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ON THE POSSIBLE APPLICATIONS OF THE STEERING OF CHARGED PARTICLES BY BENT SINGLE CRYSTALS, INCLUDING THE POSSIBILITY OF SEPARATED CHARM PARTICLE BEAMS*

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ABSTRACT

This note reviews some aspects of the steering of charged particles using the phenomenon of channeling in bent single crystals. The basic steering mechanism, first proposed by Tsyganov, is considered. Crystal angular and spatial acceptance, deflection, dechanneling and radiation damage are discussed. Examples of bent crystal applications are presented including Tevatron extraction, focusing, and the possibility of charm particle separated beams.

INTRODUCTION

When a charged particle moves in a direction close to the axes or planes in a single crystal its behavior is drastically altered. This remarkable behavior, channeling[1], has been widely studied since its discovery in the early sixties in sputtering experiments and Monte Carlo calculations[2]. Channeling behavior has now been observed at energies up to 250 GeV in an experiment carried out at Fermilab[3].

For a positive particle traveling close to a crystal plane the channeling process consists of a series of gentle transverse repulsive pushes from the positive nuclear centers in the planes so that the particle effectively glides back and forth between the two planes as it moves almost parallel to them. Since the particle remains mostly in regions of low charge density there is relatively less energy loss compared to a random direction. Similarly, scattering is reduced. There is less radiation damage since close collisions with nuclei are diminished.

The physical picture for channeling near an axis is more complicated but the same general behavior occurs.

The channeling process is characterized by a critical angle beyond which the particle tends to leave the channel. The axial critical angle at high energies is

$$\psi_c^a = \alpha \sqrt{\frac{4Ze^2}{pcd}} ,$$

where α is a constant close to 1 that depends on crystal parameters, Z is the atomic charge of the target nucleus, d is the atomic spacing along the lattice and p is the particle momentum. For the remainder of the article, is taken as 1 for convenience. The experimental determination and the "correct" theoretical definition of the critical angle leave room for this flexibility. At high energy this critical angle is small, typically on the order of 40 microradians for a 250 GeV particle channeled along the <110> axis in germanium. Characteristically planar channeling angles are about 1/3 the axial angles.

Particles in channeled trajectories can scatter out into random directions. This can occur because of scattering off of electrons, impurities or lattice imperfections. This process is characterized by a dechanneling length. There has not been a great deal of work on theoretical estimates of dechanneling lengths, but dechanneling lengths are expected to increase in proportion to the energy and this behavior is observed. At multihundred GeV energies, a typical dechanneling length is on the order of ten centimeters in germanium.

Negative particles also exhibit channeling effects. The theory of negative particle channeling is not yet well understood. As a result it is not so easy to predict what will happen in any given situation.

What would happen to a particle in a channeled trajectory if the crystal through which it moves is bent? In 1976 Tsyganov[4] reviewed this question. In essence, Tsyganov reasoned that a particle moving in a planar channel is moving in a transverse potential well. If the energy in transverse motion exceeds the height of the well, corresponding to exceeding the critical angle, the particle moves out of the channeled trajectory. Bending the crystal is equivalent to introducing a rising centrifugal potential, lowering one side of the well and raising the other. Alternatively it can be thought of as moving the particle equilibrium position from the bottom of the well. In effect this means that the critical angle diminishes and the dechanneling length shortens. Tsyganov estimated the critical radius of bending to be

$$R_T = \frac{E}{eE_c},$$

where E is the total energy of the particle, and e is the charge of the particle. E_c is the interatomic electrical field intensity (in the case of planar channeling for positive particles) at the distance from the plane of the crystal lattice where the trajectory of the particle no longer remains stable due to its interaction with individual atoms. Tsyganov uses the field value at the Thomas-Fermi screening distance to evaluate E_c . (See, for instance, Gemmell, formulas 2.22 and 2.26.) Typical values of R_T and E_c are given in Table I for 100 GeV/c particles in a (110) plane. The value of E_c is equal to $0.5 * 10^{10}$ v/cm for the (111) plane of a silicon crystal. The planar critical angle at 8.4 GeV for the (111) plane in silicon is about 45 microradians, the Tsyganov radius is equal to 2 cm.

This Tsyganov radius can be related to an equivalent magnetic field for a relativistic particle by noting that the radius of curvature of a particle in a magnetic field is $R = p/.03B$ (where $E = p$ (in GeV/c), B is in kilogauss and R in meters). A value for the equivalent magnetic field can be found if the magnetic deflection radius is equated to the Tsyganov radius. This field value is independent of energy. Table 1 contains values for silicon, germanium, and tungsten. The equivalent magnetic field for the (110) plane in tungsten is 160 megagauss.

Recently Tsyganov and his collaborators in a joint USSR-USA collaboration have studied channeling in bent crystals in an experiment at Dubna[5]. The experiments were performed in an 8.4 GeV proton beam using thin silicon crystals. The upstream ends of the individual crystals were prepared as totally depleted semiconductor detectors so that energy loss in a crystal could be measured. It was possible to align the crystals by looking for small energy losses, a technique pioneered at high energy by an Aarhus-CERN group[6]. Particle trajectories entering and leaving the crystal were monitored with a series of drift chambers with extremely good spatial resolution.

In a typical run a silicon crystal 2 cm long, 1 cm high and 1 to 2 mm thick in the direction of bending was aligned so that the proton beam was transmitted along the (111) plane of the crystal. The proton beam that was used had a wide enough range of angular divergences so that the crystal plane was fully illuminated. Channeled particles were selected by cutting on low energy losses in the intrinsic detector built into the front end of the crystal.

As the crystal was bent the channeled fraction of the beam followed the direction of the downstream end of the crystal. The crystals were bent up to 26 milliradians. The first experimental limitation occurred when the planar channeling distribution moved off of the downstream drift chamber. That chamber was then moved closer to the crystal. Eventually the bending program was limited when a crystal broke at a bending angle slightly greater than 26 milliradians. This was equivalent to a bending radius of 38 centimeters.

It should be noted that the radius of bend achieved at 26 milliradians was far larger than the critical Tsyganov radius. Thus under more propitious circumstances the beam probably could have been deflected through much larger angles to better understand the limits of bending and the Tsyganov radius. It is important that experiments be carried out at higher energies or with thinner crystals so that the limiting cases can be explored.

A complete analysis has not yet been made of the dechanneling length in the Dubna experiment and how that length depended on the amount of bending. The following observations can be made at this point. First there is relatively little dechanneling when the silicon crystal is not bent. Measurements on other silicon crystals of 1 and 2 cm thickness have shown that the phenomenological dechanneling length is on the order of a centimeter. Secondly, differential dechanneling can be seen as the crystal is bent. There is a peak in the unbent direction of the crystal due, in part, to dechanneling in the upstream portion. A second peak appears at the angle of the downstream end of the crystal due to transmission. Between the two peaks there is a small but noticeable spill as particles are lost in transmission. The integrated area of the spill approximately corresponds to the expected dechanneling loss. It should be noted that this type of observation of differential losses may in itself represent a new approach to measuring dechanneling losses.

