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Magnetic Moments of Composite Quarks and Leptons--Further Difficulties

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ABSTRACT

The previously noted difficulty of obtaining Dirac magnetic moments in composite models is combined with the observation that a "light" bound fermion state with a small size must have the Dirac moment in a renormalizable theory since its anomalous moment is determined by its excitation spectrum. New constraints on composite models are given, including the "superconfinement" condition that creation of virtual electron-positron pairs by the superstrong gluons responsible for binding the constituents of the electron must be strictly forbidden in photon electron scattering.

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The difficulty of obtaining the Dirac magnetic moment[1,2] has been suggested as a crucial test for composite models of leptons and quarks.[3] A number of papers[4-7] have subsequently argued that the Dirac moment is automatically obtained in any model which correctly describes the strong binding and small size of the leptons. This paper examines these arguments and shows that their conclusions are somewhat misleading for model builders. Obtaining the Dirac moment remains a serious test for any model because it may be easier to calculate the ground state magnetic moment in a given model than to verify that it satisfies these conditions assumed in Refs. 4-7.

The absence of low-mass photo-excitations of the leptons is always assumed in the arguments which automatically obtain the Dirac moment. Although this is true for the physical electron and muon, it may not hold in a composite model under consideration. Such models will have to be evaluated by incomplete evidence as they can be expected to be even more difficult for calculation that QCD. In models where it is much easier to calculate the magnetic moment of the ground state than to show that there are no low-mass photo-excitations, the arguments of Refs. 4-7 showing that the Dirac magnetic moment is automatically obtained are not very useful.

There is, however, interesting physics in this discussion which can be clarified by combining the approaches of Refs. 1,2 and Refs. 4,5. The results can be useful to model builders by providing intuition, constraints and guide lines. The constraints might enable them to reject unsuitable models with a minimum of wasted effect. They might also lead to no-go theorems showing that the problem cannot be solved either in general or with a wide class of models.

This paper discusses three constraints which must be satisfied by any model:

1. The magnetic moment of the bound state must depend only upon the total charge of the system and be independent of how the charge is distributed between the constituents.

2. The excitation of multilepton states in photon-lepton scattering must be completely described by QED, with no additional contributions observable at present energies from the superstrong interactions responsible for the binding of the constituents of the leptons.

3. The composite model for the spin 1/2 leptons must not have a low mass excitation with spin 3/2 analogous to the Δ in the quark model for the nucleon.

The nature of the difficulties imposed by these constraints is most strikingly illustrated in the following simple but extreme example. Consider an electron model as a composite of a neutral fermion and a scalar boson with charge -e. The naive nonrelativistic model for such a state has zero magnetic moment since the charged constituent has no angular momentum and the constituent with spin has no charge. A Dirac moment is obtained only if the charged boson has just the right peculiar value of orbital angular momentum to contribute the exact value of the moment for the combined system.

The argument of Refs. 4,5 suggest that this miracle occurs automatically if a light bound state can be constructed from a heavy scalar boson and a heavy fermion. The essential peculiar feature of the bound state is that the scale defined by its size (or the masses of the constituents) is much smaller than the scale defined by its Compton wave length (or the mass of the bound state). They show that the anomalous magnetic moment and the excitation spectrum are determined by the scale of the size of the system, whereas the Dirac moment is determined by the mass or Compton wave length.

One very remarkable feature of this argument is its complete independence of the precise coupling of the individual constituents to the electromagnetic field; e.g. their electric charges. Thus the magnetic moment of such a low mass bound state must be very close to the Dirac moment <u>regardless of the electric charges of the constituents.</u> If the argument holds for a neutral fermion and a charged boson, it must also hold, <u>with the same wave function for the composite system,</u> for a charged fermion and a neutral boson, or for a fermion with charge xe and a boson with charge -(1 + x)e, where x can have any arbitrary value. This puts extreme conditions on the model, and suggests that any composite model made from two different elementary fields cannot have a simple description in terms of constituents, like the constituent quark model for hadrons.

This argument also shows that simple relativistic models with the Dirac equation in external potentials cannot describe such superstrong binding. Although such calculations show that a bound fermion in an external potential contributes a magnetic moment corresponding to its charge and mass,[7-9] the result is misleading, since the infinitely heavy potential source is assumed to have no charge and no spin. If all the charge of the system is on the infinitely heavy source and the fermion has no charge, the magnetic moment according to this calculation is zero. If the source has no charge, but is an infinitely heavy spin one boson, the total angular momentum of the system will be in the opposite direction to the angular momentum carried by the fermion, and the magnetic moment calculated in this way will have the wrong sign compared to the Dirac moment for the composite system.

The basic nature of the problem is clarified by examining the excitation spectrum for the electron. The lowest excited states with the same quantum numbers as the electron have a single electron and several electron-positron pairs. Simple relativistic models based on the Dirac or Bethe-Salpeter equations cannot be expected to describe an excitation spectrum which contains only multiparticle excitations up to a very high energy. Any model which attempts to describe the electron from first principles as a bound state of several super-strongly interacting particles must also give a reasonable description of multielectron systems. Thus any scattering amplitude in which the electron appears as a pole must have branch points at masses of $(2n+1)m_e$ beginning with $3m_e$.

