{O}(p^4) \qquad (V.4)$$

We note that in Eq. (V.2) the coefficients of the $\ell n\,\mu^2$ terms are not exactly the negative of those of the $\ell n\,v_1^2$ terms. This is partly because some of the divergences come from diagrams with light lines only. Also, because of the trilinear coupling $\sim\ v_1 h^2 H$, we have an integral like

$$v_1^2 \int d^n k\,\frac{1}{k^2+m_h^2}\,\frac{1}{k^2+m_H^2}$$
$$\cong\ v_1^2\,i\pi^2\,[\frac{1}{2-n/2} - [(1 + \frac{m_h^2}{m_H^2})\,\ell n\,\frac{m_H^2}{\mu^2} - \frac{m_h^2}{m_H^2}\,\ell n\,\frac{m_h^2}{\mu^2}]] \qquad (V.5)$$

This gives a term $\sim -i\pi^2\,v_2^2\,\ell n\,m_H^2$ unaccompanied by $1/(2-n/2)$, or $\ell n\,\mu^2$. Note that there is another parameter in the model with mass dimension, namely v_2^2, and there are always $\ell n\,v_2^2$ terms which can occur to ensure we only consider the logarithms of dimensionless quantities. For simplicity we have suppressed these $\ell n\,v_2^2$ terms.

It is also quite noticeable that the mismatch in coefficients for $\ln v_1^2$ and $\ln \mu^2$ in Eq. (V.2) is equal to the corresponding coefficient of $\ln \mu^2$ in Eq. (V.4). This proves to be a general phenomenon in all the Green's functions. The significance of this observation will be discussed in the next section.

(2) Γ^{2A_1}

Here, the heavy mass effects cancel out completely in the mass shift calculated in the full theory as would be expected from gauge invariance. We have

$$-\Gamma^{2A_1} = e^2 v_2^2 - 4e^4 v_2^2 \frac{1}{16\pi^2} \ln \mu^2$$
$$+ p^2 \left(1 + 3e^2 \frac{1}{16\pi^2} \ln v_1^2 - \frac{8}{3} e^2 \frac{1}{16\pi^2} \ln \mu^2\right)$$
$$+ \mathcal{O}(p^4) \tag{V.6}$$

and

$$-\Gamma^{*2A_1} = e^{*2} v^{*2} - 4e^{*4} v^{*2} \frac{1}{16\pi^2} \ln \mu^2$$
$$+ p^2 \left(1 + \frac{1}{3} e^{*2} \frac{1}{16\pi^2} \ln \mu^2\right)$$
$$+ \mathcal{O}(p^4) \tag{V.7}$$

(3) $\Gamma^{2A_1,h}$ $(p_i=0)$

As in Γ^{2A_1}, here all graphs with heavy line(s) contribute in the combination $\ln v_1^2 - \ln \mu^2$. This implies that all the $\ln v_1^2$ are ultraviolet in origin. The results are

$$-\Gamma^{2A_1,h}(p_i=0) = 2e^2\, v_2 - 2e^4 v_2 \frac{1}{16\pi^2} \ell n\, v_1^2$$

$$- 4e^4\, v_2 \frac{1}{16\pi^2} \ell n\, \mu^2 \qquad (V.8)$$

and

$$-\Gamma^{*2A_1,h}(p_i=0) = 2\, e^{*2} v^* - 6e^{*4} v^* \frac{1}{16\pi^2} \ell n\, \mu^2 \qquad (V.9)$$

(4) $\Gamma^{3h}(p_i=0)$

We encounter here an integral of the kind given in Eq. (I.1), with $m \to m_h$ and $M \to m_H$. In words, some of the $\ell n\, v_1^2$'s originates from the infrared region of integration. Again, without much further ado, we write down the results

$$-\Gamma^{3h}(p_i=0) = 6(\lambda_2 - \frac{\lambda_3^2}{\lambda_1}) v_2 + (3A - 6e^2(\lambda_2 - \lambda_3^2/\lambda_1))\; v_2 \frac{1}{16\pi^2} \ell n\, v_1^2$$

$$- 3[A + 20\, (\lambda_2 - \frac{\lambda_3^2}{\lambda_1})^2 + 6e^4 - 4e^2(\lambda_2 - \frac{\lambda_3^2}{\lambda_1})]\,.$$

$$\cdot\, v_2 \frac{1}{16\pi^2}\; \ell n \mu^2 \qquad (V.10)$$

whereas

$$-\Gamma^{*3h}(p_i=0) = 6\lambda^* v^* - 3(20\lambda^{*2} + 6e^{*4} - 2\lambda^* e^{*2})$$

$$\cdot\, v^* \frac{1}{16\pi^2}\; \ell n \mu^2 \qquad (V.11)$$

(5) $\Gamma^{4h}(p_i=0)$

Besides the integral of the type of Eq. (I.1) with $m \to m_h$ and $M \to m_H$ we have a new one of the form

$$m_H^4 \int d^4k \frac{1}{(k^2+m_h^2)^2} \frac{1}{(k^2+m_H^2)^2} \cong i\pi^2\, \ell n\, v_1^2 \qquad (V.12)$$

which again gives a $\ln v_1^2$ from the infrared region. All told, the results are

$$-\Gamma^{4h}(p_i=0) = 6(\lambda_2 - \frac{\lambda_3^2}{\lambda_1}) + 3(A-4e^2(\lambda_2 - \frac{\lambda_3^2}{\lambda_1}))\ \frac{1}{16\pi^2}\ \ln v_1^2$$

$$- 3\ [A+20(\lambda_2 - \frac{\lambda_3^2}{\lambda_1})^2 + 6e^4 - 8e^2(\lambda_2 - \frac{\lambda_3^2}{\lambda_1})]$$

$$\cdot \frac{1}{16\pi^2}\ \ln\mu^2 \qquad \text{(V.13)}$$

and

$$-\Gamma^{*4h}(p_i=0) = 6\lambda^* - 3(20\lambda^{*2} + 6e^{*4} - 4e^{*2}\lambda^*)$$

$$\cdot \frac{1}{16\pi^2}\ \ln\mu^2 \qquad \text{(V.14)}$$

VI. One Loop Results

To extract the effective parameters from the calculations in the last section, we first write

$$e^* = e^*_{tree} + \delta e^* = e + \delta e^* \tag{VI.1}$$

$$v^* = v^*_{tree} + \delta v^* = v_2 + \delta v^* \tag{VI.2}$$

$$\lambda^* = \lambda^*_{tree} + \delta\lambda^* = \lambda_2 - \frac{\lambda_3^2}{\lambda_1} + \delta\lambda^* \tag{VI.3}$$

$$Z_{A_1}^{1/2} = 1 + \delta L_{A_1} \tag{VI.4}$$

and

$$Z_h^{1/2} = 1 + \delta L_h \tag{VI.5}$$

in which all the δ-quantities are from one loop.

Now we use Eq. (V.1) to relate Γ's to Γ^*'s. Thus, from Γ^{2h}, we obtain

$$\delta L_h = e^2 \frac{1}{16\pi^2} \ell n\ (v_1^2/\mu^2) \tag{VI.6}$$

and

$$2(\delta\lambda^*)\ v_2^2 + 4(\lambda_2-\lambda_3^2/\lambda_1)v_2(\delta v^*) + 4(\lambda_2-\lambda_3^2/\lambda_1)(\delta L_h)$$
$$= v_2^2\ \frac{A}{16\pi^2}\ \ell n(v_1^2/\mu^2) \tag{VI.7}$$

As for Γ^{2A}, it yields

$$\delta L_{A_1} = \frac{3}{2} e^2 \frac{1}{16\pi^2} \ell n\ (v_1^2/\mu^2), \tag{VI.8}$$

and

$$2(\delta L_{A_1})\ e^2 v_2^2 + 2\ e(\delta e^*)\ v_2^2 + 2\ e^2\ v_2(\delta v^*) = 0 \tag{VI.9}$$

$\Gamma^{2A_1,h}$ gives

$$-4\, e(\delta e^*)v_2 - 2e^2(\delta v^*) - (\delta L_h + 2\delta L_{A_1})\, 2e^2 v_2$$

$$= 2e^4 v_2 \frac{1}{16\pi^2} \ell n\ (v_1^2/\mu^2) \tag{VI.10}$$

Finally, $\Gamma^{3h}(p_i=0)$ and $\Gamma^{4h}(p_i=0)$, respectively result in

$$3(\delta L_h)(\lambda_2 - \frac{\lambda_3^2}{\lambda_1})v_2 + (\delta\lambda^*)v_2 + (\lambda_2-\lambda_3^2/\lambda_1)(\delta v^*)$$

$$= \frac{1}{2}\ (A-2e^2(\lambda_2-\lambda_3^2/\lambda_1))\ \frac{1}{16\pi^2} \ell n(v_1^2/\mu^2) \tag{VI.11}$$

and

$$4(\delta L_h)(\lambda_2-\lambda_3^2/\lambda_1) + \delta\lambda^*$$

$$= \frac{1}{2}\ (A-4e^2\ (\lambda_2-\lambda_3^2/\lambda_1))\ \frac{1}{16\pi^2} \ell n\ (v_1^2/\mu^2) \tag{VI.12}$$