There appears to be little, if any, increase in dechanneling as the crystal is bent. This observation is based on both differential and integral observations of dechanneling. However, it should be noted that the bending radii were far larger than the critical Tsyganov radius of bending so that the crystal potential suffered only a relatively small change. As a result, it seems quite reasonable that there was little increase in dechanneling.

If a planar channel in an unbent crystal is illuminated with a beam that has an angular divergence much less than the critical angle, the only loss in transmission is due to dechanneling. Typical transmissions of channeled particles were on the order of fifty percent. The transmission at 26 mrad was around 30% so that thee was only a moderate change as a result of the bending.

One bending run was made with the crystal aligned on an axis. This run has not yet been analyzed in detail. There appear to have been bending effects but the behavior is not entirely clear. Analysis is continuing on that run. One possibility that requires investigation is that particles following a bent axis would leak out into planes.

Deflection of a beam of charged particles using channeling in a bent crystal is a distinctly different process than bending with a magnet. Up to some momentum related to the Tsyganov radius a channeled particle near the plane direction should be deflected independent of its momentum. The lower momenta particles will be lost more rapidly than the higher momenta ones since their dechanneling length will be shorter. At high momentum near the critical radius, more divergent particles will be lost. The overall band pass is shown schematically in Fig. 1. Channeling with a bent crystal, then is a wide momentum band pass method of deflecting charged particles. This could be a distinct advantage for beam systems that require the deflection of a large range of momenta. On the other hand the same feature means that such a system would have no momentum selection capability. Note that magnets in many secondary beam systems at accelerators serve both the requirements of deflection and momentum selection.

There is as yet no information on the behavior of negative particles in a bent crystal. No measurements were made on negative particles in the Dubna run. The behavior of negative particles in straight single crystals is quite different than positive particles. Axial channeling effects have been observed for negative pions from 2 - 35 GeV/c[3,7]. Planar effects have been observed for electrons at 800 KeV[8]. It is distinctly possible that there would be no deflection of negative particles by a bent crystal. This is, of course, in contrast to magnetic deflection.

In the following sections the factors that effect application of bent crystals to deflecting charged beams will be discussed in more detail. In addition, some examples will be given of possible applications.

FACTORS AFFECTING CHARGE PARTICLE BENDING WITH SINGLE CRYSTALS

Materials:

Every facet of the channeling phenomenon is affected by the properties of the single crystal that is used. The critical angle, dechanneling length, Young's modulus, susceptibility to radiation damage, melting point, and temperature effects related to lattice vibrations all depend on the choice of the crystal. Technology for growing good single crystals is much better developed for some materials than others. Most channeling at high energy has been done with germanium and silicon crystals so that the crystal could also serve as a detector and thereby use low energy losses to align the crystal. The art of growing perfect germanium and silicon crystals has been highly developed over a period of many years so that extremely good crystals are available free of mosaic defects. This good feature of silicon and germanium crystals has led to their use at high energy. For many materials, the average mosaic defect structure in a crystal can easily exceed the critical angle at high energies.

In the following paragraphs the behavior of various properties with materials will be discussed. It will become apparent that a high Z material is quite desirable from any standpoint. In the last several years a technique has been perfected (electron beam zone refining) and another is evolving (low temperature epitaxy) for growing small, perfect tungsten crystals[9]. Tungsten also has a high melting point and relatively small lattice vibrations. Tungsten crystals, then, may turn out to be the ideal crystals for high energy applications. Of course, for any application all the relevant properties need to be considered.

Angular Acceptance

The angular acceptance of a channel in one angular direction in a single crystal is determined by the critical angle, that is

$$\theta_a \alpha \psi_c = K/\sqrt{p},$$

where K is determined by the crystal parameters and which plane or axis is being used for channeling. The actual acceptance may not be set exactly by the critical angle but the functional behavior with crystal parameters and momentum will be determined by it. It should be noted once more, that planar critical angles are smaller than axial critical angles, characteristically one third as large. As a result the acceptance transverse to a plane will be one third as large as the angular acceptance of an axis. (The axial and planar critical angles at 100 GeV for silicon, germanium, and tungsten are given in Table I.)

For planes, however, the angular acceptance parallel to the plane is infinite. This means that for typical applications the beam angular divergence would be arranged to be small in the plane of bending, that is transverse to the crystal planes, and large in the crystal planes (see Fig. 2).

The critical angle decreases inversely with the square root of particle momentum. For the axial case, it increases with the square root of the atomic number. Therefore, high atomic number materials give larger critical angles and more acceptance.

It is interesting to compare these angular acceptances to accelerator and secondary beam emittances. The horizontal emittance inside the Fermilab main ring at 400 GeV is 0.1 mm mrad. For a β of 70m this gives a spot of half width 1.5 mm and half angular divergence of 20 microradians. The vertical angular emittance is roughly comparable. The emittance scales inversely as the momentum. Clearly under some conditions it may be quite possible to capture all this beam inside the critical angle of a reasonably sized crystal.

The angular divergence of a typical secondary beam at 400 GeV at Fermilab is 250 microradians. These emittances are characteristically set by the magnet aperture. In a broad sense secondary beam angular divergences also scale inversely as the momentum since the particle production distribution is somewhat matched to the beam acceptance and focal strengths of the quadrupoles. For contrast the critical planar channeling angle in the (110) plane of tungsten is estimated to be seventeen microradians at 400 GeV (based on scaling the critical angle from 8 GeV).

The critical angle scales inversely as the square root of the momentum, while the beam angular acceptance scales inversely with the momentum. As the momentum of a beam increases the relative acceptance of the channel increases as the square root of the momentum, so that the possibility of adapting crystals to real beam applications also becomes more significant.

Spatial acceptance: The spatial acceptance of a crystal depends on its transverse dimensions. The amount of bending will also depend on how thick the crystal is in the bending direction. The thinner the crystal the more it can be bent. The total spatial acceptance will then be equal to the height times the thickness in the bending direction.

It is possible to obtain very large single crystals. Ingots with one dimension on the order of a meter are often produced. The essential limit is cost. It is also possible to cut very thin crystals, so thin that the crystals can be wrapped around a pencil.

For the initial channeling runs at multihundred GeV energies, there was some concern about the possibility of dislocations and mosaic structure spoiling the channeling effects. So far, there has been no problem with this in either the Dubna or the Fermilab experiments.

Modern dislocation free single crystals are significantly stronger than the single crystals used in the forties and fifties for bent crystal spectrometers. Those crystals contained substantial mosaic structure and required total containment to produce the necessary curvature. For dislocation free single crystals, a technique of bending with three point suspension holding the crystal loosely is more satisfactory and can produce larger bends. The crystal establishes its own curvature. For uniform curvature it is necessary to keep the crystal thickness uniform. It is also important to keep the surface free of irregularities.

