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SHORT RANGE CORRELATIONS AND BARYON DECAY*

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ABSTRACT

Hyperfine interactions break SU_6 symmetry in the short range correlations between quarks and thereby give significant corrections to branching ratios for proton decay from values computed in the SU_6 invariant limit.

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One of the ingredients of proton decay calculations in the quark model is the two body correlation function at short distances; i.e. the probability amplitude to find two quarks at the same point. This quantity is not relevant to many experimental observables in hadron spectroscopy. Therefore the simple SU_6 wavefunctions which have been successful in fitting other data may not be so good for baryon decay. SU_6 breaking induced by hyperfine interactions between quark pairs suggests that this may be the case. The hyperfine interaction attracts quark pairs in the spin singlet state and repels pairs in the spin triplet state, and has been shown[1] to be responsible for the charge radius of the neutron which is zero in the SU_6 symmetry limit.[2] In baryon decay this effect enhances the probability that a ud pair coupled to spin zero will be at the same point and reduces the probability that a ud or a uu pair coupled to spin one will be at the same point. Since the hyperfine interaction is short ranged its influence on proton decay which is due to a short range coupling should be even more striking than on the neutron charge radius, which is a global property. An estimate of this effect may be made using wavefunctions which have been proposed to explain the neutron charge radius. It is also possible to make some nearly model independent observations.

Let us first make the extreme assumption that the proton wave function for pairs at short distances is completely dominated by the hyperfine interaction, and that the probability of finding a ud pair in the spin singlet state at the same point is much greater than the probability of finding a spin triplet pair.

We therefore neglect the contribution from spin triplet pairs. Note that this singlet model also gives the $\Delta I = 1/2$ rule for two-quark transitions in nonleptonic hyperon decays since two quarks in the spin singlet state are required by Fermi statistics of colored quarks to be in a 3^* of flavor SU_3 if they are in a 3^* of color SU_3.

We therefore consider the contribution to proton decay from the decay of a spin triplet ud pair into a lepton ℓ and an antiquark $\bar{q}$

$$u + (ud)_{S=0} \rightarrow u + (\bar{q}\ell)_{J=0} \rightarrow (\bar{q}u) + \ell \; . \qquad (1)$$

where the $(\bar{q}\ell)$ system is required to be in a state of angular momentum $J = 0$ by angular momentum conservation under the assumption that the additional u quark is a spectator and does not exchange angular momentum during the decay. The $\bar{q}u$ state produced in this way has no spin correlation between the u and $\bar{q}$ and is a mixture of 3/4 triplet and 1/4 singlet state. Furthermore, since this mechanism can only produce $(\bar{q}u)$ and not $(\bar{q}d)$ pairs, the μ^+K^0 final state is forbidden and the isoscalar and isovector mesons which are linear combinations of $(\bar{u}u)$ and $(\bar{d}d)$ must be produced with equal probability. The $\bar{\nu}_\mu K^+$ final state, forbidden in static SU_6 models[7,8] by a cancelation between $S = 0$ and $S = 1$ amplitudes, is allowed in the singlet model. Thus in the singlet model

$$\bar{\Gamma}(p \rightarrow e^+\rho^0):\bar{\Gamma}(p \rightarrow e^+\omega):\bar{\Gamma}(p \rightarrow e^+\pi^0):\bar{\Gamma}(p \rightarrow e^+\eta_n) = 3:3:1:1 \; , \qquad (2a)$$

$$\bar{\Gamma}(p \rightarrow \mu^+K^0) = 0 \qquad (2b)$$

$$\bar{\Gamma}(p \rightarrow \bar{\nu}_\mu K^+) \neq 0 \qquad (2c)$$

where $\bar{\Gamma}$ denotes the reduced width with the kinematic, phase space and form factors taken out, and η_n denotes the nonstrange part of the η and η' wave functions. With Isgur's mixing angle[3] making both the η and η' 50-50 mixtures of strange and non-strange pairs,

$$\bar{\Gamma}(p \to e^+\eta) = \bar{\Gamma}(p \to e^+\eta') = (1/2)\bar{\Gamma}(p \to e^+\eta_n) \tag{3}$$

Table I lists the predicted branching ratios for proton decay in the singlet model, including phase space factors and using SU_5 to relate charged and neutral lepton decay modes. These results are very different from those of calculations[4-8] which neglect the SU_6 breaking effects. The predictions (2b) and (2c) actually reverse the SU_6 selection rule. Since SU_6 breaking is appreciable but not as extreme as taken in this model, one would expect the branching ratios for a realistic wave function to be somewhere in between the values (2) and those calculated with SU_6. However, the large difference between the two indicates that moderate SU_6 breaking can have large effects on branching ratios.

