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ABSTRACT

The quark-antiquark potential in QCD is discussed with particular emphasis on the related β-function. The empirical information about the potential at intermediate distances, due to the Ψ- and Υ-spectroscopies, is reviewed. Finally we examine the quantitative connection between the ζ-spectroscopy, formed by the anticipated t-quark and its antiquark, and the short distance behavior of the quark-antiquark potential.

I. THE $(Q\bar{Q})$ POTENTIAL IN QCD

More than six years ago Appelquist and Politzer[1] were led by the idea of asymptotic freedom to suggest that heavy quarks would form non-relativistic positronium-like bound states, which should be observed as narrow resonances. Since then the dynamics of heavy quark systems[2] has been extensively investigated. Theoretical efforts have been concentrated on the static quark-antiquark potential in QCD and on the development of the phenomenological potential model.

The theoretical investigations[3-7] have shown that the $(Q\bar{Q})$ potential in QCD can be defined as the binding energy of a quark-antiquark pair in the limit of infinite quark mass. It can be expressed in a manifestly gauge invariant form as a Euclidean path integral, the vacuum expectation value of a rectangular Wilson loop[8] of size R × T,

$$V(R) = -\lim_{T\to\infty} \frac{1}{T} \langle \mathrm{tr}\, P\, e^{ig\oint dx^\mu A_\mu} \rangle_0 \tag{1.1}$$

Equation (1.1) has provided the starting point for many attempts to compute the $(Q\bar{Q})$ potential from first principles, and in particular lattice gauge theory calculations[9,10] have recently led to very promising results.

Here we will be mostly interested in the short distance behavior of the potential which, due to asymptotic freedom, can be calculated perturbatively as a power series in the strong coupling constant. In momentum space the potential reads[6,11,12]

$$V(Q, \mu, \alpha_{\overline{MS}}(\mu)) = -\frac{4\pi C_2(R)\alpha_{\overline{MS}}(\mu)}{Q^2}\left\{1 + \frac{\alpha_{\overline{MS}}(\mu)}{4\pi}\left[\left(\frac{11}{3}C_2(G) - \frac{2}{3}N_f\right)\ln\frac{\mu^2}{Q^2} + \frac{31}{9}C_2(G) - \frac{10}{9}N_f\right] + O\left(\alpha^2_{\overline{MS}}(\mu)\right)\right\}; \tag{1.2}$$

Q, μ and $\alpha_{\overline{MS}}(\mu)$ are the momentum transfer, the scale in dimensional regularization and the coupling constant in the $\overline{MS}$-scheme.[13] As a physical quantity the potential satisfies a renormalization group equation without anomalous dimension[14] ($\alpha = g^2/4\pi$),

$$\left[\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g}\right] V(Q, \mu, \alpha(\mu)) = 0 \qquad , \qquad (1.3)$$

which implies the short distance behavior[15]

$$V(Q) \underset{Q \to \infty}{} - \frac{16\pi^2 C_2(R)}{Q^2} \frac{1}{b_o \ln \frac{Q^2}{\Lambda_p^2}} \left(1 - \frac{b_1}{b_0^2} \frac{\ln \ln \frac{Q^2}{\Lambda_p^2}}{\ln \frac{Q^2}{\Lambda_p^2}} + O\left(\frac{1}{\ln^2 \frac{Q^2}{\Lambda_p^2}} \right) \right) ; \qquad (1.4)$$

the relation between the scale parameters Λ_p and $\Lambda_{\overline{MS}}$ reads[16]

$$\Lambda_p = \Lambda_{\overline{MS}} \exp \left[\frac{1}{2b_0} \left(\frac{31}{9} C_2(G) - \frac{10}{9} N_f \right) \right] \qquad . \qquad (1.5)$$

Λ_p represents the characteristic scale parameter of the coupling constant $\alpha_p(\mu)$, which can be defined in terms of the $(Q\overline{Q})$ potential (cf. Eq. (1.2)),

$$\begin{aligned} \alpha_p(\mu) &\equiv - \frac{\mu^2 V(\mu, \mu, \alpha_{\overline{MS}}(\mu))}{4\pi C_2(R)} \\ &= \alpha_{\overline{MS}}(\mu) \left[1 + \frac{\alpha_{\overline{MS}}(\mu)}{4\pi} \left(\frac{31}{9} C_2(G) - \frac{10}{9} N_f \right) + O\left(\alpha^2_{\overline{MS}}(\mu) \right) \right]. \quad (1.6) \end{aligned}$$

$\alpha_p(\mu)$ is a <u>physical quantity</u>, the strength of the quark-antiquark interaction at momentum transfer μ, and therefore gauge and scheme independent. For SU(3) and 3 flavors one obtains

$$\alpha_p(\mu) = \alpha_{\overline{MS}}(\mu) \left[1 + 0.78 b_0 \frac{\alpha_{\overline{MS}}(\mu)}{4\pi} + O\left(\alpha^2_{\overline{MS}}(\mu) \right) \right] \qquad (1.7a)$$

and

$$\Lambda_p = 1.48 \Lambda_{\overline{MS}} \qquad . \qquad (1.7b)$$

The coupling constant $\alpha_p(\mu)$ satisfies the inequality[13]

$$\alpha_{\overline{MS}}(\mu) < \alpha_p(\mu) < \alpha_{MOM}(\mu) \qquad , \qquad (1.8)$$

and may be a useful expansion parameter for other processes in perturbative QCD.

The static $(Q\overline{Q})$ potential is a quantity which has the dimensions of mass. It depends on dimensionless parameters (e.g. group factors) characterizing the QCD Lagrangian and (if the masses of light quarks can be neglected) a single scale which we may choose to be Λ or the string tension k. In order to disentangle these two ingredients which determine the $(Q\overline{Q})$ interaction it is instructive to study the dimensionless β-function of the running coupling constant $\rho(Q^2) \equiv \alpha_p(Q)/4\pi$.[16] The asymptotic behavior of $\rho(Q^2)$ for large Q^2 reads (cf. Eq. (1.4)),

$$\rho(Q^2) \underset{Q\to\infty}{\sim} \frac{1}{b_0 \ln \frac{Q^2}{\Lambda_p^2}} - \frac{b_1}{b_0^3} \frac{\ln\ln \frac{Q^2}{\Lambda_p^2}}{\ln^2 \frac{Q^2}{\Lambda_p^2}} + O\left(\frac{1}{\ln^3 \frac{Q^2}{\Lambda_p^2}} \right) , \qquad (1.9a)$$

and at small Q^2 the hypothesis of linear confinement implies

$$\rho(Q^2) \underset{Q\to 0}{\sim} \frac{K}{Q^2} [1 + o(1)] , \qquad (1.9b)$$

where the parameter K is related to the string tension k and the Regge slope α' of light hadron spectroscopy,

$$\alpha' = \frac{1}{2\pi k} = \frac{1}{4\pi^2 C_2(R) K} . \qquad (1.9c)$$

The β-function of the coupling constant ρ is given by

$$\beta_p(\rho) = Q^2 \frac{\partial}{\partial Q^2} \rho(Q^2) \Big|_{Q^2=Q^2(\rho)} , \qquad (1.10)$$

and from Eqs. (1.9) we read off its asymptotic behaviors

$$\beta_p(\rho) \underset{\rho\to 0}{\sim} -b_0\rho^2 - b_1\rho^3 + O(\rho^4) , \qquad (1.11a)$$

$$\beta_p(\rho) \underset{\rho\to\infty}{\sim} -\rho [1 + o(1)] . \qquad (1.11b)$$