The consistent set of solutions to Eqs. (VI6-11) is

$$\delta e^* = -\frac{7}{2} e^3 \frac{1}{16\pi^2} \ell n\ (v_1^2/\mu^2) \tag{VI.13}$$

$$\delta v^* = 2e^2\ v_2 \frac{1}{16\pi^2} \ell n\ (v_1^2/\mu^2) \tag{VI.14}$$

$$\delta\lambda^* = (\frac{1}{2}\ A-6e^2\ (\lambda_2-\lambda_3^2/\lambda_1))\ \frac{1}{16\pi^2} \ell n\ (v_1^2/\mu^2) \tag{VI.15}$$

$$\delta L_{A_1} = \frac{3}{2}\ e^2 \frac{1}{16\pi^2}\ \ell n\ (v_1^2/\mu^2) \tag{VI.16}$$

and

$$\delta L_h = e^2 \frac{1}{16\pi^2} \ell n\ (v_1^2/\mu^2) \tag{VI.17}$$

There are several remarks we should make at this juncture:

(1) As we said earlier, there are seven equations to solve, which consistently yield the same values for the five effective parameters. This is an explicit partial verification of the decoupling theorem. The general proof was outlined in Section IV and will be elaborated on elsewhere.

(2) δv^* is proportional to v_2, which is a confirmation to this loop order that there is a natural separation of the lights from the heavies in a mass hierarchy. The violation is only logarithmic.

(3) There is one feature which stands out in the effective parameters: only the combination $\ell n(v_1^2/\mu^2)$ occurs. Algebraically, this comes about because of our previous observation that whatever the difference in coefficients is between $\ell n\ v_1^2$ and $-\ \ell n\ \mu^2$ in Γ, it is equal to the coefficient of $-\ \ell n\ \mu^2$ in Γ^*.

From purely dimensional ground, this matching need not be the case at all, because there is another scale v_2^2 in the theory. If $\ell n(v_2^2/\mu^2)$ were an active variable in the effective parameters, the coefficient of $\ell n\ v_1^2$ in general would not have matched that of $-\ \ell n\ \mu^2$. There is of course a profound reason why only $\ell n\ (v_1^2/\mu^2)$ appears. First of all, it helps to be reminded that the minimal subtraction procedure never introduces any extraneous infrared singularity into a thoery. The result here is merely a statement that the infrared behavior of the full theory is the same as that of the light theory as determined by the operator structure. We expect this remarkable result to be true to all orders in perturbation expansion.

VII. Renormalization Group Equations

Various proposals have been made in the literature to sum up the large $\ln(v_1^2/\mu^2)$ powers to relate parameters, such as masses and couplings at the unification scale to those at low energy region, where experiments are presently performed. For example, it was argued that when the external energy is increased to the unification scale, then the low energy effective couplings, up to some trivial Clebsch-Gordon coefficients, will merge into one. In such attempts, the running of the coupling constants was governed by the content of the light theory. This has its potential problems, as fully appreciated by workers in this area. When the external energy is raised, one will start producing superheavy objects. How to use an effective field theory to join in smoothly with the full theory is a subject of repeated discussion. Thus, some authors have devised methods to cross thresholds, while others have imposed high energy boundary conditions to circumvent discontinuous matching.[3,8]

Our approach to tackle this problem is different. We have shown that there is a decoupling theorem which in fact defines the low energy effective theory. All the $\ln(v_1^2/\mu^2)$ powers are just effects of radiative corrections due to heavy particles on low energy physical processes. In fact, it makes no sense for us to use the effective theory to study processes with external energy momentum comparable to the unification scale, because our approximation to arrive at the light theory restricts us to low energy. To put it abstractly, if we want to approach the unification scale, operators of higher dimensions than those we have included in the effective Lagrangian are just as important.

Let us then concentrate on low energy physics and work out a method to relate these two sets of parameters. We shall derive a set of renormalization group equations, which govern the dependence of the effective parameters on $\ln v_1^2/\mu^2$, as well as other parameters of the full theory. The solution of these equations will be carried out in the spirit of a leading logarithmic sum.

For ease of notation, let us use g to denote the generic coupling constant and Γ^n to denote the generic one light particle irreducible Green's function with n light legs in the full theory. We have the familiar renormalization group equation

$$\left(\mu \frac{\partial}{\partial\mu} + \beta \frac{\partial}{\partial g} + \gamma_{v_1} \frac{\partial}{\partial \ln v_1} + \gamma_{v_2} \frac{\partial}{\partial \ln v_2} + \gamma_\alpha \frac{\partial}{\partial \ln \alpha} - n\gamma\right) \cdot \Gamma^n = 0 \qquad \text{(VII.1)}$$

in which the bare parameters are fixed in carrying out differentiations. The various anomalous dimension coefficients are defined as usual

$$\beta = \mu \frac{d}{d\mu} g \qquad \text{(VII.2)}$$

$$\gamma_{v_1} = \mu \frac{d}{d\mu} \ln v_1 \qquad \text{(VII.3)}$$

$$\gamma_{v_2} = \mu \frac{d}{d\mu} \ell n \; v_2 \tag{VII.4}$$

$$\gamma_\alpha = \mu \frac{d}{d\mu} \ell n \; \alpha \tag{VII.5}$$

and

$$2\gamma = \mu \frac{d}{d\mu} (\ell n \; Z)_{\text{full}} \tag{VII.6}$$

There is also the renormalization group equation for the one particle irreducible Green's functions in the light theory

$$\left(\mu \frac{\partial}{\partial\mu} + \beta^* \frac{\partial}{\partial g^*} + \gamma^*_{v^*} \frac{\partial}{\partial \,\ell n\, v^*} + \gamma^*_{\alpha^*} \frac{\partial}{\partial \,\ell n\, \alpha^*} - n\,\gamma^*\right)\Gamma^{*n} = 0 \tag{VII.7}$$

in which bare parameters of the light theory are held fixed. We define

$$\beta^* = \mu \frac{d}{d\mu} g^* \tag{VII.8}$$

$$\gamma^*_{v^*} = \mu \frac{d}{d\mu} \ell n \; v^* \tag{VII.9}$$

$$\gamma^*_{\alpha^*} = \mu \frac{d}{d\mu} \ell n \; \alpha^* \tag{VII.10}$$

and

$$2\gamma^* = \mu \frac{d}{d\mu} \ell n \; Z^* \tag{VII.11}$$

Because of decoupling, we must demand that upon substituting Eq. (I.3) into Eq. (VII.1), the resulting equation for Γ^* is identical to Eq. (VII.7). For this to be so, we have the following relations:

$$\left(\mu \frac{\partial}{\partial \mu} + \beta_g \frac{\partial}{\partial g} + \gamma_{v_1} \frac{\partial}{\partial \ln v_1} + \gamma_{v_2} \frac{\partial}{\partial \ln v_2} + \gamma_\alpha \frac{\partial}{\partial \ln \alpha}\right) g^* = \beta^* \tag{VII.12}$$

$$\left(\mu \frac{\partial}{\partial \mu} + \beta_g \frac{\partial}{\partial g} + \gamma_{v_1} \frac{\partial}{\partial \ln v_1} + \gamma_{v_2} \frac{\partial}{\partial \ln v_2} + \gamma_\alpha \frac{\partial}{\partial \ln \alpha}\right) \cdot \ln v^* = \gamma^*_{v^*} \tag{VII.13}$$

$$\left(\mu \frac{\partial}{\partial \mu} + \beta_g \frac{\partial}{\partial g} + \gamma_{v_1} \frac{\partial}{\partial \ln v_1} + \gamma_{v_2} \frac{\partial}{\partial \ln v_2} + \gamma_\alpha \frac{\partial}{\partial \ln \alpha}\right) \cdot \ln \alpha^* = \gamma^*_{\alpha^*} \tag{VII.14}$$

and

$$\left(\mu \frac{\partial}{\partial \mu} + \beta_g \frac{\partial}{\partial g} + \gamma_{v_1} \frac{\partial}{\partial \ln v_1} + \gamma_{v_2} \frac{\partial}{\partial \ln v_2} + \gamma_\alpha \frac{\partial}{\partial \ln \alpha}\right) \cdot \ln Z = 2(\gamma - \gamma^*) \tag{VII.15}$$

Note that Z in the last equation is the finite wave function renormalization which must be performed to carry the full theory over to the light theory. Eqs. (VII.12-15) are just chain differention relations. They nevertheless impose very stringent conditions on how the effective parameters can depend on $\ln v_1^2/\mu^2$. As they stand, however, they are not at all useful, because both sides of the equations have large $\ln v_1^2/\mu^2$ dependence and any integration is only pro forma. We must rely on perturbation theory to solve them.