As an illustration of a more elaborate and possibly more realistic calculation we discuss here the case of harmonic oscillator wavefunctions which is known to give a reasonable estimate for the charge radius of the neutron.[1,9] The SU_6 breaking caused by the hyperfine interaction can then be described as an admixing of a $[70, 0^+]$ component into the ground state $[56, 0^+]$, with a small amplitude ($\sin\theta = -1/4$), as discussed elsewhere.[9] Therefore we take

$$|N_\uparrow\rangle = \cos\theta\,|N_\uparrow^2 S_S\rangle + \sin\theta\,|N_\uparrow^2 S_M\rangle \tag{4}$$

$$|N_\uparrow^2 S_S\rangle \equiv |\text{'}56\text{'}\rangle = \Psi_{00}^S \frac{1}{\sqrt{2}} (\phi_N^\lambda \chi_\uparrow^\lambda + \phi_N^\rho \chi^\rho)$$

$$|N_\uparrow^2 S_M\rangle \equiv |\text{'}70\text{'}\rangle = \frac{1}{2}\left[\Psi_{00}^\lambda \phi_N^\rho \chi_\uparrow^\rho + \Psi^\rho \phi^\lambda \chi^\rho + \Psi^\rho \phi^\rho \chi^\lambda - \Psi^\lambda \phi^\lambda \chi^\lambda\right]$$

$$\sin\theta \simeq -1/4$$

$$\Psi^{s}_{00} = (\alpha^3/\pi^{3/2})\,e^{-\frac{1}{2}\alpha^2(\rho^2+\lambda^2)}$$

$$\Psi^{\lambda}_{00} = \frac{1}{\sqrt{3}}\,(\alpha^5/\pi^{3/2})(\rho^2-\lambda^2)\,e^{-\frac{1}{2}\alpha^2(\rho^2+\lambda^2)}$$

$$\Psi^{\rho}_{00} = \frac{1}{\sqrt{3}}\,(\alpha^5/\pi^{3/2})\,2\vec{\lambda}\cdot\vec{\rho}\;e^{-\frac{1}{2}\alpha^2(\rho^2+\lambda^2)}$$

where the symbols have their usual[9] meaning. We take the wavefunction (4) and compute the expectation values of $\delta^3(\vec{r}_{u_\uparrow u_\uparrow})$, $\delta^3(\vec{r}_{u_\uparrow u_\downarrow})$ etc., where $\vec{r}_{u_\uparrow u_\uparrow}$ is the distance between a pair of u quarks whose spin is pointing in the plus z direction. In this way we obtain, <u>to first order in $(\sin\theta)$</u>, in a proton of spin up:

$$\begin{aligned}\langle\delta^3(\vec{r}_{u_\uparrow u_\uparrow})\rangle_{p_\uparrow} &= \langle\delta^3(\vec{r}_{u_\uparrow u_\downarrow})\rangle_{p_\uparrow} = \langle\delta^3(\vec{r}_{u_\uparrow d_\uparrow})\rangle_{p_\uparrow}\\ &= \text{const} * \left[1-\sqrt{2}\tan\theta\,\frac{\langle\Psi^{s}_{00}|\delta^3(\vec{\rho})|\Psi^{\lambda}_{00}\rangle}{\langle\Psi^{s}_{00}|\delta^3(\vec{\rho})|\Psi^{s}_{00}\rangle}\right] \qquad (5)\\ &= \text{const} * \left[1-\frac{\sqrt{6}}{8}\right]\end{aligned}$$