The differential equation (1.10) can be integrated using the boundary conditions Eqs. (1.9) for large or small values of Q^2. The requirement, that both solutions are identical, leads, as a consistency condition, to a relation between the dimensionless quantity K/Λ_p^2, or equivalently $\alpha'\Lambda_{\overline{MS}}^2$, and the β-function,[16]

$$\ln (\alpha' \Lambda_{\overline{MS}}^2) = -\ln(4\pi C_2(R)) - \frac{1}{b_0}\left(\frac{31}{9} C_2(R) - \frac{10}{9} N_f \right) - \frac{1}{b_0} - \frac{b_1}{b_0^2} \ln b_0$$
$$- \int_0^1 dx \left[\frac{1}{b_0 x^2} - \frac{b_1}{b_0^2 x} + \frac{1}{\beta_p(x)} \right] - \int_1^\infty dx \left[\frac{1}{x} + \frac{1}{\beta_p(x)} \right] . \qquad (1.12)$$

We note that precisely the subtraction of the leading term at large ρ and the one- and two-loop contributions at small ρ are required in order to render the integrals over the β-function in Eq. (1.12) finite.

Obviously, the β-function determines the relation between α' and $\Lambda_{\overline{MS}}$. Setting the scale of the theory by fixing α' or $\Lambda_{\overline{MS}}$ determines the $(Q\bar{Q})$

potential in terms of its β-function. In principle, the β-function can be evaluated directly in QCD. At present, however, this has not been achieved. Yet there exists a simple empirical β-function which has the following properties:

(i) at small and large values of ρ, $\beta_p(\rho)$ conforms to the theoretically required asymptotic behavior of Eqs. (1.11);

(ii) at intermediate values of ρ, $\beta_p(\rho)$ is empirically correct, as it leads to an accurate description of the Ψ and Υ spectroscopies, which probe the quark-antiquark coupling strength at intermediate distances;

(iii) the integral over $\beta_p(\rho)$ yields $\alpha' \Lambda^2_{\overline{MS}} = 0.27$, i.e. a Regge slope of $\alpha' \backsim 1\ \mathrm{GeV}^{-2}$ corresponds to a scale parameter $\Lambda_{\overline{MS}} \backsim 0.5$ GeV. These values for α' and $\Lambda_{\overline{MS}}$ are consistent with results obtained from light hadron spectroscopy and deep inelastic scattering processes.[16]

This β-function is given by the following Ansatz:

$$\frac{1}{\beta_p(\rho)} = -\frac{1}{b_0\rho^2\left(1 - e^{-1/b_0\rho}\right)} + \frac{b_1}{b_0^2}\,\frac{1}{\rho}\,e^{-\ell\rho} \quad , \tag{1.13}$$

where the parameter ℓ is determined from the Ψ and Υ spectroscopies, which yield $\ell = 24$.

The β-function Eq. (1.13) is closely related to the $(Q\bar{Q})$ potential proposed by Richardson.[17] The β-function, which corresponds to Richardson's running coupling constant, is obtained from Eq. (1.13) in the limit $\ell \to \infty$ (or by setting $b_1 = 0$),

$$\beta^{\mathrm{Rich}}(\rho) = -b_0\rho^2\left[1 - e^{-1/b_0\rho}\right] \quad . \tag{1.14}$$

Its intriguing feature is the essential singularity at $\rho = 0$. Precisely this structure is expected if classical field configurations are important for the transition between weak and strong coupling regimes. Yet the two-loop contribution to the β-function is required in order to relate the short distance behavior of the $(Q\bar{Q})$ potential to a well-defined QCD scale parameter, say $\Lambda_{\overline{MS}}$. These considerations led to the β-function Eq. (1.13).

The discussion of this section may be summarized as follows:

(i) The $(Q\bar{Q})$ potential can be defined in QCD as the binding energy of a quark-antiquark pair in the infinite-mass-limit; it can be expressed in terms of the Wilson loop integral.

(ii) The short distance part of the potential has been evaluated in perturbation theory. The physical coupling constant $\alpha_p(\mu)$, which is defined in terms of the quark-antiquark potential, may be a convenient expansion parameter for other processes in perturbative QCD.

(iii) The $(Q\bar{Q})$ potential can be expressed in terms of the dimensionless β-function $\beta_p(\rho)$ and a dimensionful constant which one may choose to be the scale parameter Λ or the string tension k. A simple empirical Ansatz for $\beta_p(\rho)$ has been obtained which conforms to the theoretical expectations for small and large values of ρ; at intermediate coupling strengths $\beta_p(\rho)$ is empirically correct as the resulting potential describes successfully the Ψ- and Υ-spectroscopies.

II. THE ($Q\bar{Q}$) POTENTIAL AT INTERMEDIATE DISTANCES

Over the past six years the potential model[2] for heavy quarkonia has been extensively developed. As we have outlined in the previous section one expects theoretically the static ($Q\bar{Q}$) potential to be Coulombic at short distances and to become linear at large quark-antiquark separation. The "Coulomb plus linear" potential, which is obtained by a simple superposition of both asymptotic limits, therefore represents the prototype of a QCD-like potential model, and its detailed study by the Cornell group[18] has led to a successful description of the Ψ and Υ families.

More recently, various authors have investigated the effects of logarithmic modifications[19] of the Coulombic part of the potential which are expected as a result of vacuum polarization corrections in QCD. Richardson's[17] potential, in particular, yields an excellent description of the ($c\bar{c}$)- and ($b\bar{b}$)-spectra. In order to relate the short distance behavior of the ($Q\bar{Q}$) potential to a well-defined QCD scale parameter, say $\Lambda_{\overline{MS}}$,[13] the two-loop contribution to the β-function and the one-loop correction to the potential have to be incorporated consistently. These considerations led to a new potential model[16] and, within this framework, to a value of $\Lambda_{\overline{MS}} = 0.5$ GeV which, determined from quarkonium spectroscopy, is consistent with analyses[20] of deep inelastic scattering experiments.

QCD-like potential models have achieved a successful description of the Ψ and Υ spectroscopies, in particular with respect to leptonic widths[16] and hyperfine splittings which are most sensitive to the short distance part of the ($Q\bar{Q}$) potential. However, this success is not unique.[21] It is shared with the class of logarithmic and small power potentials, investigated in detail by Quigg and Rosner[22] and Martin,[23] which do not conform to the theoretical expectations at either small or large distances. Thus, so far quarkonia have not led to any conclusive evidence for the theoretical preconceptions based on QCD. Yet the Ψ and Υ families have determined the quark-antiquark potential at intermediate distances: the four phenomenologically successful potentials, shown in Fig. 1, all coincide numerically at distances r with 0.1 fm $\leq r \leq$ 1.0 fm, although their functional forms are very different. At large and small distances a variety of asymptotic behaviors appear to be compatible with present experimental data.