Let us see how a leading logarithm sum is performed. As an example, we write the effective gauge charge

$$e^* = e\ f(e^2\ \ln v_1/\mu) \tag{VII.16}$$

in which e is regarded as a small quantity, but $e^2 \ln v_1/\mu$ is taken to be of order unity. It should be noted that we have used the suggestive result of Section VI which states that the effective parameters have no $\ln\ v_2^2/\mu^2$ dependence. Furthermore, it has been verified that e^* has no gauge parameter α dependence in the leading logarithm approximation. Equating terms of the same order in e, we find that the leading logarithm sum satisfies the equation

$$(\mu \frac{\partial}{\partial \mu} + \beta_e \frac{\partial}{\partial e})\ e^* = \beta^*_{e^*} \tag{VII.17}$$

in which β_e and $\beta^*_{e^*}$ need to be calculated only to the lowest non-trivial order, i.e.

$$\beta_e = -\frac{20}{3}\frac{e^3}{16\pi^2} \tag{VII.18}$$

$$\beta^*_{e^*} = \frac{1}{3}\frac{e^{*3}}{16\pi^2} \tag{VII.19}$$

We rewrite (VII.17) as

$$(\frac{\partial}{\partial \kappa} - \beta_e \frac{\partial}{\partial e})\ \frac{24\pi^2}{e^{*2}} = 1 \tag{VII.20}$$

where

$$\kappa \equiv \ln(v_1/\mu) \tag{VII.21}$$

The solution of this linear equation(VII.20) can be arrived at if we expand $f(e^2\kappa)$ of Eq. (VII.16) in an infinite series of its agrument and substitute into Eq. (VII.20) to obtain recurrence relations of the coefficients. However, a more elegant and customary way to solve it is to introduce a running coupling constant $\bar{e}$, which satisfies the equation

$$\frac{d}{dk}\bar{e} = \beta_e(\bar{e}) = -\frac{20}{3}\frac{\bar{e}^3}{16\pi^2} \tag{VII.22}$$

with the solution

$$\bar{e}^2 = e^2/(1 + \frac{40}{3}\frac{e^2}{16\pi^2}\kappa) \tag{VII.23}$$

We wish to emphasize the point that this intermediary coupling constant 'runs' according to the content of the full theory.

Now, the standard method yields

$$\frac{24\pi^2}{e^{*2}} = \frac{24\pi^2}{\bar{e}^2} + \kappa \tag{VII.24}$$

or

$$e^{*2} = \frac{e^2}{1 + 14\frac{e^2}{16\pi^2}\kappa} \tag{VII.25}$$

This solution is consistent with Eq. (VI.13)[13], which is the one loop result.

We can generalize this method to perform the next to leading logarithm sum as well. In general, the mass insertion term $\gamma_{v_1} \dfrac{\partial}{\partial \ln v_1}$ and the gauge term $\gamma_\alpha \dfrac{\partial}{\partial \ln \alpha}$ cannot be neglected. We are looking into more realistic models to carry out this exercise, however.

Before we go on to the next section, we want to reiterate that our renormalization group equations are formulated entirely with respect to physics in the low energy sector. Information of the full theory is fed into these equations through the β's and the γ's, while that of the light theory is given via the β^*'s and the γ^*'s. To us, this is the cleanest and most unambiguous way to interrelate parameters of the two theories.

VII. Concluding Remarks

In this article and a sequel, we have taken a gauge model with O(3) symmetry and analyzed the problem of the gauge hierarchy and decoupling. We have shown to all orders that the separation of particles into light and heavy sectors as in the grand unification scheme is stable in a perturbation expansion; the heavy mass effects give only logarithmic correction to the light effective masses. Also, we have shown that if we confine ourselves to low energy light particle processes only, there exists an effective Lagrangian with light fields alone that can be used to reproduce all the one light particle irreducible Green's functions to an accuracy of $\mathcal{O}(1)$. All the heavy particle effects again appear only as logarithmic corrections and can be absorbed by finite wave function, mass and coupling renormalizations. In the specific model, the limiting effective Lagrangian is the O(2) Higgs model. Last but not the least, we have proposed a set of renormalization group equations to systematically sum the large $\ln(v_1^2/\mu^2)$ terms. Here, our approach is to stay in the low energy region to perform this task to avoid the imposition of extra boundary conditions.

We have explicitly verified all we have said at the one loop level.

It is expected that the method and approach advocated here to be independent of the symmetry group. A model was chosen mainly to make our discussion concrete. We are extending this program to other cases where the symmetry group better accommodates nature, such as SU(5).

Acknowledgments

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APPENDIX

In this appendix we display diagram by diagram the one loop, leading order contributions to Γ^{2h}, Γ^{2A_1}, $\Gamma^{2A_1,h}$, Γ^{3h} and Γ^{4h}. By considering the diagrams that consist of only light fields, one can also easily extract Γ^{*2h}, Γ^{*2A_1}, $\Gamma^{*2A_1,h}$, Γ^{*3h} and Γ^{*4h}. For notational simplicity in the following tables we have defined

$$\frac{1}{\varepsilon'} \equiv \frac{1}{16\pi^2}\left[\frac{1}{\varepsilon} - \ell n\left(\frac{v_1^2}{\mu^2}\right)\right] \tag{A.1}$$

$$\frac{1}{\varepsilon''} \equiv \frac{1}{16\pi^2}\left[\frac{1}{\varepsilon} + \ell n\ \mu^2\right] \tag{A.2}$$

where

$$\varepsilon \equiv 2-n/2 \tag{A.3}$$

with n being the dimension of space-time. To obtain the results given in Section V from the tables we have employed minimal subtraction renormalization.

The "tadpole" contributions given in the figures and tables are determined from Eqs. (III.6, 10 and 11) and just contribute to the h, H and h-H propagators. In particular the "tadpole" contributions to the h-line, H line and h-H line (ρ_{hh}, ρ_{HH} and ρ_{hH} respectively) are given by

$$\rho_{hh} = R_1\left(\frac{1}{v_1}\cos\theta\,\sin^2\theta - \frac{1}{v_2}\sin\theta\,\cos^2\theta\right) + R_2\left(\frac{1}{v_1}\sin^3\theta + \frac{1}{v_2}\cos^3\theta\right) \tag{A.4}$$

$$\rho_{HH} = R_1 \left(\frac{1}{v_1} \cos^3\theta - \frac{1}{v_2} \sin^3\theta\right) + R_2 \left(\frac{1}{v_1} \cos^2\theta \sin\theta + \frac{1}{v_2} \sin^2\theta \cos\theta\right) \tag{A.5}$$

$$\rho_{hH} = \sin\theta \cos\theta\left[R_1\left(\frac{1}{v_1} \cos\theta + \frac{1}{v_2} \sin\theta\right) + R_2\left(\frac{1}{v_1} \sin\theta - \frac{1}{v_2} \cos\theta\right)\right] \tag{A.6}$$

where R_1 and R_2 are the radiative corrections to the H and h tadpoles and θ is given by Eq. II.22. In the tables the "tadpole" contributions are indicated by an x where the letter above it indicates the particle in the tadpole loop.

References and Footnotes

1. K. Symanzik, Comm. Math. Phys. 34, 7 (1973); T. Appelquist and J. Carrazone, Phys. Rev. D11, 2856 (1975).
2. The original gauge hierarchy problem was pointed out by E. Gildener, Phys. Rev. D14, 1667 (1976). For further disucssion and references see I. Bars, invited talk at Orbis Scientiae 1979, Coral Gables, Florida; E. Gildner, Phys. Letts. 92B, 111 (1980); K.T. Mahanthappa and D.G. Unger, preprint COLO-HEP-14 and UM HE 79-31, (revised 1980).
3. There has been a flurry of activity in demonstrating decoupling at the one loop or two loop level. A partial list is: B. Ovrut and H. Schnitzer, Phys. Rev. D21, 3369 (1980), D22, 2518 (1980) and Brandeis preprints; T. Hagiwara and N. Nakazawa, preprint HUTP-80/A012; F.T. Hadjioannou, A.B. Lahanas and C.E. Vayonakis, Phys. Lett. 84B, 427 (1979); G. Senjanovic and A. Sokorac, Nucl. Phys. B164, 305 (1980); M. Yoshimura, Prog. Theor. Phys. 64, 353 and 594 (1980). For different approaches, see S. Weinberg, Phys. Letts. 91B, 51 (1980), N.P. Chang, A. Das and J. Perez-Mercader, Phys. Rev. 22, 1414 (1980); P. Binétruy and T. Schücker, CERN preprints TH-2802-CERN, TH 2857-CERN; L. Hall, preprint HUTP-80/A024.
4. M. Veltman, Acta Physica Polonica B8, 475 (1978); G. Senjanovic and A. Sokorac, Phys. Rev. D18, 2708 (1978); D. Toussaint, Phys. Rev. D18, 1626 (1978); T. Appelquist and R. Shankar, Nucl. Phys. B158, 317 (1979);

T. Appelquist and C. Bernard, Phys. Rev. D22, 200 (1980); A. Longhitano, Phys. Rev. D22, 1166 (1980).

5. J.C. Pati and A. Salam, Phys. Rev. D8, 1240 (1973); H. Georgi and S.L. Glashow, Phys. Rev. Letts. 32, 438 (1974); H. Georgi, H.R. Quinn and S. Weinberg, Phys. Rev. Letts. 33, 451 (1974).

