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Semileptonic Decay of Charmed Particles and Weak Form Factors

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ABSTRACT

We present a phenomenological analysis of semileptonic decays of charmed particles based on SU(4) symmetry and (axial-) vector-meson-dominance of weak form factors. The modified monopole form factors, which have been recently discussed by Sehgal, are extensively applied for charmed-particle decays throughout this work. These form factors contain an as yet undetermined parameter. We find a precise quantitative relation, depending on this parameter, between semileptonic decay rates and production cross sections of charmed baryons in the neutrino-induced quasi-elastic scatterings. The preliminary data on latter processes suggests that either (i) form factors decrease in the space-like region much faster than previously expected, or (ii) the Cabibbo factor is considerably smaller than the conventional value assumed for production processes of charmed baryons, i.e., $\sin^2\theta << 0.05$. We also discuss a few methods to test basic hypotheses underlying this work.

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I. INTRODUCTION

Since the discovery of charmed mesons, a lot of experimental and theoretical works has been devoted to the study of charmed particles.[1] One of the important phenomena in this respect is the semileptonic decay, which is the most probable source of (a) the multileptonic events observed in high energy neutrino scatterings (b) prompt lepton events in NN scatterings and so on. To describe these phenomena, we have a theoretical framework originally due to Cabibbo[2] and generalized by Glashow, Iliopoulos and Maiani (GIM).[3] When a new flavor of hadrons is discovered, our immediate interest is therefore to know whether it can be understood within the generalized Cabibbo-scheme or not. This paper is concerned with a particular aspect of the problem, i.e., semileptonic decay rates of charmed particles. Several authors have suggested that all charmed particles should have approximately the same semileptonic decay rates and even the same life time.[4] This is a little surprise to us, if taken literally, because then limitations due to the phase space, which are so important in hyperon semileptonic decays, do not seem to play a significant role for charmed particle decays. We will study several important problems related to semileptonic decays including the above-mentioned one in a phenomenological way. Let us first discuss D mesons for which there is a considerable amount of evidence for semileptonic decays.

The data from the DELCO detector at SPEAR[5] has shown that the main contribution to inclusive processes $D \to e\nu X$ comes from $X = K$ and K^* if the resonant production of $K\pi$ is assumed. The situation for the decay $D \to e\nu\pi$ is still unclear at present. Combining the branching ratio for $D \to e\nu K$, which is obtained from an analysis of the electron energy spectrum, with a theoretical estimate on the absolute decay rate for this mode, one obtains a life time of D mesons. It is equal to several times 10^{-13} seconds. Although the analysis may be complicated by a mixture of neutral and charged D mesons produced in e^+e^- collisions, and direct measurements of the life time still widely spread,[6] we learn one important lesson: the dominant semileptonic-decay mode of D mesons satisfies $\Delta C = \Delta S = 1$ as far as we follow the conventional scheme. The data from SPEAR[5] is clearly in contradiction with the dominance of $D \rightarrow e\nu\pi$ and consequently favors the GIM picture. Indeed this is one of the few reasons to identify D mesons as charmed particles.

Next we notice that the theory of semileptonic decays for ordinary strange-particles has been elaborately tested experimentally. Semileptonic amplitudes are expressed by a set of form factors in these cases from a general invariance principle. These form factors, which are Lorentz-invariant functions of momentum-transfer squared to the lepton pair, are assumed to satisfy a simple SU(3)-symmetry relation at zero momentum-transfer limit. In order to calculate the semileptonic decay rates, one needs to know the momentum dependence of form factors. For baryons, these are usually conjectured from the electron scattering data or the quasi-elastic neutrino-scattering data on nucleons. Theoretical ideas often utilized in this connection are the (axial-) vector-meson-dominance, PCAC hypothesis, and so on.[7] Phenomenologically, however, one important difference arises at this point between hyperons and charmed baryons. In hyperon semileptonic decays, apart from a recently studied decay $\Omega^- \rightarrow \Xi^0 e^- \nu$,[8] the largest momentum-transfer to the lepton pair occurs in the decay $\Sigma^- \rightarrow n e^- \bar{\nu}$, where $t_{max} = 0.066\ (\mathrm{GeV}/c)^2$. In contrast, if masses of charmed baryons are greater than 2.2 GeV as experimental indications suggest,[9] then t_{max} can be as large as $0.8\ (\mathrm{GeV}/c)^2$ even for the dominant (i.e., Cabibbo-favored) semileptonic decays into hyperons. This remark applies also to semileptonic decays of D mesons. Consequently one may expect that semileptonic decay rates of charmed particles depend on the behavior of form factors in a time-like region much more heavily than those of hyperons and kaons do. The quantitative study of this effect is still fragmented, as we believe, and is indeed one motivation for this work. We shall follow one of the simplest ways to introduce form factors. It is tantamount to assuming monopole form factors with an additional factor multiplied. The latter depends on a real parameter α, which is related to the weak-charge radius. This possibility has been recently discussed by Sehgal[10] and also by other authors previously.[11-14] It provides us with a very convenient way to parametrize the existing data. More importantly, it allows us to speculate on possible form factors for charm changing processes in a natural way.

In the following sections, we shall calculate two basic quantities: (i) semi-leptonic decay rates of charmed particles and (ii) production cross sections of charmed baryons in neutrino-induced quasi-elastic scatterings. Specifically, it is found for charmed baryons that these two quantities are correlated in a simple way depending on the choice of the parameter α and the Cabibbo factor. So precise experimental data for one of these processes will greatly facilitate an understanding of the other. Our approach to the semileptonic decay is essentially the same as conventionally assumed for hyperon β decays and also adopted by Buras[15] for charmed baryons, although details are considerably different.

This paper is organized as follows. In Sec. 2, we describe a general formalism of charmed-particle decays and related neutrino-induced processes. In Sec. 3, various form factors are studied. It is found that existing form factors for $\nu p \to \mu^- \Delta^{++}$ in Adler's model and Bijtebier's model can be conveniently reparametrized by our formula with a slightly larger charge-radius than that of nucleons. In Sec. 4, we present numerical results for semileptonic decay rates of charmed baryons. In Sec. 5, the neutrino-induced production cross sections of typical charmed baryons are presented. Finally Sec. 6 contains concluding remarks. Comparisons with existing data and other theoretical approaches are also undertaken in this section. Appendix I contains new analytic formulas for semileptonic decay rates of mesons.

II. FORMALISM OF SEMILEPTONIC DECAYS OF CHARMED PARTICLES

The decay $D \to e\nu X$ where $X = K$ or K^* has been widely discussed in the literature and excellent descriptions are already available.[16] We begin with the decay $D \to e\nu K$, for which a pure vector interaction is usually assumed in analogy with the decay $K \to e\nu\pi$. The relevant matrix element is given by

$$M = i\frac{G\cos\theta}{\sqrt{2}}\left[(q_D + q_K)_\lambda f_+ + (q_D - q_K)_\lambda f_-\right]\bar{\nu}(q_\nu)(1-\gamma_5)\gamma_\lambda e(q_e) \tag{1}$$

where q_D and q_K are momenta of D and K respectively, and $f_\pm$ are functions of $s = -(q_\nu + q_e)^2 > 0$ only. [Our metric is such that $p\cdot q = \vec{p}\cdot\vec{q} + p_4 q_4$.] The f_--term always appears in a final result with m_e, the lepton mass, and its contribution to the decay rate is negligible for the electron mode. Then we have

$$\frac{d^2\Gamma}{ds\,dE_e} = \frac{G^2\cos^2\theta}{2(2\pi)^3}\left[\frac{2E_e}{m_D}(s + m_D{}^2 - m_K{}^2) - s - 4E_e{}^2\right]|f_+(s)|^2 \tag{2}$$

in an obvious notation. From this we obtain the total decay rate:

$$\Gamma(D \to e\nu K) = \frac{G^2\cos^2\theta}{192\pi^3 m_D{}^3}\int_0^{\Delta^2} ds\left(\lambda(s, m_D{}^2, m_K{}^2)\right)^{3/2}|f_+(s)|^2 \tag{3}$$

with $\Delta = m_D - m_K$ and $\lambda(a, b, c) \equiv a^2 + b^2 + c^2 - 2ab - 2bc - 2ca$. The values of the form factor $|f_+(s)|$ at $s = 0$ are 1 for $K_L{}^o \to \pi^\pm e^\mp \nu$, $D^o \to K^- e^+ \nu$, $D^+ \to \bar{K}^o e^+ \nu$, $\sqrt{2}$ for $\pi^+ \to \pi^o e^+ \nu$, $1/\sqrt{2}$ for $D^+ \to \pi^o e^+ \nu$ and $K^+ \to \pi^o e^+ \nu$, respectively with suitable changes of masses and Cabibbo factors understood. For a constant form factor, explicit formulas for both the electron energy spectrum $d\Gamma/dE_e$ and the total decay rate Γ are well known. In the next section, we consider a modified monopole form factor for this decay. In particular we find analytical expressions of $d\Gamma/dE_e$ and Γ for an ordinary monopole form factor. The decay $D \to K^* e\nu$ will not be pursued further in this work, although it is actually important. So we will concentrate the rest of our attention to the semileptonic decays of charmed baryons.

Let us consider the decay:

$$A(p_1, m_A) \to B(p_2, m_B) + \ell^+(q_\ell) + \nu(q_\nu) \quad , \tag{4}$$

where ℓ^+ stands for either e^+ or μ^+. In (4), A is a charmed baryon with $J^P = \frac{1}{2}^+$ and B represents a hadronic state with baryon number 1. The mass of A (B) is denoted by m_A (m_B). Because of high mass of A, $J^P = 1/2^\pm$, $3/2^\pm$, $5/2^\pm$, ... may be permissible for B. However, we restrict ourselves to $J^P = 1/2^+$ and $3/2^+$, partly because form factors are known for related processes: $\nu n \to \mu^- p$ and $\nu p \to \mu^- \Delta^{++}$ in a space-like region. An obvious restriction to be made here is that only weak decays should be actually allowed by various conservation laws. Otherwise our results for semileptonic decay rates are practically useless. The contribution of τ-lepton mode is negligible, even if allowed kinematically. This is due to a very small phase space which is available to the decay. Some of the useful materials are found in ref. 17 and papers quoted therein.

(I) The decay $\frac{1}{2}^+ \to \frac{1}{2}^+ + \ell^+ \nu$

We write the matrix element for this process as

$$M = \frac{Gc}{\sqrt{2}} \bar{\nu}(q_\nu)(1 - \gamma_5)\gamma_\lambda \ell(q_\ell) \langle \tfrac{1}{2}^+, p_2, m_B | J_\lambda | \tfrac{1}{2}^+, p_1, m_A \rangle \quad , \tag{5}$$

where the second factor on the right-hand side is equal to

$$\langle p_2 | J_\lambda | p_1 \rangle = i\bar{u}(p_2)\Big[F_1(q^2)\gamma_\lambda - F_2(q^2)\sigma_{\lambda\,p} q_p/\Sigma \quad ,$$

$$+ (F_3(q^2)\gamma_\lambda + iF_4(q^2))\gamma_5\Big]u(p_1) \tag{6}$$

with $g = p_2 - p_1 = -(q_\ell + q_\nu)$, and $\Sigma = m_A + m_B$ as before. The constant c is a combination of an SU(4) factor and a Cabibbo factor and will be detailed later. F_1 is normalized to 1 at $t = 0$. We neglected two possible terms, i.e., q_λ-term and $\sigma_{\lambda\,p} q_p \gamma_5$-term as usual. An elementary manipulation gives the standard formula for the semileptonic decay rates ($s \equiv -q^2$)

$$\Gamma = \frac{G^2 c^2}{384\,\pi^3 m_A{}^3} \int_{m_\ell{}^2}^{\Delta^2} ds(1 - m_\ell{}^2/s)^2 \sqrt{\lambda(s, m_A{}^2, m_B{}^2)} \times \Big[|F_1|^2 \{\Delta^2(4s - m\ ^2)$$

$$+ 2\Sigma^2\Delta^2(1 + 2m_\ell{}^2/s) - (\Sigma^2 + 2s)(2s + m_\ell{}^2)\} + \frac{|F_2|^2}{\Sigma^2}(2s + m_\ell{}^2)(2\Sigma^2 + s)(\Delta^2 - s)$$

$$+ |F_3|^2 \{\Sigma^2(4s - m_\ell{}^2) + 2\Delta^2\Sigma^2(1 + 2m_\ell{}^2/s) - (\Delta^2 + 2s)(2s + m_\ell{}^2)\}$$

$$+ 6\,R_e(F_1 \cdot F_2{}^*)(\Delta^2 - s)(2s + m_\ell{}^2) + 2m_\ell{}^2(\Delta^2 - s)\{-2\Sigma R_e(F_3 \cdot F_4{}^*) + s|F_4|^2\}\Big]$$

$$(\Delta = m_A\ \ m_B) \quad . \tag{7}$$

(II) The decay $1/2^+ \to 3/2^+ + \ell^+\nu$

This process contributes only to the decay $\underline{6}(C = 1) \to \underline{10}(C = 0)$ of SU(3) representations. Following ref. 18, we write the matrix element for the hadronic weak current as

$$\langle \tfrac{3}{2}^+, p_2, m_B | J_\lambda | \tfrac{1}{2}^+, p_1, m_A \rangle = i\bar{U}_\alpha(p_2)\Big\{ \delta_{\alpha\lambda}(F_1{}^A + F_1{}^V\gamma_5) + i\,\frac{p_{1\alpha}}{\Sigma}\gamma_\lambda(F_2{}^A + F_2{}^V\gamma_5)$$

$$+ \frac{1}{\Sigma^2}\, p_{1\alpha}(p_1 + p_2)_\lambda(F_3{}^A + F_3{}^V\gamma_5)$$

$$+ \frac{1}{\Sigma^2}\, p_{1\alpha}(p_1 - p_2)_\lambda(F_4{}^A + F_4{}^V\gamma_5)\Big\}U(p_1) \quad . \tag{8}$$