The evidence for a flavor independent[25,26] ($Q\bar{Q}$) potential has also been established in a model independent way by use of the inverse scattering method. Using mass differences and leptonic widths of the S-states in the Ψ or Υ families as input and assuming different "correction factors"[26] in the van Royen-Weisskopf formula (which reflect uncertainties of unknown relativistic and higher order radiative corrections), the ($Q\bar{Q}$) potential has been constructed. Again, as shown in Fig. 2, it appears to be uniquely determined at distances between 0.1 fm and 1.0 fm where it coincides with the specific models shown in Fig. 1. Direct evidence for the flavor independence of the static potential is provided by Fig. 3, where the potentials constructed from the Ψ and Υ families are compared; they agree remarkably well in the distance range 0.1 fm - 1.0 fm. The accuracy to which potential models can account for the properties of quarkonia is demonstrated by table I, where the predictions of various models for the three narrow S-states of the Υ family have been compiled. We emphasize that the non-relativistic potential model of heavy quark systems works much better than one might expect on the basis of coupled channel calculations[18] or estimates of relativistic corrections.[27]

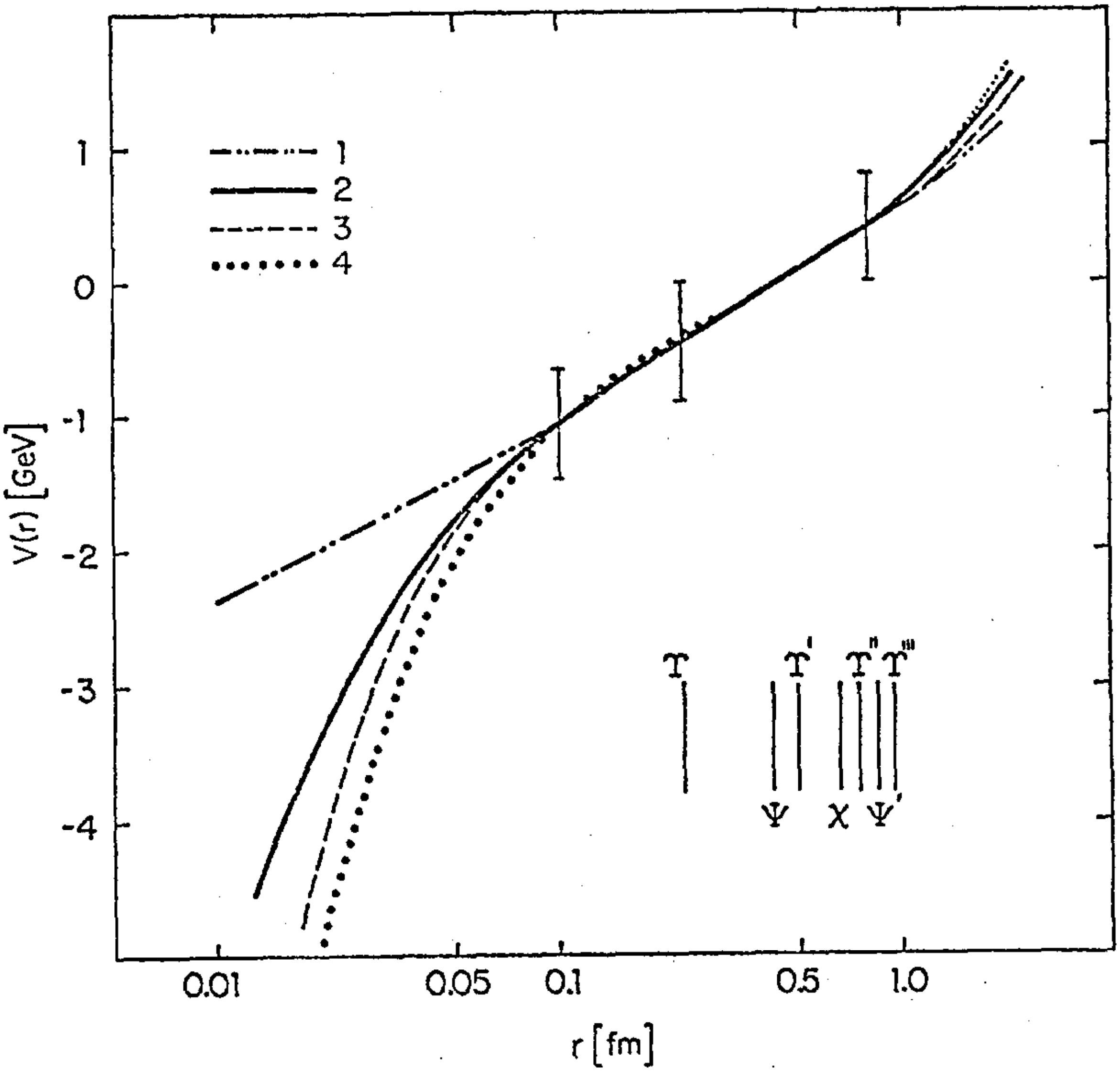

Fig. 1. Various successful potentials are shown. The numbers refer to the following references: (1) Martin, Ref. 23; (2) Buchmüller, Grunberg and Tye, Ref. 16; (3) Bhanot and Rudaz, Ref. 24; (4) Cornell group, Ref. 18. The potentials (1), (3) and (4) have been shifted to coincide with (2) at r = 0.5 fm; the "error bars" indicate the uncertainty in absolute, r-independent normalization. States of the Ψ and Υ families are displayed at their mean square radii. From Ref. 16.