The maximum elastic deflection of a crystal of length L and thickness t supported at three points is

$$y = \frac{S_m L^2}{6E_y t},$$

where S_m is the maximum stress (typically read off the first maximum of a stress-strain curve) and E_y is Young's modulus, the initial slope of a stress-strain curve[10]. For an oriented single crystal these numbers can be derived from the stress tensor[11]. For cubic crystals the actual mechanical properties show only weak anisotropic effects. All of these properties are sensitive to the quality of the crystal and the temperature. Information on germanium and silicon is contained in articles by Patel and Chaudhuri[12] and Alexander and Haasen[13]. More recent information on mechanical properties is contained in papers by Sumino and Kalman and Weissmann[14]. The situation with a metal is somewhat different. The material can be bent well beyond the elastic limit by introducing dislocations. These dislocations will result in mosaic structure. The impact of this mosaic structure on channeling in any bent geometry would have to be evaluated. Under some circumstances it is possible that it might not cause significant beam degradation.

The maximum deflection can be linked to a minimum radius of curvature by noting that $y = L^2/8R$. The minimum radius of curvature is then

$$R = \frac{3}{4} \frac{E_y t}{S_m}.$$

Clearly the minimum radius is directly related to thickness.

In the Dubna experiment a number of silicon crystals were bent to the breaking point. A relatively simple three point holding device was employed and no special surface treatment was used for the crystals. The experimenters found empirically that the breaking angle was inversely proportional to the thickness of the crystal in the bending directions, that is:

$$\theta_b = a/t,$$

where $a = 13$ milliradian-mm, and t is the thickness in mm. This is functionally equivalent to the dependence on t of the minimum radius of curvature.

As noted earlier, E_y and S_m are dependent on temperature properties and the orientation of the crystal. For characteristic silicon values[12] of $E_y = 13 \times 10^{10}$ Nt/m^2 and $S_m = 1 \times 10^8$ Nt/m^2, this gives a minimum bending radius of 90 cm for a thickness of 1 mm. This is quite close to the value of 76 cm found by Tsyganov from observation for the (111) plane. Other minimum bending radii are given in Table I. The tungsten radius is an estimated value for elastic deformation.

The beam in the Fermilab main ring is several mm high and several mm wide at 400 GeV. This could be easily contained in the transverse dimensions of one crystal bent up to an angle of the order of ten milliradians. The size of a secondary beam depends on the location in the beam line. Characteristically, the spot at a focus is 1 mm wide. At parallel sections the beam width is set by the magnet aperture and can be 5 to 10 cm wide. Thus there would be no difficulty in accepting the full beam at a focus (where however, the beam angular divergence is largest). However in a parallel section it would be impossible to fully contain an ordinary beam in one crystal. However, a large spatial acceptance in the direction of a bend could possibly be achieved by lumping together several thin laminnae of crystals.

Deflection: As noted earlier, Tsyganov derives a critical bending radius that is proportional to the momentum at high energies. At 400 GeV/c for the (111) plane in silicon, the Tsyganov radius is 90 cm. From the result of the Dubna experiment it should be possible to bend a crystal 2 cm long and 1 mm thick in the direction of bending on a radius of 90 cm to give a bending angle of 22 mrad. The effective bending power of the crystal corresponds to an extraordinary 15 megagauss magnetic field. Since the dechanneling length is on the order of centimeters at this energy such a bend could be achieved with a minimum of beam degradation. A high Z crystal such as tungsten has an even smaller Tsyganov radius and an even higher equivalent magnetic field. It should be stressed that bending in the Dubna experiment was roughly a factor of 20 away from the maximum set by the critical radius so that it is not yet clear that the critical radius could even be closely approached. Nevertheless even in the Dubna experiment the equivalent magnetic field was 800 kilogauss.

Clearly the limit for maximum deflection is linked to the requirement that the deflection angle be less than the angle of bend at which the crystal breaks. For a given length of crystal exactly at its breaking angle there will be some maximum particle momentum that will be channeled. Thus there is a linear relationship between momentum and deflection. If the crystal is thinner and the breaking angle doubles the maximum momentum that can be channeled will be halved so that there is a clear and inverse relationship between maximum deflection angle and maximum momentum that is transmitted. The situation is reminiscent of a charged particle bending in a magnetic field. The difference is that every momentum below the maximum momentum will also be channeled in the crystal.

The amount of deflection should also be roughly proportional to the length of the crystal. (End constraints would modify an exact relationship.) The maximum useful length of the crystal would probably be set by dechanneling considerations. Thus the maximum deflection that can be achieved for a given set of circumstances will also depend on the dechanneling length.

Dechanneling: Theoretical and experimental understanding of dechanneling is not particularly well developed. Theoretical calculations give only a rough guide to the magnitude of dechanneling lengths and there are few experimental measurements of dechanneling lengths. For bending applications, however, it is necessary to have some appreciation of how dechanneling behaves with crystal parameters and some feeling for the magnitude of the effect.

In a perfect crystal, axial dechanneling arises because of scatterings from the electrons away from the nuclear axes and from near collisions with the nuclear centers. Nuclear collisions are sensitive to lattice vibrations and increase at higher temperatures. Crystal imperfections such as interstitial atoms and discloations also contribute to dechanneling.

The basic dechanneling process has been studied for the axial case by Bonderup et al[15]. It is a diffusion mechanism in which on-axis trajectories are first scattered by electronic collisions and then increasingly scattered by nuclear collisions as the particles approach the critical angle. The nuclear scattering eventually always dominates the process. Bonderup et al. define electron and nuclear diffusion lengths:

$$Z_e = \frac{E}{2\pi e^2 L_e \, Nd}$$

$$Z_n = \frac{3}{8\,\rho 2} \quad \frac{E\ (.885a_o)^2}{\pi Nd\ Z^{5/3} e^2}$$

where E is the relativistic total energy of the particle, d is the lattice spacing along the string, $L_e = \log\,(2mv^2/I)$, N is the atomic density of the material, a_o is the Bohr radius, ρ^2 is the average square of the lattice vibrations and Z is the atomic number of the material.

Clearly these diffusion lengths go as the energy of the particle so that dechanneling lengths increase with energy. The nuclear lengths depend on temperature because the lattice vibrations increase with temperature and thus decrease the diffusion length. Table I gives these lengths for the <110> axis in silicon, germanium, and tungsten at room temperature. For germanium and tungsten the nuclear length is shorter. Note that these lengths should not be added in quadrature; the diffusion process is more complicated. Rather the nuclear length should be used to gain some sense of scale. For some crystals it should be possible to increase the axial dechanneling lengths by cooling the crystals.

In the Fermilab experiment, only one thickness of germanium crystal was used. However, it is possible to define a phenomenological dechanneling length and verify that it scales properly with energy. If the definition of the length is based on finding the fraction of the particles initially inside the critical angle that scatter to greater than three times the critical angle, the phenomenological length at 100 GeV along the <110> axis is 13.5 cm. This is approximately twice the nuclear diffusion length at liquid nitrogen temperature calculated from the Bonderup et al. formula.