The treatments of Refs. 4-7 do not consider these branch points and assume that the dominant contribution above the electron pole to photon electron scattering in lowest order in α comes from states at very high mass. This effectively assumes that narrow bound states exist at a mass many orders of magnitude above the masses of millions of open decay channels allowed by all known conservation laws. Some drastically new type of conservation law is needed to prevent the coupling and mixing of such high mass states with multiparticle states of an electron and a number of charge-conjugate electron position pairs with vacuum quantum numbers. Such mixing would introduce unwanted low-mass contributions into the dispersion relations and sum rules which obtain an anomalous moment having a mass scale determined by the masses of the intermediate states coupled to an electron and a photon. The neglect of all the contributions of all multielectron states in these treatments assumes that the superstrong "gluons" which bind the constituents into a single electron are somehow forbidden to be emitted by an electron and to create electron-positron pairs.

The problem of obtaining a magnetic moment independent of the manner in which the charge is distributed between the constituents can be formulated more generally without assuming simple constituent models and be stated relativistically. Consider a composite model constructed from two basic fields, denoted by Ψ_1 and Ψ_2. These may be either Bose or fermi fields, but at least one fermi field is necessary to make a composite fermion. We assume that the electromagnetic current is additive in the two fields,

$$J^\mu = q_1 \bar{J}_1^\mu(\Psi_1) + q_2 \bar{J}_2^\mu(\Psi_2), \tag{1}$$

where $\bar{J}_1^\mu$ and $\bar{J}_2^\mu$ are "reduced" currents, depending respectively only on Ψ_1 and Ψ_2 respectively and the coupling constants q_1 and q_2 for each field are factored out.

The angular momentum carried by each field can be defined relativistically by observing the behavior of each field separately under rotations. Thus we can define the total angular momentum $\vec{J}$ as the sum of the contributions from the two fields,

$$\vec{J} = \vec{J}_1 + \vec{J}_2 \ . \tag{2}$$

The magnetic moment operator for each field can be defined separately using the specific form of the electromagnetic current. We can thus write the magnetic moment operator for the system as

$$\vec{\mu} = q_1\vec{\mu}_1 + q_2\vec{\mu}_2, \tag{3}$$

where $\vec{\mu}_1$ and $\vec{\mu}_2$ are reduced magnetic moment operators with the coupling constants factored out.

Let us now assume that a bound state exists, formed from these constituent fields ψ_1 and ψ_2. The total electric charge of this bound state is given by

$$Q = \langle n_1\rangle q_1 + \langle n_2\rangle q_2, \tag{4}$$

where n_1 and n_2 are the "reduced charges" of each field. In the simple constituent model these are just the number of constituents of type 1 and type 2 in the bound state. In the Harari Rishon model,[3] for example, n_1 and n_2 are the numbers of V and T particles in the state and take on integral values from -3 to +3 for the quarks and leptons. Note that the wave function can contain an orbitrary number of particle-antiparticle pairs in addition to the n_1 of type 1 and the n_2 of type 2, and may not be eigenfunctions of n_1 and n_2 if charge exchange is possible between the two fields. The magnetic moment of this state is given by the expectation value of the operator (3) in this state,

$$\langle\vec{\mu}\rangle = q_1 \langle\vec{\mu}_1\rangle + q_2 \langle\vec{\mu}_2\rangle \tag{5}$$

Assuming that the state has a well defined angular momentum, e.g. J=1/2 for quarks and leptons, Eq. (5) can be rewritten

$$<\vec{\mu}> = q_1<\vec{\mu}_1\cdot\vec{J}>+q_2<\vec{\mu}_2\cdot\vec{J}> <\vec{J}>/[J(J+1)]. \tag{6}$$

From the argument of Refs. (4,5), the magnetic moment (6) must be the Dirac moment if the found state has a much lower mass than the constituents, independent of the values of q_1 and q_2. The ratio of the magnetic moment (6) to the total charge Q must then be independent of Q_1 and Q_2. This gives following condition,

$$<\vec{\mu}_1\cdot\vec{J}>/<n_1> = <\vec{\mu}_2\cdot\vec{J}>/<n_2>. \tag{7}$$

Thus the reduced magnetic moments $\vec{\mu}_1$ and $\vec{\mu}_2$ carried by the two fields in the bound state wave function must satisfy the condition (7). This result (7) is a precise quantitative constraint which must be satisfied by any model which makes a light bound state out of two heavy fields with a non-electromagnetic superstrong interaction.

Note that if $<\vec{\mu}_1\cdot\vec{J}_1>$ and $<\vec{\mu}_2\cdot\vec{J}_2>$ have the same sign, as is the case is all simple models, where the magnetic moment of a positively charged field is parallel to the direction of the angular momentum, then $<\vec{J}_1\cdot\vec{J}>$ and $<\vec{J}_2\cdot\vec{J}>$ must also have the same sign. This means that for J=1/2 the projections of $\vec{J}_1$ and $\vec{J}_2$ in the direction of the total angular momentum are parallel and are both less than 1/2. This proves that the state cannot be an eigenfunction of both J_1 and J_2 and must have components both with $J_1=J_2+1/2$ and $J_1=J_2-1/2$.