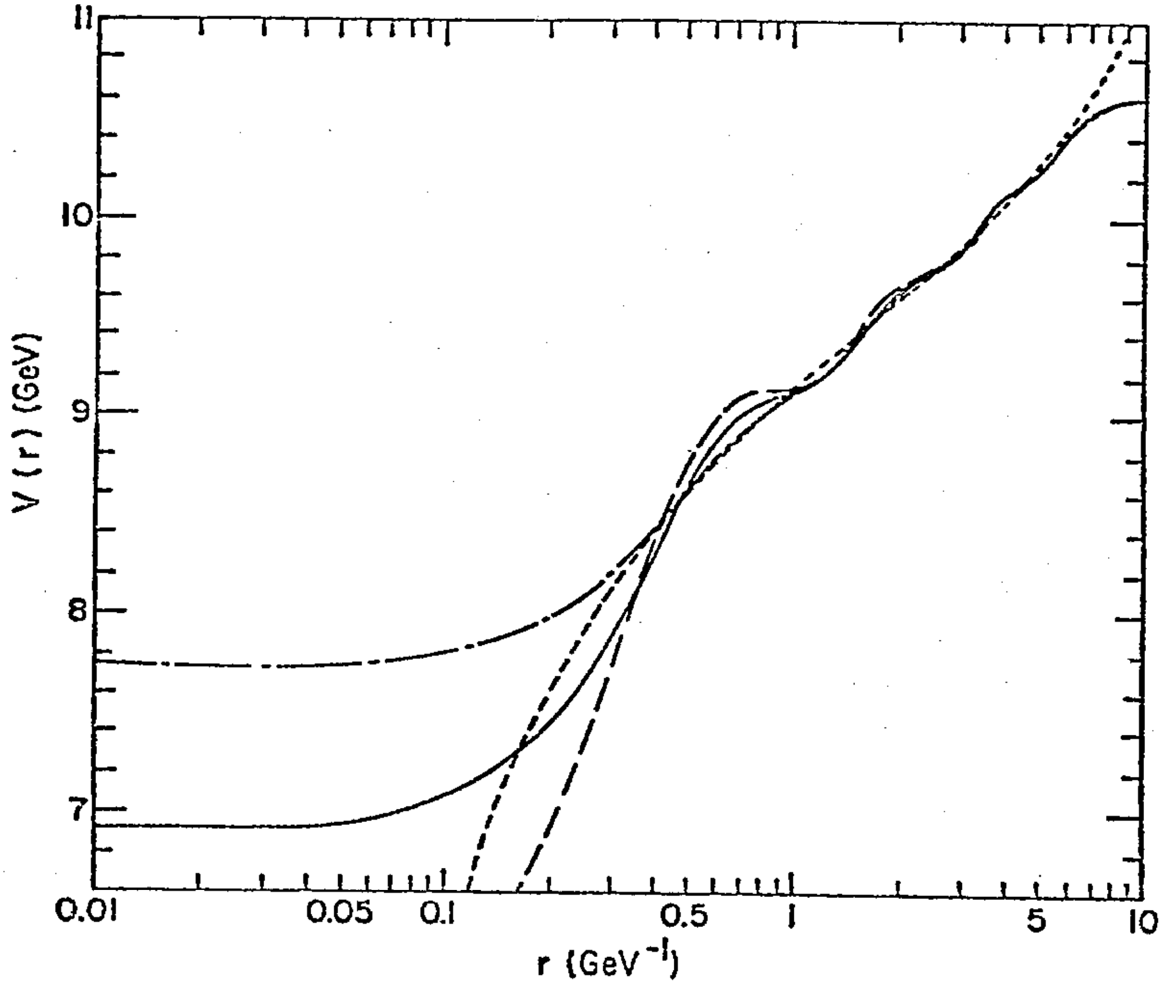

Fig. 2. Three potentials constructed from Υ data by means of the inverse scattering method, corresponding to three different "correction factors" ρ in the van Royen-Weisskopf formula, $\Gamma_{ee}(nS) = 1/\rho\ (16\pi\alpha^2 e_Q^2)/M_n^2\ |\phi_n(0)|^2$. Dot-dashed line: $\rho = 1.0$; solid line: $\rho = 1.4$; long-dashed line: $\rho = 2$. The short-dashed line is the QCD-like potential of ref. 16, with the scale parameter chosen as $\Lambda_{\overline{MS}} = 0.5$ GeV. From Ref. 26.

Table I. Predictions of various potential models for the Υ family, compared with experiment. Model 1: Martin[23]; model 2: Buchmüller, Grunberg and Tye,[16] $\Lambda_{\overline{MS}} = 0.5$ GeV; model 3: Richardson[17]; model 4: Bhanot and Rudaz[24] (the range of predictions, which are dependent on the b-quark mass, is given); model 5: Cornell group.[18] The first column contains the leptonic widths in keV, the second and third columns the excitation energies in MeV and, in brackets, the ratios of the leptonic widths with respect to the Υ leptonic width. From Ref. 16.

	Υ	Υ'	Υ"
Experiment			
a) Ref. 36	1.29 ± 0.22	553 ± 10 (0.45 ± 0.08)	---
b) Ref. 37, 38	1.02 ± 0.22 1.10 ± 0.17	560 ± 3 (0.45 ± 0.07)	889 ± 4 (0.32 ± 0.06)
Model 1 (Martin)	---	560 (0.43)	890 (0.28)
Model 2 (Buchmüller, Grunberg & Tye)	1.07	555 (0.46)	890 (0.32)
Model 3 (Richardson)	---	555 (0.42)	886 (0.30)
Model 4 (Bhanot and Rudaz)	1.07 - 1.77	561 - 566 (0.47 - 0.76)	881 - 879 (0.34 - 0.51)
Model 5 (Cornell group)	---	560 (0.48)	898 (0.34)

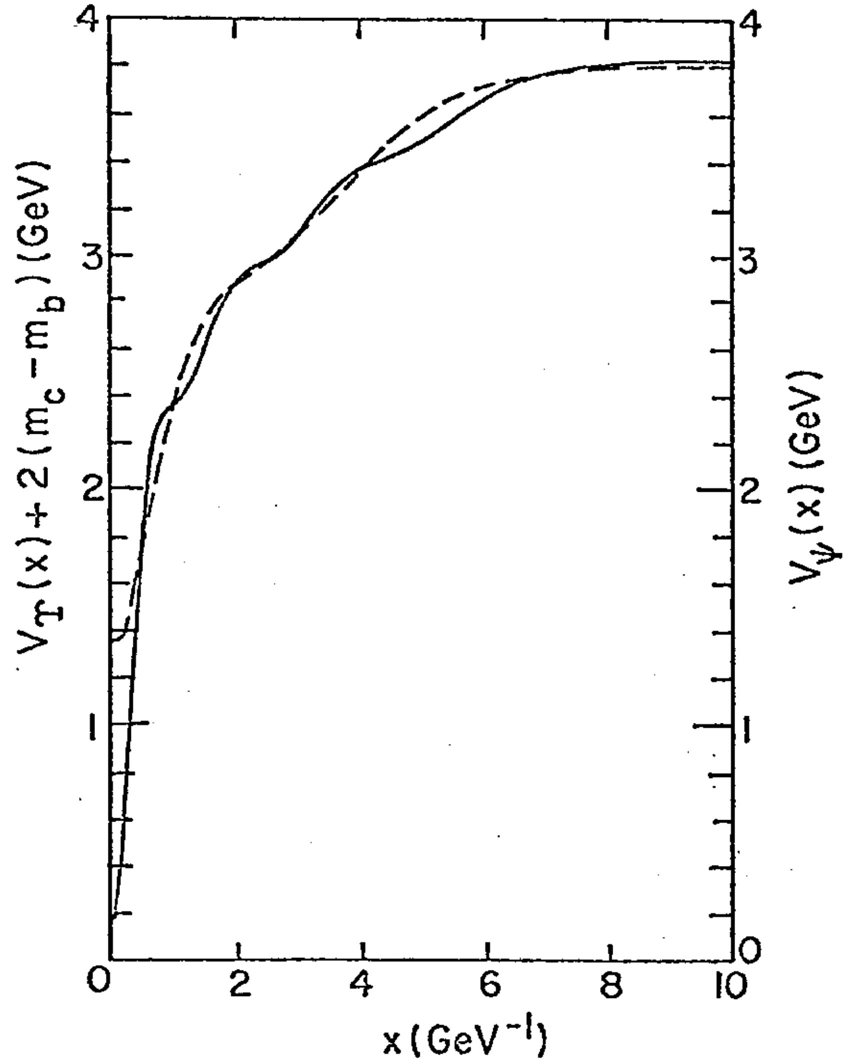

Fig. 3. Comparison of potentials deduced from the Ψ and Υ families. The energy scale is appropriate for the Ψ spectrum. The label on the left-hand ordinate refers to the potential constructed using Υ data (solid curve). The label on the right-hand ordinate refers to the potential constructed using Ψ data (dashed curve). $\rho = 1.4$ (cf. figure caption of Fig. 2). From Ref. 26.

The ultimate theory of strong interactions will have to explain why the corrections to the non-relativistic limit are so small.

We conclude:

(i) QCD-like potential models provide an accurate description of the Ψ and Υ families. However, this success is shared with power potentials, which disagree with theoretical expectations based on QCD. Thus, no direct unequivocal evidence for asymptotic freedom has been obtained so far on the basis of quarkonia.