Particles, then, are retained near channels for lengths on the order of, but somewhat longer than, the Bonderup et al. diffusion lengths.

Based on this, it seems reasonable to consider the use of crystals up to tens of cm long at multihundred GeV energies.

Planar dechanneling lengths are expected to behave in somewhat the same way. Planar dechanneling is principally due to electronic processes (see for instance, Gemmell[1], p.154). This implies that there will be little temperature dependence in the planar dechanneling lengths. Low energy measurements around 1 MeV suggest that planar dechanneling lengths for silicon are thirty to forty percent as large as axial dechanneling lengths[16].

<u>Crystal Integrity and Radiation Damage:</u> For bent crystals to serve a useful role in manipulating charged particle beams it is important that they stand up under both bending and the radiation damage induced in the crystals by charged particle beams.

The Dubna experiment already gives a baseline for trouble free operation in connection with both of these possibilities.

In that experiment the crystals returned to their original shapes when they were released from the bending fixture. This indicates that relatively few dislocations were introduced into the crystals. In addition, since there was no significant indication of additional dechanneling when the crystals were bent almost to the breaking point, it is plausible to assume that any crystal imperfections that were introduced were not significant enough to cause appreciable dechanneling.

Of course this does not mean that dislocation problems will not occur for other types of crystals or for bends on a smaller radius of curvature. Note, however, that the 40 cm bending radius used at 8 GeV (for a 0.5 mm silicon crystal) is approaching the Tsyganov critical radius at 250 GeV. Thus for multihundred GeV experiments using silicon it is already clear that the introduction of dislocations from bending probably constitutes no problem.

The Dubna experiment also gives a limit on the integrated beam intensity that will cause no appreciable radiation damage. It is estimated that the crystals were typically exposed to between 10^9 and 10^{10} protons/cm^2. In the Fermilab experiment the crystal was probably exposed to substantially more than 10^{10} particles/cm^2. In neither case was there any indication of crystal degradation with exposure either from degraded energy resolution or broadening of channeling distributions.

However these limits are rather low. For most applications it would be desirable to have substantially larger integrated doses.

Radiation damage studies of crystals indicate that a dose of 10^{17} protons/cm^2 on a randomly oriented single crystal causes significant damage. How the damage affects the crystal and the magnitude of the effects constitutes an entire field of study. For situations in which crystals will be exposed to intense integrated fluxes, it is helpful to review the relevant literature[17].

An oriented crystal in a well collimated beam (that is a beam with all particles inside the critical angle) should be able to survive higher doses. Further, there is an approach that could be taken to ameliorate radiation damage to a crystal. If the crystal is gently heated, some radiation damage sites are annealed out. This opens the possibility of heating an operating crystal in a beam either with an external heat source or with the beam itself to continuously anneal out the damage.

This process has already worked in some applications. For example[18] the damage to a randomly oriented, thin nickel crystal from a continuous beam of 10^2 nanoamps/mm^2 (equivalent to $6 * 10^{13}$ particles/cm^2-sec) has been annealed out by heating the crystal to a temperature of 550^{o} . This number was for a continuous irradiation. In accelerator applications a duty factor of ten to twenty percent would be introduced. The thermal cycling would probably not present a problem except for really high doses.

These doses should be compared to average beam intensities at high energy accelerators:

Extracted beam at 400 GeV/c: 10^{12} - 10^{13} particles/sec.

Extracted beam at the Tevatron: 10^{12} particles/sec.

High intensity pion beam: 10^{9} particles/sec.

Of course a crystal can't melt and still serve as a single crystal. However it appears that it may be possible to expose crystals to rather significant beams and still have them behave usefully. It is certainly also possible to consider changing crystals rather frequently.

BENT CRYSTALS AS EXTRACTION ELEMENTS

In modern accelerators the circulating beam is usually extracted from the machine and transported to external experimental areas. Characteristically this is done by incorporating an electrostatic or magnetic septum in the accelerator. At extraction the beam is expanded by a resonant perturbation so that most of the beam hops across the active septum element and passes through the septum. Typical electrostatic septum elements are on the order of a tenth of a millimeter thick and the beam can be expanded several millimeters in one turn around the machine. The inefficiency, or beam loss due to extraction, is approximately determined by the ratio of the septum thickness divided by the resonant jump. One of the desiridata for extraction is high efficiency. The septum must also provide enough of a change in direction to the beam so that it clears the accelerator. Electrostatic septa can only give a small kick. In practice they often deflect on to magnetic septa.

For the Fermilab main ring extraction is no particular problem. There are long straight sections and it is only necessary to deflect the beam four milliradians to clear the accelerator. At some other machines less space is available and extraction is more difficult. At Serpukhov it is necessary to deflect the beam more than 100 milliradians.

A bent single crystal can easily provide just such angular deflections. The physical element of the septum then becomes the equivalent of two atomic lattice vibrations thick. The beam can be expanded on to the crystal much more gradually so that the requirements on the resonant extraction elements are reduced.

There will be losses on a crystal due to dechanneling and direct collision losses on the nuclear centers. Collision losses on the nuclear centers set a minimum efficiency roughly equal to the interaction probability for the crystal times the square of the ratio of the lattice vibations divided by the interatomic spacing. In practical cases this number can be rather small. Dechanneling decreases linearly with energy. It will be shown that dechanneling losses can be made quite competitive with conventional septum operation.

In order to use a bent crystal as an extraction element it is necessary to match the acceptance of the crystal, essentially two times the critical angle for channeling, with the angular emittance of the accelerator at extraction. This becomes increasingly easier for high energies and higher atomic numbers for the crystal.

Bent crystal extraction elements could have an additional virtue besides providing relatively large angular deflections with weaker resonant extraction elements. For single crystal extraction the external beam phase space could be substantially reduced. The effective width of the beam would be reduced since the average orbit jump could be much smaller. The angular emittance at extraction would be preserved to within a factor of two or three. This feature of a smaller phase space could be exploited in many ways throughout the entire experimental process.

Design Considerations: Design of a bent crystal extraction system would involve several factors: the phase space acceptance of the bent crystal, the beam losses in the crystal, and the radius of curvature or angular deflection needed for the system.

The portion of the extraction emittance of the beam that exceeds twice the critical angle will not be collected in channeling trajectories and will be lost. Relatively larger critical angles occur with higher atomic numbers. For example the planar critical angle of tungsten is twice that of silicon. The examples below show that these larger critical angles are desirable. If the raw extracted beam angular emittance exceeds twice the critical angle it might still be possible to match the emittance and acceptance by using well established accelerator matching techniques. For the cases illustrated below there are no special matching requirements.

