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STATUS OF THE TOP-QUARK

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A brief review of current predictions and bounds on the mass of the t-quark is presented.

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INTRODUCTION

Exactly two years ago at the 1979 Lepton, Photon Symposium at Fermilab, Mary K. Gaillard presented a histogram showing the number of papers predicting a given value of toponium mass as a function of mass.[1] I've translated this into the t-quark mass and presented it in Fig. (1) including several other bounds arising from an assortment of physical principles and arguments. These include Veltman's ρ-parameter bound, a well-known bound from grand unification due to Cabibbo, Maiani, Parisi and Petronzio, and considerations of Higgs potential stability and unitarity.[2] The central popular mechanism for generating t-quark mass predictions, you will recall, was to employ discrete symmetries in the Higgs sector of left-right symmetric models, leading to a central popular prediction of $m_t \sim 15$ GeV. The then current Petra lower bound.[3] It should be noted that discrete symmetries can lead to larger values (and smaller ones), but clearly with diminished unanimity. We should also mention independent ideas which presume a basic universal ratio of $m^{(+2/3)}/m^{(-1/3)}$ for all generations and lead to a prediction of $m_t \sim 26$ GeV.[4]

In the intervening two years physicists have kept busy generating new t-quark mass predictions and Petra has pushed the lower bound up to ~ 18.5 GeV.[5] New physical ideas have emerged which now focus more directly upon dynamical aspects of grand-unification. U.V. versus I.R. behavior of field theory, and nonlinear renormalization group equations with quasi-fixed point behavior. Some of these ideas make statements about hypothetical fourth generation fermion masses. Also, Buras has produced a new bound by considering rare weak Kaon processes which R. Oakes and I have recently extended to include the presence of a fourth generation (the Kaon system remains a useful probe of the fermion spectrum!). The results of these are presented in Fig. (2) on the same scale as Fig. (1). Presently, I will briefly review these ideas with an eye to statements one can make about mass scales beyond the third generation, as well.

SOME NEW IDEAS

Veltman[6] has recently attempted to formulate conditions under which the low energy spectrum of a field theory is protected from high energy dynamics. As is well-known, the most severe problems