In Eq. (8) $U_\alpha(p_2)$ represents a Rarita-Schwinger field for spin 3/2 and satisfies $\gamma_\alpha U_\alpha(p_2) = P_{2\alpha}U_\alpha(p_2) = (i\gamma\cdot p_2 + m_B)U_\alpha(p_2) = 0$. We treat physical particles with spin 3/2 as if they are stable. The matrix element for semileptonic decays is given by

$$M = \frac{Gc}{\sqrt{2}} \langle \tfrac{3}{2}^+, p_2, m_B | J_\lambda | \tfrac{1}{2}^+, p_1, m_A \rangle \bar{\nu}(q_\nu)(1 - \gamma_5)\gamma_\lambda\, \ell(q_\ell) \quad . \tag{9}$$

The coefficient c stands for a combination of SU(4) factors and Cabibbo factors as before and will be determined later. After taking a summation over spins, the Lorentz-invariant matrix-element squared is concisely written as

$$T \equiv \sum_{\text{spin}} |M|^2 m_A m_B m_\ell q_{\nu o} = \frac{4}{3} G^2 c^2 \sum_{i=1}^{5} R_i(s) X_i \tag{10}$$

in terms of Lorentz-invariant functions R_i, and X_i introduced by Albright and Liu[18] (All R_i's were recalculated by us). We record here only X_i:

$$X_1 = p_2\cdot q_\ell p_1\cdot q_\nu + p_2\cdot q_\nu p_1\cdot q_\ell \quad ,$$

$$X_2 = p_2\cdot q_\ell p_1\cdot q_\nu - p_2\cdot q_\nu p_1\cdot q_\ell \quad ,$$

$$X_3 = p_2\cdot q_\ell p_2\cdot q_\nu \quad ,$$

$$X_4 = p_1\cdot q_\nu p_1\cdot q_\ell \quad ,$$

$$X_5 = m_A m_B q_\nu\cdot q_\ell \quad . \tag{11}$$

From these X_i's, we define $Y_i(s)$'s by

$$\int \frac{d^3q_\ell}{2E_\ell}\frac{d^3q_\nu}{2E_\nu}\delta(Q - q_\nu - q_\ell)X_i = \frac{\pi}{24}(1 - m_\ell^2/s)Y_i(s) \quad (i = 1,2,\ldots,5) \tag{12}$$

with $s = -Q^2$. In terms of functions $R_i(s)$ and $Y_i(s)$ we obtain a general decay-rate formula ($\Delta = m_A - m_B$ as before):

$$\Gamma(\tfrac{1}{2}^+ \to \tfrac{3}{2}^+ + \ell^+\nu) = \frac{G^2c^2}{288\pi^3 m_A^3}\int_{m_\ell^2}^{\Delta^2} ds\sqrt{\lambda(s, m_A^2, m_B^2)}(1 - m_\ell^2/s)^2$$

$$\times \sum_{i\neq 2} R_i(s)Y_i(s) \quad . \tag{13}$$

We notice that $R_2(s)$ does not contribute to the total decay rate Γ, although it does to the lepton energy spectrum. By defining variable:

$$\Sigma^2 - s = x_1 \qquad s - m_\ell^{\,2} = x_5 \quad ,$$

$$\Delta^2 - s = x_2 \qquad s + 2m_\ell^{\,2} = x_6 \quad ,$$

$$s + \Sigma\Delta = x_3 \qquad m_A^{\,2} + m_B^{\,2} - s = x_7 \quad ,$$

$$s - \Sigma\Delta = x_4 \qquad m_A + m_B = \Sigma \quad . \tag{14}$$

$Y_i(s)$ are explicitly written as follows:

$$Y_1(s) = x_5 x_7 - \frac{1}{s} x_3 x_4 x_6 \quad ,$$

$$Y_2(s) = 0 \quad ,$$

$$Y_3(s) = m_B^{\,2} x_5 + \frac{1}{2s} x_4^{\,2} x_6 \quad ,$$

$$Y_4(s) = m_A^{\,2} x_5 + \frac{1}{2s} x_3^{\,2} x_6 \quad ,$$

$$Y_5(s) = -6 m_A m_B s \quad . \tag{15}$$

It is also very convenient to rewrite Eq. (9) by introducing a different set of form factors in the following way:

$$M = \frac{Gc}{\sqrt{2}} U_\alpha(p_2) \left\{ - \left(\frac{C_3^{\,V}}{m_A} i\gamma_\alpha + \frac{C_4^{\,V}}{m_A^{\,2}} p_{2\lambda} + \frac{C_5^{\,V}}{m_A^{\,2}} p_{1\lambda} \right) \gamma_5 F_{\lambda\alpha} + C_6^{\,V} j_\alpha \gamma_5 \right.$$
$$\left. + \left(\frac{C_3^{\,A}}{m_A} i\gamma_\lambda + \frac{C_4^{\,A}}{m_A^{\,2}} p_{2\lambda} \right) F_{\lambda\alpha} - C_5^{\,A} j_\alpha + \frac{C_6^{\,A} p_{1\alpha} q\, j}{m_A^{\,2}} \right\} u(p_1) \tag{16}$$

where $F_{\lambda\alpha} = q_\lambda j_\alpha - q_\alpha j_\lambda$, $j_\alpha = i\bar{\nu}(q_\nu)(1-\gamma_5)\gamma_\lambda \ell(q_\ell)$, and $q = p_2 - p_1$.

Form factors C_i^V and C_i^A $(i = 3,4,5,6)$ are identical to those used by Llewellyn Smith,[19] although the metric is different (we used Pauli's). All C_i^V and C_i^A are relatively real, if the time-reversed invariance is assumed. The relation between two sets of form factors introduced in Eq. (8) and Eq. (16) is found in ref. 19. In the formula for decay rates, contributions to the integral come from the time-like region of the momentum $q_\nu + q_\ell$. The available data on form factors are extracted from scattering experiments, i.e., from space-like regions of $q_\nu - q_\ell$, for which we have a formula $(t = -(q_\nu - q_\ell)^2 < 0)$:

$$\frac{d\sigma}{dt}(\nu N \to \mu^- + \text{charmed baryon with } J^P = \tfrac{1}{2}^+) =$$

$$= \frac{G^2c^2}{2\pi} \times \frac{1}{(s - m_A{}^2)^2}\Bigg[\frac{t}{2}\ \{(t-\Delta^2)\,|F_1 + F_2|^2 + (t-\Sigma^2)\,|F_3|^2\}$$

$$+ \{ts + (s - m_A{}^2)(s - m_B{}^2)\}\ \{|F_1|^2 - \frac{t}{\Sigma^2}|F_2|^2 + |F_3|^2\}$$

$$- t(t + 2s - m_A{}^2 - m_B{}^2)\mathrm{Re}\,[(F_1 + F_2)]\,F_3{}^*\Bigg] \tag{17}$$

where $s = -(p_1 + q_\nu)^2$ and we neglected the lepton mass along with the F_4 term. The corresponding formula for $\nu p \to \mu^- \Delta^{++}$ has been given in ref. 18.

Let us turn our attention to SU(4)- and Cabibbo-factors. Among many possibilities, we follow the Cabibbo-GIM scheme of $|\Delta C| = 1$ weak current and write[3]

$$J_\lambda(\Delta C = 1) = J_\lambda{}^D \sin\theta - J_\lambda{}^F \cos\theta \quad . \tag{18}$$

The J^D (J^F) represents a vector- or axial-vector-current with internal quantum numbers of the charged D (F) meson. The classification of charmed baryons is done in an ordinary way (see table I).[20] In particular, charmed baryons with $J^P = 1/2^+$ which belong to $\underline{6}$ of SU(3) can decay into either $\underline{8}(J^P = 1/2^+)$ or $\underline{10}(J^P = 3/2^+)$ of non-charmed baryons through semileptonic interactions, whereas those charmed baryons which belong to $\underline{3}^*(J^P = 1/2^+)$ of SU(3) can decay into $\underline{8}(J^P = 1/2^+)$ only. It is straightforward to read off the published table all the relevant Clebsch-Gordan coefficients for processes of our interest.[21] At $s = 0$, decay amplitudes for $\underline{6}(C = 1) \to \underline{8}(C = 0)$ are always proportional to $\sqrt{3/2}(F - D)$ or the amplitude for the decay $\Sigma^- \to ne^-\nu$, while those amplitudes for $\underline{3}^*(C = 1) \to \underline{8}(C = 0)$ are proportional to $\sqrt{3/2}(F + D/3)$ or the amplitude for the decay $\Lambda \to pe^-\bar{\nu}$. These coefficients have been tabulated in ref. 15 and readers are referred to it. Formulas collected above are sufficient for our purposes in this work. Some alternative formulas for physically interesting quantities will be mentioned in sec. 5. Before concluding this section, we make two remarks.

Firstly, the analyses of hyperon β decays and semileptonic decays of kaons already suggested, for the Cabibbo angle in an ordinary sector ($\Delta C = 0$), that $\cos^2\theta + \sin^2\theta < 1$, if both $\cos^2\theta$ and $\sin^2\theta$ are extracted from the experimental data on Cabibbo-favored and Cabibbo-suppressed processes separately.[22] In order to obtain numerical results in this work, we shall temporarily assume conventional values of these parameters (i.e., $\cos^2\theta = 0.95$, $\sin^2\theta = 0.05$). However, our formulation of charmed-particle decays can accommodate a very general possibility including the above-mentioned one by just accepting these two parameters as independent quantities.

Secondly, we notice that our use of SU(4) symmetry is equivalent to the quark-counting rule. Therefore the ratio of two physical amplitudes which are directly related by this symmetry is always a rational number if absolutely squared, and is in no case an irrational (or transcendental) number. This is also our basis to define Cabibbo factors at zero momentum-transfer limit and has been successfully applied to hyperon β decays. The mass differences between various particles are treated separately.

III. WEAK FORM FACTORS

The central problems in this section are (a) to extrapolate weak form factors from a space-like region to a time-like region and (b) to generalize them to charm-changing processes. For the semileptonic decay of kaons, the form factor in a time-like region of the momentum of the lepton pair can be directly obtained from $K_{\ell 3}$ decays. Experiments showed that a monopole form factor with $K^*(892)$-dominance fits the data well. The corresponding form factor for the decays of D or F mesons is presumably obtained by replacing K^* with D^* or F^* depending on the value of $|\Delta S|$. Above problems (a) and (b) are then solved for charmed mesons. However, as will be shown in the following, we should be a little more cautious about these assumptions. One reason is this; the existing data on semileptonic decays of kaons still allows a wide variety of form factors in such a way that does not affect semileptonic decays of kaons appreciably but can change those of D mesons substantially.

Let us start with electromagnetic form factors of nucleons. By introducing form factors for the electromagnetic current in analogy with Eq. (6), we write them

$$F_1(t) = \frac{G_E(t) - \omega G_M(t)}{1 - \omega}$$

$$F_2(t) = \frac{G_M(t) - G_E(t)}{1 - \omega} \qquad \omega = t/(4m_p^2) \tag{19}$$

by Sachs form factors $G_E(t)$ and $G_M(t)$. We have also $F_1{}^p(0) = 1$, $F_2{}^p(0) = \mu_p = 1.793$, $F_1{}^n(0) = 0$, $F_2{}^n = \mu_n = -1.913$. One may assume

$$G_E{}^p(t) = \frac{G_M{}^p(t)}{1+\mu_p} = \frac{G_M{}^n(t)}{\mu_n} \quad , \; G_E{}^n \approx 0 \qquad . \tag{20}$$

The conventional dipole fit for the proton electromagnetic form factor is given by

$$G_E{}^p(t) = \left(1 - \frac{t}{0.71}\right)^{-2} \quad [\, t \text{ in } (\mathrm{GeV}/c)^2 \,] \qquad . \tag{21}$$

By assuming the same t-dependence, we obtain the matrix element of the weak vector currents responsible for hyperon semileptonic decays. In contrast to the case of mesons, one cannot expect from (21) an obvious way to generalize it to charm changing processes. This is because the parameter 0.71 in (21) does not seem to have any simple interpretations in terms of a physical particle's mass. One can formally replace it by 4.02 (4.58) for charm-changing processes with $\Delta S = 0$ ($\Delta S = 1$). But it certainly lacks a justification in spite of the fact that resultant form factors are not definitely excluded on the present experimental knowledge (see Sec. 6). One way out of this problem is to adopt a modified monopole form factor. Following recent remarks by Sehgal,[10] we found it a very convenient way to introduce weak form factors for $\frac{1}{2}^+ \to \frac{1}{2}^+$ transitions in the following way ($\Delta C = \Delta S = 0$):

$$f_V(t) = \frac{1}{1 - \dfrac{t}{m_p{}^2}} \exp\left(\alpha_V \frac{t}{1 - t/\Sigma^2}\right) \qquad , \tag{22}$$

$$f_A(t) = \frac{1}{1 - \dfrac{t}{m_{A_1}{}^2}} \exp\left(\alpha_A \frac{t}{1 - t/\Sigma^2}\right) \qquad , \tag{23}$$

where $\Sigma = m_A + m_B$, $m_{A_1}^{\ 2} = 2m_\rho^{\ 2} = 1.20\ (\mathrm{GeV})^2$. In these equations, α_V and α_A are real parameters of the dimension $(\mathrm{GeV}/c)^{-2}$ and depend on the processes under consideration. Several interesting consequences of these form factors have been discussed. The choice $\alpha = 1.0\ (\mathrm{GeV}/c)^{-2}$ can well describe both the observed vector-and axial-vector-form factors of nucleons for quasi-elastic scatterings. For charm-changing processes, we simply replace $m_\rho^{\ 2}$ and $m_{A_1}^{\ 2}$ with corresponding charmed meson masses, $m_{D^*}^{\ 2}$ and $m_{D_A}^{\ 2}$ ($m_{F^*}^{\ 2}$ and $m_{F_A^*}^{\ 2}$ for $\Delta C = \Delta S = 1$) respectively to obtain proper form factors. The parameter α remains to be undetermined. This is not a disadvantage, for we can then adjust it as a free parameter to the scattering data ($t < 0$), followed by an extrapolation into a time-like region to obtain desired form factors for decay processes. Exponential factors in (22), (23) are of a gaussian type, modified by a relativistic correction factor.[23] We assume it as a basis of our phenomenological analysis of semileptonic decays. Therefore, the rest of this section is devoted to the determination of the parameter α for various processes from existing data. We will also apply our parametrization to mesons and NΔ transitions.

