Fermi National Accelerator Laboratory

FERMILAB-Pub-80/43-THY
May 1980

Higher Order QCD Corrections to Quarkonium Decays

ANDRZEJ J. BURAS
Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

ABSTRACT

We point out that the large next-to-leading-order QCD corrections to para-quarkonium decays found by Barbieri et al. are substantially suppressed when two-loop corrections to the renormalization group β function are included in the analysis. For the Υ family the full effect of the next-to-leading order corrections amounts to roughly 20%. The importance of two-loop β effects in other quarkonium decays, in exclusive processes, and in large $p_\perp$ inclusive processes is emphasized.

Operated ersities Research Association Inc. under contract with the United States Department of Energy

During the past two years great effort has been made by various groups to calculate next-to-leading-order QCD corrections to various processes.[1] In particular it has been found[2] that the next-to-leading-order corrections to paraquarkonium decays are so large that the perturbative calculations for charmonium systems and even for the Υ family cannot be trusted. In this note we want to point out that the large corrections found in ref. 2 are substantially suppressed when additional QCD corrections, not considered in ref. 2, are included in the analysis. These are the two-loop contributions to the β function[3] which generally tend to suppress the value of the QCD effective coupling constant $\alpha_s(Q^2)$, as compared to its leading order value. Since the perturbative expansion for paraquarkonium decays begins in order $\alpha_s^2(Q^2)$, the effect in question is much stronger than e.g., in deep-inelastic scattering where the perturbative expansion for the relevant moments of structure functions begins with $[\alpha_s(Q^2)]^p$, $(p \leq 1)$. We would like to emphasize that the effect of the two-loop contributions to the β function is a part of next-to-leading order corrections to quarkonium decays and should be included in the analysis together with the corrections calculated in ref. 2.

Let us first write the leading-order and next-to-leading-order expressions for paraquarkonium decays.

For paraquarkonium ($^1S_0$) decays we consider the ratio

$$P \equiv \frac{9e_Q^4\alpha_{EM}^2}{2} \frac{\Gamma(^1S_0 \to \text{hadrons})}{\Gamma(^1S_0 \to \gamma\gamma)} \tag{1}$$

where α_{EM} is the electromagnetic coupling constant and e_Q is the charge of the constituent quark. As discussed in ref. 2, and recently more generally in ref. 4, the ratio P should be insensitive to bound state effects and should be calculable in perturbation theory in $\alpha_s(Q^2)$ (in what follows we shall drop the index s). In the leading order we have[5]

$$P_{LO} = [\alpha_{LO}(Q^2)]^2 \tag{2}$$

where

$$\alpha_{LO}(Q^2) = \frac{12\pi}{25 \ln \frac{Q^2}{\Lambda_{LO}^2}} \quad , \tag{3}$$

and $Q^2 = 4m^2$, with m being the mass of the constituent quark. The numerical factors in Eq. (3) correspond to four effective flavors. Using next the results of ref. 2 and taking into account the two-loop corrections to $\alpha(Q^2)$, we obtain the following expression for P (through order $\alpha^3(Q^2)$)

$$P = [\alpha_i(Q^2)]^2 \left[1 + H_i \frac{\alpha_i(Q^2)}{\pi} \right] \tag{4}$$

$$= \left[\frac{12\pi}{25 \ln \frac{Q^2}{\Lambda_i^2}} \right]^2 \left[1 + 0.48 \frac{\bar{H}_i(Q^2)}{\ln \frac{Q^2}{\Lambda_i^2}} \right] \tag{5}$$

where

$$\bar{H}_i(Q^2) = H_i - 3.08 \ln\ln \frac{Q^2}{\Lambda_i^2} \quad . \tag{6}$$

In obtaining Eq. (5) we have used the following formula for $\alpha(Q^2)$:

$$\alpha_i(Q^2) = \frac{12\pi}{25 \ln \frac{Q^2}{\Lambda_i^2}} \left[1 - 0.74 \frac{\ln\ln \frac{Q^2}{\Lambda_i^2}}{\ln \frac{Q^2}{\Lambda_i^2}} \right] \tag{7}$$

with numerical factors corresponding to four flavors. The index i in Eqs. (4-7) distinguishes between various definitions of the effective coupling constant. Λ_i are the corresponding scale parameters. The negative terms involving $\ln\ln(Q^2/\Lambda^2)$ in Eqs. (6) and (7) come from the two-loop contribution to the β function.