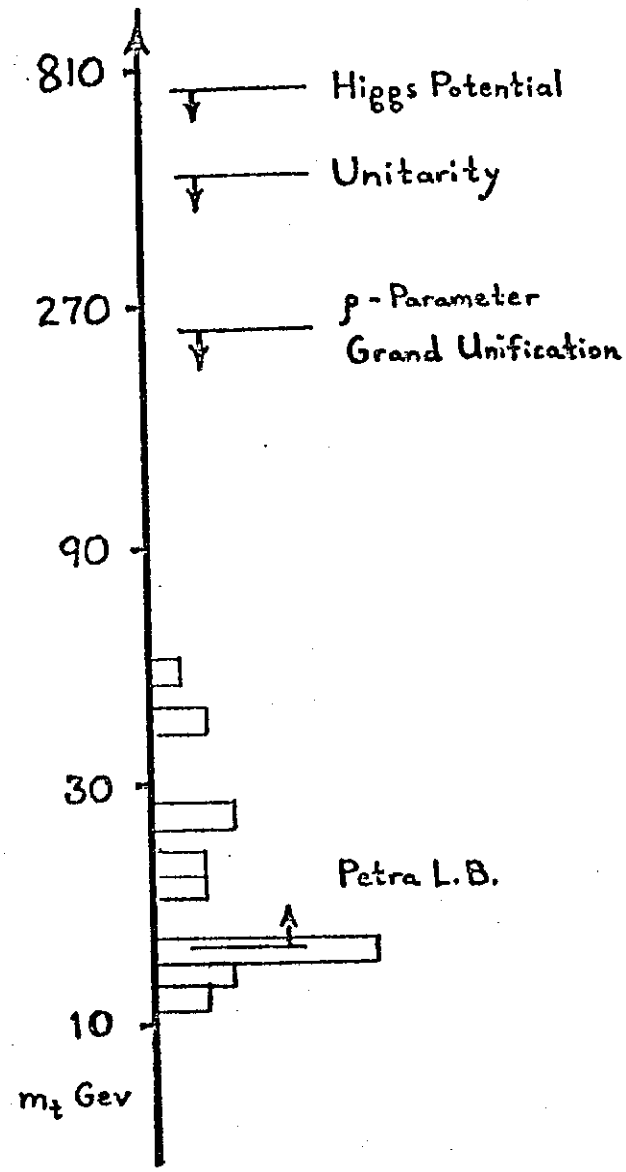

Fig. 1

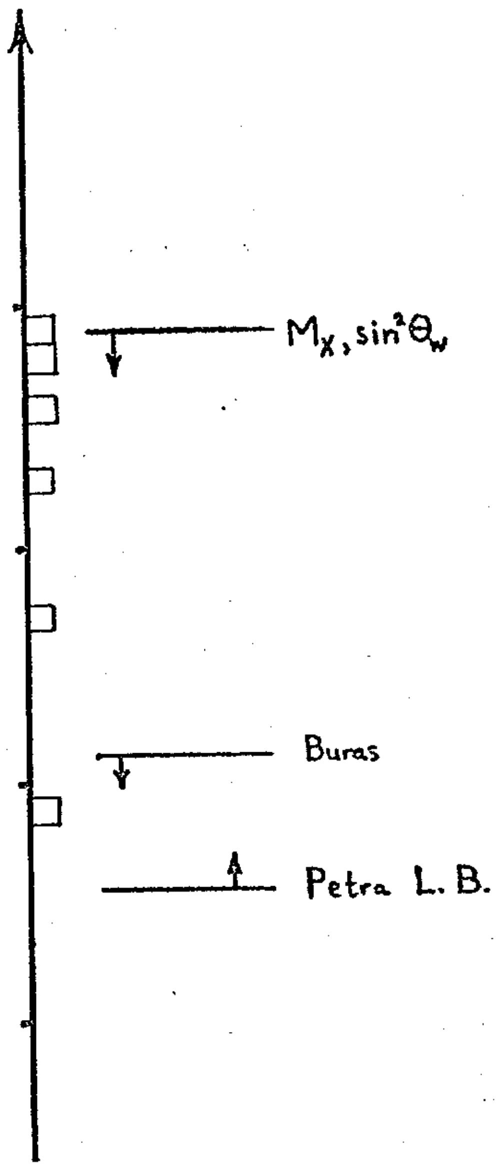

Fig. 2

Comparison of old results of predictions and bounds on m_{top} from refs. 1,2 and 3 in Fig. 1 with recent results presented in Fig. 2.

occur in the mass scales of spin-zero bosons which receive normally additive (quadratic divergences) renormalization corrections, viz. the case of fermions which can receive only multiplicative corrections due to the chiral symmetries which become present in the absence of the fermion mass. For the ordinary Higgs boson whose mass must be of order the weak interaction scale or less, $m_H \sim M_W$ (to prevent various disasters such as strong coupling problems or violation of unitarity bounds), this problem becomes severe. For example, if there is compositeness of Higgs bosons, or other fields for which the standard model becomes an effective Lagrangian for momenta scales less than some Λ, then one would expect a Higgs boson mass scale of order Λ (or conservatively $\sim g\Lambda/4\pi$) which is a serious and seemingly unnatural constraint, if, for example, the compositeness scale is >>TeV. The problems of naturalness surrounding spin-0 particle masses have largely motivated the developments of technicolor and supersymmetric G.U.T.'S.

One of Veltman's observations is that one can remove the quadratic divergences (poles at d=2) if a certain relationship amongst the coupling constants of the theory is satisfied. This relationship involves the Higgs-Yukawa coupling constant of the top-quark and thus translates into a definite mass prediction. For example, neglecting the quartic couplings, the additive quadratic divergence correcting m_H^2 is found to be

$$\delta m_H^2 = \{g_2^2\ (1-d)(\frac{1}{2} + \frac{1}{2\cos^2\theta_W}) + \sum_f g_f^2\}\frac{\Lambda^2}{16\pi^2} \tag{1}$$

Demanding that this vanish one obtains (neglecting light quarks and noting that $\sum_f$ includes a color sum):

$$g_f^2 = g_2^2(\frac{1}{2} + \frac{1}{2\cos^2\theta_W}) \tag{2}$$

or $m_f = g_f \cdot V \simeq 69$ GeV, after putting d=4, $V \sim 175$ GeV.