It is very useful to consider at first the semileptonic decay of kaons. It also serves us for an illustrative purpose. As we noticed before, a monopole form factor with $K^*(892)$-dominance is a good approximation for this process. Therefore let us consider a form factor:

$$f_+^{K^+\to\pi^0 e^+\nu}(t) = \frac{1}{\sqrt{2}}\,\frac{1}{1 - t/m_{K^*}^{\ 2}}\,\exp\left(\frac{\alpha_V t}{1 - t/\Sigma^2}\right) \tag{24}$$

where $\Sigma = m_{K^*} + m_{\pi^0}$. The conventional monopole form factor is reproduced by choosing $\alpha = 0$. If we assume $\alpha = 0$, then several experimental results suggest that the corresponding mass of vector mesons is definitely smaller than 0.892 GeV.

From existing data,[24] we estimate $\alpha \simeq 0.2\,(\mathrm{GeV}/c)^{-2}$.[25] The consequence of this observation on semileptonic decays of D mesons can be perhaps best seen by plotting the decay rate as a function of the parameter α by using Eq. (3). For D^o, we write

$$f_+^{D^o \to K^- e^+ \nu} = \frac{1}{1 - t/m_{F^*}^{\;2}} \exp\left(\frac{\alpha_V t}{1 - t/\Sigma^2}\right) \tag{25}$$

with $\Sigma = m_{D^o} + m_{K^-} = 2.357$ GeV, $m_{F^*}^{\;2} = 4.58\ \mathrm{GeV}^2$.[26] The ratio $\Gamma(\alpha)/\Gamma(\alpha = 0)$ is shown in Fig. 1 for each case. An interesting feature is a strong dependence of $\Gamma(D^o \to K^- e^+ \nu)$ and $\Gamma(D^o \to \pi^- e^+ \nu)$ on the parameter α. Although α's for $K^+ \to \pi^o e^+ \nu$ and $D^o \to K^- e^+ \nu$ need not be equal to each other, a small positive value of α, $\approx 0.2\,(\mathrm{GeV}/c)^{-2}$ can increase the semileptonic decay rate by about 40% as compared with the conventional monopole form factor. Accordingly, a precise determination of the parameter α for D-decays can be an important subject for future experiments. Incidentally, we can estimate the finite-width effect of K^* on $\Gamma(K \to \pi e \nu)$. This width is 49.5 ± 1.5 MeV[27] and the total semileptonic decay rate decreases only 0.3% and $\Gamma(\alpha)/\Gamma(\alpha = 0)$ remains practically unchanged. The similar effect for D mesons or F mesons is perhaps negligible because of very small width of D^* and F^*. Absolute decay rates for $K \to \pi e \nu$, $D \to K e \nu$, and $D \to \pi e \nu$ are fixed only when Cabibbo factors are given. A tentative value $\cos^2\theta = 0.95$, ($\sin^2\theta = 0.05$) leads at $\alpha = 0$ to $\Gamma(D^o \to K^- e^+ \nu) = 1.46 \times 10^{11}\,\mathrm{sec}^{-1}$, $\Gamma(D^o \to \pi^- e^+ \nu) = 1.53 \times 10^{10}\,\mathrm{sec}^{-1}$, $\Gamma(K^+ \to \pi^o e^+ \nu) = 4.13 \times 10^6\,\mathrm{sec}^{-1}$, and $\Gamma(K_L^{\;o} \to \pi^{\pm} e^{\mp} \nu) = 8.33 \times 10^6\,\mathrm{sec}^{-1}$ respectively. The latter two are slightly larger than observed values (6% and 11% respectively) and thus favor a smaller $\sin^2\theta$. In connection with the monopole form factor ($\alpha = 0$), it is often useful to make use of analytical formulas for decay rates. They are found in Appendix I. We will see that most of our arguments on charmed mesons can be also applied to charmed baryons.

Now we turn to semileptonic decays of charmed baryons.

$$(\mathrm{I})\ \tfrac{1}{2}^+ \rightarrow \tfrac{1}{2}^+ + \ell^+\nu$$

We noticed before that the dipole form factor:

$$f_V(t) = (1 - t/m_{D^*}^2)^{-2} \tag{26}$$

is often used in literature (use m_{F^*} for $|\Delta C| = |\Delta S| = 1$ process). If this form factor is parametrized according to (22) in a time-like region, we obtain a rather low value of α, i.e., about 0.25 $(\mathrm{GeV}/c)^{-2}$ for $m_A = 0.938$ GeV and $m_B = 2.4$ GeV. This value is insensitive to a small change of the charmed baryon mass. We shall discuss later that such a small value of α is unlikely from an experimental point of view if conventional Cabibbo factor is assumed. Furthermore the sudden decrease of α from 1.0 $(\mathrm{GeV}/c)^{-2}$ of nucleons down to 0.25 $(\mathrm{GeV}/c)^{-2}$ for $N \rightarrow C_1^{++}$ transitions is somewhat unnatural. So we take it as a practical lower bound considered in our calculation. An interesting observation here is that the original formula for $f_V(t)$ by Licht and Pagnamenta[13] gives, if applied to a charm-changing process with a natural choice of α' (i.e., $\alpha' = 2/\Sigma^2$ in Eq. (27) below), a remarkably close value both to the dipole form factor (26) and to (22) with $\alpha = 0.25\ (\mathrm{GeV}/c)^{-2}$. It is given as follows:

$$f_V^{\mathrm{L.P.}}(t) = \frac{1}{1 - t/m_{D^*}^2}\,\frac{1}{1 - t/\Sigma^2}\,\exp\left(\frac{\alpha' t}{1 - t/\Sigma^2}\right) \quad . \tag{27}$$

In a space-like region ($t < 0$), if applied to nucleons ($m_{D^*}^2 \rightarrow m_\rho^2$, $\Sigma^2 \rightarrow 4m_p^2$, $\alpha' \rightarrow (2m_p^2)^{-1}$), it is in an excellent agreement with the existing data up to

$-t = 20\,(\mathrm{GeV/c})^2$. Therefore we shall also consider this form factor in later applications. In semileptonic decays of charmed baryons we are interested only in $|t| << \Sigma^2$, and then (27) can be included in (22) by renormalizing the parameter α'. So, for most cases, we use (22) and (23). For a general framework of our calculations, we follow the standard procedure for fixing form factors at $t = 0$. Electromagnetic form factors of nucleons are sufficient to determine the vector form factors for charm-changing processes at $t = 0$. For $t \neq 0$, one can assume (22) with suitable choices of masses of relevant particles.

For axial-vector form factors, it is necessary to know the conventional parameters F and D ($F + D = 1.25$). In order to test the validity of various form factors before applying them to charm-changing processes, we calculated all the known branching-ratios of hyperon semileptonic decays by using (i) conventional dipole form factors for both vector- and axial-vector-contributions, (ii) modified monopole form factors with $\alpha_V = \alpha_A = 1.0\,(\mathrm{GeV/c})^{-2}$, (iii) Licht-Pagnamenta form factors with $\alpha_V' = \alpha_A' = 2/\Sigma^2$ ($\Sigma = m_A + m_B$). We used $F = 0.44$, $D = 0.81$, $\sin^2\theta = 0.05$, and $\cos^2\theta = 0.95$ respectively, which were partly suggested by the dipole fit. The contribution of $|F_4|^2$ term (see Eq. (7)) together with an interference term $\mathrm{Re}(F_3 \cdot F_4^*)$ is only 0.6 to 0.7% even in the decay $\Sigma^- \to n\mu^-\nu$ if we assume the standard PCAC hypothesis. It is difficult to say which set of form factors is really favored in a time-like region. This is mainly due to relatively small mass differences between ordinary baryons as compared with their rest masses. The consequence of the large mass difference between charmed baryons and ordinary (non-charmed) baryons is therefore particularly interesting. Next we consider

(II) $1/2^+ \to 3/2^+ + \ell^+\nu$

The available experimental data related to this process comes from electro-productions and neutrino interactions on nucleons. Vector form factors of Dufner-Tsai's are among the most frequently used ones.[28] A characteristic feature is that $C_3{}^V(t)/C_3{}^V(0)$ decreases much faster than the dipole form factor of corresponding elastic processes in a space-like region. It is possible to reparametrize their vector form factors in the same way as (22). We find

$$\alpha_V(\tfrac{1}{2}^+ \to \tfrac{3}{2}^+) = 1.33 \pm 0.10\,(\mathrm{GeV/c})^{-2} \tag{28}$$

from the same data used in ref. 28 for nucleons with $0 \leq |t| \leq 2.35\,(\mathrm{GeV/c})^{-2}$. As was noted, the fast decrease of their form factors with increasing $q^2 = -t$ can be attributed to a larger spacial extension of excited baryons (Δ^{++}) as compared with nucleons. Equation (28) is its quantitative expression. As usual, we assume the same t-dependence for weak vector-form-factors of NΔ transitions.

Axial-vector form factors are extracted from the data on single-pion productions in neutrino scatterings. Isobar models are frequently used. Some of the recent analyses summarize the data in the following formula ($q^2 = -t$):

$$C_i{}^A(q^2) = C_i{}^A(0)\,\frac{1}{(1 + q^2/M_A{}^2)^2}\exp\left(aq^2/(1 + bq^2)\right) \quad . \tag{29}$$

The parameters a and b are real and depend on the choice of theoretical models. We immediately notice a similarity between (23) and (29). In Eq. (23), m_{A_1} is the mass of physical axial-vector meson A_1, while M_A in (29) is a free parameter. Following Shreiner and von Hippel,[29] Bell et. al.,[30] we take $a = -0.61$ and $b = 0.19$ which correspond to Adler's model[31] and Bijtebier's model.[32] It is not intended here to reparametrize the existing data with a proper estimation of errors. So we tried to approximate (29) with the above choices of a and b by our formula (23). Both expressions (23) and (29), normalized at $t = 0$, agree within 1% for $0 \leq -t \leq 1\ (\mathrm{GeV/c})^2$ if we choose $\alpha = 1.36\ (\mathrm{GeV/c})^{-2}$ ($M_A = 1.10$ GeV), 1.26 (1.15), 1.20 (1.18), 1.17 (1.20), 1.09 (1.25) respectively. Values of α are rather sensitive to the parameter M_A. The choice $M_A = 1.25^{+0.15}_{-0.13}$ GeV of ref. 30 (Adler 75) gives $\alpha = 1.09^{+0.23}_{-0.24}\ (\mathrm{GeV/c})^{-2}$ for the fit $|t| \leq 1\ \mathrm{GeV}^2$. $M_A = 1.00^{+0.14}_{-0.11}$ GeV of the same referenece corresponds to a much higher value of $\alpha \approx 1.6\ (\mathrm{GeV/c})^{-2}$. The result of ABCMO collaboration,[33] $M_A = 0.98 \pm 0.08$ GeV also corresponds to a large α, although the definition of M_A is slightly different here. So it is not unreasonable to assume for $N\Delta$ transitions that

$$\alpha_V = \alpha_A = 1.3\ (\mathrm{GeV/c})^{-2} \quad \text{for} \quad |t| \leq 1\ (\mathrm{GeV/c})^2 \tag{30}$$

in our discussion (see Eq. (28)). This is perhaps not valid in general because high energy $\nu p \to \mu^- \Delta^{++}_{1236}$ data already suggests $\alpha_A > \alpha_V$ for $|t| \geq 2\ (\mathrm{GeV/c})^2$.[34] The above value of α is clearly larger than the corresponding value for quasi-elastic processes. It is possible to generalize this result to charm changing processes by saying $\alpha(1/2^+ \to 3/2^+) > (1/2^+ \to 1/2^+)$. Therefore we choose $0.4 \leq \alpha \leq 1.2$ for $1/2^+ \to 1/2^+ + \ell^+\nu$, and $0.6 \leq \alpha \leq 1.4$ for $1/2^+ \to 3/2^+ + \ell^+\nu$ respectively in numerical calculations. The Licht-Pagnamenta form factors with $\alpha' = 2/\Sigma^2$ for the former process are also included, which are equivalent to our (23) and (24) with $\alpha \simeq 0.25\ (\mathrm{GeV/c})^{-2}$.