The authors of ref. 2 have performed the calculation in the minimal subtraction scheme[6] (MS) for which they obtained the value $H_{MS} = 22.14$. Here we prefer to discuss the $\overline{MS}$ scheme[7] and the momentum subtraction scheme[8] (MOM) which turned out[1] to be useful in the study of higher order corrections in deep-inelastic scattering. Using the relations[7,8] between α_{MS}, $\alpha_{\overline{MS}}$ and α_{MOM}, we obtain

$$H_i = \begin{cases} 14.0 & i = \overline{MS} \\ 7.6 & i = MOM \end{cases} \quad . \tag{8}$$

Using next $\Lambda_{MOM} = 0.55$ GeV and $\Lambda_{\overline{MS}} = 0.30$ GeV (typical values for Λ_i to be discussed below) we find for the paraquarkonium in the Υ family ($Q^2 \approx 100$ GeV^2) the following values for $\bar{H}_i$:

$$\bar{H}_i = \begin{cases} 8.0 & i = \overline{MS} \\ 2.1 & i = MOM \end{cases} \quad . \tag{9}$$

We note that (a) H_i and $\bar{H}_i$ are sensitive to the choice of the scheme for α (this is due to the fact that the perturbative expansion for the ratio P begins in α^2), (b) the two-loop contribution to the β function substantially suppress the corrections found in ref. 2 (in the absence of these two-loop effects $\bar{H}_i = H_i$). In spite of the fact that the parameters H_i and $\bar{H}_i$ are sensitive to the choice of the scheme for α the ratio P should be scheme independent because it is a physical quantity. In order to check

it and at the same time to get a better estimate of the next-to-leading order corrections to the ratio P we shall extract the parameters Λ_{LO}, Λ_{MOM} and $\Lambda_{\overline{MS}}$ from deep-inelastic scattering for which the next-to-leading order corrections are known.[1,7,9] For the moments of the deep-inelastic non-singlet structure functions (e.g. F_2^{NS}) the analogs of Eqs. (2-5) are given as follows. In the leading order we have

$$M_n^{NS} \equiv \int_0^1 dx\, x^{n-2} F_2^{NS}(x, Q^2) = A_n^{NS} \left[\frac{12\pi}{25 \ln \frac{Q^2}{\bar{\Lambda}_{LO}^2}} \right]^{d_n^{NS}} , \tag{10}$$

whereas after the inclusion of next-to-leading-order effects M_n^{NS} is given by

$$M_n^{NS} = A_n^{NS} [\, \alpha_i(Q^2) \,]^{d_n^{NS}} \left[1 + R_{n,i}^{NS} \frac{\alpha_s(Q^2)}{4\pi} \right] \tag{11}$$

$$= A_n^{NS} \left[\frac{12\pi}{25 \ln \frac{Q^2}{\Lambda_i^2}} \right]^{d_n^{NS}} \left[1 + \frac{3\bar{R}_{n,i}^{NS}(Q^2)}{25 \ln \frac{Q^2}{\Lambda_i^2}} \right] . \tag{12}$$

In obtaining Eq. (12) the formula (7) has been used. The parameters d_n^{NS}, $R_{n,i}^{NS}$ and $\bar{R}_{n,i}^{NS}(Q^2)$ are known and can be found in ref. 1. A_n^{NS}, which are independent of Q^2, have to be found from the data. As in Eqs. (4-7) also here the index i distinguishes between various definitions of the effective coupling constant.

While in general there is no reason[9] for the scale parameters Λ_{LO} and $\bar{\Lambda}_{LO}$ (Eqs. 3 and 10) to be equal, it is justified to use the same values of Λ_i in Eqs. (5) and (12) if the parameters H_i, $\bar{H}_i$, $\bar{R}_{n,i}^{NS}$ and $\bar{R}_{n,i}^{NS}(Q^2)$ are calculated in the same renormalization scheme. Thus once the parameters Λ_i are extracted by means of

Eq. (12) from scaling violations in deep-inelastic scattering, the ratio P can be calculated.

Since the present deep-inelastic data are not accurate enough to determine very precisely the parameters Λ_i and because the numerical values of these parameters depend on the assumptions about the higher twist contributions,[11] we have performed instead the following simple exercise.