Beam losses from dechanneling can be estimated in terms of a phenomenological dechanneling length. For the phenomenological length defined earlier the function lost to dechanneling is

$$f = (1 - e^{-z/\lambda_p}),$$

where λ_p is the phenomenological dechanneling length for particles scattering beyond three times the critical angle and z is the crystal thickness. Of course accepting scatters out to two or three times the critical angle does produce some phase space dilution. For low Z materials the planar dechanneling length is shorter than the axial length. However, since planar dechanneling is mostly due to electrons there is little atomic number dependence. There is also no temperature dependence. In the examples that follow the planar dechanneling length is assumed to scale with atomic number as the Bonderup et al. electron diffusion length in Table I. In addition it is assumed that the phenomenological dechanneling length is twice the diffusion length based on the axial case in germanium at Fermilab energies.

With the dechanneling length known and a desired loss rate specified the crystal length is established. That length of crystal can be bent down to the larger of two radii - the Tsyganov critical radius or the radius for breaking. In practice the radius for breaking can be changed by using a different thickness of crystal. The mimimum thickness for a crystal will also depend on the minimum rate at which the beam can be expanded during extraction. The bending strategy is successful if the bend is sufficient for the extraction application. In any case the breaking limit becomes less and less of a problem with higher energies since the Tsyganov critical radius is relatively larger while the breaking radius remains constant.

Extraction at Serpukhov: As an example consider beam extraction at the 76 GeV proton accelerator at Serpukhov. The straight sections are about 5 m long and the beam must be deflected about 0.5 m vertically to clear extraction obstacles[19]. This requires a deflection of 0.1 radians. The emittance of the circulating beam is 1 mm-mrad. For a betatron wave length of 40 m the beam half width is 6 mm, resulting in an angular emittance of 300 microradians. Experience at Fermilab indicates that the angular emittance at resonant extraction might be one third of this or 100 microradians. Two times the planar critical angle of tungsten at this energy is about 80 microradians. Thus there is little difficulty in matching the acceptance of the tungsten channel to the internal emittance at extraction. Silicon, with the smaller planar critical angle, would present more of a challenge.

The circulating beam at Serpukhov is quoted at $2 * 10^{12}$ protons. Experience at Fermilab indicates that loss rates on the order of $1 * 10^{11}$ protons are acceptable. Assume that the phenomenological dechanneling length is twice the electron diffusion length given in Table I scaled down to 76 GeV, that is 16 cm. An acceptable loss rate due to dechanneling can be achieved for a crystal 0.7 cm long. To obtain a deflection of 0.1 radians this must be bent on a radius of 7 cm. A tungsten crystal on the order of 0.1 mm thick should be able to be bent to that radius without introducing dislocations. As noted earlier it is not completely clear how badly dislocations would effect the performance. One tenth of a millimeter might be rather thin from the standpoint of controlling the extraction expansion. If necessary it might be possible to increase the thickness by laminating.

This radius is four times the Tsyganov radius so that it would seem to be quite practical from the standpoint of maintaining the beam in channeled trajectories. Thus this scheme seems quite practical from the standpoint of emittance matching, beam losses and the Tsyganov radius. The crystal must be rather thin to accommodate the bending.

Extraction at Fermilab at 1 TeV:

At Fermilab the straight sections are 50 m long. The beam must be deflected about 20 cm to clear downstream beam elements in the Doubler, corresponding to an angular deflection of 4 milliradians. The circulating beam emittance in the Doubler is expected to approximate that in the present main ring so that it will be 0.1 π mm-mrad. For a betatron wavelength of 70 m the beam half width is 2.5 mm corresponding to an angular emittance of 40 microradians. However at extraction this is expected to be $\pm$ 15 microradians. The planar critical angle at 1 TeV in tungsten is 11 microradians so the crystal is roughly matched to the angular emittance.

The crystal length can be set by requiring 1% losses for a circulating beam of 10^{13} protons. The phenomenological de-channeling length at 1 TeV is 216 cm. Therefore the crystal can be 2 cm long. The radius of bending for this length will be 500 cm for a 4 mrad deflection. A crystal up to 5 millimeters thick could be bent on this radius. This Tsyganov critical radius at 1 TeV is 20 cm, so that the actual bend is far from approaching the limit. Indeed this bend is well within the limit of the bending experiments that have already been carried out at Dubna.

Radiation damage of the crystal is a consideration. It might be desirable to spread the beam vertically to minimize the damage per unit area. If 10^{17} particles/cm^2 constitutes an indicator of radiation damage the crystal would last for 10^4 pulses or 1 week of Doubler operation. The crystal is 0.4

collision lengths long and six radiation lengths long. Tungsten targets do suffer heating effects because of the radiation length consideration. Note however that nuclear interactions producing the neutral pion, gamma ray-conversion electron chain will be minimized because of the aligned geometry. Ionization heating from the beam alone is expected to be reduced by a factor of two from that in an amorphous target. For a ten second spill it corresponds to a 4 watt heat load.

Certainly radiation damage and target heating would have to be carefully considered in any design. Perhaps the matching problems for a low atomic number crystal would be more than made up for by other considerations. In any case, at 1 TeV the possibilities of a bent crystal extraction channel would seem to merit some consideration.

BENT CRYSTALS AS FOCUSING ELEMENTS

A stack of planar crystals bent to different radii can serve as a focusing element in one dimension. Figure 3 shows a schematic illustration of a system in which a solid single crystal aligned with a plane along the beam axis is sliced to make, in effect, a comb. The teeth of the comb are then pressed together perpendicular to the aligned plane. If a parallel beam with angular dispersion no greater than the planar critical angle is incident on the array it will be focused to a point. There will be a transverse aberration at the focus equivalent to the width of one of the teeth of the comb. There is a loss for non-focused parts of the beam approximately equal to the ratio of the space between the teeth of the comb over the width of a tooth.

This system only affects the beam in one dimension. The perpendicular direction will be unaffected. A second element placed after the first one could be used to focus the perpendicular direction. The fact that the beam has already been focused in one direction by the first element will not disturb the operation of the second crystal since planes are being used to bend the beam and motion along the plane will be unaffected.

The focal condition for such a system is

$$f\theta \cong y$$

where y is the transverse distance from the center of the lens and f is the focal length. For the mth tooth a distance $y_m = (m-1)\ (\ \delta\ + w_o)$ from the center of the lens the necessary deflection to focus is

$$d_m = \frac{L}{2f}\,(m - 1)(\delta + \omega_o),$$

where δ is the separation between the teeth, ω_o is the width of the tooth, and L is the length of the tooth. The deflection from the interstitials between the teeth is just $d_m = (m-1)\,\delta$ Substituting gives the focal length of the lens:

$$f = L\,\frac{(\delta + \omega_o)}{2\delta} \approx \frac{L\omega_o}{2\delta}\,.$$

As an example, for a tooth to interstial ratio of 1000 and a tooth length of 1 cm the focal length would be 5 m. Since there is a relatively small interstial space the loss to non-focused trajectories would be 0.1%. As noted earlier a crystal length of 1 cm at multihundred GeV energies, would produce relatively little dechanneling. In some cases it might not even be necessary to physically remove an interstitial region. One end of the crystal might be compressed or the crystal could be differentially cooled to change the lattice spacing at one end.