6. Fine tuning here means the order by order adjustment of parameters in the scalar potential to achieve a large gauge hierarchy.

7. These Green's functions were discussed by, e.g., B.W. Lee in Chiral Dynamics (Gordon and Breach, New York, 1972).

8. A partial list is: D.A. Ross, Nucl. Phys. B140, 1 (1978); A.J. Buras, J. Ellis, M.K. Gaillard, and D.V. Nanopoulos, Nucl. Phys. B135, 66 (1978); T. Goldman and D.A. Ross, Phys. Letts. 84B, 208 (1979); C.H. Llewellyn Smith, G.G. Ross and J.F. Wheater, Oxford Preprint 1980; I. Antoniadis, C. Bouchiat, J. Iliopoulos, LPTENS 80/21 (1980). For a review and further references, see J. Ellis, M.K. Gaillard, D.V. Nanopoulos and S. Rudaz, Nucl. Phys. B176, 61 (1980).

9. Yoichi Kazama and York-Peng Yao, in preparation.

10. For example, B.W. Lee, Les Houches, 1975, R. Bailian and J. Zinn-Justin, editors, (North Holland 1976).

11. C. Becchi, A. Rouet and R. Stora, CPT report, Marseille, 1974 (unpublished) and Ann. Phys. (N.Y.) 98, 287 (1976).

12. W. Zimmermann, in Lectures on Elementary Particles and Quantum Field Theory, edited by S. Deser et al. (MIT Press, Cambridge, Mass., 1971), Vol. I, p. 397.

13. This result, when generalized to other bigger groups such as SU(5), agrees with that given by H. Georgi, H.R. Quinn and S. Weinberg, Phys. Rev. Letts. 33, 451 (1974).

TABLE CAPTIONS

Table I. Contributions to Γ^{2h} corresponding to the diagrams in Figure 4.

Table II. Contributions to Γ^{2A_1} corresponding to the diagrams in Figure 5.

Table III. Contributions to $\Gamma^{2A_1,h}$ corresponding to the diagrams in Figure 6. A multiplicative factor of v_2 is suppressed for each entry.

Table IV. Contributions to Γ^{3h} corresponding to the diagrams in Figure 7. A multiplicative factor of v_2 is suppressed for each entry.

Table V. Contributions to Γ^{4h} corresponding to the diagrams in Figure 8.

FIGURE CAPTIONS

Figure 1. Diagrams that contribute to Γ^{4h} in the tree approximation. The dashed lines represent h-lines while the solid lines represent H-lines.

Figure 2. An example of the algebraic rearrangement. The dashed lines represent the light (mass m) lines while the solid line represents the heavy (mass M) line. Diagram (ii) indicates parts 1 and 2.

Figure 3. Diagrams that result after the algebraic rearrangement. The blob indicates the effective light theory vertex.

Figure 4. Diagrams that contribute to Γ^{2h}. The external lines are h-lines. Diagram (iii) represents the tadpole contributions.

Figure 5. Diagrams that contribute to Γ^{2A_1}. The external lines are A_1 lines.

Figure 6. Diagrams that contribute to $\Gamma^{2A_1,h}$. The wavy external lines are A_1-lines while the dashed external line is an h-line.

Figure 7. Diagrams that contribute to Γ^{3h}. All the external lines are h-lines. In diagram (v) the x represents the tadpole contributions.

Figure 8. Diagrams that contribute to Γ^{4h}. The external lines are h-lines. In diagram (viii) the x represents the tadpole contributions.

TABLE I

Diagram Type	α	β	Contribution to Γ^{2h}
(i)	H	H	$\frac{1}{\varepsilon'} v_2^2 \left[8\lambda_3^2-16\lambda_3^3/\lambda_1+8\lambda_3^4/\lambda_1^2\right]$
	η	η	$\frac{1}{\varepsilon'} v_2^2\left[2\lambda_2^2+2\lambda_4^2+2\lambda_3^2\lambda_4^2/\lambda_1^2+2\lambda_3^4/\lambda_1^2+4\lambda_2\lambda_4 -4\lambda_2\lambda_3\lambda_4/\lambda_1-4\lambda_2\lambda_3^2/\lambda_1-4\lambda_3\lambda_4^2/\lambda_1-4\lambda_3^2\lambda_4/\lambda_1 +4\lambda_3^3\lambda_4/\lambda_1\right]$
	h	h	$\frac{1}{\varepsilon''} v_2^2\left[18(\lambda_2-\lambda_3^2/\lambda_1)^2\right]$
	ψ_3	ψ_3	$\frac{1}{\varepsilon''} v_2^2 \left[2(\lambda_2-\lambda_3^2/\lambda_1)^2\right]$
	h	H	$\frac{1}{\varepsilon'} \{v_1^2[4\lambda_3^2]+v_2^2[-32\lambda_3^4/\lambda_1^2+28\lambda_2\lambda_3^2/\lambda_1+8\lambda_3^3/\lambda_1]\} - \frac{1}{\varepsilon''} v_2^2 \left[4\lambda_2\lambda_3^2/\lambda_1-4\lambda_3^4/\lambda_1^2\right]$
	ξ	η	$\frac{1}{\varepsilon'} v_1^2[\lambda_4^2]+v_2^2[-4\lambda_2\lambda_4+2\lambda_3\lambda_4^2/\lambda_1-\lambda_3^2\lambda_4^2/\lambda_1^2 + 4\lambda_3^2\lambda_4/\lambda_1]\}$
	A_1	A_1	$\frac{1}{\varepsilon''} v_2^2 \left[8e^4\right]$
	A_2	A_2	$\frac{1}{\varepsilon'} v_2^2 \left[8e^4\lambda_3^2/\lambda_1^2\right]$
	A_3	A_3	$\frac{1}{\varepsilon'} v_2^2 \left[8e^4-16\lambda_3 e^4/\lambda_1+8\lambda_3^2 e^4/\lambda_1^2\right]$
	C_1	C_1	$\frac{1}{\varepsilon''} v_2^2 \left[-e^4\right]$
	C_2	C_2	$\frac{1}{\varepsilon'} v_2^2 \left[-e^4\lambda_3^2/\lambda_1^2\right]$
	C_3	C_3	$\frac{1}{\varepsilon'} v_2^2 \left[-e^4 +2\lambda_3 e^4/\lambda_1-\lambda_3^2 e^4/\lambda_1^2\right]$
	A_1	ψ_3	$\frac{1}{\varepsilon''} \{v_2^2[-2e^4]+p^2[2e^2]\}$

Table I con't.

Diagram Type	α	β	Contribution to Γ^{2h}
	A_3	η	$\frac{1}{\varepsilon'}\{v_1^2[-e^4-e^2\lambda_4]+v_2^2[e^4\lambda_3^2/\lambda_1^2+e^2\lambda_3^2\lambda_4/\lambda_1^2$ $-2e^4\lambda_3/\lambda_1-2e^2\lambda_3\lambda_4/\lambda_1]+p^2[2e^2]\}$
	A_3	ξ	$\frac{1}{\varepsilon'}v_2^2[-2e^4+4e^4\lambda_3/\lambda_1-2e^4\lambda_3^2/\lambda_1^2]$
	A_2	π_3	$\frac{1}{\varepsilon'}v_2^2[-2e^4\lambda_3^2/\lambda_1^2]$
(ii)	H		$\frac{1}{\varepsilon'}\{v_1^2[2\lambda_1\lambda_3]+v_2^2[6\lambda_3^2-10\lambda_3^3/\lambda_1+6\lambda_2\lambda_3^2/\lambda_1]\}$
	η		$\frac{1}{\varepsilon'}\{v_1^2[\lambda_2\lambda_4]+v_2^2[\lambda_4^2+\lambda_3\lambda_4+\lambda_3^3\lambda_4/\lambda_1^2-\lambda_2\lambda_3^2\lambda_4/\lambda_1^2$ $+\lambda_3^2\lambda_4^2/\lambda_1^2-2\lambda_3\lambda_4^2/\lambda_1]\}$
	ξ		$\frac{1}{\varepsilon'}\{v_1^2[\lambda_3e^2+\lambda_4e^2]+v_2^2[e^2\lambda_2+e^2\lambda_3^2/\lambda_1-e^2\lambda_3^3/\lambda_1^2$ $-e^2\lambda_3^2\lambda_4/\lambda_1^2+2e^2\lambda_3\lambda_4/\lambda_1]\}$
	π_3		$\frac{1}{\varepsilon'}\{v_1^2[\lambda_3e^2]+v_2^2[-e^2\lambda_3^3/\lambda_1^2+e^2\lambda_3^2/\lambda_1]\}$
	h		$\frac{1}{\varepsilon''}v_2^2[6\lambda_2(\lambda_2-\lambda_3^2/\lambda_1)]$
	ψ_3		$\frac{1}{\varepsilon''}v_2^2[\lambda_2e^2]$
	A_1		$\frac{1}{\varepsilon''}v_2^2[4e^4]$
	A_2		$\frac{1}{\varepsilon'}v_2^2[4e^4\lambda_3^2/\lambda_1^2]$
	A_3		$\frac{1}{\varepsilon'}\{v_1^2[4e^4]+v_2^2[4e^4]\}$
(iii)	H		$\frac{1}{\varepsilon'}\{v_1^2[-2\lambda_1\lambda_3-4\lambda_3^2]+v_2^2[8\lambda_3^4/\lambda_1^2+2\lambda_3^3/\lambda_1$ $-10\lambda_2\lambda_3^2/\lambda_1-6\lambda_3^2]\}$

Table I con't.