Other assumptions about form factors are

$$C_4{}^V(t) = -(m_A/m_B)C_3{}^V(t) \quad ,$$

$$C_5{}^V(t) = 0 \quad ,$$

$$C_6{}^V(t) = 0 \quad , \tag{31}$$

where m_A and m_B are masses of particles with $J^P = 1/2^+$ and $J^P = 3/2^+$ respectively. For axial-vector form factors

$$C_3{}^A(t) = 0 \quad ,$$

$$C_5{}^A(t) = \text{const.} \times C_4{}^A(t) \text{ (Adler's model, Bijtebier's model)} \quad ,$$

$$C_6{}^A(t) : \text{neglected (see below)} \quad . \tag{32}$$

In a final result $C_6{}^A$ always appears with the lepton mass $m_\ell{}^2$. It is further expected to have a pion- (D^- or F-meson for $\Delta C = 1$) pole.[29] We neglected this term because our typical momentum $|t| << m_D{}^2$ and thus it cannot have a large contribution even in a time-like region. The constant in (32) for $N\Delta$ transitions is equal to -4 for Adler's model and 0.4 for Bijtebier's model. At $t = 0$, we have $(C_3{}^V(0))^2 = 2.05$, $C_5{}^A(0) = 1.2$. For definiteness we follow Adler's model. In order to apply these results to charm-changing processes, they have to be related to appropriate ones for the latter processes by SU(4) symmetry at $t = 0$. The relation $C_4{}^V(t) = -(m_A/m_B)C_3{}^V(t)$ which comes from the absence of Q2 transitions in electroproduction and which greatly simplifies calculations, cannot be trivially transferred to charm-changing processes. This is because the mass ratio m_A/m_B is not always the same for relevant particles. To find out a natural generalization, we write for the vector contribution (see Eq. (13)):

$$\left(\sum_i R_i Y_i(s)\right)_{\text{vector}} = \frac{x_1^2 x_2}{4m_A^4}(x_5 + x_6)(C_4^V(t))^2 \tag{33}$$

in our approximation. A more symmetric form of (33) about masses is obtained by introducing rescaled form factors in (16) as follows:

$$\frac{C_3^V(t)}{m_A} = \frac{C_3^{V'}(t)}{\Sigma} \quad ,$$

$$\frac{C_4^A(t)}{m_A^2} = \frac{C_4^{A'}(t)}{\Sigma^2} \quad , \tag{34}$$

where $\Sigma = m_A + m_B$ as before. The absence of Q2 transitions then implies $C_3^{V'} = -(m_B/\Sigma)C_4^{V'}$, and (33) now becomes

$$\left(\sum_i R_i Y_i(s)\right)_{\text{vector}} = \frac{x_1^2 x_2}{4\Sigma^4}(x_5 + x_6)(C_4^{V'}(t))^2 \tag{35}$$

with $C_4^{V'}(0) = -8.4$ for $n\Delta^+$ transitions. We assume SU(4) symmetry on $C_4^{V'}(0)$. In the same way we write

$$\frac{C_4^A(t)}{m_A^2} = \frac{C_4^{A'}(t)}{\Sigma^2} \quad . \tag{36}$$

The axial-vector contribution to (13) now reads

$$(\Sigma R_i Y_i(s))_{\text{axial-vector}} = \frac{x_1}{4\Sigma^4}\Big\{ \tfrac{1}{2}(x_1 + x_2)Y_1(s) + 2m_A{}^2 Y_3(s) + 6m_A{}^2 m_B{}^2 s +$$

$$+ (d - 1)m_B{}^2\Big[2Y_1(s) + 2(d - 1)Y_3(s) + 3s(x_1 + x_2 + 2(d - 1)m_B{}^2)\Big]\Big\}$$

$$\times (C_4{}^{A'}(t))^2 \tag{37}$$

where a constant d is defined by

$$C_5{}^A(t) = -\left(\frac{m_\Delta}{m_p}\right)^2 dC_4{}^A(t) \quad . \tag{38}$$

We find d = 2.34. The SU(4) symmetry is assumed on $C_4{}^{A'}(0)$. Equations (35) and (37) are actually used in our calculations of $1/2^+ \to 3/2^+ + \ell^+\nu$ decays.

IV. SEMILEPTONIC DECAY RATES OF CHARMED BARYONS

In numerical calculations we used $m_{D^*} = 2.01$ GeV, and $m_{F^*} = 2.14$ GeV.[36] The masses $m_{D_A{}^*}$ and $m_{F_A{}^*}$ of axial-vector counterparts of these particles are unavailable at present and therefore were assumed to be the same as m_{D^*} and m_{F^*} respectively. We show in Fig. 2 the semileptonic decay rates of C = 1 charmed baryons for $\alpha = 0.4\ (\mathrm{GeV}/c)^{-2}$. Lepton masses are included in these calculations. Curves represent the summation over e- and μ- mode. Semileptonic decay rates of A^+ and A^o are approximately the same for an entire range of masses under consideration. Indeed the decay rate of A^o is smaller than that of A^+ by only 5%. So we showed only A^+ in Fig. 2 and most of the subsequent figures by a dotted line. Fig. 3 shows the semileptonic decay rates for $\alpha = 0.8\ (\mathrm{GeV}/c)^{-2}$. Although this case is similar to Fig. 2, we notice that all curves are considerably steeper than the previous case of $\alpha = 0.4\ (\mathrm{GeV}/c)^{-2}$.

Figure 4 shows the semileptonic decay rates for $\alpha = 1.0\ (\mathrm{GeV/c})^{-2}$. This case corresponds to the simplest generalization of nucleon form factors (22) and (23) to charm-changing processes. In Fig. 5 we show the case of $\alpha = 1.2\ (\mathrm{GeV/c})^{-2}$. This corresponds to a rather large weak-charge radius. Decay rates of every particle increase rapidly with an increasing parameter α and also with increasing masses of decaying baryons. Figure 6 shows the same decay rates for $\frac{1}{2}^+ \to \frac{1}{2}^+ + \ell^+\nu$ by using Licht-Pagnamenta form factors (see (27)). At low masses, there is little difference between this case and the results in Fig. 2. As was noted before, Fig. 6 corresponds to $\alpha \simeq 0.25\ (\mathrm{GeV/c})^{-2}$. The semileptonic decay rate of $C_0^{\,+}$ changes from $1.3 \times 10^{12}\mathrm{sec}^{-1}$ (Licht-Pagnamenta with $\alpha' = 2/\Sigma^2$) to $9.6 \times 10^{12}\mathrm{sec}^{-1}$ ($\alpha = 1.2\ (\mathrm{GeV/c})^{-2}$) if $m_{C_0^{\,+}} = 2.3$ GeV. In all these calculations, induced-pseudo-scalar terms $|F_4|^2$ and $|F_3 \cdot F_4^{\,*}|$ can contribute appreciably only to the muonic decay mode as is expected. However, even in this case, the typical contribution of these terms is of the order of 0.1% of this decay mode if PCAC with D- or F-dominance is assumed. Therefore it is negligible. Much larger contributions of lepton masses come from terms which are proportional to $m_\ell^{\,2}$ and are explicitly written in the integrand of Eq. (7). For the muonic decay mode, the phase space is clearly a little smaller than the corresponding one for the electronic decay mode. Indeed this caused a considerable reduction of semileptonic decay rates of hyperons for the former. Notwithstanding this, it is found in some cases of charmed baryons that muonic decay rates exceed electronic decay rates by a few percent. One example is $\Gamma(C_1^{\,++} \to pe^+\nu) < \Gamma(C_1^{\,++} \to p\mu^+\nu)$ at $\alpha = 1.0\ (\mathrm{GeV/c})^{-2}$ for all mass-values of $C_1^{\,++}$ (> 2 GeV), by about 1% ($m_{C_1^{\,++}} = 2.0$ GeV) to 3% ($m_{C_1^{\,++}} = 3.0$ GeV). Another example is, at same α, $\Gamma(C_1^{\,++} \to \Sigma^- e^+\nu) < \Gamma(C_1^{\,++} \to \Sigma^-\mu^+\nu)$ for $m_{C_1^{\,++}} > 2.15$ GeV. Similar tendencies are also manifest for $C_1^{\,+}$ and $C_1^{\,0}$, but not for $C_0^{\,+}$. However, the difference between these two decay rates is at most a few percent for our mass range and is not substantial.

In the calculation of decay rates for $1/2^+ \to 3/2^+ + \ell^+\nu$, one of the largest uncertainties comes from our choice of form factors C_i^V and C_i^A at $t = 0$. We recall that these decays can contribute only to the transition $\underline{6}(C = 1) \to \underline{10}(C = 0)$ of SU(3) and therefore they do not introduce any uncertainties into semileptonic decay rates of $\underline{3}^*$. Our experience in $N\Delta$ transitions tells us that for the decay $1/2^+ \to 3/2^+ + \ell^+\nu$ the parameter α should be a little larger than the corresponding α for $1/2^+ \to 1/2^+ + \ell^+\nu$. So we give in Fig. 7 our numerical result for $\alpha = 0.6\ (\mathrm{GeV}/c)^{-2}$. One immediately notices that they are of comparable magnitude and therefore very important. This has been observed previously with dipole form factors. Another feature is that curves have approximately the same slope as previous cases. Figure 8 and Fig. 9 correspond to the choices $\alpha = 1.0$ and $1.2\ (\mathrm{GeV}/c)^{-2}$ respectively. They show only a minor change from Fig. 7. These cases are perhaps the simplest choices in view of (22). We stress that in our calculations of the decay $1/2^+ \to 3/2^+ + \ell^+\nu$, all lepton masses were neglected. This is justified from the nature of our approximation. Figure 10 shows the semileptonic decay rates for a large value of α, i.e., $1.4\ (\mathrm{GeV}/c)^{-2}$. The dependence of decay rates on the parameter α is less manifest in the decay $1/2^+ \to 3/2^+ + \ell^+\nu$ than in the decay $1/2^+ \to 1/2^+ + \ell^+\nu$. This is evidently due to smaller phase space available because final-state baryons are more massive in the decay $1/2^+ \to 3/2^+ + \ell^+\nu$ (cf. Fig. 1). In table II, we present detailed branching ratios for particular choices of parameters. From this table, it is also possible to estimate the decay rates when Cabibbo factors are changed.

V. PRODUCTION CROSS SECTIONS IN NEUTRINO SCATTERINGS

In the previous section we presented semileptonic decay rates of charmed baryons depending on a parameter α. Unless this parameter is obtained from an independent experiment, they are not very useful. As the semileptonic decay is intimately related to the production process of the same particles in neutrino scatterings, this parameter is also expected to play a significant role in the latter reaction. Therefore we calculated the production cross sections of C_0^+ (2.3 GeV) and C_1^{++} (2.3 GeV, 2.4 GeV) which are supposed to have a best chance of decaying weakly. We restrict ourselves to quasi-eleastic processes, although several experiments have indicated multiparticle productions to be more likely at high neutrino energies.[29]

Figure 11 shows differential production cross sections in quasi-elastic processes. We find (i) they are strongly dependent on the choice of the parameter α, although (ii) dependences on the charmed-baryon masses and incident neutrino energies are almost negligible for our choice of masses. One recalls that form factors at $t = 0$ were determined theoretically by SU(4) symmetry and that overall multiplicative constants were fixed by choosing Cabibbo factors. Thus an increasing α means increasing semileptonic-decay rates and decreasing production-cross-sections in neutrino scatterings at the same time. On the other hand, a change of the SU(4) factor and/or Cabibbo factor means a simultaneous increase (or decrease) of these two quantities.

It is at this point that the distinct role of these two sets of parameters in fitting the data is clearly understood. In numerical results of this paper, however, we used several conventional values for them. In Fig. 12, we show the total quasi-elastic cross sections for production of typical charmed baryons. The contribution to $\sigma(E_\nu)$ comes mostly from $d\sigma/dt$ with a small-$|t|$ region. This is apparent due to a rapid decrease of $d\sigma/dt$ with an increasing $|t|$ at a given E_ν. Indeed, in our case, $\sigma(E_\nu)$ is virtually determined from $d\sigma/dt$ with $|t| \lesssim$ several $(\mathrm{GeV}/c)^2$. The $d\sigma/dt$ is in turn completely controlled by a parameter α. Thus we can easily understand the strong dependence on the parameter α of $\sigma(E_\nu)$. If the statistics allow one to determine $d\sigma/dt$ experimentally, it is possible to know both α and $(d\sigma/dt)_{t=0}$. The latter can pinpoint the combination of an SU(4) factor and a Cabibbo factor. In connection with this, it is found that a very useful quantity is the differential production cross section of charmed baryons in a laboratory frame. Let θ^{lab} be the angle between the incident neutrino-beam and the produced charmed baryon in a laboratory frame. Then it is given by

$$\frac{d\sigma}{d\cos\theta^{lab}} = \frac{d\sigma}{dt} \times \frac{(s - m_A{}^2)\left[\lambda(t, m_A{}^2, m_B{}^2)\right]^{3/2}}{2m_A{}^2\left[t(s + m_B{}^2) + (m_B{}^2 - m_A{}^2)(s - m_B{}^2)\right]} \quad . \tag{39}$$

Although this quantity suffers from an unpleasant singularity at t corresponding to the maximum production angle θ^{lab}_{max} of charmed baryons, we have at $t = 0$ (see comment (ii) below),

$$\left(\frac{d\sigma}{d\cos\theta^{lab}}\right)_{t=0} = \frac{G^2c^2}{4\pi}\frac{(m_B{}^2 - m_A{}^2)^2}{m_A{}^2}\left(|F_1|^2 + |F_3|^2\right)_{t=0} \quad , \tag{40}$$

which is independent of incident neutrino energy. We neglected lepton masses here. With a previous choice of parameters, the right-hand side is equal to

$$(40) = \begin{array}{ll} 5.42 \times 10^{-39}\mathrm{cm}^2 & (\nu p \to \mu^- C_1{}^{++},\ m_{C_1{}^{++}} = 2.3\ \mathrm{GeV}) \\ 6.63 \times 10^{-39}\mathrm{cm}^2 & (\nu p \to \mu^- C_1{}^{++},\ m_{C_1{}^{++}} = 2.4\ \mathrm{GeV}) \\ 1.06 \times 10^{-38}\mathrm{cm}^2 & (\nu n \to \mu^- C_0{}^{+},\ m_{C_0{}^{+}} = 2.3\ \mathrm{GeV}) \end{array} \quad . \tag{41}$$

The constant c^2 is equal to $\sin^2\theta$, $\sin^2\theta$, and $3/2\sin^2\theta$ for these processes respectively and $F_1(0) = 1$ by definition. We make two technical remarks: (i) At a given production angle $\theta^{lab} < \theta^{lab}_{max}$, where θ^{lab}_{max} is 21.1^o (23.5^o) for $E_\nu = 20$ GeV (100 GeV) for the first case above, there are two possible magnitudes of three momentum of charmed baryons corresponding to two possible directions of secondary muons. Formula (40) refers to "slow" baryons produced at $\theta^{lab} = 0$. "Fast" baryons, which are also produced at $\theta^{lab} = 0$, come from the backward production in the center-of-mass frame. However these events are extremely rare because $d\sigma/dt$ at $t = -|t|_{max} = -(s - m_A^2)(s - m_B^2)/s$ is very small and can be practically neglected. So we may pick up all quasi-elastic events near $\theta^{lab} = 0$. (ii) The limit $t = 0$ can be achieved only if $m_\ell^2 = 0$.