(ii) The $(Q\bar{Q})$ potential has emerged as a measurable quantity, which can be directly compared with predictions derived from any fundamental theory of strong interactions. The Ψ and Υ spectroscopies have determined the quark-antiquark potential at distances between 0.1 fm and 1.0 fm.

III. THE $(Q\bar{Q})$ POTENTIAL AT SHORT DISTANCES

As we have discussed in Sec. I, the $(Q\bar{Q})$ potential has been computed perturbatively in QCD. In coordinate space the result[6,11] reads (for 4 flavors)

$$V^{QCD}(r) \underset{r\to 0}{\sim} -\frac{4}{3}\frac{\alpha_s(r)}{r} \quad ,$$

$$\alpha_s(r) = \frac{12\pi}{25t}\left[1 - \frac{462}{625}\frac{\ln t}{t} + \left(\frac{53}{75} + 2\gamma_E\right)\frac{1}{t} + O\left(\frac{1}{t^2}\right)\right] \quad ,$$

$$t = \ln\frac{1}{r^2\Lambda^2_{\overline{MS}}} \quad , \qquad (3.1)$$

where $\gamma_E = 0.5772\ldots$ is Euler's constant. In order to compare this perturbative short distance behavior with a phenomenological potential, one has to specify at what distances corrections to Eq. (3.1) are expected to be negligible. In analyses of deep inelastic scattering processes comparison with perturbative QCD is considered to be justified for momentum transfers Q, which satisfy $Q^2/\Lambda^2 > 100$. Correspondingly, at distances r, with

$$r < r_c \quad , \quad \frac{1}{r_c^2\Lambda^2_{\overline{MS}}} = 100 \quad , \qquad (3.2)$$

$V^{QCD}(r)$ should be a good approximation to the $(Q\bar{Q})$ potential. Indeed, for distances $r < r_c$, the corrections of relative order $1/t$ in Eq. (3.1) are less than $\sim 15\%$ and the perturbation series is self-consistent. Nonperturbative effects, such as gluonic vacuum fluctuations, characterized by a nonvanishing expectation value $\phi \equiv \langle 0|\frac{\alpha_s}{\pi}G_{\mu\nu}G^{\mu\nu}|0\rangle$,[28] appear to be negligible; a dimensional analysis[16] suggests corrections less than 1%.

Figure 4 shows $V^{QCD}(r)$ for different values of $\Lambda_{\overline{MS}}$ and distances $r < r_c$; for comparison the potentials of Martin[23] and Ref. 16 are also given. For $\Lambda_{\overline{MS}} = 0.1$ GeV, $V^{QCD}(r)$ and the empirical potential, determined by the Ψ and Υ families, overlap for distances between 0.1 fm and 0.2 fm. The two potentials appear to be clearly different in this region, and a quantitative analysis[16] shows that values of $\Lambda_{\overline{MS}}$ less than or equal to 0.1 GeV appear indeed incompatible with quarkonium spectroscopy. For values $\Lambda_{\overline{MS}} > 0.2$ GeV perturbation theory becomes unreliable already at distances $r < 0.1$ fm. Therefore present quarkonia cannot distinguish in a model independent way between values of $\Lambda_{\overline{MS}}$ larger than 0.2 GeV.

Obviously, the ζ-spectroscopy,[29] formed by the anticipated t-quark and its antiquark, will be sensitive to larger values of the scale parameter. Figure 5 shows two potentials whose asymptotic behaviors at short distances are characterized by the scale parameters $\Lambda_{\overline{MS}} = 0.2$ GeV and $\Lambda_{\overline{MS}} = 0.5$ GeV. The indicated mean square radii illustrate down to which distances the $(Q\bar{Q})$ potential will be probed by $(t\bar{t})$ bound states of a given mass. The properties of the 1S-state of the ζ-spectroscopy will be most sensitive to the short distance part of the potential. Figure 6 shows the 1S-leptonic widths as a function of the t-quark mass for $\Lambda_{\overline{MS}} = 0.2$ GeV, $\Lambda_{\overline{MS}} = 0.5$ GeV and Martin's potential[23]; for a t-quark mass of 30 GeV various predictions of the different

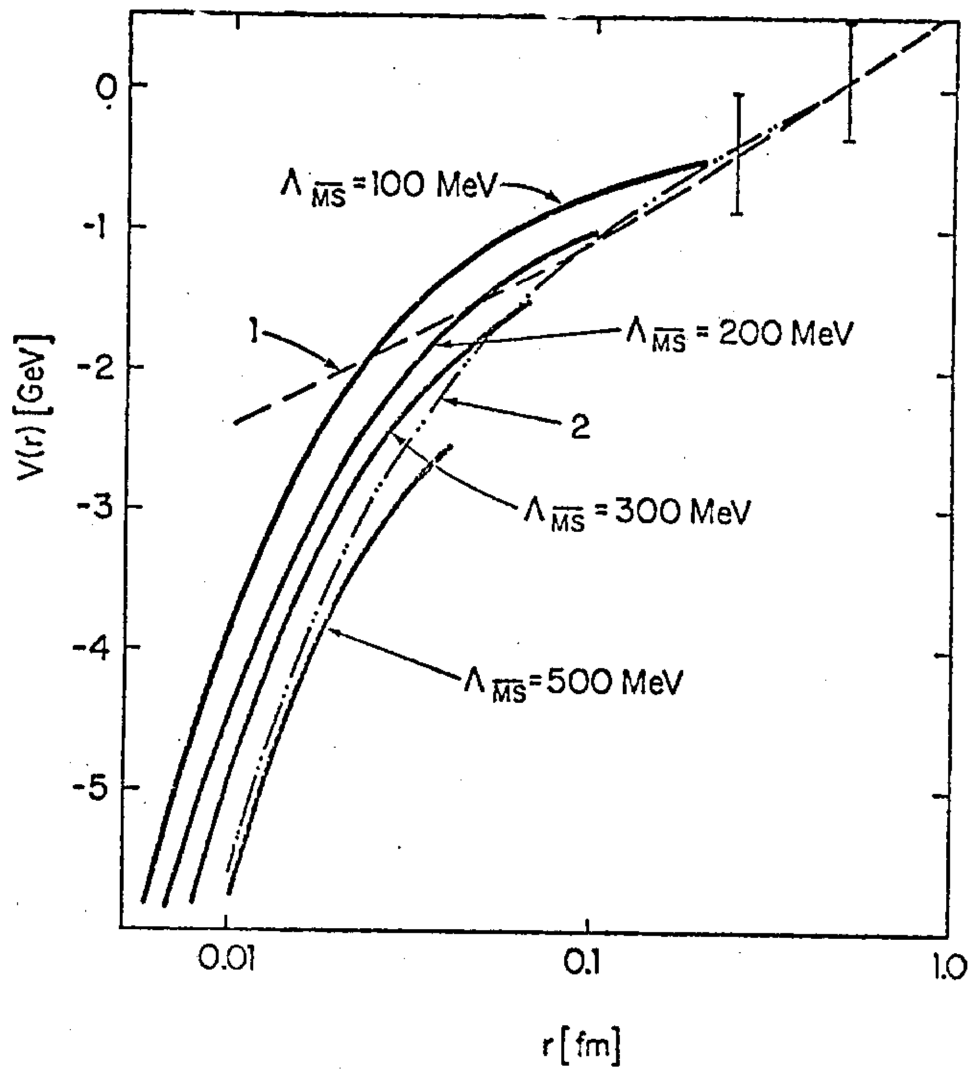

Fig. 4. 2-loop "asymptotic freedom" potentials for 4 flavors and different values of $\Lambda_{\overline{MS}}$ at distances $r \leq r_c$, $r_c^2 = 1/(100\,\Lambda^2_{\overline{MS}})$. For comparison the potentials (1) and (2) of Fig. 1 are also displayed. The "error bars" indicate the uncertainty with respect to absolute normalization. From Ref. 16.

models are listed in table II. It appears obvious that a $(t\bar{t})$ system with $m_\zeta \geq 40$ GeV will clearly distinguish between power potentials and QCD-like models as well as between different values of $\Lambda_{\overline{MS}}$.