As "theoretical" data we have taken Eq. (12) for the $\overline{MS}$ scheme with $\Lambda_{\overline{MS}}$ in the range from 0.1 GeV to 0.3 GeV.[F1] Subsequently we have found the best values for Λ_{MOM} and $\overline{\Lambda}_{LO}$ from the fit to our theoretical data for n = 2,4,6 and 8 in the range $10\ GeV^2 \leq Q^2 \leq 200\ GeV^2$. The extracted values of Λ_{MOM} and $\overline{\Lambda}_{LO}$ as functions of $\Lambda_{\overline{MS}}$ are shown in Fig. 1a. The corresponding values of the effective coupling constants (Eqs. 3 and 7) are shown for $Q^2 = 100\ GeV^2$ in Fig. 1b. The latter value of Q^2 is relevant for paraquarkonium in the T family. We observe that $\alpha_{\overline{MS}}$ and α_{MOM} are smaller than α_{LO} by 30% and 20% respectively.

Having the values of the coupling constants and of the corresponding scale parameters at hand, we can now systematically discuss the next-to-leading-order corrections to paraquarkonium decays. In order to exhibit the effect of the second term in Eq. (6) we plot in Fig. 2 the values for the square brackets in Eqs. 4 and 5 as functions of $\Lambda_{\overline{MS}}$. The values of Λ_i shown in Fig. 1a have been used. We observe that in both the $\overline{MS}$ and MOM schemes the expansion in inverse powers of logarithms looks much better than the expansion in α. This is mainly due to the second term in Eq. (6). The final result for P and P_{LO} (Eqs. 2 and 5) with $\overline{\Lambda}_{LO} = \Lambda_{LO}$ and Λ_i given in Fig. 1a is shown in Fig. 3. Although there is <u>a priori</u> no reason that the same value of Λ should be used in the leading order expressions for paraquarkonium decays and deep-inelastic scattering, the difference between the

"LO" curve and $\overline{MS}$ and MOM curves in Fig. 3 indicates the size of next-to-leading-order effects in paraquarkonium decays as compared to next-to-leading-order effects in deep-inelastic scattering. The latter effects are known to be small at $Q^2 = 100\ GeV^2$. We observe that the predictions of the $\overline{MS}$ and MOM schemes are compatible with each other[F2] and differ from the leading-order prediction by 18% and 25% respectively for the largest values of $\Lambda_{\overline{MS}}$ considered. The corresponding differences are smaller for smaller values of $\Lambda_{\overline{MS}}$. We conclude that the next-to-leading-order corrections to paraquarkonium decays are much smaller than previously claimed.

In this note we have emphasized that the two-loop contributions to the β function may have an important effect for the final result in the case of quantities for which the perturbative expansion begins with a high power in α. Therefore we expect the two loop β effects discussed here to play an important part in the interpretation of higher order corrections to other quarkonia decays, in exclusive processes and in large $p_\perp$ processes. In particular we have checked that the large next-to-leading-order corrections to large $p_\perp$ hadron-hadron scattering found in ref. 14 are substantially suppressed when the two-loop effects discussed here are taken into account. We hope to discuss these and related questions elsewhere.

ACKNOWLEDGMENTS

I would like to thank W.A. Bardeen, L. Baulieu, A. Duncan, C.H. Llewellyn-Smith, A. Mueller, F.E. Paige, C. Quigg and N. Sakai for useful discussions.

FOOTNOTES AND REFERENCES

[F1] The value $\Lambda_{\overline{MS}} = 0.3$ GeV is consistent with the BEBC[12] and CDHS[13] data in the range $10\ \mathrm{GeV}^2 \leq Q^2 \leq 100\ \mathrm{GeV}^2$.

[F2] The 7%-10% difference between $\overline{MS}$ and MOM scheme predictions for P might indicate the size of α^4 corrections to Eq. (4).

[1] See reviews by A. Peterman, Phys. Rep. 53C (1979) 157; A.J. Buras, Rev. of Modern Physics, Vol. 52 (1980) 199; R.D. Field, CALT-68-739; J. Ellis and C.T. Sachrajda, CERN preprint TH.2782-CERN.

[2] R. Barbieri, E. d'Emilio, G. Curci and E. Remiddi, Nucl. Phys. B154 (1979) 535.

[3] W. Caswell, Phys. Rev. Lett. 33 (1974) 244; D.R.T. Jones, Nucl. Phys. B75 (1974) 531.

[4] A. Duncan and A.H. Mueller, CU-TP-175.

[5] T. Appelquist and H.D. Politzer, Phys. Rev. D12 (1975) 1404.

