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BL Parity, A New Conserved Quantity in
Weak Interactions of Quarks and Leptons

HARRY J. LIPKIN*
Fermi National Accelerator Laboratory, Batavia, Illinois 60510
and
Argonne National Laboratory, Argonne, Illinois 60439

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ABSTRACT

The parity quantum number $\pi_{BL} = (-1)^{(B-L-H)/2}$ where H is defined as ± 1 for right and left-handed states is shown to be rigorously conserved in all transitions which conserve electric charge and weak isospin and whose external particles are any combination of quarks and leptons with the conventional electric charges and weak isospin classification. Implications for conservation of B-L are given.

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Investigations of possible baryon-number-violating interactions in unified models of quarks and leptons[1,2] show a mysterious tendency[3,4] to conserve B-L. A detailed analysis of the general S-matrix for processes conserving electric charge and weak isospin[5] shows how many features of higher symmetries and the mysterious conservation of B-L arise without explicit assumptions beyond SU(2) × U(1). Many of these results are explained by noting the existence of a new conserved quantum number, which we call "BL parity"

$$\pi_{BL} = (-1)^{(B-L-H)/2} \tag{1}$$

where B is the baryon number, L is the lepton number and H is the helicity, defined as +1 for right-handed and -1 for left-handed states.

We first prove that this BL parity is conserved in all transitions which conserve electric charge and weak isospin for any combination of external particles which are quarks or leptons with the conventional weak isospin classification (doublets for left-handed particles and singlets for right-handed) and the conventional electric charges (0 and -1 for leptons, +2/3 and -1/3 for quarks).

Consider the linear combination of quantum numbers

$$\Phi = \frac{3(B-L)-H}{2} - 6(Q-I_3) \tag{2a}$$

$$\Phi = (B-L-H)/2 + (B-L) - 6(Q-I_3) \tag{2b}$$

where Q is the electric charge and $I_3$ is the third component of weak isospin. This quantity $\Phi$ is seen by direct calculation to have only even integer values for all quarks and leptons with the conventional classification, as shown in Table I. Any multifermion state of these particles has an even value of $\Phi$. Thus $\phi$ can change only by an even integer in any transition between two such states. In any transition in which Q and $I_3$ is conserved, the third term of eq. (2b) does not change. The second term can only change by an even integer. Thus the first term can only change by an even integer, and the parity $\pi_{BL}$ is conserved.

This conservation law immediately gives the following results:

1. B-L is conserved modulo 4 in all transitions which have an even number of helicity flips, i.e. in which H changes by a multiple of 4.

2. B-L is conserved in all 4-point functions in which helicity is conserved, or if there is a double helicity flip. This includes all transitions mediated by exchange of gauge vector bosons (helicity-conserving) and all transitions mediated by the exchange of scalar bosons (double-flip), if there are no derivative couplings.

3. B-L must be violated in all transitions with an odd number of helicity flips. The change in B-L is 4n+2.

4. B-L must change by two units in any four-point function with a single helicity flip. However, such transitions cannot occur with simple exchanges unless there are derivative couplings, since the exchanged object must flip helicity at one vertex and conserve it at the other.

5. A six-point function which could give rise to neutron oscillations[6] via the $\delta(B-L) = 2$ $n \to \bar{n}$ transition must have an odd number of helicity flips. It is therefore impossible to construct such a function with only gauge boson exchanges and local couplings. At least one helicity-flip, e.g. Higgs exchange, is necessary.

The two essential conditions underlying this conservation law are the conservation of Q and $I_3$ and the conventional classification. Thus the conservation law holds in any model with arbitrarily complex diagrams if these two conditions are respected. The conservation law will be violated if these two conditions are violated. This will occur, for example, if:

1. Dynamical mechanisms are introduced which violate weak isospin conservation. Mass terms which couple left and right-handed states are an example of such mechanisms.