The main problem in the determination of Λ by means of the $(Q\bar{Q})$ potential is the uncertainty in the absolute normalization of the potential, i.e. the uncertainty in our knowledge of the c-quark and b-quark masses. The ζ-spectroscopy will measure the $(Q\bar{Q})$ potential down to distances of about 0.04 fm, where a change of $\Lambda_{\overline{MS}}$ by 100 MeV will change the "asymptotic freedom" potential by about 300 MeV. The uncertainty in the absolute normalization of the empirical potential of about $\pm$ 400 MeV (cf. Fig. 1) will lead to an uncertainty of about $\pm$ 150 MeV in the determination of Λ. This situation would be improved through a better theoretical understanding of finite

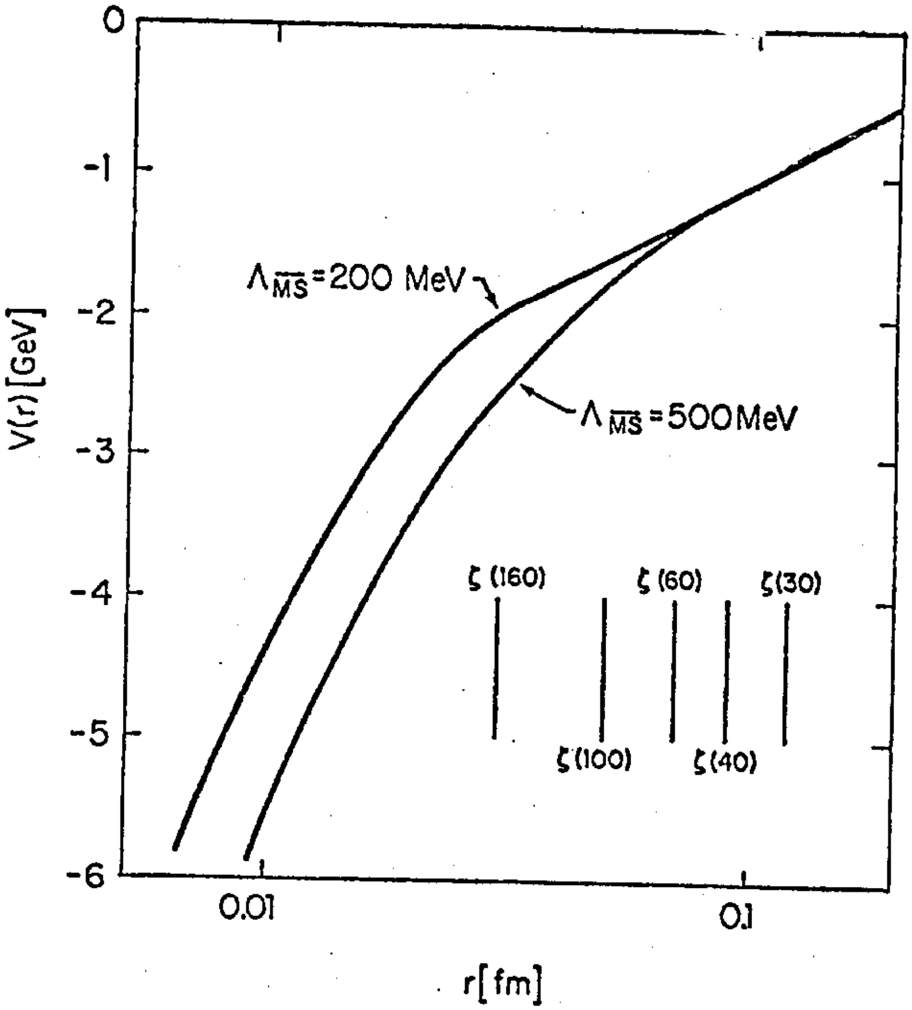

Fig. 5. Two $(Q\bar{Q})$ potentials which approach "asymptotic freedom" potentials with $\Lambda_{\overline{MS}}$ = 200 MeV and $\Lambda_{\overline{MS}}$ = 500 MeV at short distances. Mean square radii of $(t\bar{t})$ ground states (denoted as $\zeta(2m_t)$) are shown for $\Lambda_{\overline{MS}}$ = 500 MeV and different quark masses m_t. From Ref. 16.

Table II. Comparison of $(t\bar{t})$ spectra for different potential models, with m_t = 30 GeV.

	Martin[23]	$\Lambda_{\overline{MS}}$=0.2 GeV[16]	$\Lambda_{\overline{MS}}$=0.5 GeV[16]	Richardson[17]
$E_2 - E_1$ [MeV]	512	610	762	801
$\Gamma_{ee}(2S)/\Gamma_{ee}(1S)$	0.48	0.53	0.30	0.29
$E_3 - E_1$ [MeV]	814	913	1090	1136
$\Gamma_{ee}(3S)/\Gamma_{ee}(1S)$	0.34	0.28	0.18	0.17