Once the d=2 pole has been removed it does not recur in higher orders of perturbation theory, but poles at d=4-(2/m) do occur in m-loops. To satisfy the cancellation to all orders of all poles with d<4 may require a supersymmetry. Veltman has thus suggested 69 GeV as a prediction for the t-quark mass and has proposed this as a natural outcome of a model built upon a realistic symmetry to enforce eq. (2).

Equation (2) is much like a fixed point condition (vanishing β-function). If one demands similarly that certain poles about d=4 vanish then one is led to the ideas of Pendleton and Ross.[7] These authors consider the evolution equations of the Higgs-Yukawa coupling constant for a single heavy t-quark and simultaneously the evolution equation of g_3 (the QCD coupling constant). One has:

$$16\pi^2 \frac{d}{dt} \ln(g_{top}) = \frac{9}{2} g_{top}^2 - 8g_3^2$$

$$16\pi^2 \frac{d}{dt} \ln(g_3) = -(11 - \frac{2}{3} n_f) g_3^2 \qquad (3)$$

or, upon combining:

$$16\pi^2 \frac{d}{dt} \ln(g_{top}/g_3) = \frac{9}{2} g_{top}^2 - g_3^2 \Big|_{n_f=6} \qquad (4)$$

Pendleton and Ross demand the vanishing of the right-hand side of eq. (4), which locks g_{top} into a fixed, stable ratio with g_3. One obtains $m_{top}=g_{top}\cdot V \simeq 110$ GeV from (4), but the inclusion of electroweak radiative corrections results in $m_{top} \simeq 135$ GeV.

One can pose a slightly more physical question in the same vein: given an arbitrary initial value for g_{top} at some scale, e.g., $M_x \simeq 10^{15}$ GeV (which may be either a grand-unified scale or a compositeness scale; we demand effective pointlikeness for masses below M_x), then what is the most likely result for g_{top} at low energies $\simeq M_W$? One might think that this is just the result of Pendleton and Ross, but in fact there is an intermediate fixed point behavior which sets in below scale $M \ll M_x$ and which persists down to scales of order 1 GeV. One can see this by solving eq. (3) for g_{top} directly:

$$g_{top}^2(\mu) = \frac{g_{top}^2(M_x)(g_3^2(\mu)/g_3^2(M_x))^{8/b_0}}{1+(9g_{top}^2(M_x)/2g_3^2(M_x))((g_3^2\mu)/g_3^2(M_x))^{1/b_0}-1)} \qquad (5)$$

where $b_0 = 11-(2/3)n_f$. In the limit $(g_3^2\mu)/g_3^2(M_x))^{8/b_0} \gg 1$ we reach the Pendleton-Ross result.

$$g_{top}^2(\mu) \to \frac{2}{9}\left(\frac{g_3^2(\mu)}{g_3^2(M_x)}\right)^{7/b_0} g_3^2(M_x) = \frac{2}{9} g_3^2(\mu) \Big|_{n_f=7} . \qquad (6)$$

However, to be at a fixed point in the sense that $g_{top}(M_x)$ no longer influences $g_{top}(\mu)$ it is sufficient that:

$$\frac{9}{2}\,\frac{g_{top}^2(M_x)}{g_3^2(M_x)}\left(\frac{g_3^2(\mu)}{g_3^2(M_x)}\right)^{1/b_0} - 1 \;\gg 1 \qquad (7)$$

which can easily occur long before the limit leading to the Pendleton-Ross result. Setting $R=g_3^2(\mu)/g_3^2(M_x)$ one finds:

$$g_{top}^2(\mu) = \frac{2b_0}{9}\,\frac{g_3^2(\mu)}{\ln R}\left\{1 + \frac{1}{2b_0}\ln R + \frac{7}{12b_0^2}(\ln R)^2 + \ldots\right\} \qquad (8)$$

Including the effects of electroweak interactions one finds $m_{top}=g_{top}(m_{top})\cdot V$ which yields $m_{top}\simeq 240$ GeV. This is the same as the upper bound of Cabibbo, et al.[2] and we see presently that it is the most probable result for the mass of a quark that is relatively strongly coupled ($g_{Higgs} \gtrsim 1$ is still perturbative!) at M_x.