A *SEPARATED BEAM FOR SHORT-LIVED PARTICLES

The ability to dramatically deflect charged particles in a short distance offered by bent channeling crystals raises an intriguing possibility, the construction of separated particle beams of very short lived particles such as charm particles. Charm particle lifetimes are so short that it has been extremely difficult to detect charm particle tracks, let alone separate charm particles from other long lived particles. If charm particles could be separated in direction from other particles, it might be possible to observe many of the missing decay modes of the charm particles. For example, the current Particle Data Group tables list decay modes for only 15% of the branching ratio of the D^+ because trigger systems frequently rely on detecting a kaon or an electron.

The bends that are potentially possible with single crystals are significant enough that such a possibility should exist at sufficiently high energies. Figure 4 shows a schematic of a possible separation arrangement. The single crystal consists of a short straight portion called the production region, followed by a bent section with a bent plane called the decay region. Consider the case where the production and decay angles of the high momentum charm particles are much smaller than the bend angle of the decay region. (It will be shown later that this must be approximately possible at energies that are sufficiently large.) In that case some of the particles will be captured in the angular acceptance of a crystal planar channel - plus-minus the critical angle parallel to the plane of bending and infinity parallel to the crystal plane. This is shown in figure 5. All of the charged particles, both short lived and long lived ones, that are trapped in the crystal channel will be deflected away from the production cone. Charm particles will decay, effectively flying out of the channel like water spinning off a wet tire. Of course it is necessary that the angle that at least one of the decay products makes with the local crystal axis be greater than the critical angle. Long lived particles will continue on out the end of the channel. In effect three regions in angle space will be created - the depleted production cone diffused somewhat by the decay of short lived particles, the band of decays for the charm particles decaying in the bending channel and the line of long lived particles coming out the end.

A cleaner arrangement could be constructed in which there was a second straight section followed by another bend. The length of the production section and first bend would have to be somewhat shorter than the charm particle lifetime in the laboratory. The second straight section would be several times the charm particle lifetime. The final bend would be through a large angle. This arrangement is shown in figure 6. The charm particles are separated by the first bend and then all decay in the same direction, so that three collimated regions of particles are created rather than two spots and a band.

Of course some production would occur downstream of the production straight section. After the crystal had curved with the plane outside of the production cone no more short lived particles would be captured into the crystal channel. Thus the production downstream of the separation region only acts as a background producing mechanism.

This type of arrangement would permit the study of charm particle decays separated from other particles. By controlling the nature of the first bend it might also be possible to measure charm particle lifetimes.

To see that this is a real possibility it is necesaary to consider three sets of angles - the production and decay angles of the short lived particles - here grouped together as θ_D , the channeling critical angle θ_C and the average bend angle θ_B , for some particle with a finite lifetime.

Charm particle production distributions are not yet well understood experimentally. Further these distributions will vary depending on the production mechanism. Neutrino charm particle production may now be the best understood but because of low rates would be inappropriate for this approach. Charm particle production is generally considered to come from a gluon-gluon interaction. For photoproduction the requirements of color dressing mean that less momentum will be transferred at the lower vertex. In effect that means that there will be more forward peaking in the production distribution so that photoproduction might be a more favorable place to try this technique. For 100 GeV charm particle production it is expected that the production distribution would begin to fall at angles on the order of 20 mrad.

The x distribution may go something like $(1-x)^5$ so that relatively few charm mesons will be produced at high energy. This is somewhat unfavorable since high momentum charm particles will have longer lifetimes in the laboratory and narrower production and decay distributions. The lower momentum particles contribute to the background.

Decay distributions for charm particles have transverse momenta averaging 0.5 GeV/c so that for a 100 GeV particle the decay distribution will be characterized by an angular width around 5 milliradians. Both the production for particles with x near 1 and decay widths scale as $\theta_D \propto 1/p$.

Next consider the critical angle for the channel. For 100 GeV particles in tungsten in a planar channel the critical angle is expected to be about 34 microradians. As noted earlier this angle scales as $\theta_C \propto 1/\sqrt{p}$. Since the angular acceptance is infinite along the plane the channel acceptance for particles with x near 1 will be on the order of θ_C/θ_D and will scale as $\sqrt{p}$. At 100 GeV it would be approximately $0.3 * 10^{-2}$ so that 0.3% of the charm particles would be accepted into the channel. Note that materials other than tungsten might be found with larger critical angles. (However tungsten is one of the more favorable cases.)

At some momentum θ_C will exceed θ_D and the acceptance will be 100%. This momentum is large by present standards (on the order of 1000 TeV).

Finally it is necessary to consider the bending angle. If the crystal is bent at the Tsyganov radius and if the length of the first bending section is set equal to the mean particle lifetime, $<\tau>$, the bending angle of the first section is

$$\theta_B = \frac{eE_c <\tau>}{mc} .$$

This angle is independent of the charm particle momentum. For tungsten with a Tsyganov radius of 2.1 cm and a D^+ lifetime of $8 * 10^{-13}$ sec.[20] θ_B = 611 milliradians.

Clearly even at 100 GeV this angle could be substantially larger than the angle of the production cone for high momentum particles. In practice a radius perhaps twice as large as the Tsyganov radius would be necessary to keep the channeling acceptance large. In addition the first bend would be made the equivalent of about half the lifetime so that the deflection angle would be 150 milliradians. This would produce a very clear separation of production and decay. For this process to be realized at any energy that could be obtained in the next decade, it would be necessary that deflections would have to approach the critical radius of bending.

A process called blocking occurs in channeling experiments. At low momentum this process makes it difficult to trap particles produced on nuclear centers into channeled trajectories. Charged particles produced at a nucleus and moving close to an atomic row or plane are strongly deflected by multiple coulomb scatters so that no outgoing particles appear along the crystal planes or axes. The nuclear density is especially high along these planes and axes so that the scattering is in effect supercoulomb scattering. Roughly the density in a plane increases by the ratio of the atomic spacing between the planes divided by the projected rms lattice vibration. For axes the density increase is the square of this ratio.

In the high energy régime (1 GeV and greater) there appears to be no real information on blocking. This is because the axes of the crystals have generally been oriented so near the beam direction that outgoing particle distributions have been dominated by donut effects, that is the tendency of particles on the order of a critical angle away from an axis tending to spiral around the axis in angle space. In effect, this leaves a hole along the axis. Both the Aarhus-CERN and Fermilab experiments indicate such a behavior but it cannot be considered as blocking. To study blocking at high energy it would be necessary to use outgoing produced particles at angles many times the critical angle from the beam scattered into the direction of the axis.

On the basis of standard channeling potential theory there is no reason to believe that blocking should not occur at high energy. Note however that multiple scattering angles scale as 1/p while critical angles scale as $1/\sqrt{p}$. This suggests that some aspect of the underlying mechanism could change with momentum but in a gradual way. Below 1 GeV it is essentially impossible for a particle to scatter out of the high charge nuclear region without exceeding the critical angle. Above 1 GeV the situation may change.