[6] G. 't Hooft, Nucl. Phys. B61 (1973) 455.

[7] W.A. Bardeen, A.J. Buras, D.W. Duke and T. Muta, Phys. Rev. D18 (1978) 3998.

[8] R. Barbieri, L. Caneschi, G. Curci and E. d'Emilio, Phys. Lett. 81B (1979) 207; W. Celemaster and R.J. Gonsalves, Phys. Rev. Lett. 42 (1979) 1435.

[9] E.G. Floratos, D.A. Ross, and C.T. Sachrajda, Nucl. Phys. B129 (1977) 66; Nucl. Phys. B152 (1979) 493; G. Curci, W. Furmanski, and R. Petronzio, CERN preprint TH.2815-CERN.

[10] M. Bace, Phys. Lett. 78B (1978) 132.

[11] L.F. Abbott and R.N. Barnett, SLAC-PUB-2325 (1979).

[12] P.C. Bosetti, et al., Nucl. Phys. B142 (1978) 1.

[13] J.G.H. de Groot, et al., Z. Phys. C1 (1979) 143.

[14] R.K. Ellis, M. Furman, H. Haber and I. Hinchliffe, LBL-10304.

FIGURE CAPTIONS

Fig. 1: Values of Λ_i and α_i ($Q^2 = 100\ \mathrm{GeV}^2$) as functions of $\Lambda_{\overline{MS}}$.

Fig. 2: The values for the square brackets in Eqs. 4 and 5 as functions of $\Lambda_{\overline{MS}}$ for $Q^2 = 100\ \mathrm{GeV}^2$.

Fig. 3: The ratio P calculated in the $\overline{MS}$ and MOM as functions of $\Lambda_{\overline{MS}}$ for $Q^2 = 100\ \mathrm{GeV}^2$. For comparison also the leading order (LO) prediction is shown.

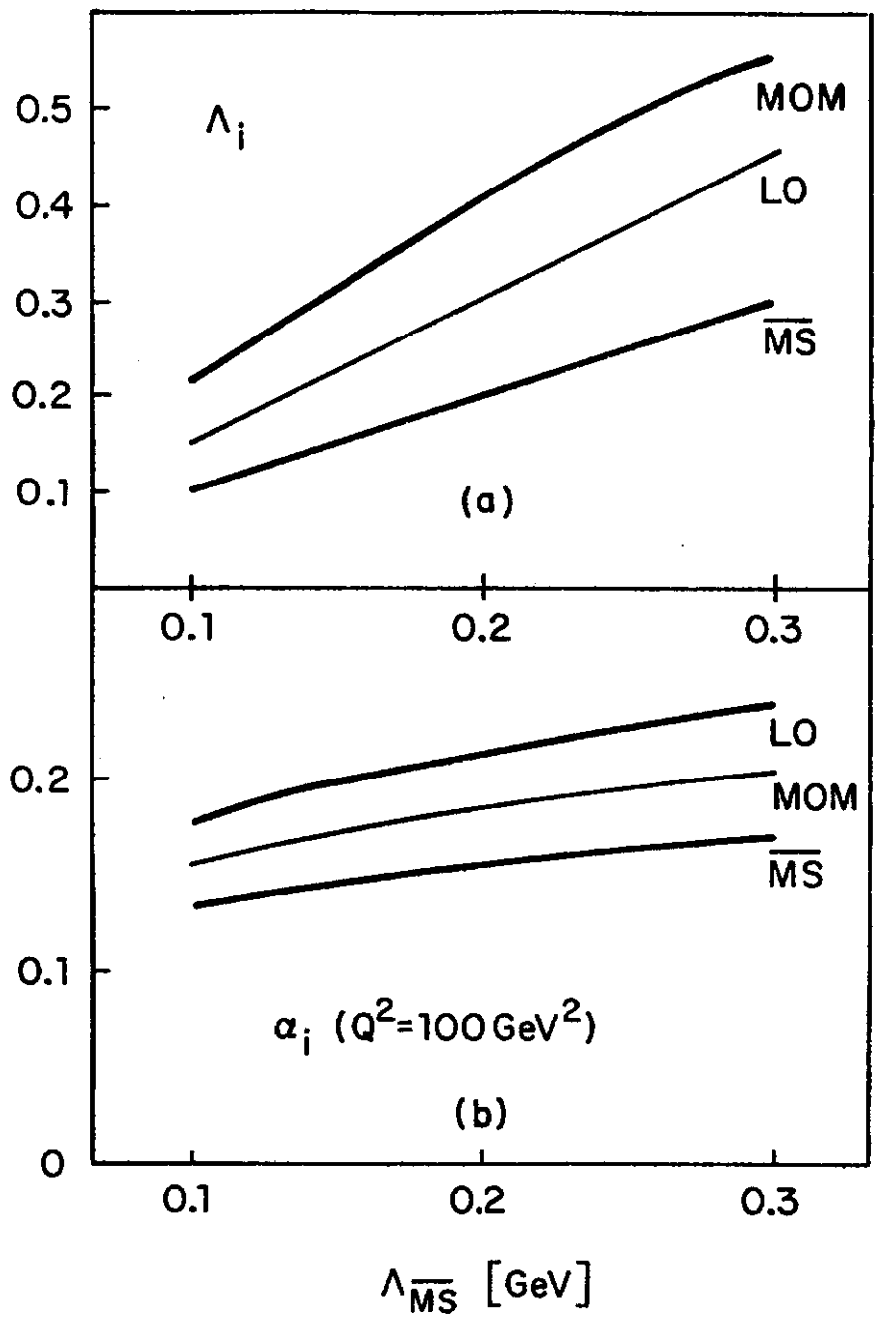
0.5
0.4
0.3
0.2
0.1
Λ_i
MOM
LO
$\overline{MS}$
(a)
0.1
0.2
0.3
0.2
0.1
0
LO
MOM
$\overline{MS}$
α_i (Q^2=100 GeV^2)
(b)
0.1
0.2
0.3
$\Lambda_{\overline{MS}}$ [GeV]