The fixed point scenerio is quite interesting from our point of view though it may have no bearing on the t-quark mass. However, for a heavy standard SU(5) fourth generation, it gives predictions for the masses of each of the elements of the generation:

$$m_{+2/3} \simeq 220\ \text{GeV},\ m_{-1/3} \simeq 215\ \text{GeV},\ m_{-1} \simeq 60\ \text{GeV} \qquad (9)$$

where the neutrino is assumed to be massless. These have been obtained numerically, but can be understood analytically, as in ref. (8). Also, the effects of such objects on the standard SU(5) scenario have been investigated and are found to be negligible.[9] We mention that the results quoted in eq. (9) differ slightly from those quoted by the authors of ref. (10) who have simply reproduced the Pendleton-Ross arguments for a fourth generation. The results of eq. (9) are the actual intermediate fixed points, or most probable values of the masses, assuming arbitrary coupling strength at M_x. These predictions are essentially the ultra-heavy quark-lepton analogues of the BEGN results for m_b/m_τ.[11]

Recently, Buras has investigated the effects of a heavy t-quark upon the standard rare Kaon processes:[13] $K_L \to \mu^+\mu^-$ and $\Delta m_{K_L K_S}$. We will give here only a very schematic outline of the analysis. The $K_L K_S$ mass difference is obtained by an exact calculation of the box diagrams and by estimating the matrix element, which Buras parametrizes as follows:

$$\langle K_L|\bar{s}\gamma_\mu d\ \bar{s}\gamma^\mu d|K_S\rangle = \frac{1}{R}\cdot(\text{MIT bag model}) \quad . \qquad (10)$$

Thus R=1 corresponds to the usual MIT bag model result[12] and R=.42 is the vacuum insertion value of Lee and Gaillard.[13] Similarly one can calculate the short-distance contribution to $K_L\to\mu^+\mu^-$ which involves a known current matrix element.

For $R \gtrsim .5$ the charm contribution, with $m_c = 1.5$ GeV, cannot account for all of the $K_L K_S$ mass difference, nor does it saturate the bound on the short-distance $K_L \to \mu^+\mu^-$ amplitude. Presently we simply ignore the charm contribution altogether to see what emerges. One has:

$$\begin{aligned} (K_L - K_S): &\quad |\mathrm{Re}\ A_t|^2 (m_t^2/M_W^2)\eta = .44 \times 10^{-4} R \\ (K_L \to \mu^+\mu^-): &\quad |(\mathrm{Re}\ A_t)|(m_t^2/M_W^2)\eta' \leq .19 \times 10^{-2} \end{aligned} \tag{11}$$

where Re A_t is a combination of Kobayshi-Miskawa mixing angles and η, η' are QCD renormalization effects. Combining the above conditions to eliminate $|\mathrm{Re}\ A_t|$ one finds:

$$\frac{m_t^2}{M_W^2} \leq \frac{1}{R} \frac{\eta}{\eta'^2}(.09) \quad . \tag{12}$$

Of course eq. (12) is merely the extreme case of neglecting the charm contribution and assuming $m_t^2/M_W^2 \ll 1$. Buras actually carries out a search for the maximum m_t^2/M_W^2 over all KM angle combinations using exact expressions. For $R=1$ he finds $m_{top} \lesssim 40$ GeV, whereas for $R = .5$ there is no bound since the charmed quark saturates the $K_L K_S$ mass difference and one could imagine that the t-quark simply decouples. (see Fig. 3).