In any case blocking at high energy requires more investigation. If blocking were a problem particles produced on a nucleus in a bending crystal could not enter a channel directly. However particles not produced in a channel could ease into channels as their trajectory touched a bending circle of the crystal. Another way to overcome any consequence of blocking would be to use a small amorphous target directly upstream of the crystal or introduce interstitial impurities.

In summary, it is possible that blocking might not constitute any problem at high energy. If it does there are straightforward ways available to overcome the problem.

In view of the interesting possibility of substantially separating charm particle decays from other processes it seems important to now investigate bending down to the critical radius and perhaps study true blocking at multihundred GeV energies. A Fermilab-Albany group is now proposing to do this at Fermilab with existing apparatus. The logical next step would be to incorporate a bent single crystal in a large aperture spectrometer with a good charm trigger.

The possibilities discussed here have been developed by a numer of people. E. N. Tsyganov's ideas on bending are seminal. I have incorporated many suggestions and comments from C. R. Sun and W. Gibson. I would like to express my thanks to L. Teng and R. DesLattes for helpful guidance.

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TABLE I

Some Channeling Properties for 100 GeV Particles

	Silicon	Germanium	Tungsten
Z	14	32	74
A	28.09	72.59	183.85
ρ	2.35	5.33	19.3
Axial Critical Angle (microradian) <110>	46	68	138
Planar Critical Angle (microradian) (110)	16	20	34
Minimum Elastic Bending Radius (1 mm bar, cm, energy independent)	76 cm	∿76 cm	94 cm* (est.)
Critical Field ((110), v/cm)	0.61×10^{10}	1.28×10^{10}	4.73×10^{10}
Tsyganov Radius (planar, (110), cm)	16.3	7.8	2.1
Equivalent Magnetic Field (Megagauss, (110), plane)	20	43	160
Nuclear Diffusion Length (axial, <110>, cm) (room temperature)	10.3	2.2	1.7
Electronic Diffusion Length (axial, <110>, cm)	6.44	7.7	10.8

*Tungsten could probably be bent to a smaller radius if dislocations are introduced.

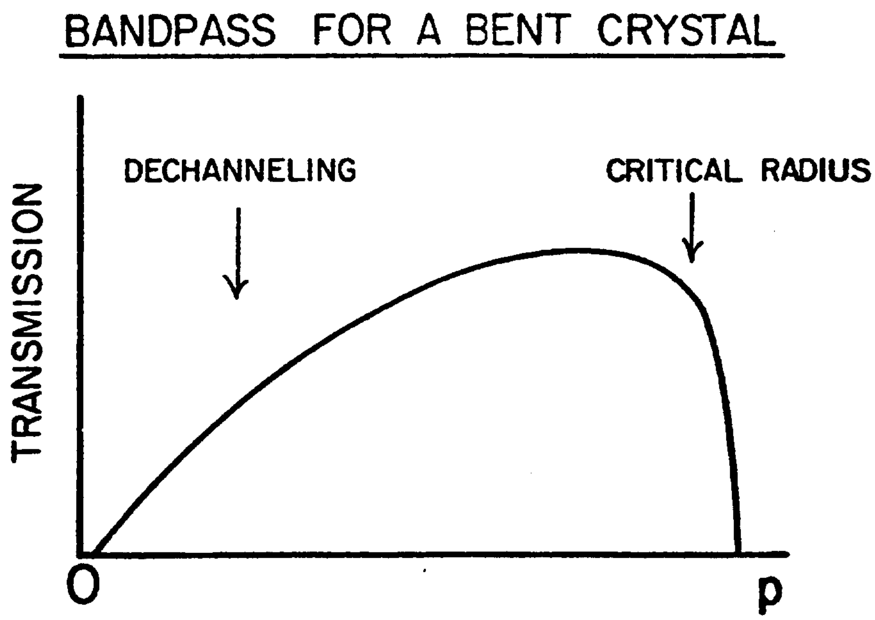
BANDPASS FOR A BENT CRYSTAL
TRANSMISSION
DECHANNELING
CRITICAL RADIUS
O
p

Fig. 1

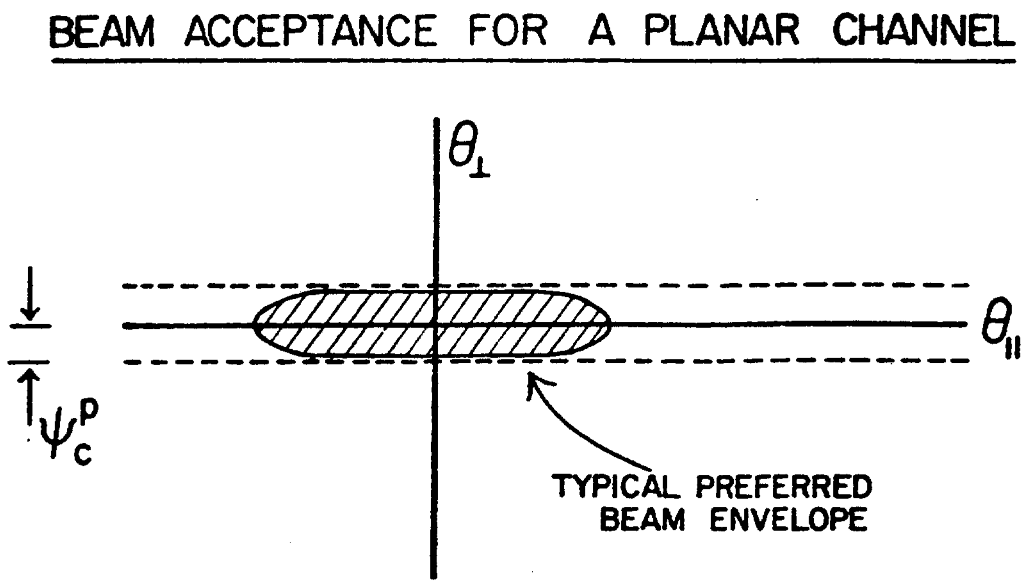
BEAM ACCEPTANCE FOR A PLANAR CHANNEL
$\theta_\perp$
$\theta_\parallel$
ψ_c^p
TYPICAL PREFERRED
BEAM ENVELOPE