Fig. 1

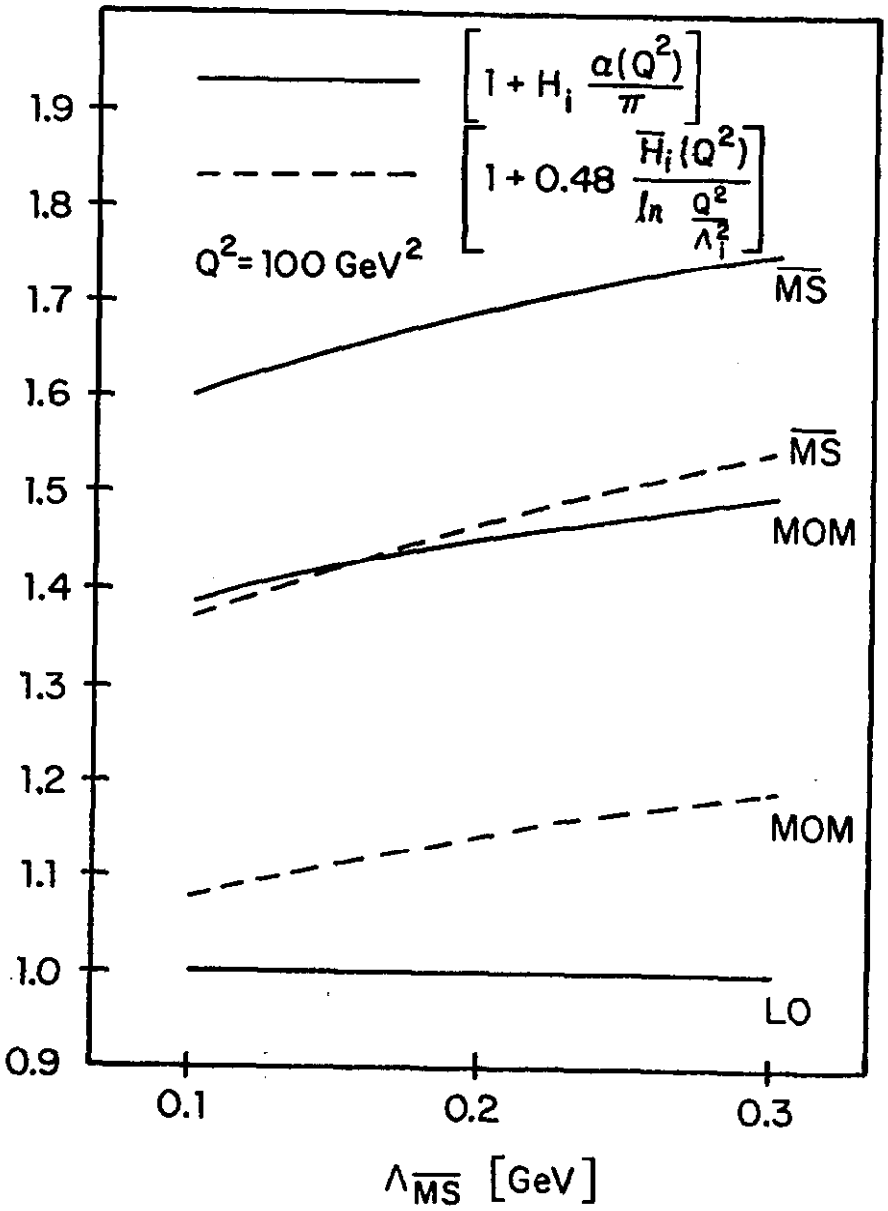
$\left[1 + H_i \frac{\alpha(Q^2)}{\pi}\right]$
$\left[1 + 0.48 \frac{\overline{H}_i(Q^2)}{\ln \frac{Q^2}{\Lambda_i^2}}\right]$
$Q^2 = 100\ \mathrm{GeV}^2$
$\overline{MS}$
$\overline{MS}$
MOM
MOM
LO
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.1
0.2
0.3
$\Lambda_{\overline{MS}}$ [GeV]