This bound can be criticized from other points of view that exploit the uncertainties in the large distance contributions to $K_L \to \mu^+\mu^-$ and that perhaps the underlying mechanism for either the $K_L K_S$ mass difference or the $K_L \to \mu^+\mu^-$ process involves Higgs bosons.[14] Surprisingly, the large distance contributions to the $K_L K_S$ mass difference seem to reinforce the bound.[12] One further point is the question of what are the effects of the fourth generation on this bound and is there a joint bound on m_{top} and $m_{top'}$?

Recently Oakes and I have addressed this problem.[15] The appearance of a large number of KM angles requires one to resort to the computer except for the (unrealistic?) case $m_t \simeq m_{t'}$, which can be treated analytically as in eq. (11). We find that a further constraint must be employed beyond those considered by Buras, namely that CP violation must not be too large. We assume then that $|\varepsilon|$ be less than 10^{-2}, which it clearly is provided that $|\varepsilon'|/|\varepsilon| \lesssim 10^{-3}$. However, the possibility of a large $|\varepsilon'|$ (e.g., penguins) requires that we allow for the possibility that $|\varepsilon|$ be larger than its superweak value of 2×10^{-3}. Hence, we assume that the observed CP violation is not small because of a miraculous cancellation.

In that case an excluded region in the m_t, $m_{t'}$ plane emerges as in Fig. (4) with $R=1.0$ in the free quark model. We see for a very heavy t' quark that a heavy t-quark is permitted, though for any $m_{t'}$, m_t is always bounded. For $m_{t'} = 220$ GeV, we see that $m_t \lesssim 140$ GeV.