The wave function defined in Ref. 2, Eq. (6) satisfies these constraints, since it was constructed to give a Dirac moment for all values of the charges of the constituents. However, as noted there, it can only be achieved with a peculiar relation between spin and statistics for the fundamental fields. More realistic models, if they exist, must have wave functions very different from those of simple constituent models; e.g. they could contain additional particle-antiparticle pairs with non-trivial angular momenta and significant contributions to the magnetic moment.

One way out of this difficulty is to note that the electromagnetic current for a non-relativistic moving bound system can be split into two components, one due to its center-of-mass motion and the other due to its internal degrees of freedom. In non-relativistic physics the center-of-mass motion is eliminated by going to the center-of-mass system. In the relativistic quantum mechanics this is no longer possible. The magnetic moment of a Dirac electron which has no internal structure comes from the operators describing its motion as a whole. The electron might have a composite structure on a very small scale which would not affect its motion required by relativistic quantum mechanics within a range of its Compton wave length. This fits into the picture of Refs. 4-5 in which the Dirac moment is always present and the anomalous moment comes from the structure. However, it is not obvious that the internal motions of a composite electron are separable and completely decoupled from the motions giving rise to the Dirac moment. This is equivalent to requiring a complete decoupling of the low-lying multiparticle excitations, and may be very difficult to prove.

An alternative way out of this difficulty is to assume that electromagnetism plays an important role in the superstrong binding force and that therefore the bound state wave function depends upon the values of q_1 and q_2. In that case the condition (7) does not hold.

Another difficulty to be faced by composite models for the electron is the absence of a low-lying excitation with spin 3/2 which can be excited by a photon on the electron, like the Δ is excited from the nucleon. Such a spin 3/2 state is expected to arise in many models. Some way must be found to get rid of it or to push it up to a very high mass if the model is to describe the leptons of the real world. In simple constituent models where the electron spin of 1/2 is obtained by coupling several non-trivial constituent spins to a total spin of 1/2, the spin 3/2 state arises from recoupling the constituent spins. In the general two-field model described by Eqs. (1-7) the same problem arises even though there may not be well defined constituents. Equations (2) and (7) show that the angular momenta $\vec{J}_1$ and $\vec{J}_2$ carried by the two fields are both finite and can therefore be coupled to a total angular momentum of 3/2 as well as to 1/2.

If such a spin 3/2 state exists in a given model, the sum rule arguments break down and the anomalous moment is not small. This is clear in the case of the nucleon. The N-Δ transition for example gives a large contribution to any sum rule for the anomalous moment of the nucleon. In a particular model it may be easier to calculate the ground state magnetic moment than to prove the absence of a low-lying spin 3/2 state. Estimates or bounds on the magnetic moment might be obtainable from models with approximate ground state wave functions. But if the excitation spectrum is exceedingly difficult to calculate, particularly for higher spin states, the masses of the lowest spin 3/2 excitations may be unknown and the argument of Refs. 4,5 completely useless.

This discussion can be summarized as requiring any composite model describing the electron to be "superrelativistic" with "superconfined" constituents.

Super-relativistic goes beyond both nonrelativistic and simple relativistic models. A non-relativistic composite model is characterized by constituent velocities $v \ll c$. Relativistic potential models using Dirac or Bethe-Salpeter equations are useful when velocities are no longer small, but when an excitation spectrum exists with energies smaller than the energy required to produce many bound state pairs. The composite model needed to describe the electron can be called superrelativistic because it must have a rich low-lying spectrum of multiparticle states. Models where it is much easier to create many pairs than to excite the original constituents to a radial or orbital excitation cannot be described in any simple way by potential models.

Superconfinement goes beyond the ordinary confinement of QCD. Quarks in QCD are not observable as free particles, but are observable as hadrons jets produced in collisions, are emitted in pairs in hadron decays by interactions arising from QCD gluons, and give rise to forces and scattering between hadrons resulting from quark or gluon exchange. If leptons are composites of constituents bound by some superstrong gauge field, these constituents are confined much more than in the sense of QCD. There must not be any observable effects in lepton-lepton and lepton-photon scattering which reveal the existence of additional interactions beyond QED. There can be no lepton jets produced by deep inelastic photon absorption on a charged constituent of the electron, and no observable electron-electron interactions resulting from superstrong gluon or constituent exchange. The superstrong interactions which bind the constituents of the electron must not only confine the constituents from being observed as free particles. They must also confine all the secondary effects of these superstrong interactions normally observed in QCD.

Such a superrelativistic superconfined system will naturally have the Dirac magnetic moment to a very good approximation. The Dirac moment is obtained from the electromagnetic current density due to the motion of the entire bound system. If all effects of the composite structure are superconfined, the anomalous moment must be very small with a mass scale determined by the excitation energy of the composite structure. But superconfinement will undoubtedly be harder to tests and prove in any proposed model than confinement in QCD. Thus magnetic moment calculations may prove to be highly significant test of such models.

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FOOTNOTES AND REFERENCES

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