However, even for muons, the exact lower limit of $|t|$ at a given neutrino energy E_ν is very close to zero. Indeed, in the reaction $\nu p \to \mu^- C_1^{++}$ with $m_{C_1^{++}} = 2.3$ GeV, $|t|_{min}$ is equal to 10^{-2} (GeV/c)^2 at $E_\nu = 5$ GeV and is less than 10^{-3} (GeV/c)^2 for $E_\nu > 30$ GeV. It is a rapid decreasing function of E_ν and we are justified to assume $|t|_{min} = 0$. The precise measurement of $(d\sigma/d\cos\theta^{lab})_{t=0}$ then gives the Cabibbo factor directly through Eq. (40).

VI. CONCLUDING REMARKS

In this work we tried to clarify the role of weak form factors in semileptonic decays of heavy particles. For mesons, the precise study of form factors for $K_{\ell 3}$ is found very important and useful to understand decays of D- and F-mesons. As for baryons, a general relation was obtained between two basic quantities, i.e., semileptonic decay rates and production cross sections in neutrino scatterings. We found considerable differences between semileptonic decay rates of various charmed particles depending on physical masses and strangeness. However these results are, after all, assumptions. At present, experimental evidences for the semileptonic processes of charmed baryons are scarce and, therefore, quantitative comparisons with theoretical models are tentative. Nevertheless it is very interesting to look into the existing data here. The only known candidate for the quasi-elastic production of charmed baryons in neutrino reactions are Λ_C^+ (2260 ± 10 MeV) and Σ_C^{++} (mass ≈ 2.43 GeV) from bubble chamber experiments.[35] The cross section for the former event is estimated to be $(2.8 \pm 2.0) \times 10^{-40} \mathrm{cm}^2$ if $E_\nu = 4$ GeV and $\sigma_{CC}/E_\nu = 0.7 \times 10^{-38} \mathrm{cm}^2/\mathrm{GeV}$ are assumed. As for the latter, Σ_C^{++} (2.43 GeV), only one event has been attributed to the quasi-elastic production in 10^5 charged-current events. If $E_\nu = 20$ GeV,[36] then the production cross section is estimated to be $4 \times 10^{-40} \mathrm{cm}^2/\mathrm{BR}(\Sigma_C^{++} \to \Lambda_C^+ \pi^+)$, which becomes $0.4 \times 10^{-40} \mathrm{cm}^2$ with the assumption: B.R. = 10%. These events therefore suggest a rather small production rate as compared with theoretical estimates. This favors a very large α if $\sin^2\theta = 0.05$ is assumed (or $\sin^2\theta \ll 0.05$ if $\alpha = 1.0\ (\mathrm{GeV}/c)^{-2}$). Meanwhile a possible candidate of doubly-charged particles which decay only weakly, has been observed by emulsion chambers, with the mass = 2290^{+285}_{-84} MeV and the lifetime $\tau = 5.4^{+0.7}_{-0.3} \times 10^{-14}$ sec.[37] If this event is identified with our C_1^{++}, then our theoretical prediction for its semileptonic-decay branching ratio is about

10% with the choice: mass = 2.3 GeV, $\alpha(1/2^+ \to 1/2^+ + \ell^+\nu) = 1.0\ (\mathrm{GeV}/c)^{-2}$, and $\alpha(1/2^+ \to 3/2^+ + \ell^+\nu) = 1.2\ (\mathrm{GeV}/c)^{-2}$. An experimental group at CERN SPS reports a proper decay time $(7.3 \pm 0.1) \times 10^{-13}$ sec for the candidate of Λ_C^+ (2.295 ±0.015 GeV)[39], which suggests a rather long lifetime from our point of view.

We make also a few brief comments on related theoretical works. As for semileptonic decays of charmed baryons, the work of Buras[15] approximately corresponds to our minimum α (Licht-Pagnamenta), owing to the use of dipole form factors. Decay rates of Gavela[40] are about one-half of our minimum value if $m_{C_0^+} = 2.3$ GeV and $m_{C_1^{++}} = 2.4$ GeV are assumed. As for production cross sections in neutrino interactions, results by Alivez, et al.[41] and those of Shrock-Lee[17] are roughly the same and correspond to our maximum values (i.e., our minimum α). The cross sections of the Orsay group[14] are approximately equal to our values at $\alpha = 1.2\ (\mathrm{GeV}/c)^{-2}$ if $m_{C_0^+} = 2.3$ GeV. Finjord and Ravndal[42] predict a very small production cross section for typical charmed baryons, which correspond to a very large α ($\gg 1\ (\mathrm{GeV}/c)^{-2}$) in our formulation. These arguments on production processes are valid only when the Cabibbo factor and the SU(4) factor for charm-changing processes with $\Delta S = 0$ are assumed to be conventional values, i.e., $\sin^2\theta = 0.05$. If a good statistic becomes available both for semileptonic decays and quasi-elastic productions in neutrino experiments, then the Cabibbo factor and α will be determined without ambiguities. Related works on form factors will be done in subsequent papers.

ACKNOWLEDGMENTS

Our interest in this subject has been stimulated by talks with experimenters. We thank them and numerous theorists at Fermilab for continual advice and encouragement throughout this work. The hospitality extended to us by Dr. L. Lederman and Dr. J.D. Bjorken is greatly acknowledged.

APPENDIX

We describe here analytical formulas for semileptonic decay rates of psuedo-scalar mesons in the case of ordinary monopole form factors. The lepton mass is neglected. By writing $f_+(t) = (1 = t/m_{F^*}^2)^{-1} f_+(0)$ for the decay $D \to Ke\nu$, we find from Eq. (2), (i) electron energy spectrum:

$$\frac{d\Gamma}{dE_e} = \frac{G^2 \cos^2\theta}{2(2\pi)^3} \left| f_+^{D\to Ke\nu}(0) \right|^2 (2E_e/m_D - 1) m_{F^*}^4$$

$$\times \left\{ \ln(1-y) + y \right\} \qquad , \qquad \text{(A1)}$$

where

$$y = 2E_e(m_D^2 - m_K^2 - 2m_D E_e) / \left[m_{F^*}^2 (m_D - 2E_e) \right]$$

and (ii) total semileptonic decay rate:

$$\Gamma(D \to Ke\nu) = \frac{G^2 \cos^2\theta}{4(2\pi)^3} \left| f_+^{D\to Ke\nu}(0) \right|^2 m_D^5 A \qquad , \qquad \text{(A2)}$$

where

$$A = \frac{(r_2 - r_1)}{12 r_1^4} \left\{ 6 - 9(r_1 + r_2) + 2(r_1 - r_2)^2 \right\} + \frac{1}{4r_1^4}(1 - 2r_1 - 2r_2 + r_1^2 + r_2^2) \ln(r_1/r_2)$$

$$+ \frac{(1 - r_1 - r_2)}{r_1^3} BI \qquad ,$$

$$B = -\frac{1}{4r_1^2} \lambda(1, r_1, r_2) \qquad ,$$

$$r_1 = (m_D/m_{F^*})^2, \quad r_2 = (m_K/m_{F^*})^2 \qquad ,$$

$$I = F(Z_2) - F(Z_1) ,$$

$$F(Z) = \frac{1}{\sqrt{B}} \tan^{-1} (Z/\sqrt{B}) \qquad (B > 0) ,$$

$$= \frac{1}{2\sqrt{|B|}} \ln \left| \frac{Z - \sqrt{|B|}}{Z + \sqrt{|B|}} \right| \quad (B < 0) ,$$

$$Z_1 = (1 - r_1 + r_2)/(2r_1),\ Z_2 = (1 + r_1 - r_2)/(2r_1) .$$

The symbol λ was defined in the text (see (3)). In the limit of the constant form factor ($m_{F^+} \to \infty$), above A reduces to the familiar expression:

$$A = \frac{1}{24} (1 - 8a + 8a^3 - a^4 - 12a^2 \ln a)$$

with $a = r_2/r_1$.

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TABLE CAPTIONS

Table I: Charmed $\frac{1}{2}^+$ baryon states in 20 dimensional representation of SU(4) (C = 1).

Table II: Semileptonic branching ratios of charmed baryons with $J^P = 1/2^+$. $\alpha_V = \alpha_A = 1.0\ (\mathrm{GeV/c})^{-2}$ for $1/2^+ \to 1/2^+ + \ell^+\nu$ and $\alpha_V = \alpha_A = 1.2\ (\mathrm{GeV/c})^2$ for $1/2^+ \to 3/2^+ + \ell^+\nu$ were assumed. Other parameters are: $\cos^2\theta = 0.95$, $\sin^2\theta = 0.05$, $m_{C_1} = m_{C_0} = 2.3$ GeV, $m_S = m_A = 2.5$ GeV, $m_{T^0} = 2.7$ GeV. Both e- and μ- modes are included. F = 0.44, D = 0.81. (A) and (B) stand for initial and final baryons respectively. Cf. ref. 15.

Table I

SU(3)	Label	I	I_3	S
	C_1^{++}		1	
	C_1^{+}	1	0	0
	C_1^{o}		-1	
$\underline{6}$	S^{+}	½	½	-1
	S^{o}		-½	
	T^{o}	0	0	-2
$\underline{3}^{*}$	A^{+}	½	½	-1
	A^{o}		-½	
	C_o^{+}	0	0	0

Table II

(A)	(B)	Decay rates (%)	(A)	(B)	Decay rates (%)
C_1^{++}	Σ^+	44.8	S^+	Σ^*	5.5
	P	19.8	S^o	Ξ^-	26.6
	$\Sigma^{*(+)}$	29.6		Σ^-	4.1
	Δ^+	5.8		$\Xi^{*(-)}$	58.0
C_1^+	Σ^o	46.2		$\Sigma^{*(-)}$	11.4
	n	10.2	T^o	Ξ^-	9.6
	$\Sigma^{*(o)}$	31.2		Ω^-	85.1
	Δ^o	12.3		$\Xi^{*(-)}$	5.3
C_1^o	Σ^-	48.0	A^+	Ξ^o	83.5
	$\Sigma^{*(-)}$	32.1		Σ^o	7.0
	Δ^-	19.9		Λ	9.5
S^+	Ξ^o	25.7	A^o	Ξ^-	85.8
	Λ	11.8		Σ^-	14.2
	Σ^o	2.0	C_o^+	Λ	70.0
	$\Xi^{*(o)}$	55.0		n	30.0

FIGURE CAPTIONS

Fig. 1: Semileptonic decay rates of D- and K-mesons plotted as functions of parameters α_V defined in Eqs. (24) and (25). Decay rates are normalized at $t = 0$ and therefore independent of Cabibbo factors. The figure includes the electronic decay mode only. For absolute decay rates, see text. We used $m_{K^*} = 0.892$ GeV, $m_{D^*} = 2.01$ GeV and $m_{F^*} = 2.14$ GeV.

Fig. 2: Semileptonic decay rates of charmed baryons for $\frac{1}{2}^+ \to \frac{1}{2}^+ + \ell^+\nu$ with $\alpha = 0.4\ (\mathrm{GeV}/c)^{-2}$. Lepton masses were retained in calculations. Both the e- and μ-modes are included and summed in the figure. The dotted curve stands for the decay rate of A^+. We used $m_{D^*}^2 = m_{D_A^*}^2 = 4.03\ (\mathrm{GeV})^2$, $M_{F_I^*}^2 = m_{F_A^*}^2 = 4.58\ (\mathrm{GeV})^2$, $\cos^2\theta = 0.95$, and $\sin^2\theta = 0.05$ (θ is the Cabibbo angle). The horizontal axis shows the masses of decaying baryons.

Fig. 3: Semileptonic decay rates of charmed baryons for $\frac{1}{2}^+ \to \frac{1}{2}^+ + \ell^+\nu$ with $\alpha = 0.8\ (\mathrm{GeV}/c)^{-2}$. Both the e- and μ-modes are included in the figure. The dotted curve represents A^+.

Fig. 4: Semileptonic decay rates of charmed baryons for $\frac{1}{2}^+ \to \frac{1}{2}^+ + \ell^+\nu$ with $\alpha = 1.0\ (\mathrm{GeV}/c)^{-2}$. Both the e- and μ-modes are included in the figure. The dotted curve represents A^+.

Fig. 5: Semileptonic decay rates of charmed baryons for $\frac{1}{2}^+ \to \frac{1}{2}^+ + \ell^+\nu$ with $\alpha = 1.2\ (\mathrm{GeV}/c)^{-2}$. Both the e- and μ-modes are included in the figure. The dotted curve represents A^+.

Fig. 6: Semileptonic decay rates of charmed baryons for $\frac{1}{2}^+ \to \frac{1}{2}^+ + \ell^+\nu$ with Licht-Pagnamenta's form factors. See Eq. (27). We assumed $\alpha'_V = \alpha'_A = 2/\Sigma^2$ where Σ is the sum of initial- and final-baryon masses. Other parameters are the same as before.

Fig. 7: Semileptonic decay rates of charmed baryons for $1/2^+ \to 3/2^+ + \ell^+\nu$ with $\alpha = 0.6\ (\mathrm{GeV}/c)^{-2}$, summed over e- and μ-modes. These decays can contribute only to $\underline{6}(C = 1)$ of SU(3). Lepton masses were neglected for all our calculations of $1/2^+ \to 3/2^+ + \ell^+\nu$ processes. Masses and Cabibbo factors are the same as before. The horizontal axis shows the masses of decaying baryons as before.

Fig. 8: Semileptonic decay rates of charmed baryons for $1/2^+ \to 3/2^+ + \ell^+\nu$ with $\alpha = 1.0\ (\mathrm{GeV}/c)^{-2}$. Both the e- and μ-modes are included and summed in the figure.

Fig. 9: Semileptonic decay rates of charmed baryons for $1/2^+ \to 3/2^+ + \ell^+\nu$ with $\alpha = 1.2\ (\mathrm{GeV}/c)^{-2}$. Both the e- and μ-modes are included.