Fig. 2

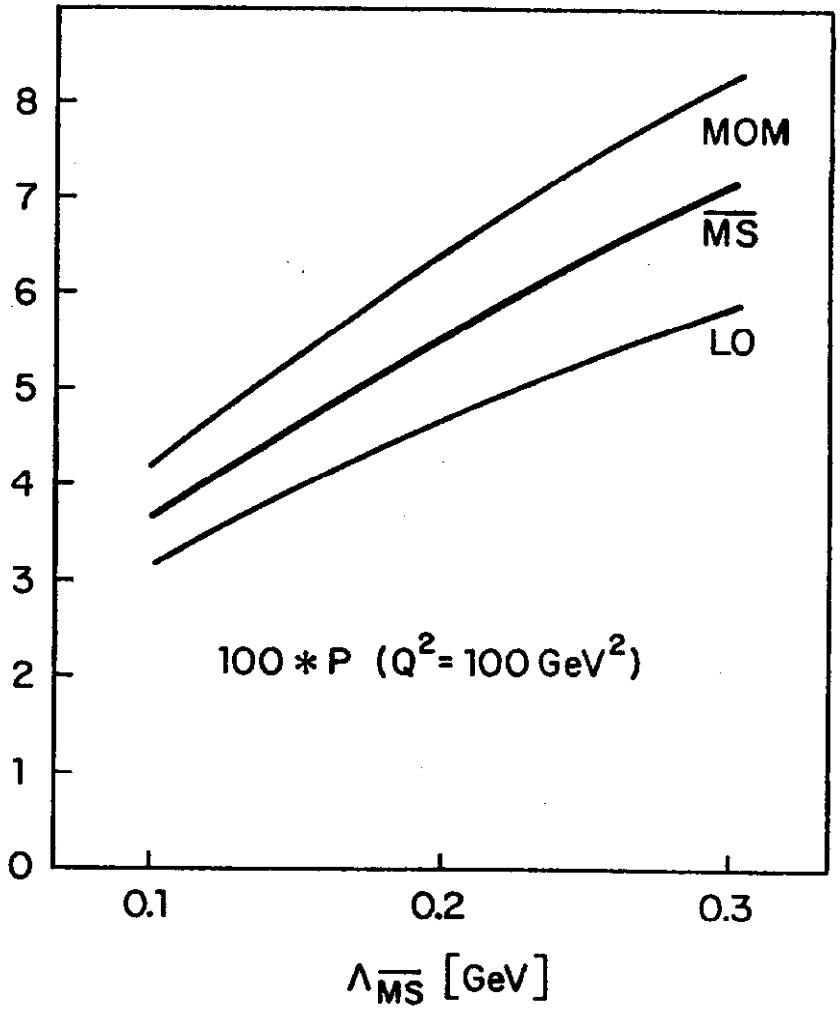
8
7
6
5
4
3
2
1
0
MOM
$\overline{MS}$
LO
100 * P (Q^2= 100 GeV^2)
0.1
0.2
0.3
$\Lambda_{\overline{MS}}$ [GeV]

Fig. 3