Diagram Type	α	β	Contribution to Γ^{2h}
	h		$\frac{1}{\varepsilon''} v_2^2[-6\lambda_2^2+10\lambda_2\lambda_3^2/\lambda_1-4\lambda_3^4/\lambda_1^2]$
	η		$\frac{1}{\varepsilon'} \{v_1^2[-\lambda_2\lambda_4-\lambda_4^2]+v_2^2[-\lambda_4^2-\lambda_3\lambda_4+\lambda_2\lambda_3^2\lambda_4/\lambda_1^2$ $-\lambda_3^3\lambda_4/\lambda_1^2]\}$
	ξ		$\frac{1}{\varepsilon'} \{v_1^2[-\lambda_3 e^2]+v_2^2[-e^2\lambda_2+e^2\lambda_3^3/\lambda_1^2-e^2\lambda_3^2/\lambda_1]$
	π_3		$\frac{1}{\varepsilon'} \{v_1^2[-\lambda_3 e^2]+v_2^2[\lambda_3^3 e^2/\lambda_1^2-\lambda_3^2 e^2/\lambda_1]\}$
	ψ_3		$\frac{1}{\varepsilon''} v_2^2\ [-e^2\lambda_2]$
	A_1		$\frac{1}{\varepsilon''} v_2^2[-4e^4]$
	A_2		$\frac{1}{\varepsilon'} v_2^2[-4e^4\lambda_3^2/\lambda_1^2]$
	A_3		$\frac{1}{\varepsilon'} \{v_1^2[-4e^4]+v_2^2[-4e^4]\}$
	C_1		$\frac{1}{\varepsilon''} v_2^2\ [e^4]$
	C_2		$\frac{1}{\varepsilon'} v_2^2\ [e^4\lambda_3^2/\lambda_1^2]$
	C_3		$\frac{1}{\varepsilon'} \{v_1^2[e^4]+v_2^2[e^4]\}$

TABLE II

Diagram Type	α	β	Contribution to Γ^{2A_1}
(i)	A_2	A_3	$\frac{1}{\varepsilon'}\{v_1^2[-9e^4]+v_2^2[-\frac{9}{2}e^4]+p^2[\frac{19}{6}e^2]\}$
	A_2	ξ	$\frac{1}{\varepsilon'}\{v_1^2[e^4] + v_2^2[-3e^4]\}$
	A_2	η	$\frac{1}{\varepsilon'} v_2^2 [4e^4]$
	A_3	π_3	$\frac{1}{\varepsilon'} v_1^2 [e^4]$
	H	ψ_3	$\frac{1}{\varepsilon'} v_2^2 [-2e^2\lambda_3^2/\lambda_1]$
	ξ	π_3	$\frac{1}{\varepsilon'}\{v_1^2[-2e^4] + v_2^2[e^4] + p^2[-e^2/3]\}$
	η	π_3	$\frac{1}{\varepsilon'} v_2^2 [-\lambda_4 e^2 - e^4]$
	C_2	C_3	$\frac{1}{\varepsilon'}\{v_1^2[e^4] + v_2^2[e^4/2] + p^2[e^2/6]\}$
	h	ψ_3	$\frac{1}{\varepsilon''}\{v_2^2[-2e^2(\lambda_2-\lambda_3^2/\lambda_1)-e^4] + p^2[-e^2/3]\}$
	A_1	h	$\frac{1}{\varepsilon''} v_2^2 [4e^4]$
(ii)	A_2		$\frac{1}{\varepsilon'} v_1^2 [3e^4]$
	A_3		$\frac{1}{\varepsilon'}\{v_1^2[3e^4] + v_2^2[3e^4]\}$
	η		$\frac{1}{\varepsilon'} v_2^2 [\lambda_4 e^2]$
	ξ		$\frac{1}{\varepsilon'} v_1^2 [e^4]$
	π_3		$\frac{1}{\varepsilon'} v_1^2 [e^4]$
	H		$\frac{1}{\varepsilon'} v_2^2 [2e^2\lambda_3^2/\lambda_1]$
	h		$\frac{1}{\varepsilon''} v_2^2 [2(\lambda_2-\lambda_3^2/\lambda_1)e^2]$
	ψ_3		$\frac{1}{\varepsilon''} v_2^2 [e^4]$

TABLE III

Diagram Type	α	β	γ	Contribution to $\Gamma^{2A_1,h}$
(i)	C_2	C_3	C_3	$\frac{1}{\varepsilon'}\left[\frac{1}{2}e^4(1-\lambda_3/\lambda_1)\right]$
	C_3	C_2	C_2	$\frac{1}{\varepsilon'}\left[-\frac{1}{2}e^4\lambda_3/\lambda_1\right]$
	A_2	A_3	A_3	$\frac{1}{\varepsilon'}\left[-9e^4(1-\lambda_3/\lambda_1)\right]$
	A_3	A_2	A_2	$\frac{1}{\varepsilon'}\left[9e^4\lambda_3/\lambda_1\right]$
	π_3	η	ξ	$\frac{1}{\varepsilon'}\left[-2e^2\lambda_4\right]$
	ψ_3	h	H	$\frac{1}{\varepsilon'}\left[-4\lambda_3^2e^2/\lambda_1\right]$
	π_3	A_3	ξ	$\frac{1}{\varepsilon'}\left[-e^4(1-\lambda_3/\lambda_1)\right]$
	ξ	A_2	π_3	$\frac{1}{\varepsilon'}\left[\lambda_3e^4/\lambda_1\right]$
	π_3	A_3	η	$\frac{1}{\varepsilon'}\left[e^4\right]$
	A_3	A_2	π_3	$\frac{1}{\varepsilon'}\left[-\frac{3}{2}e^4\lambda_3/\lambda_1\right]$
	A_2	A_3	η	$\frac{1}{\varepsilon'}\left[-3e^4\right]$
	A_2	A_3	ξ	$\frac{1}{\varepsilon'}\left[\frac{3}{2}e^4(1-\lambda_3/\lambda_1)\right]$
	ψ_3	h	h	$\frac{1}{\varepsilon''}\left[6(\lambda_3^2/\lambda_1-\lambda_2)e^2\right]$
	h	ψ_3	ψ_3	$\frac{1}{\varepsilon''}\left[2(\lambda_3^2/\lambda_1-\lambda_2)e^2\right]$
	h	A_1	ψ_3	$\frac{1}{\varepsilon''}\left[-2e^4\right]$

Table III con't.