Fig. 10: Semileptonic decay rates of charmed baryons for $1/2^+ \to 3/2^+ + \ell^+\nu$ with $\alpha = 1.4\ (\mathrm{GeV}/c)^{-2}$. Both the e- and μ-modes are included.

Fig. 11: Differential production cross sections of charmed baryons in quasi-elastic neutrino-scatterings plotted as functions of -t at $E_\nu = 20$ GeV for (a) $\nu n \to \mu^- C_0^{\;+}$ with $m_{C_0^{\;+}} = 2.3$ GeV, (b) $\nu p \to \mu^- C_1^{\;++}$ with $m_{C_1^{\;++}} = 2.3$ GeV, and (c) $^{0}\nu p \to \mu^- C_1^{\;++}$ with $m_{C_1^{\;++}} = 2.4$ GeV, respectively. The parameter α is shown in the unit of $(\mathrm{GeV}/c)^{-2}$. Other parameters are the

same as before. Lepton masses were neglected. At $E_\nu = 40$ GeV, the curves remain almost unchanged.

Fig. 12: Total production cross sections of charmed baryons in quasi-elastic neutrino-scatterings plotted against incident neutrino energy E_ν in a laboratory frame: (a) $m_{C_0^+} = 2.3$ GeV, (b) $m_{C_1^{++}} = 2.3$ GeV, (c) $m_{C_1^{++}} = 2.4$ GeV. Other details are the same as those described in the preceding figure.

Figure 1

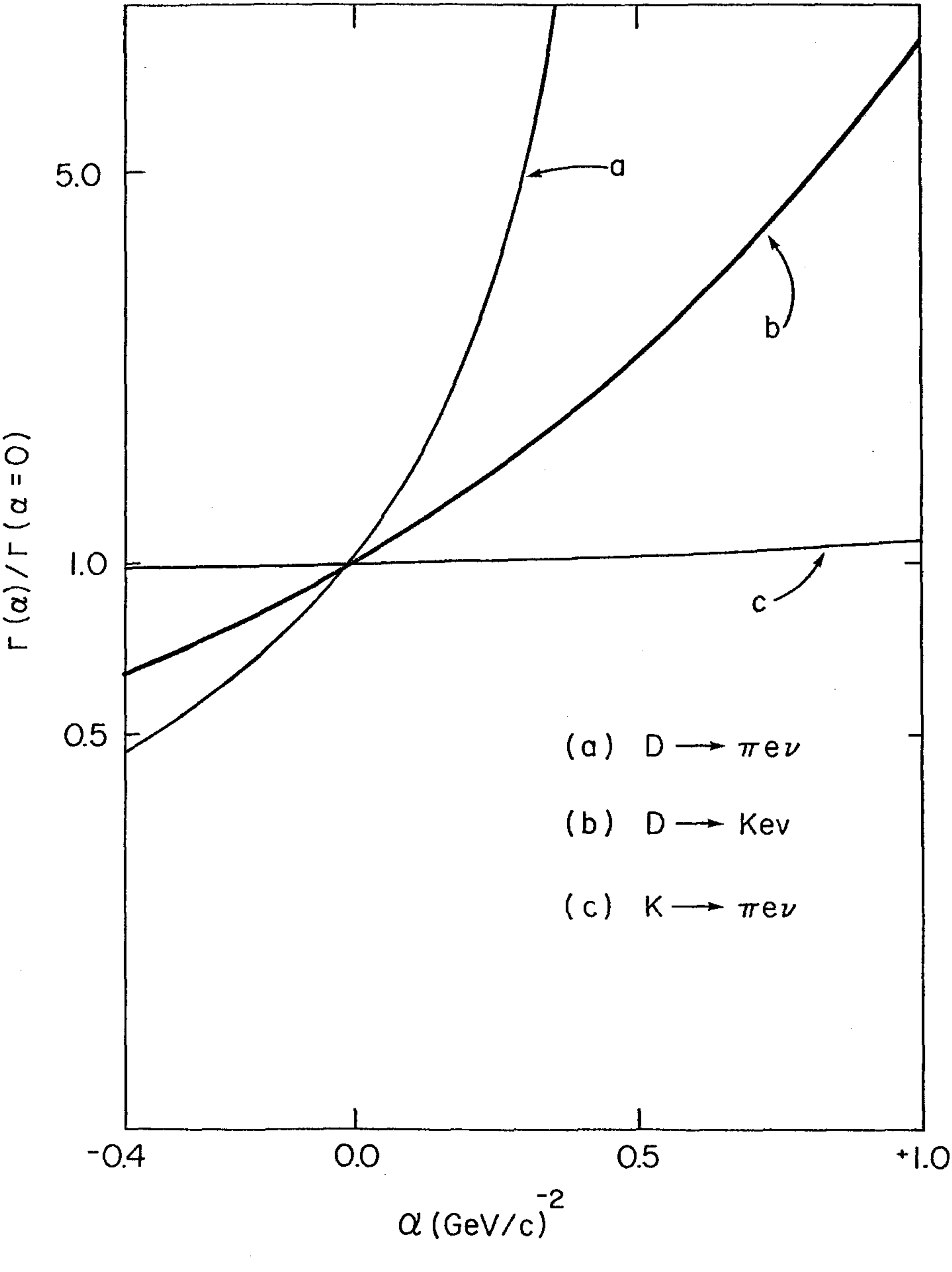
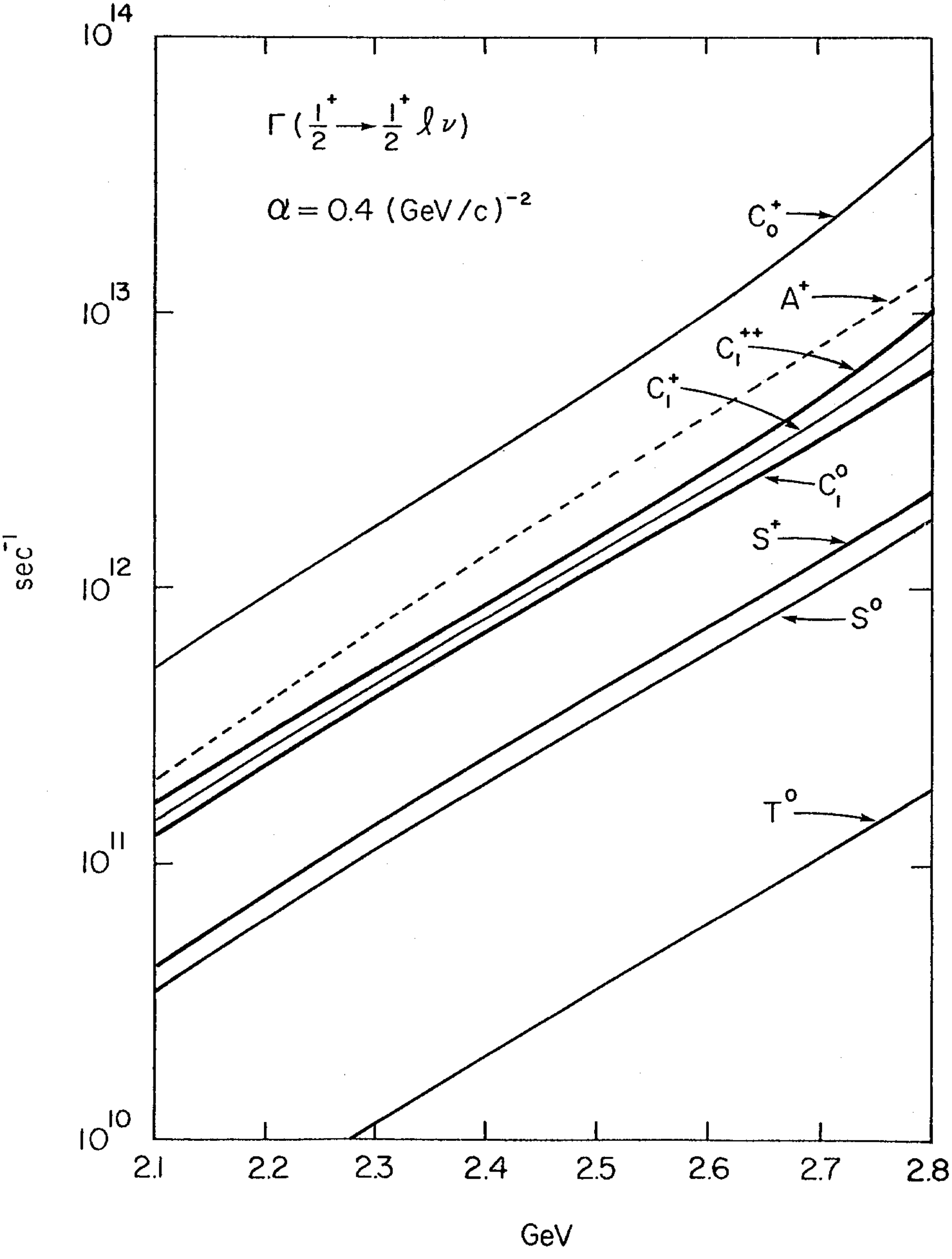
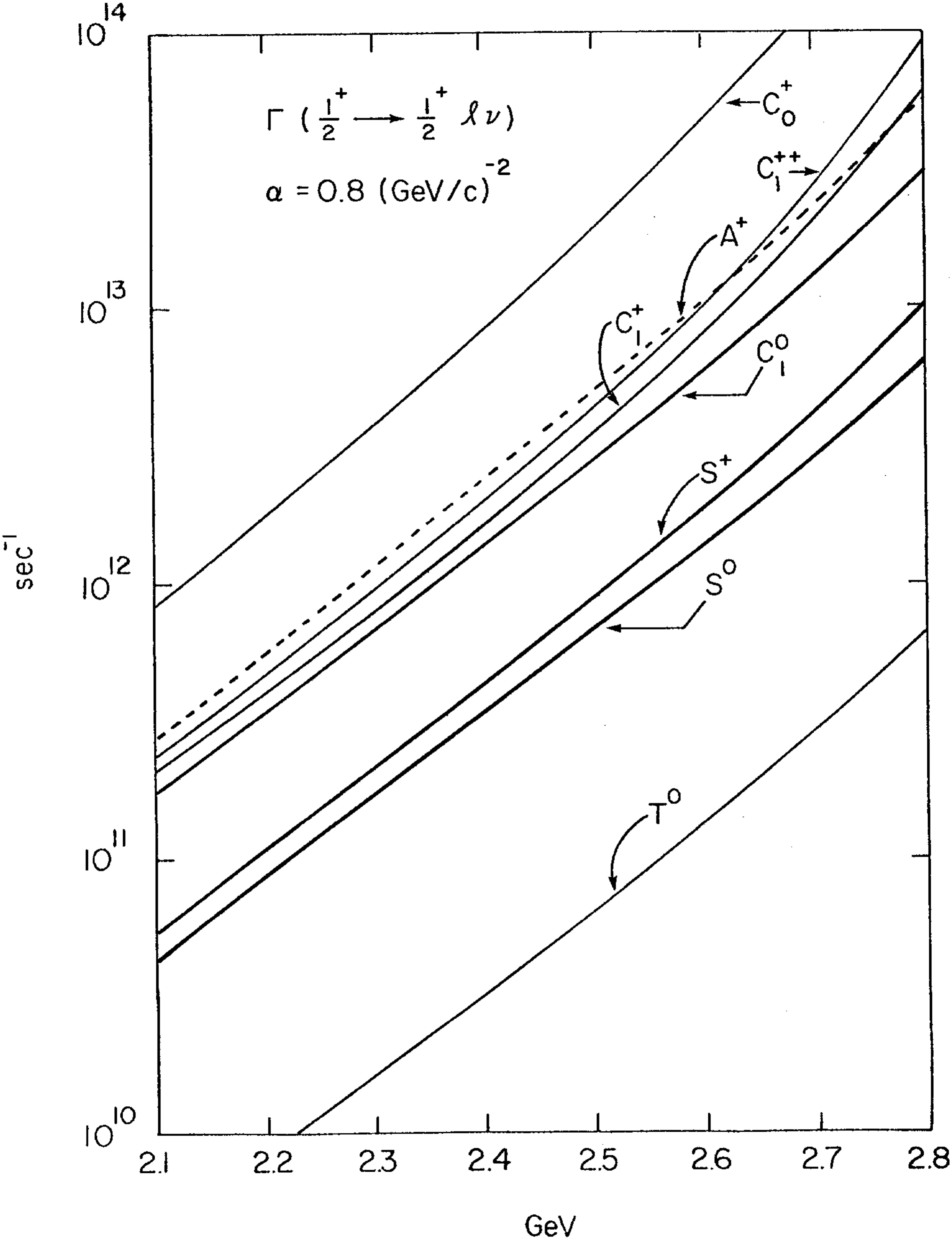