2. New exotic external particles are introduced which do not have even eigenvalues of $\phi$. One example of such a particle would be a quark or lepton with the opposite correlation between helicity and weak isospin; e.g. a left-handed quark or lepton which is a weak isospin singlet. However, the above derivation of the conservation law breaks down only for transitions where such particles appear explicitly as external particles.

These conditions for conservation of $\pi_{BL}$ can also be stated and proved in a compact form by defining a $\phi$-parity quantum number

$$\pi_\phi = (-1)^\Phi \quad . \tag{3}$$

All particles with the conventional classification are seen from Table I to have even $\phi$-parity. Thus $\pi_\phi$ is trivially conserved in any transition involving only such particles. Since $\phi$-parity differs from BL parity only by a factor $(-1)^{B-L}$ and a phase depending on $Q-I_3$, BL parity must also be conserved in any transition involving only such particles which conserves Q and $I_3$. However BL parity conservation is non-trivial since particles exist with both even and odd values of $\pi_{BL}$.

Equations (2) and (3) show that reversing the helicity of any fermion while leaving B, L, Q and $I_3$ unchanged reverses the sign of $\pi_\phi$. Thus particles of "anomalous helicity" which have the opposite helicity from conventional particles for given allowed values of B, L, Q and $I_3$ have negative values of $\pi_\phi$. Thus $\pi_\phi$ is not conserved in transitions involving an odd number of such "anomalous helicity" fermions, and $\pi_{BL}$ is also not conserved.

This argument can easily be extended to treat new bosons such as gauge vector or Higgs bosons. These can be given B, L and H quantum numbers defined by any fermion pair state to which they are coupled. If B, L and H are not conserved, these quantum numbers may not be unique and can depend upon which fermion pair state is chosen. However, the values of $\pi_\phi$ and $\pi_{BL}$ will be independent of the particular classification if Q and $I_3$ are conserved in all couplings of these bosons.

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## FOOTNOTES AND REFERENCES

[*] On leave from Department of Physics, Weizmann Institute of Science, Rehovot, Israel.

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TABLE I

QUANTUM NUMBERS OF FIRST GENERATION FERMIONS

| Fermion | $H$ | $Q$ | $I_3$ | $6(Q-I_3)$ | $\frac{3(B-L)-H}{2}$ | $\phi$ |
|---|---|---|---|---|---|---|
| $\bar{d}_R$ | 1 | +1/3 | +1/2 | -1 | -1 | 0 |
| $e_R^-$ | 1 | -1 | 0 | -6 | -2 | +4 |
| $u_R$ | 1 | +2/3 | 0 | +4 | 0 | -4 |
| $\bar{u}_R$ | 1 | -2/3 | -1/2 | -1 | -1 | 0 |
| $e_L^+$ | -1 | +1 | 0 | +6 | 2 | -4 |
| $\bar{u}_L$ | -1 | -2/3 | 0 | -4 | 0 | +4 |
| $u_L$ | -1 | +2/3 | +1/2 | +1 | +1 | 0 |
| $d_L$ | -1 | -1/3 | -1/2 | +1 | +1 | 0 |
| $e_R^+$ | 1 | +1 | +1/2 | +3 | +1 | -2 |
| $\bar{\nu}_R$ | 1 | 0 | -1/2 | +3 | +1 | -2 |
| $d_R$ | 1 | -1/3 | 0 | -2 | 0 | +2 |
| $e_L^-$ | -1 | -1 | -1/2 | -3 | -1 | +2 |
| $\nu_L$ | -1 | 0 | +1/2 | -3 | -1 | +2 |
| $\bar{d}_L$ | -1 | +1/3 | 0 | +2 | 0 | -2 |
| RIGHT HANDED NEUTRINOS | | | | | | |
| $\bar{\nu}_L$ | -1 | 0 | 0 | 0 | 2 | 2 |
| $\nu_R$ | +1 | 0 | 0 | 0 | -2 | -2 |