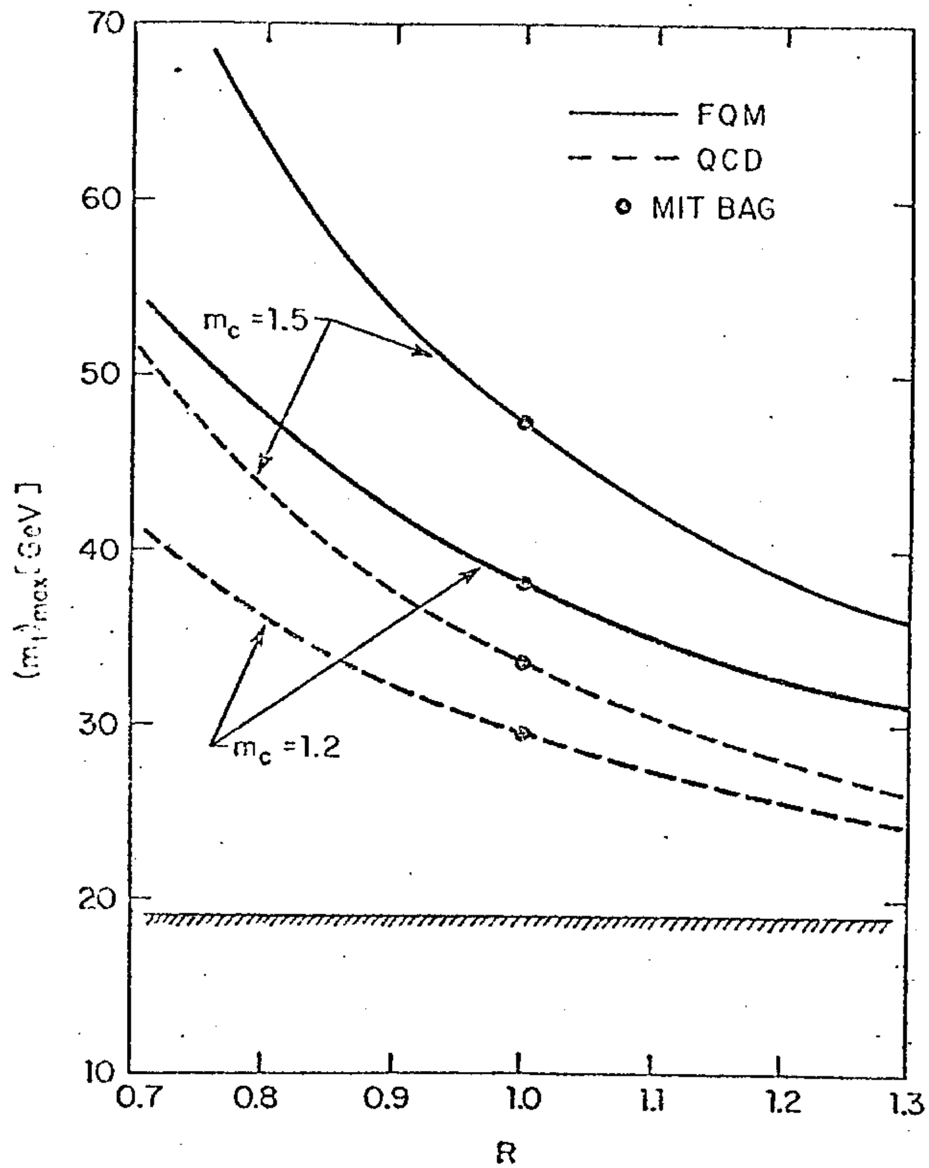

Fig. 3

Buras' results bounding m_{top} from $\Delta m_{K_L K_S}$ and $K_L \to \mu^+ \mu^-$ (from ref. 12).

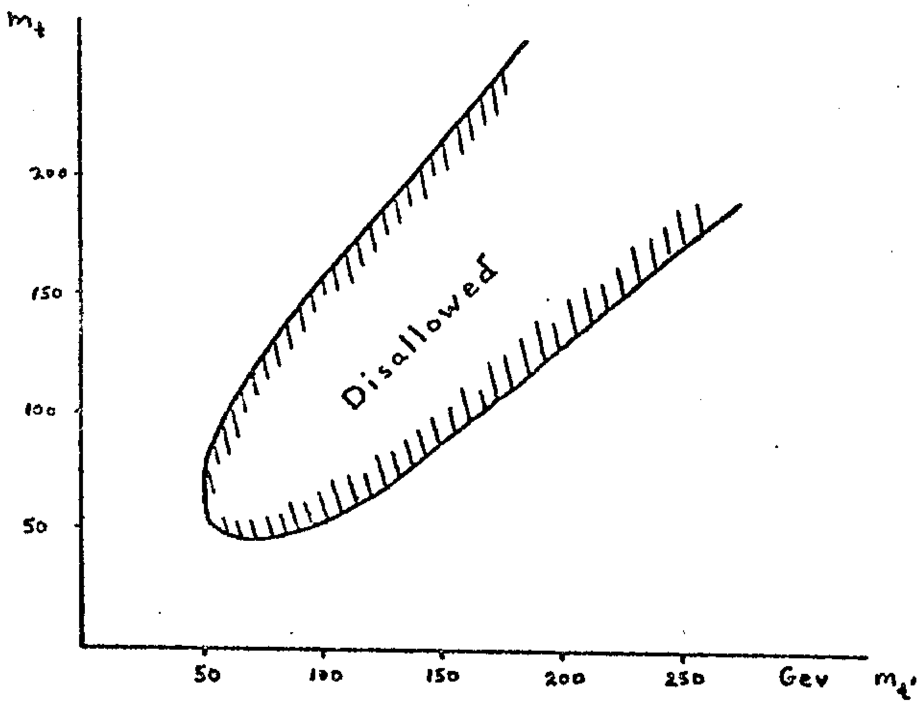

Fig. 4

Joint bound on simultaneously m_{top} and $m_{top'}$, consists of the disallowed region above, where all QCD effects have been neglected and m_c=1.5 GeV. Using exact expressions, we search numerically for allowed pairs $(m_{t'}, m_t)$ for all possible KM angle combinations. In addition to $\Delta m_{K_L K_S}$ and $K_L \to \mu^+\mu^-$, we use CP-violation as a constraint: $|\varepsilon| < 10^{-2}$ (ref. 15).

Turning this around, a very heavy t-quark with, say, the Pendleton-Ross mass would imply the existence of a fourth generation with typical intermediate fixed point mass results; or, if m_t=70 GeV as in Veltman's predictions we would require $m_{t'} \geq 130$ GeV. Hence, the t-quark through rare Kaon processes may be heavy, mandating the existence of a fourth generation, modulo the uncertainties of the MIT bag model matrix element.

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