Figure 2



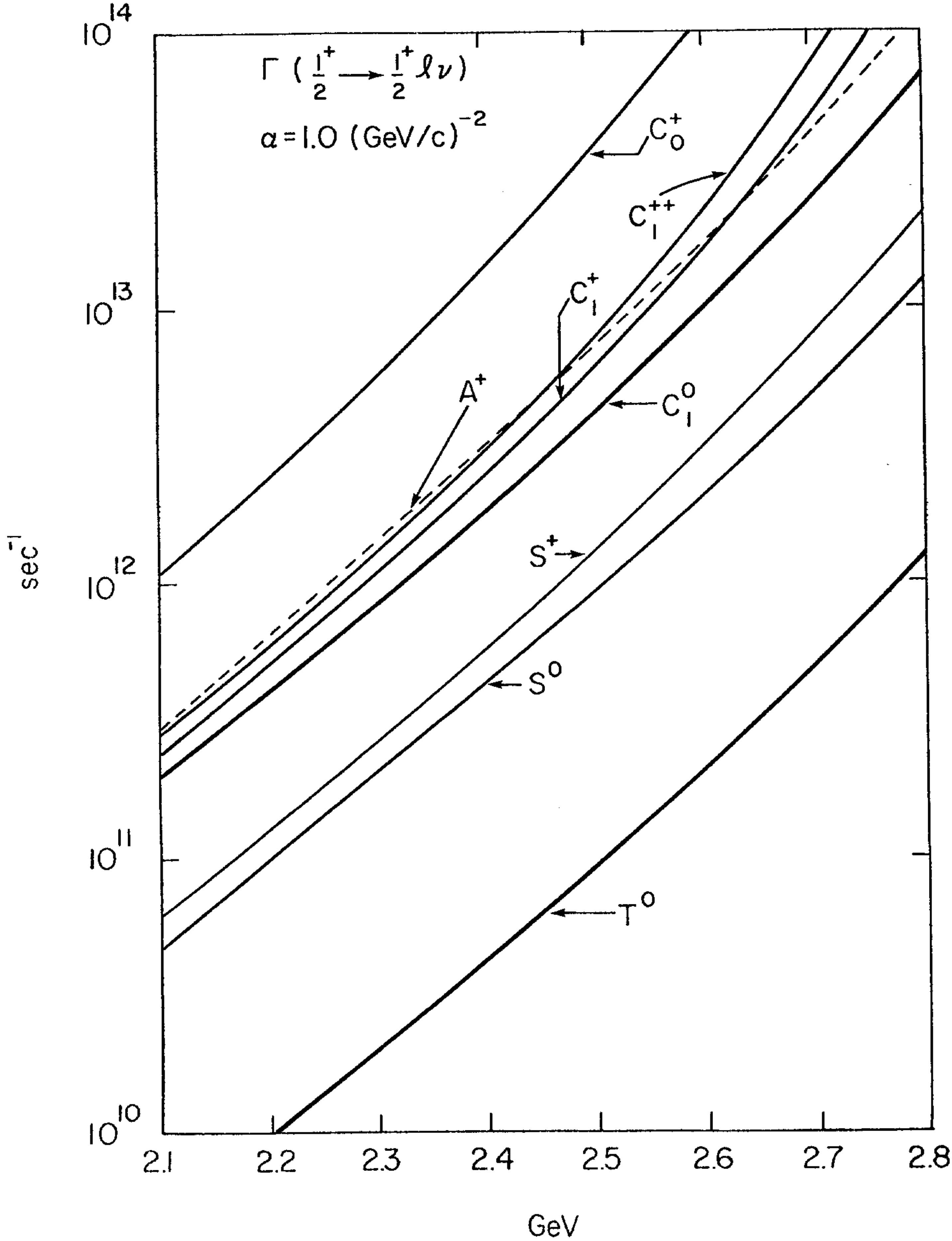
$\Gamma(\frac{1}{2}^+ \rightarrow \frac{1}{2}^+ \ell \nu)$
$\alpha = 0.4\ (\mathrm{GeV}/c)^{-2}$
C_0^+
A^+
C_1^{++}
C_1^+
C_1^0
S^+
S^0
T^0
10^{14}
10^{13}
10^{12}
10^{11}
10^{10}
$\sec^{-1}$
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
GeV

Figure 3



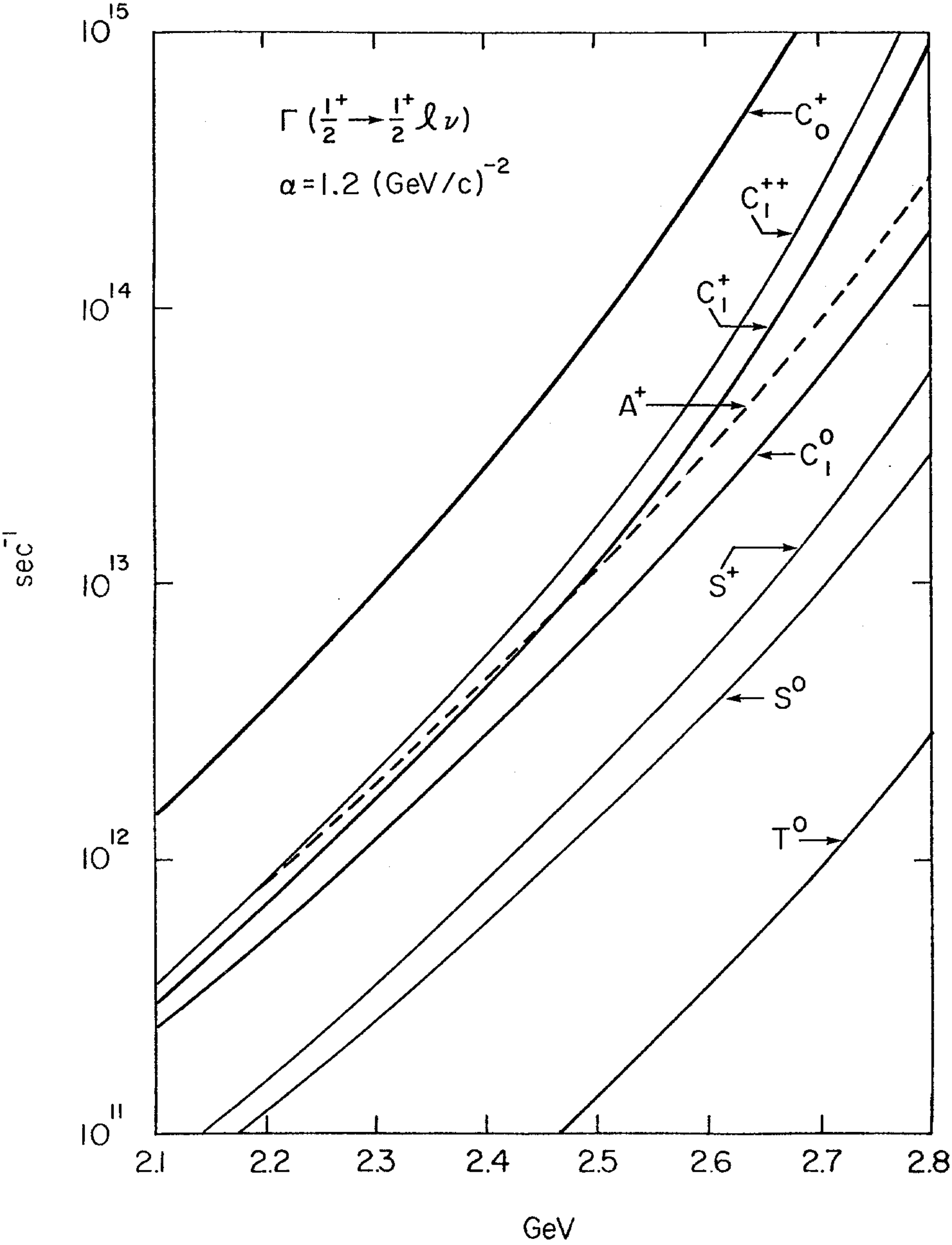
$\Gamma\ (\frac{1}{2}^{+} \longrightarrow \frac{1}{2}^{+}\ \ell\nu)$
$\alpha = 0.8\ (\mathrm{GeV}/c)^{-2}$
C_0^+
C_1^{++}
A^+
C_1^+
C_1^0
S^+
S^0
T^0
10^{14}
10^{13}
10^{12}
10^{11}
10^{10}
sec^{-1}
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
GeV

Figure 4



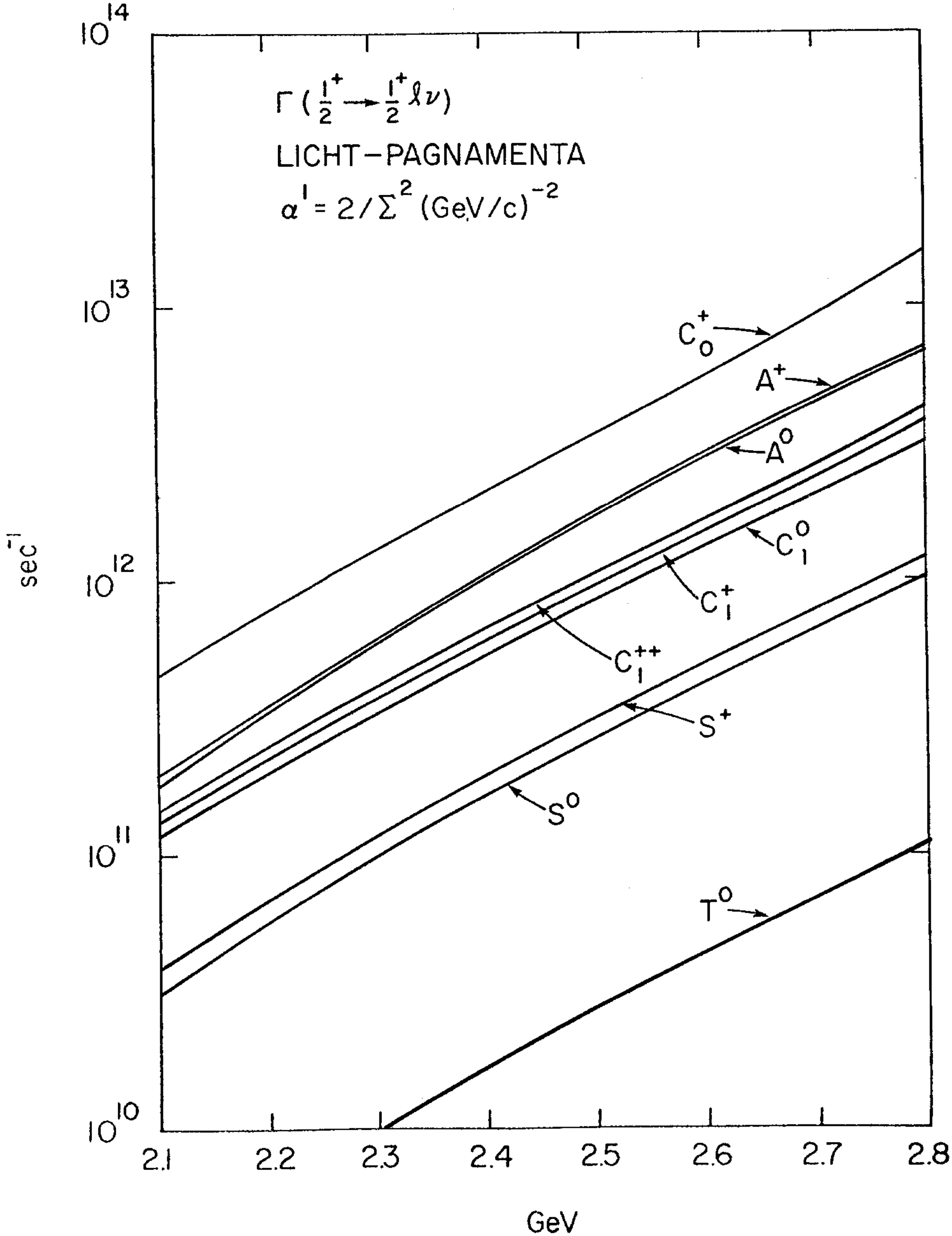
$\Gamma\left(\frac{1}{2}^+ \longrightarrow \frac{1}{2}^+ \ell\nu\right)$
$\alpha = 1.0\ (\mathrm{GeV}/c)^{-2}$
C_0^+
C_1^{++}
C_1^+
A^+
C_1^0
S^+
S^0
T^0
10^{14}
10^{13}
10^{12}
10^{11}
10^{10}
sec^{-1}
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
GeV

Figure 5



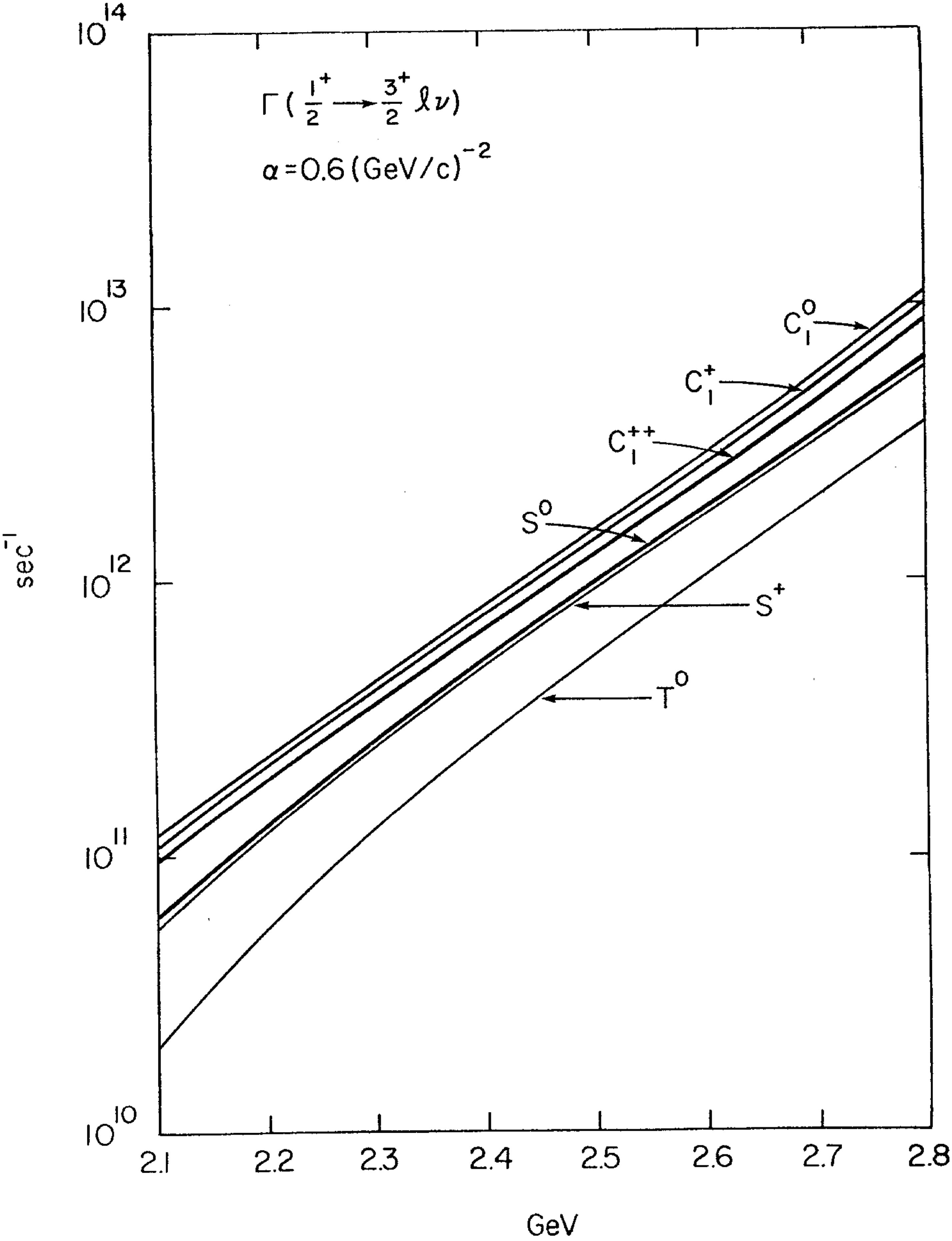
$\Gamma(\frac{1}{2}^+ \to \frac{1}{2}^+ \ell\nu)$
$\alpha = 1.2\ (\mathrm{GeV}/c)^{-2}$
C_0^+
C_1^{++}
C_1^+
A^+
C_1^0
S^+
S^0
T^0
10^{15}
10^{14}
10^{13}
10^{12}
10^{11}
sec^{-1}
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
GeV