Diagram Type	α	β	γ	Contribution to $\Gamma^{2A_1,h}$
(ii)	η	ξ		$\frac{1}{\varepsilon'}\left[2e^2\lambda_4\right]$
	H	h		$\frac{1}{\varepsilon'}\left[4e^2\lambda_3^2/\lambda_1\right]$
	A_3	A_3		$\frac{1}{\varepsilon'}\left[6e^4(1-\lambda_3/\lambda_1)\right]$
	A_2	A_2		$\frac{1}{\varepsilon'}\left[-6e^4\lambda_3/\lambda_1\right]$
	h	h		$\frac{1}{\varepsilon''}\left[6(\lambda_2-\lambda_3^2/\lambda_1)e^2\right]$
	ψ_3	ψ_3		$\frac{1}{\varepsilon''}\left[2(\lambda_2-\lambda_3^2/\lambda_1)e^2\right]$
(iii)	A_2	η		$\frac{1}{\varepsilon'}\left[4e^4\right]$
	A_3	π_3		$\frac{1}{\varepsilon'}\left[-2e^4\lambda_3/\lambda_1\right]$
	A_2	ξ		$\frac{1}{\varepsilon'}\left[-2e^4(1+\lambda_3/\lambda_1)\right]$
	A_1	h		$\frac{1}{\varepsilon''}\left[8e^4\right]$

TABLE IV

Diagram Type	α	β	γ	Contribution to Γ^{3h}
(i)	h	h	H	$\ell n(v_1^2)[-36\ \lambda_3^2(\lambda_2-\lambda_3^2/\lambda_1)/\lambda_1]$
	A_3	A_3	H	$\frac{1}{\varepsilon'}\ [-6e^4 + 6e^4\lambda_3/\lambda_1]$
	η	η	A_3	$\frac{1}{\varepsilon'}[-6\lambda_2 e^2-6\lambda_4 e^2+6\lambda_3\lambda_4 e^2/\lambda_1+6\lambda_3^2 e^2/\lambda_1]$
	η	ξ	A_3	$\frac{1}{\varepsilon'}\ [6e^2\lambda_4-6e^2\lambda_3\lambda_4/\lambda_1]$
	ψ_3	A_1	A_1	$\frac{1}{\varepsilon''}\ [-6e^4]$
	A_1	ψ_3	ψ_3	$\frac{1}{\varepsilon''}\ [-6e^2(\lambda_2-\lambda_3^2/\lambda_1)]$
(ii)	H	H	H	$\frac{1}{\varepsilon'}\ [36\lambda_3(\lambda_3-\lambda_3^2/\lambda_1)]$
	h	H	H	$\frac{1}{\varepsilon'}\ [24\ \lambda_3^3/\lambda_1-24\lambda_3^4/\lambda_1^2]$
	η	η	H	$\frac{1}{\varepsilon'}[-6\lambda_2\lambda_3^2/\lambda_1-6\lambda_2\lambda_3\lambda_4/\lambda_1-6\lambda_3^2\lambda_4/\lambda_1$ $-6\lambda_3\lambda_4^2/\lambda_1+12\lambda_3^3\lambda_4/\lambda_1^2+6\lambda_3^2\lambda_4^2/\lambda_1^2+6\lambda_3^4/\lambda_1^2]$
	η	ξ	H	$\frac{1}{\varepsilon'}[-6\lambda_3\lambda_4+6\lambda_3^2\lambda_4/\lambda_1+3\lambda_3\lambda_4^2/\lambda_1-3\lambda_3^2\lambda_4^2/\lambda_1^2]$
	A_2	A_2	H	$\frac{1}{\varepsilon'}\ [24e^4\lambda_3^2/\lambda_1^2]$
	A_3	A_3	H	$\frac{1}{\varepsilon'}\ [-24e^4\lambda_3/\lambda_1+24e^4\lambda_3^2/\lambda_1^2]$
	C_2	C_2	H	$\frac{1}{\varepsilon'}\ [-3e^4\lambda_3^2/\lambda_1^2]$
	C_3	C_3	H	$\frac{1}{\varepsilon'}\ [3e^4\lambda_3/\lambda_1-3e^4\lambda_3^2/\lambda_1^2]$
	h	h	H	$\frac{1}{\varepsilon''}\ [-18\lambda_3^2(\lambda_2-\lambda_3^2/\lambda_1)/\lambda_1]$
	ψ_3	ψ_3	H	$\frac{1}{\varepsilon''}\ [-6\lambda_3^2(\lambda_2-\lambda_3^2/\lambda_1)/\lambda_1]$

Table IV con't.

Diagram Type	α	β	γ	Contribution to Γ^{3h}
	A_2	π_3	H	$\frac{1}{\varepsilon'}\left[-6e^4\lambda_3^2/\lambda_1^2\right]$
	A_3	η	H	$\frac{1}{\varepsilon'}\left[-3e^4\lambda_3/\lambda_1+3e^4\lambda_3^2/\lambda_1^2-3e^2\lambda_3\lambda_4/\lambda_1+3e^2\lambda_3^2\lambda_4/\lambda_1^2\right]$
	A_3	ξ	H	$\frac{1}{\varepsilon'}\left[6e^4\lambda_3/\lambda_1-6e^4\lambda_3^2/\lambda_1^2\right]$
(iii)	H	H		$\frac{1}{\varepsilon'}\left[12\,\lambda_3(\lambda_3^2/\lambda_1-\lambda_3)\right]$
	h	h		$\frac{1}{\varepsilon''}\left[54\,\lambda_2(\lambda_2-\lambda_3^2/\lambda_1)\right]$
	η	η		$\frac{1}{\varepsilon'}\left[6\,\lambda_2^2+6\lambda_2\lambda_4-6\lambda_2\lambda_3\lambda_4/\lambda_1-6\lambda_2\lambda_3^2/\lambda_1\right]$
	A_3	A_3		$\frac{1}{\varepsilon'}\left[24e^4-24e^4\lambda_3/\lambda_1\right]$
	ψ_3	ψ_3		$\frac{1}{\varepsilon''}\left[6\lambda_2(\lambda_2-\lambda_3^2/\lambda_1)\right]$
	A_1	A_1		$\frac{1}{\varepsilon''}\left[24e^4\right]$
	h	H		$\frac{1}{\varepsilon'}\left[36\lambda_3^2(\lambda_2-\lambda_3)/\lambda_1\right]$
	η	ξ		$\frac{1}{\varepsilon'}\left[6\lambda_3\lambda_4-6\lambda_2\lambda_4+6\lambda_4^2-6\lambda_3\lambda_4^2/\lambda_1\right]$
(iv)	H	H		$\frac{1}{\varepsilon'}\left[18\lambda_3(\lambda_3-\lambda_3^2/\lambda_1)\right]$
	η	H		$\frac{1}{\varepsilon'}\left[-3\lambda_2\lambda_3^2\lambda_4/\lambda_1^2+3\lambda_3^3\lambda_4/\lambda_1^2+3\lambda_3^2\lambda_4^2/\lambda_1-3\lambda_3\lambda_4^2/\lambda_1\right]$
	ξ	H		$\frac{1}{\varepsilon'}\left[3\lambda_3^2e^2/\lambda_1-3\lambda_3^3e^2/\lambda_1^2-3\lambda_3^2\lambda_4e^2/\lambda_1^2+3\lambda_3\lambda_4e^2/\lambda_1\right]$
	π_3	H		$\frac{1}{\varepsilon'}\left[-3\lambda_3^3e^2/\lambda_1^2+3\lambda_3^2e^2/\lambda_1\right]$

Table IV con't.

Diagram Type	α	β	γ	Contribution to Γ^{3h}
	A_2	H		$\frac{1}{\varepsilon'}\left[12\lambda_3^2 e^4/\lambda_1^2\right]$
(v)	H	H		$\frac{1}{\varepsilon'}\left[-18\lambda_3^2 + 12\lambda_3^4/\lambda_1^2+6\lambda_3^3/\lambda_1\right]$
	η	H		$\frac{1}{\varepsilon'}\left[3\lambda_2\lambda_3^2\lambda_4/\lambda_1^2-3\lambda_3^3\lambda_4/\lambda_1^2\right]$
	ξ	H		$\frac{1}{\varepsilon'}\left[3e^2\lambda_3^3/\lambda_1^2-3e^2\lambda_3^2/\lambda_1\right]$
	π_3	H		$\frac{1}{\varepsilon'}\left[-3\lambda_3^2e^2/\lambda_1+3\lambda_3^3e^2/\lambda_1^2\right]$
	A_2	H		$\frac{1}{\varepsilon'}\left[-12e^4\lambda_3^2/\lambda_1^2\right]$
	C_2	H		$\frac{1}{\varepsilon'}\left[3e^4\lambda_3^2/\lambda_1^2\right]$

TABLE V

Diagram Type	α	β	γ	δ	Contribution to Γ^{4h}
(i)	H	h	H	h	$\ell n(v_1^2)[+24\lambda_3^4/\lambda_1^2]$
	η	A_3	ψ_3	A_3	$\frac{1}{\varepsilon'}[6e^4]$
	ψ_3	A_1	ψ_3	A_1	$\frac{1}{\varepsilon''}[6e^4]$
(ii)	A_3	η	η	H	$\frac{1}{\varepsilon'}[12e^2\lambda_3^2/\lambda_1 + 12\lambda_3\lambda_4 e^2\lambda_1]$
	η	A_3	A_3	H	$\frac{1}{\varepsilon'}[12e^4\lambda_3/\lambda_1]$
	η	ξ	A_3	H	$\frac{1}{\varepsilon'}[-12\lambda_3\lambda_4 e^2/\lambda_1]$
	A_1	ψ_3	ψ_3	H	$\frac{1}{\varepsilon''}[12\lambda_3^2 e^2/\lambda_1]$
	H	h	h	H	$\ell n(v_1^2)\ [24\lambda_3^4/\lambda_1^2]$
(iii)	H	H	H	H	$\frac{1}{\varepsilon'}[54\lambda_3^2]$
	H	η	η	H	$\frac{1}{\varepsilon'}[6\lambda_3^4/\lambda_1^2+12\lambda_3^3\lambda_4/\lambda_1^2+6\lambda_3^2\lambda_4^2/\lambda_1^2]$
	H	π_3	π_3	H	$\frac{1}{\varepsilon'}[6\lambda_3^2]$
	H	A_2	A_2	H	$\frac{1}{\varepsilon'}[24e^4\lambda_3^2/\lambda_1^2]$
	H	A_3	A_3	H	$\frac{1}{\varepsilon'}[24e^4\lambda_3^2/\lambda_1^2]$
	H	C_2	C_2	H	$\frac{1}{\varepsilon'}[-3\lambda_3^2 e^4/\lambda_1^2]$
	H	C_3	C_3	H	$\frac{1}{\varepsilon'}[-3\lambda_3^2 e^4/\lambda_1^2]$
	H	h	h	H	$\frac{1}{\varepsilon''}[6\ \lambda_3^4/\lambda_1^2]$
	H	ψ_3	ψ_3	H	$\frac{1}{\varepsilon''}[6\lambda_3^4/\lambda_1^2]$
	H	A_2	π_3	H	$\frac{1}{\varepsilon'}[-6e^4\lambda_3^2/\lambda_1^2]$

Table V con't.