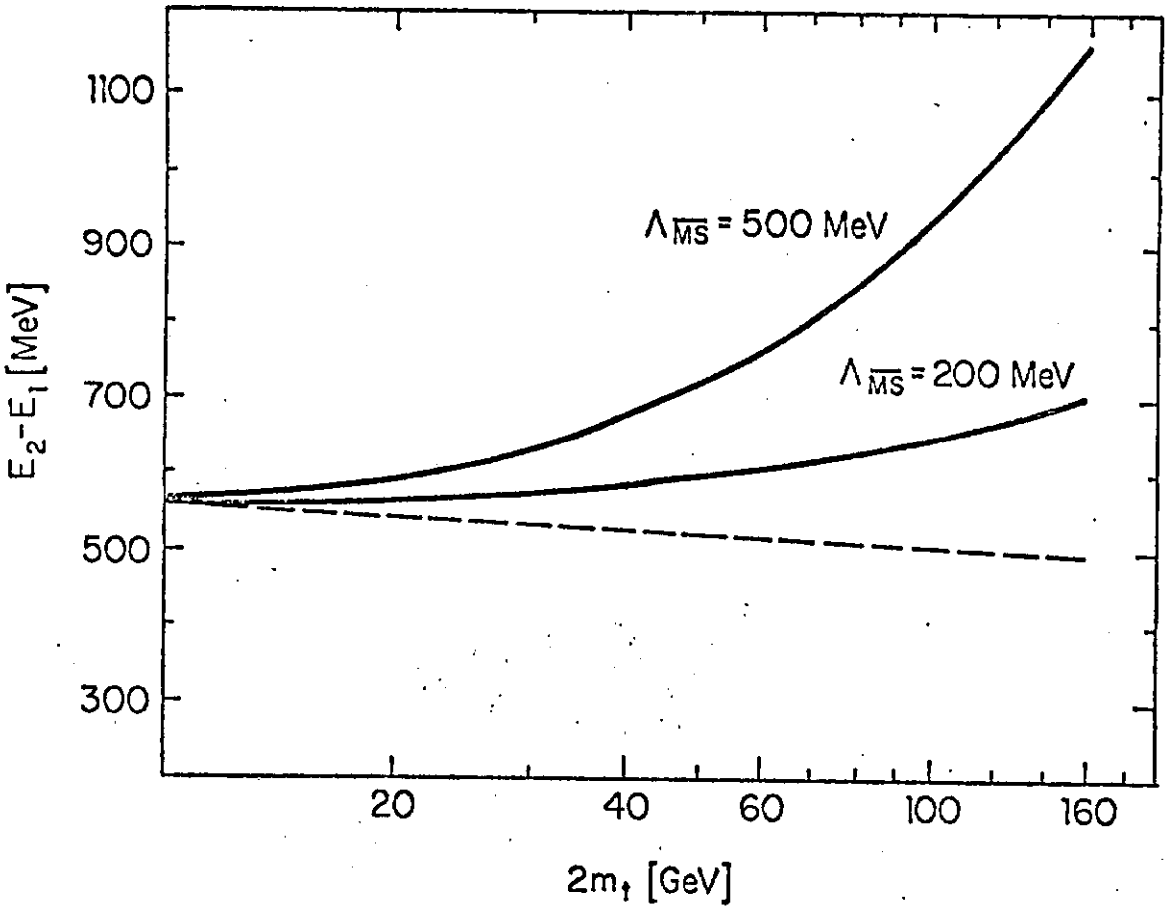

Fig. 6. Ground state leptonic widths as function of t-quark mass m_t. The solid lines correspond to the potentials of Fig. 5. The dashed line shows the results of Martin's potential.[23] Here we have ignored weak interaction effects, which would only enhance the differences. From Ref. 16.

structure, hyperfine structure and E1 transitions which would lead to a more precise determination of the quark masses.

It is also conceivable that the scale parameter will be determined more accurately through the measurement of electromagnetic and hadronic decay widths, where next to leading order QCD radiative corrections have recently been computed.[30-32] For instance, a measurement of the hadronic width of a 60 GeV toponium state with an accuracy of $\sim$20% would determine the strong coupling constant of α_s (60 GeV) within $\sim$7% and thereby measure $\Lambda_{\overline{MS}}$ with an uncertainty of about $\pm$100 MeV. This, in turn, would fix the normalization of the $(Q\bar{Q})$ potential up to $\pm$300 MeV and thereby determine the c-quark and b-quark masses within $\pm$150 MeV!

Thus the ζ-spectroscopy will not just determine the $(Q\bar{Q})$ potential at short distances and the QCD scale parameter Λ, it will also have consequences for the Ψ and Υ spectroscopies: we can expect a better determination of the c- and b-quark masses and a very accurate test of the flavor-independence of the potential at intermediate distances due to the large number of ζ-states with mean square radii in this region. For instance,

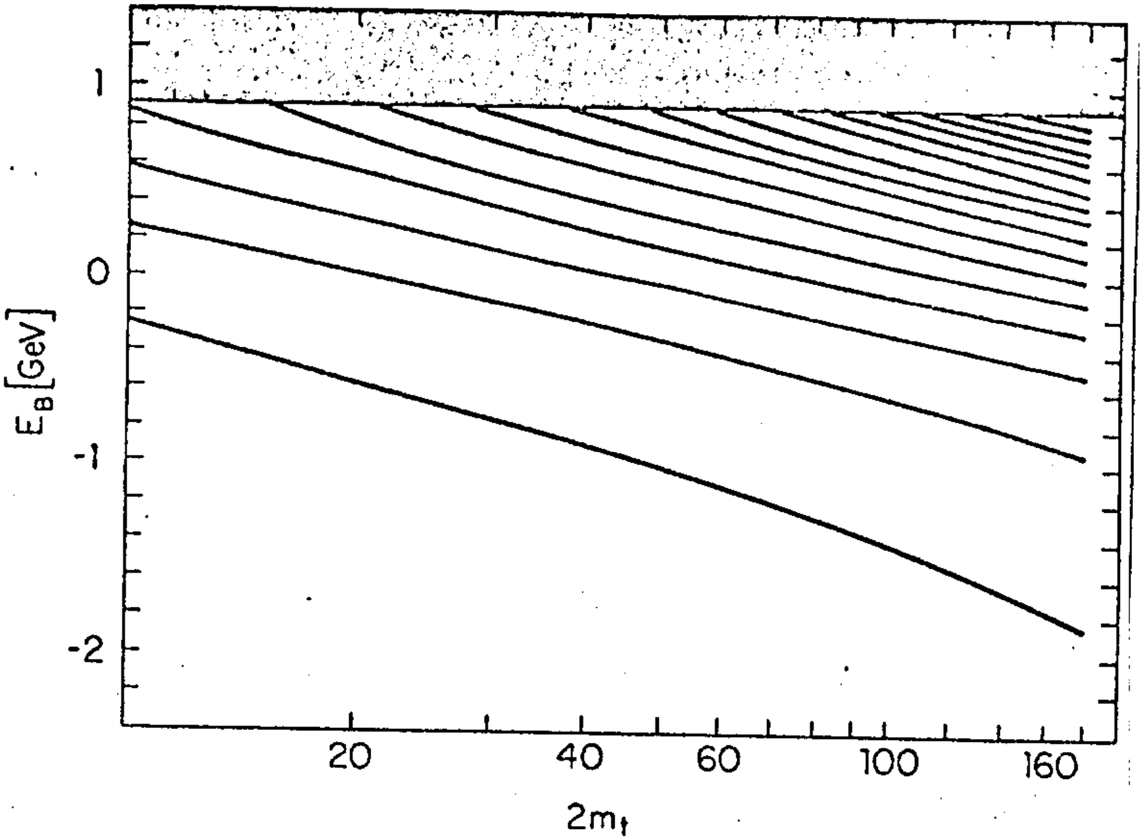

Fig. 7. $(t\bar{t})$ S-wave bound states below threshold as function of the t-quark mass. The binding energies have been computed for a potential which corresponds to $\Lambda_{\overline{MS}} = 300$ MeV; it satisfies $V(\Lambda_{\overline{MS}} = 200\text{ MeV}) \geq V(\Lambda_{\overline{MS}} = 300\text{ MeV}) \geq V(\Lambda_{\overline{MS}} = 500\text{ MeV})$. From Ref. 16.

as shown in Fig. 7, a $(t\bar{t})$ system of 60 GeV will have 8-9 narrow S-states, in accord with the semiclassical estimate[33]

$$n \approx 2\left(\frac{m_t}{m_c}\right)^{1/2} , \qquad (3.3)$$

and a corresponding numer of P-, D-, F-,... states which will lead to an extremely rich spectrum of electromagnetic and hadronic transitions.[34]

Predictions for the ζ-spectroscopy up to ground-state masses of 60 GeV have also been made in a model independent way based on inverse scattering methods.[35] The three potentials, shown in Fig. 2, which are constructed from the masses and leptonic widths of the Υ family, lead to a range of predictions for toponium, thus reflecting the degree to which the ζ-spectroscopy can already be anticipated from our understanding of the Υ-spectroscopy. The leptonic decay widths of the first four S-states are shown in Fig. 8. The main conclusion is that the properties of the 1S ground state, in particular its leptonic width, will be most important for the determination of the short distance behavior of the $(Q\bar{Q})$ potential.