where the last line corresponds to the choice[9] $\tan\theta \simeq -\frac{1}{4}$ and the use of the harmonic oscillator wavefunctions displayed above to evaluate the two expectation values. The constant in front is immaterial for our purposes here; it is the value of $|\Psi(0)|^2$ in the SU_6 limit, and is the same in (5), (6), and (7). Similarly we obtain

$$\langle\delta^3(\vec{r}_{u_\uparrow d_\downarrow})\rangle_{p_\uparrow} = \text{const} * \left[1+\frac{\sqrt{6}}{16}\right] \qquad (6)$$

and

$$\langle \delta^3(\vec{r}_{u_\downarrow d_\uparrow}) \rangle_{p_\uparrow} = \text{const} * \left[1 + \frac{\sqrt{6}}{4}\right] \tag{7}$$

For protons with spin down, the right hand sides of (6) and (7) are interchanged. The physical interpretation of the correction factors due to SU_6 breaking is simple if we recall[1] that a pair of quarks in a spin triplet state has an additional repulsion due to hyperfine interactions while in a spin singlet state there is attraction. The symmetric quark model correlates the uu pairs with spin triplet and the ud pairs with a mixture of singlet and triplet. We now recall that these expectation values $\langle \delta^3(r_{ij}) \rangle$ stand for $|\Psi(0)|^2$ which appears in the <u>rate</u> (or width) of the annihilation; therefore when computing <u>amplitudes</u> we need the square root of the expectation values which we approximate, for ease of computation, to first order in the SU_6 breaking, for example:

$$\sqrt{\langle \delta^3(\vec{r}_{u_\uparrow d_\downarrow}) \rangle_{p_\uparrow}} = \text{const}' * \left[1 + \left(\frac{\sqrt{6}}{32}\right)\right] \tag{8}$$

Finally we compound the amplitudes for the annihilation of a specific quark pair in the proton, e.g. $u_\uparrow d_\downarrow$, with the orbital amplitude (8) corresponding to that pair. The resulting amplitudes for the case of proton decay with <u>right-handed positron emission</u>, are given below:

$$P^\uparrow(\bar{u}_\uparrow u_\downarrow) = \frac{2}{\sqrt{6}}\left(1 - \frac{\sqrt{6}}{16}\right) \tag{9a}$$

$$P^\uparrow(\bar{u}_\downarrow u_\uparrow) = \frac{2}{\sqrt{6}}\left(1 + \frac{\sqrt{6}}{8}\right) \tag{9b}$$

$$P^\uparrow(\bar{d}_\uparrow d_\downarrow) = \frac{4}{\sqrt{6}}\left(1 - \frac{\sqrt{6}}{16}\right) \tag{9c}$$

$$P^{\uparrow}(\bar{d}_{\downarrow}d_{\uparrow}) = -\frac{2}{\sqrt{6}}\left(1-\frac{\sqrt{6}}{16}\right) \tag{9d}$$

$$P^{\downarrow}(\bar{d}_{\downarrow}d_{\downarrow}) = \frac{2}{\sqrt{6}}\left(1-\frac{\sqrt{6}}{16}\right) \tag{9e}$$

$$P^{\downarrow}(\bar{u}_{\downarrow}u_{\downarrow}) = \frac{4}{\sqrt{6}}\left(1+\frac{\sqrt{6}}{32}\right) \tag{9f}$$

where we use the notation $P^{\uparrow}(\bar{d}_{\downarrow}d_{\uparrow})$ for the amplitude to emit in the positive y-direction a right-handed positron from a proton of spin component $+\frac{1}{2}$ and leave behind a color singlet $\bar{d}_{\downarrow}d_{\uparrow}$ state. The brackets display the effect of SU_6 breaking and have been obtained by multiplying with the appropriate factor from Eq. (8) [or its analogues] the SU_6 invariant amplitude. For example, in (9c) the final state is $\bar{d}_{\uparrow}d_{\downarrow}$; therefore the annihilating pair is $u_{\uparrow}u_{\uparrow}$ and the appropriate factor [from (2) after taking a square root] is $1-(\sqrt{6}/16)$. The SU_6 invariant amplitudes have been computed here in a static model with both the initial quarks and the final antiquark at rest.[7,8]