Fig. 2

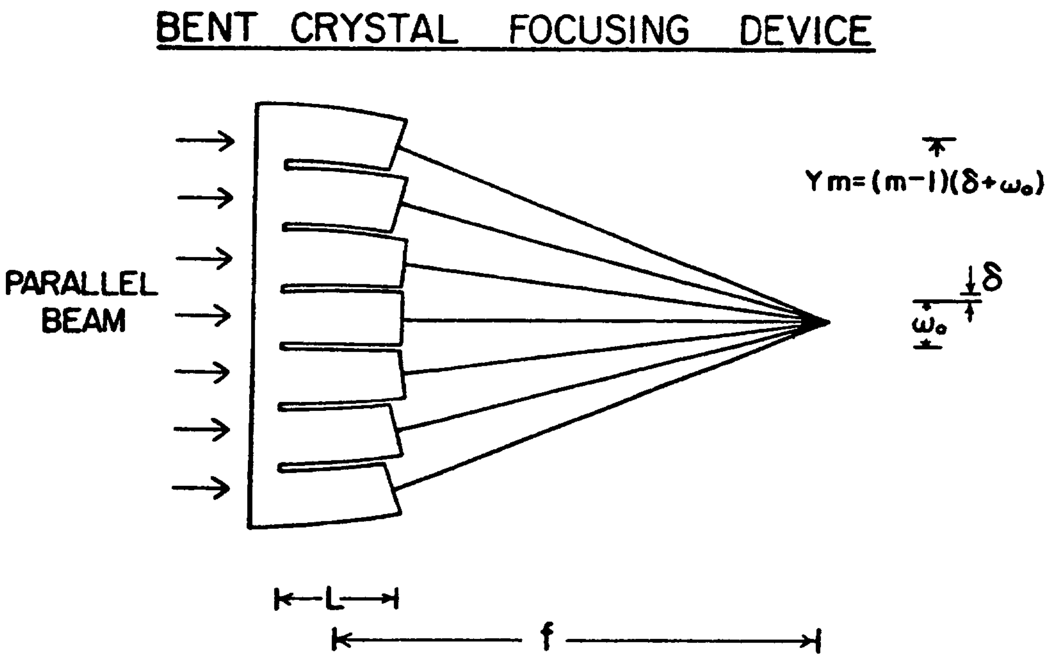

Fig. 3

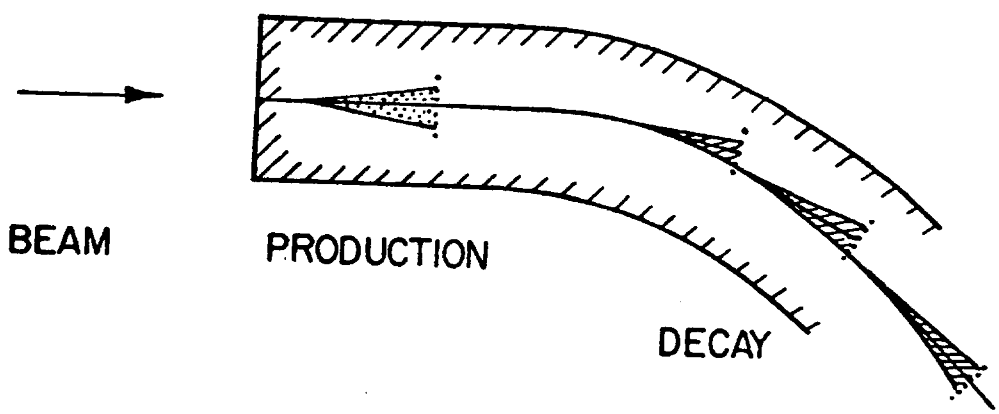

Fig. 4

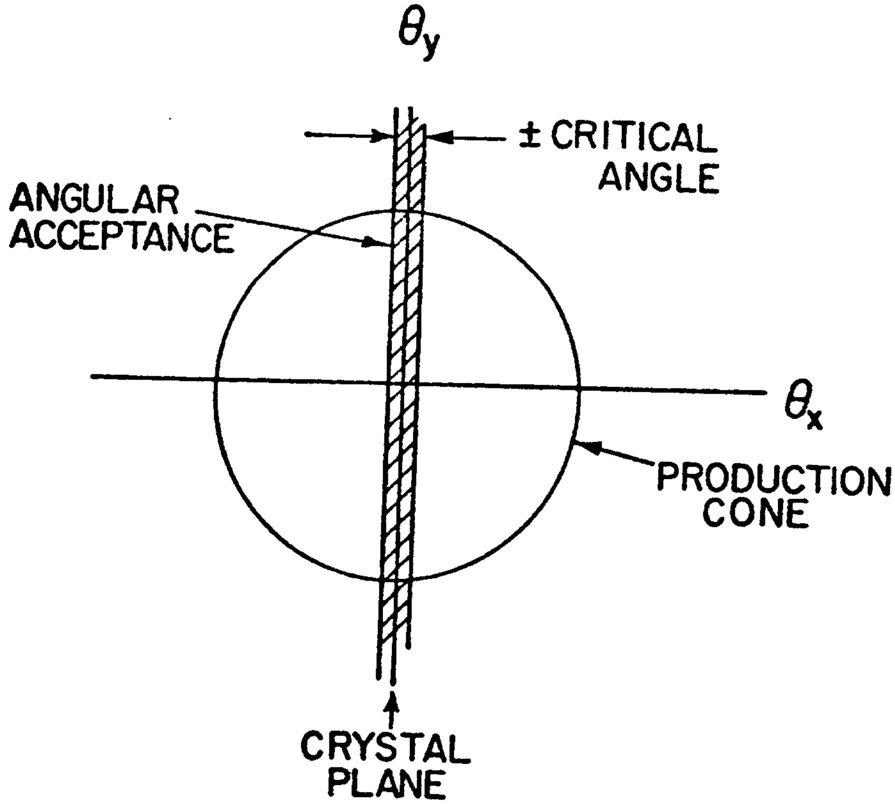
θ_y
± CRITICAL ANGLE
ANGULAR ACCEPTANCE
θ_x
PRODUCTION CONE
CRYSTAL PLANE

Fig. 5

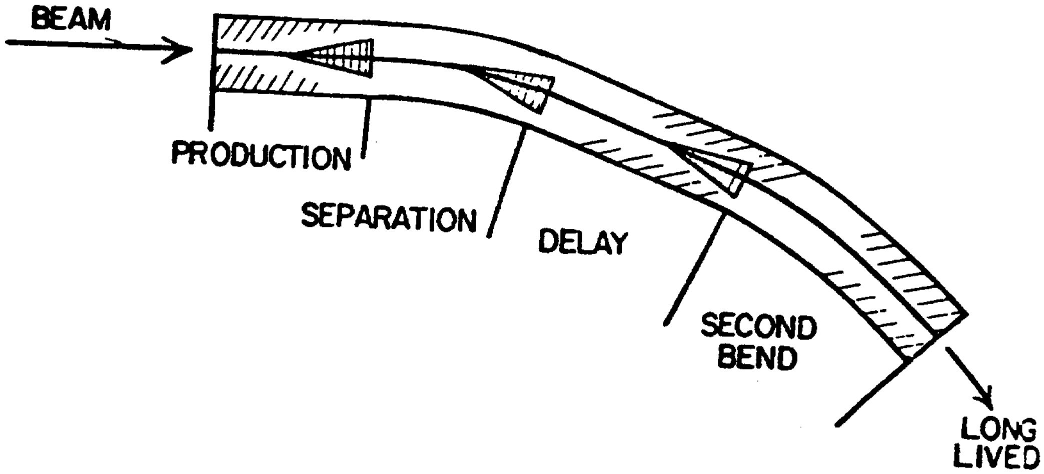

Fig. 6