We conclude:

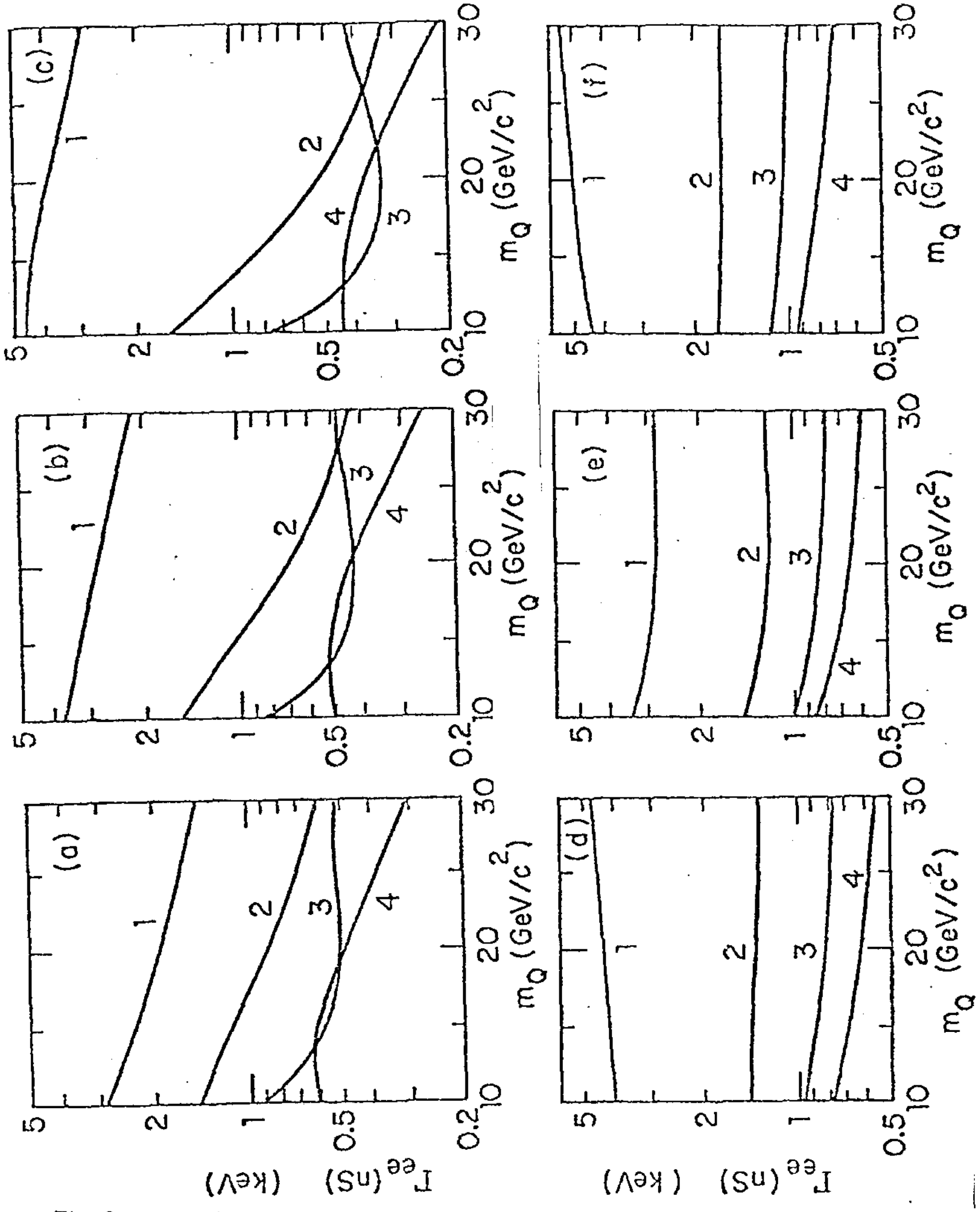

Fig. 8. Leptonic widths of the 1S - 4S levels of the ζ-spectroscopy are displayed as functions of heavy-quark mass for six potentials: (a) Inverse scattering, $\rho = 1$, $m_b = 4.5$ GeV; (b) inverse scattering, $\rho = 1.4$, $m_b = 4.75$ GeV; (c) inverse scattering, $\rho = 2$, $m_b = 5$ GeV (cf. Fig.2); (d) Richardson potential, Ref. 17; (e) Ref. 16, $\Lambda_{\overline{MS}} = 0.2$ GeV; (f) Ref. 16, $\Lambda_{\overline{MS}} = 0.5$ GeV. From Ref. 35.

(i) $V^{QCD}(r)$, the "asymptotic freedom" potential calculated in perturbative QCD, is expected to coincide with the empirical $(Q\bar{Q})$ potential at distances $r < r_c$, where $1/r_c^2 \Lambda_{\overline{MS}}^2 = 100$;

(ii) the Ψ and Υ spectroscopies lead to the lower bound on the QCD scale parameter Λ, $\Lambda_{\overline{MS}} > 0.1$ GeV;

(iii) the ζ-spectroscopy, with $m_\zeta > 40$ GeV, will determine the scale parameter Λ, if $\Lambda_{\overline{MS}} < 0.5$ GeV;

(iv) the ζ-spectroscopy may lead to a better determination of the c-quark and b-quark masses;

(v) the properties of the 1S toponium ground state will most conclusively determine the short distance behavior of the $(Q\bar{Q})$ potential.

IV. SUMMARY

The main conclusions are as follows:

(1) The $(Q\bar{Q})$ potential can be defined in QCD as the binding energy of a quark-antiquark pair in the infinite-mass-limit. The physical coupling constant $\alpha_p(\mu)$, which is defined in terms of the potential, may be a convenient expansion parameter for other processes in perturbative QCD.

(2) The $(Q\bar{Q})$ potential can be expressed in terms of the dimensionless β-function $\beta_p(\rho)$ and the scale parameter Λ_p (or the string tension k). A simple empirical Ansatz for $\beta_p(\rho)$ has been obtained which satisfies the theoretically required boundary conditions at large and small values of ρ and provides an accurate description of the Ψ and Υ families.

(3) The $(Q\bar{Q})$ potential has emerged as a measurable quantity, which allows a comparison with QCD at all coupling strengths. The Ψ and Υ spectroscopies have determined the potential at distances between 0.1 fm and 1.0 fm.

(4) Comparison of the "asymptotic freedom" potential of perturbative QCD with the empirical $(Q\bar{Q})$ potential, determined by Ψ and Υ data, leads to a lower bound on the QCD scale parameter, $\Lambda_{\overline{MS}} > 0.1$ GeV. The ζ-spectroscopy, with $m_\zeta > 40$ GeV, will determine the $(Q\bar{Q})$ potential down to distances of ~ 0.04 fm. If $\Lambda_{\overline{MS}} < 0.5$ GeV, as expected on the basis of deep inelastic scattering processes, this will lead to a determination of the QCD scale parameter Λ.

(5) The ζ-spectroscopy will have important consequences for Ψ and Υ physics. It will provide a very accurate test of the flavor-independence of the $(Q\bar{Q})$ potential at intermediate distances and may also lead to a more precise determination of the c-quark and b-quark masses.

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