Diagram Type	α	β	γ	δ	Contribution to Γ^{4h}
	H	A_3	ξ	H	$\frac{1}{\varepsilon'}\ [-6e^4\lambda_3^2/\lambda_1^2]$
	H	ξ	ξ	H	$\frac{1}{\varepsilon'}[6\lambda_3^2]$
(iv)	H	h	h		$\ell n\ (v_1^2)[-72\lambda_2\lambda_3^2/\lambda_1]$
	η	A_3	A_3		$\frac{1}{\varepsilon'}\ [-12e^4]$
	A_3	η	η		$\frac{1}{\varepsilon'}\ [-12\lambda_2 e^2]$
	A_1	ψ_3	ψ_3		$\frac{1}{\varepsilon''}\ [-12\lambda_2 e^2]$
	ψ_3	A_1	A_1		$\frac{1}{\varepsilon''}\ [-12e^4]$
(v)	h	H	H		$\frac{1}{\varepsilon'}\ [-48\lambda_3^3/\lambda_1]$
	η	ξ	H		$\frac{1}{\varepsilon'}[-12\lambda_3\lambda_4^2/\lambda_1]$
(vi)	H	H	H		$\frac{1}{\varepsilon'}[-36\lambda_3^2]$
	η	η	H		$\frac{1}{\varepsilon'}\ [-12\lambda_2\lambda_3^2/\lambda_1-12\lambda_2\lambda_3\lambda_4/\lambda_1]$
	ξ	ξ	H		$\frac{1}{\varepsilon'}\ [-12\lambda_3^2-12\lambda_3\lambda_4]$
	π_3	π_3	H		$\frac{1}{\varepsilon'}[-12\lambda_3^2]$
	A_3	A_3	H		$\frac{1}{\varepsilon'}\ [-48e^4\lambda_3/\lambda_1]$
	h	h	H		$\frac{1}{\varepsilon''}\ [-36\lambda_2\lambda_3^2/\lambda_1]$
	ψ_3	ψ_3	H		$\frac{1}{\varepsilon''}\ [-12\lambda_2\lambda_3^2/\lambda_1]$
(vii)	H	H	H		$\frac{1}{\varepsilon'}\ [18\ \lambda_3^2]$
	η	H	H		$\frac{1}{\varepsilon'}\ [3\lambda_3^3\lambda_4/\lambda_1^2+3\lambda_3^2\lambda_4^2/\lambda_1^2]$

Table V con't.

Diagram Type	α	β	γ	δ	Contribution to Γ^{4h}
	ξ	H	H		$\frac{1}{\varepsilon'}\left[3e^2\lambda_3^2/\lambda_1\right]$
	π_3	H	H		$\frac{1}{\varepsilon'}\left[3e^2\lambda_3^2/\lambda_1\right]$
	A_2	H	H		$\frac{1}{\varepsilon'}\left[12e^4\lambda_3^2/\lambda_1^2\right]$
	A_3	H	H		$\frac{1}{\varepsilon'}\left[12e^4\lambda_3^2/\lambda_1^2\right]$
(viii)	H	H	H		$\frac{1}{\varepsilon'}\left[-18\lambda_3^2\right]$
	η	H	H		$\frac{1}{\varepsilon'}\left[-3\lambda_3^3\lambda_4/\lambda_1^2-3\lambda_3^2\lambda_4^2/\lambda_1^2\right]$
	ξ	H	H		$\frac{1}{\varepsilon'}\left[-3\lambda_3^2e^2/\lambda_1\right]$
	π_3	H	H		$\frac{1}{\varepsilon'}\left[-3\lambda_3^2e^2/\lambda_1\right]$
	A_2	H	H		$\frac{1}{\varepsilon'}\left[-12e^4\lambda_3^2/\lambda_1^2\right]$
	A_3	H	H		$\frac{1}{\varepsilon'}\left[-12e^4\lambda_3^2/\lambda_1^2\right]$
	C_2	H	H		$\frac{1}{\varepsilon'}\left[3e^4\lambda_3^2/\lambda_1^2\right]$
	C_3	H	H		$\frac{1}{\varepsilon'}\left[3e^4\lambda_3^2/\lambda_1^2\right]$
(ix)	H	H			$\frac{1}{\varepsilon'}\left[6\lambda_3^2\right]$
	η	η			$\frac{1}{\varepsilon'}\left[6\lambda_2^2\right]$
	ξ	ξ			$\frac{1}{\varepsilon'}\left[6\lambda_3^2+12\lambda_3\lambda_4+6\lambda_4^2\right]$
	π_3	π_3			$\frac{1}{\varepsilon'}\left[6\lambda_3^2\right]$
	A_3	A_3			$\frac{1}{\varepsilon'}\left[24e^4\right]$

Table V con't.

Diagram Type	α	β	γ	δ	Contribution to Γ^{4h}
	h	h			$\frac{1}{\varepsilon''}\left[54\lambda_2^2\right]$
	A_1	A_1			$\frac{1}{\varepsilon''}\left[24e^4\right]$
	ψ_3	ψ_3			$\frac{1}{\varepsilon''}\left[6\lambda_2^2\right]$

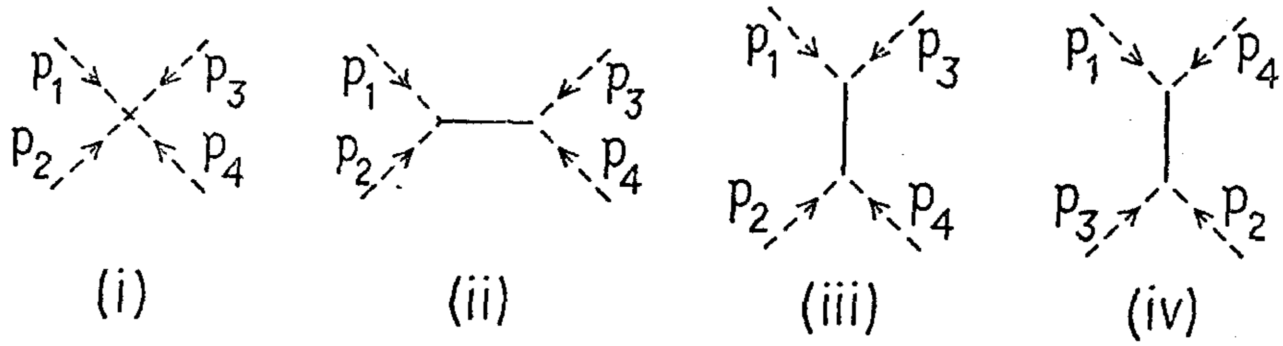


Figure 1

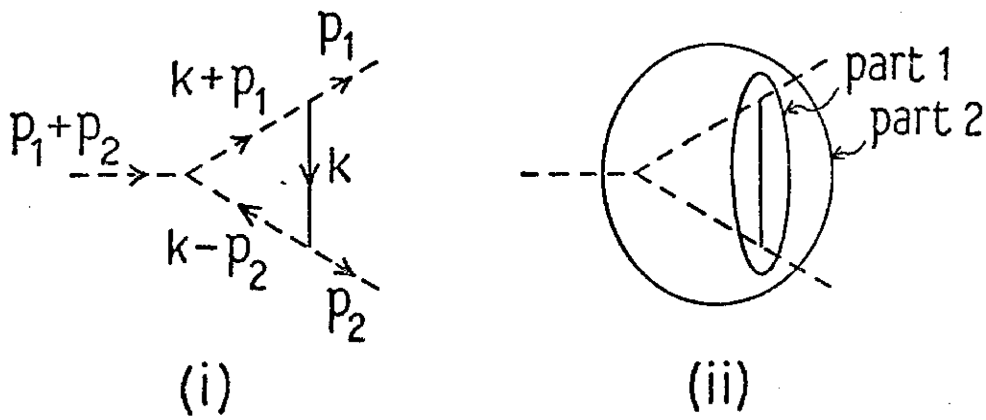
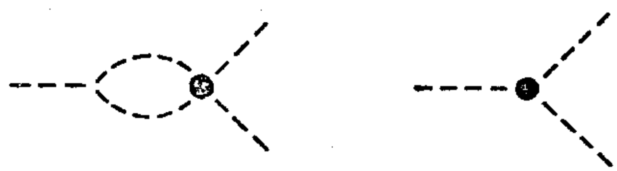