From the basic amplitudes (6) we can compute the amplitudes to decay into the physical states ρ^0, ω^0, π^0 by taking appropriate linear combinations. For example,

$$\begin{aligned} A(P^{\uparrow}\to e_R^+\omega_0^0) &= \frac{1}{2}\left[P^{\uparrow}(\bar{u}_{\uparrow}u_{\downarrow}) + P^{\uparrow}(\bar{u}_{\downarrow}u_{\uparrow}) + P^{\uparrow}(\bar{d}_{\uparrow}d_{\downarrow}) + P^{\uparrow}(\bar{d}_{\downarrow}d_{\uparrow})\right] \\ &= 3/\sqrt{6} \end{aligned} \tag{10}$$

$$A(P^{\uparrow}\to e_R^+\rho_0^0) = \frac{1}{\sqrt{6}} + \frac{2}{16} \tag{11}$$

$$A(P^{\uparrow}\to e_R^+\pi^0) = \frac{3}{\sqrt{6}} - \frac{3}{8} \tag{12}$$

$$A(P^{\downarrow}\to e_R^+\omega_{-1}^0) = \sqrt{3} \tag{13}$$

$$A(P^{\downarrow}\to e_R^+\rho_{-1}^0) = \frac{1}{\sqrt{3}} + \frac{2\sqrt{2}}{16} \tag{14}$$

where in each equation the first term is the SU_6 invariant contribution and the second the effect of SU_6 breaking. We note that the amplitudes for producing ω^0 are unaffected to first order, the amplitudes for ρ^0 are increased from the SU_6 limit and the amplitude for π^0 production is decreased from its SU_6 limit.

These results are best understood if we look at ratios of amplitudes squared, which up to phase space factors equal the ratios of rates. We thus compute for the ratio of ω to ρ

$$\frac{\Gamma(\omega)}{\Gamma(\rho)} = \frac{4.50}{0.85} \cong 5.3 \tag{15}$$

whereas in the SU_6 limit one finds[7,8] a ratio of 9:1. Similarly for the ratio of π to ρ we find

$$\frac{\Gamma(p \to e_R^+\pi)}{\Gamma(p \to e_R^+\rho)} = \frac{.722}{.850} \approx .85 \tag{16}$$

whereas in the SU_6 limit this ratio has the value 3, as can be checked directly from the amplitudes (8), (9), (11). The deviations from the SU_6 limit found here are all such as to bring the branching ratios towards the extreme limits indicated in (2).

The effects of these deviations from SU(6) on the total proton lifetime have not been considered and are expected to be small. This can be understood by inspection of eqs. (9-14). The total proton lifetime depends upon the sum of the squares of the six amplitudes (9), whereas the partial decay width into a particular channel; e.g. eq. (10) is very sensitive to the relative magnitudes and phases of these amplitudes. The sum of the squares of the amplitudes (9) is not appreciably affected by the contributions of the correction factors in the brackets which are small to begin with and tend to cancel one another in the sum. The branching ratios (15) and (16) are changed by factors of order 3. Since changes in the models and calculations of the weak interactions can easily change estimates of the proton lifetime by several orders of magnitude,[4-10] it seems safe to neglect the effects of the SU(6) breaking on the lifetime at this stage.

We conclude that the deviations from SU_6 discussed here are sufficiently important to warrant their incorporation in all cases if one wants to make realistic predictions of branching ratios for proton decay. It should be mentioned that in addition to the corrections discussed here one should also consider to first order in SU_6 breaking possible corrections to matrix elements which result directly from the wavefunctions[4] when computing the matrix elements of decay.

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TABLE I

Branching Ratios for Proton Decay in SU_5 + Singlet Model

Channel	Singlet Model Factor	SU_5 Factor	Phase Space Factor	Overall Factor	Branching Factor
$e^+\rho^0$	3	5	0.2	3	18.8%
$e^+\omega^0$	3	5	0.2	3	18.8%
$e^+\pi^0$	1	5	1.0	5	31.3%
$e^+\eta^0$	1	5	0.58	1.45	9.1%
$\nu\rho^+$	3	2	0.2	1.2	7.5%
$\nu\pi^+$	1	2	1.0	2	12.5%
μ^+K^0	0	–	–	0	0
$\bar{\nu}K^+$	¼	2	0.65	0.32	2.0%