Figure 2



(i) (ii)

Figure 3

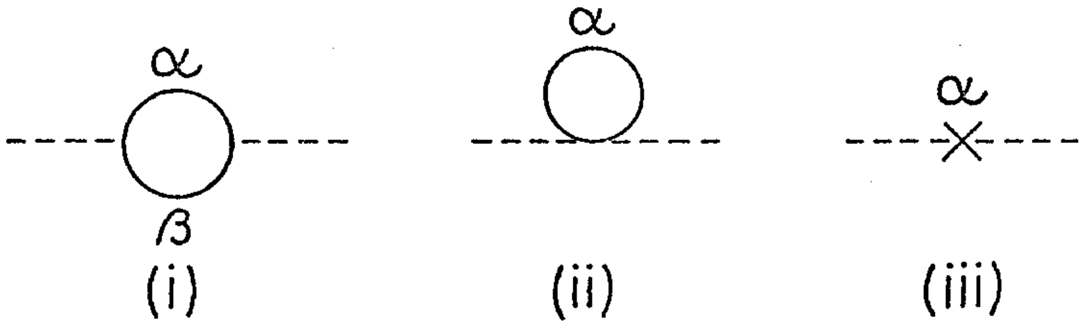


Figure 4

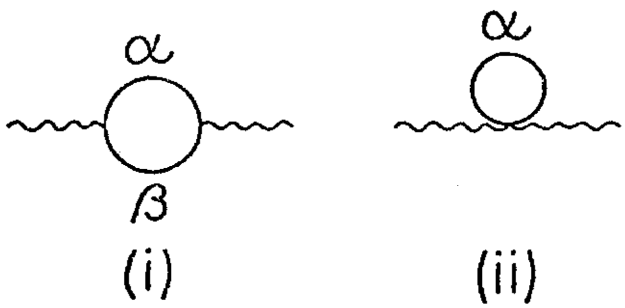


Figure 5

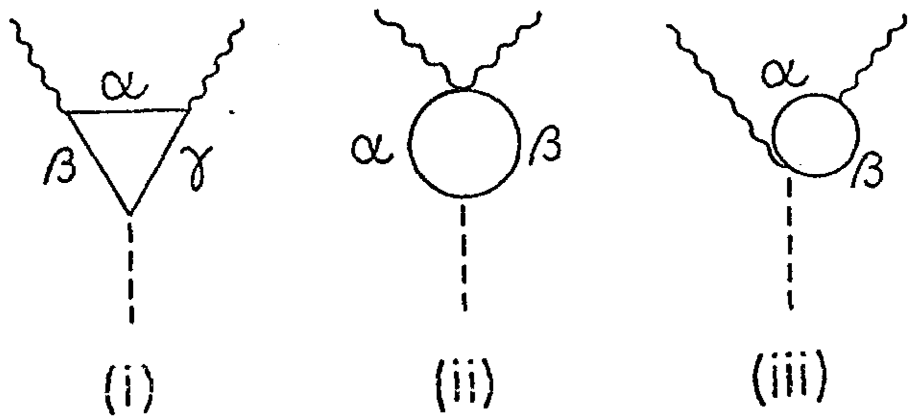


Figure 6

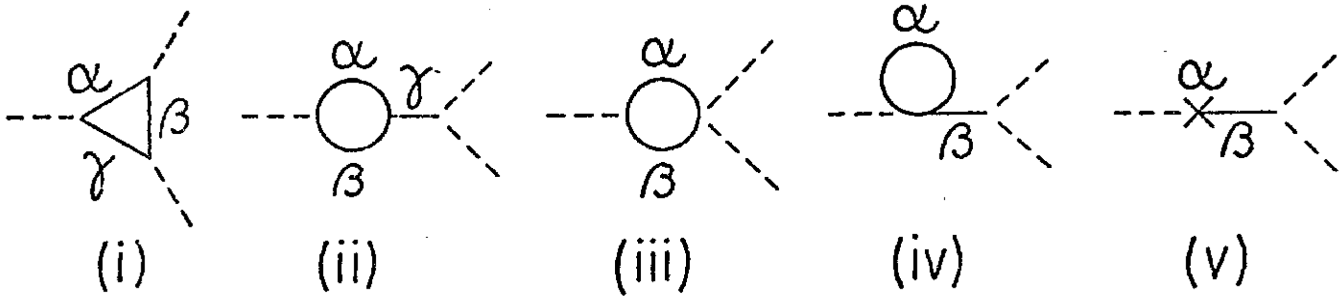


Figure 7

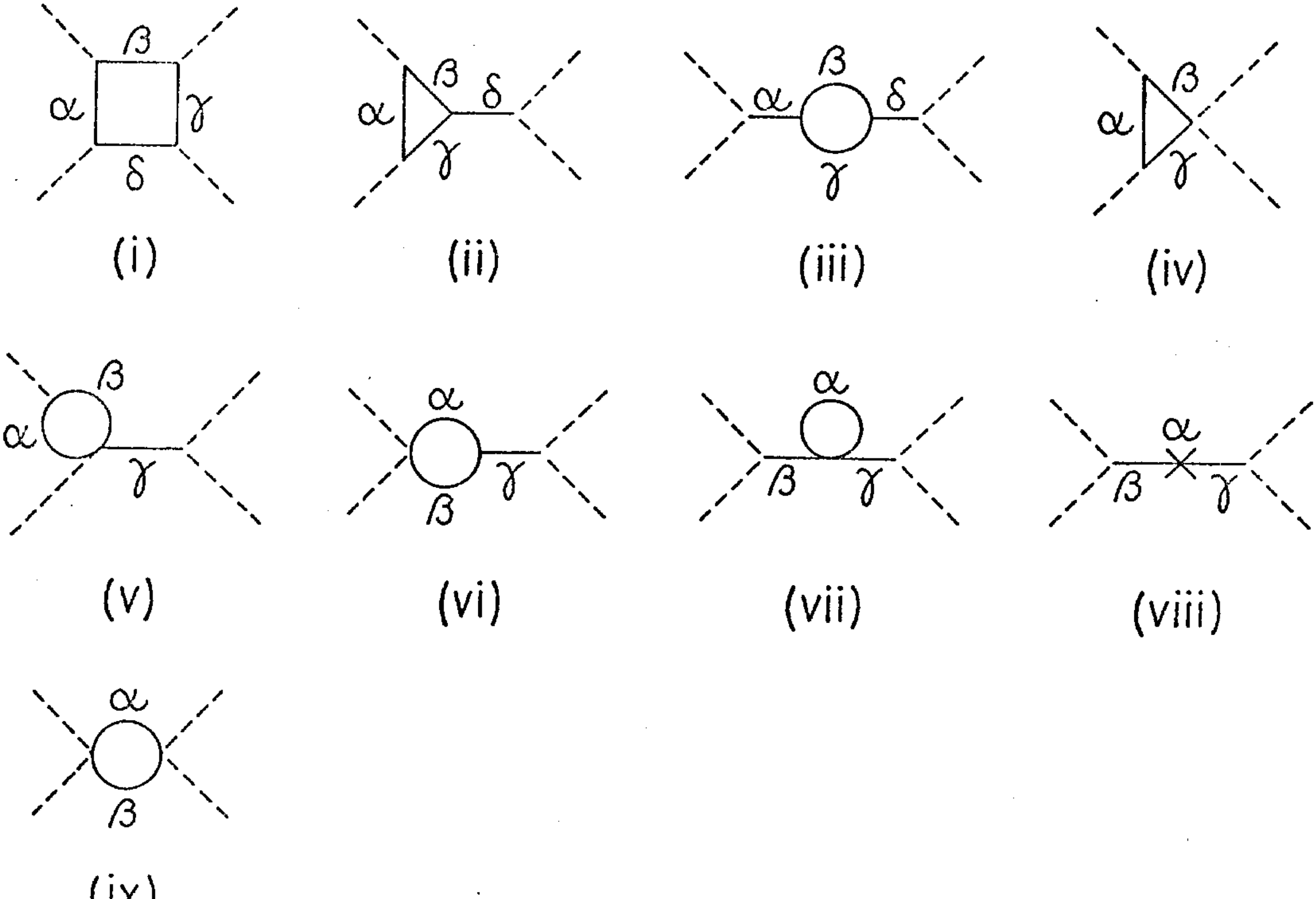


Figure 8