Figure 6



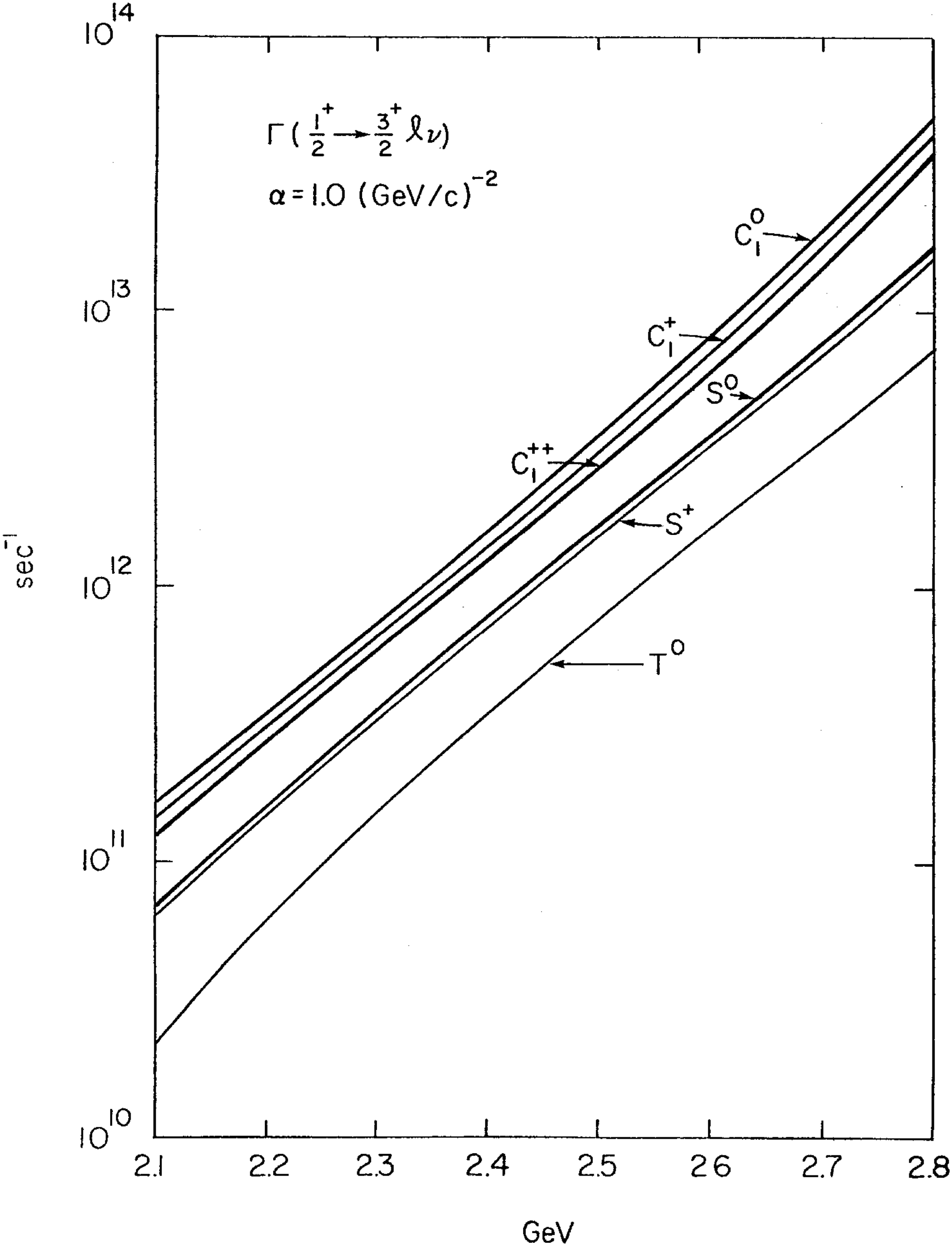
10^{14}
10^{13}
10^{12}
10^{11}
10^{10}
$\Gamma\left(\frac{1}{2}^{+} \rightarrow \frac{1}{2}^{+} \ell \nu\right)$
LICHT-PAGNAMENTA
$\alpha' = 2/\Sigma^2\,(\mathrm{GeV}/c)^{-2}$
C_0^+
A^+
A^0
C_1^0
C_1^+
C_1^{++}
S^+
S^0
T^0
$\sec^{-1}$
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
GeV

Figure 7



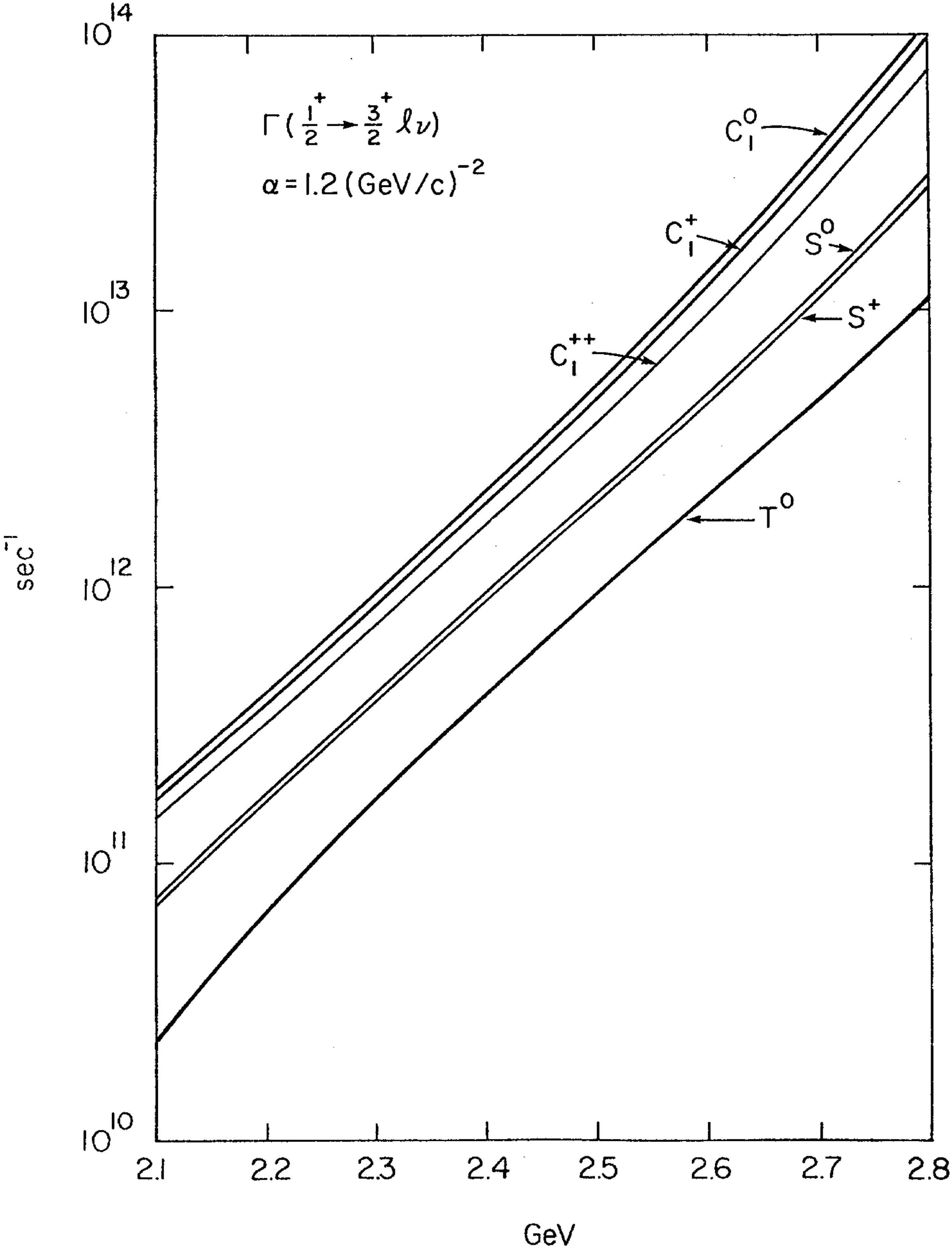
$\Gamma(\frac{1}{2}^+ \to \frac{3}{2}^+ \ell\nu)$
$\alpha = 0.6\,(\mathrm{GeV}/c)^{-2}$
10^{14}
10^{13}
10^{12}
10^{11}
10^{10}
$\sec^{-1}$
C_1^0
C_1^+
C_1^{++}
S^0
S^+
T^0
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
GeV

Figure 8



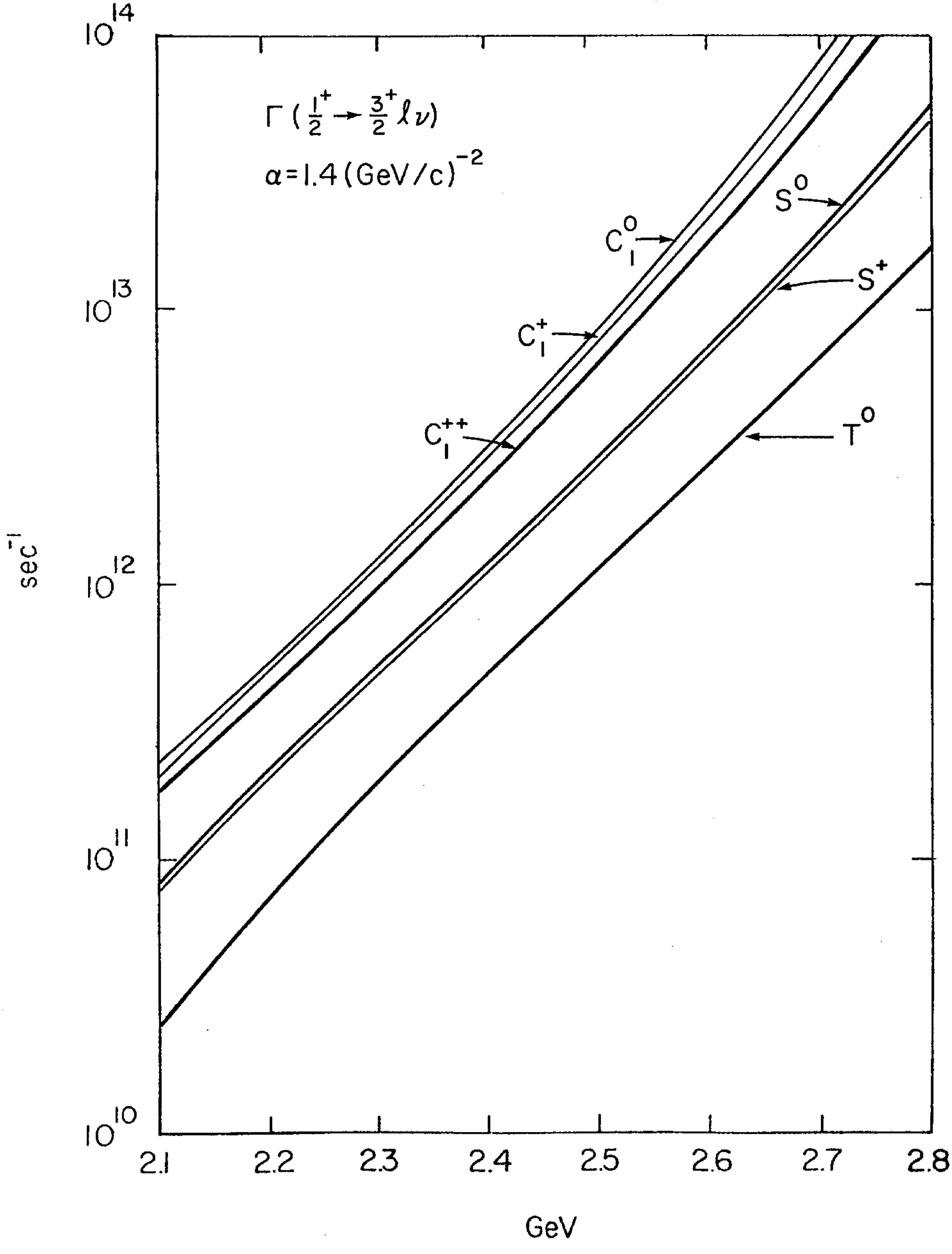
$\Gamma(\frac{1}{2}^+ \rightarrow \frac{3}{2}^+ \ell\nu)$
$\alpha = 1.0\ (\mathrm{GeV/c})^{-2}$
C_1^0
C_1^+
C_1^{++}
S^0
S^+
T^0
10^{14}
10^{13}
10^{12}
10^{11}
10^{10}
sec^{-1}
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
GeV

Figure 9



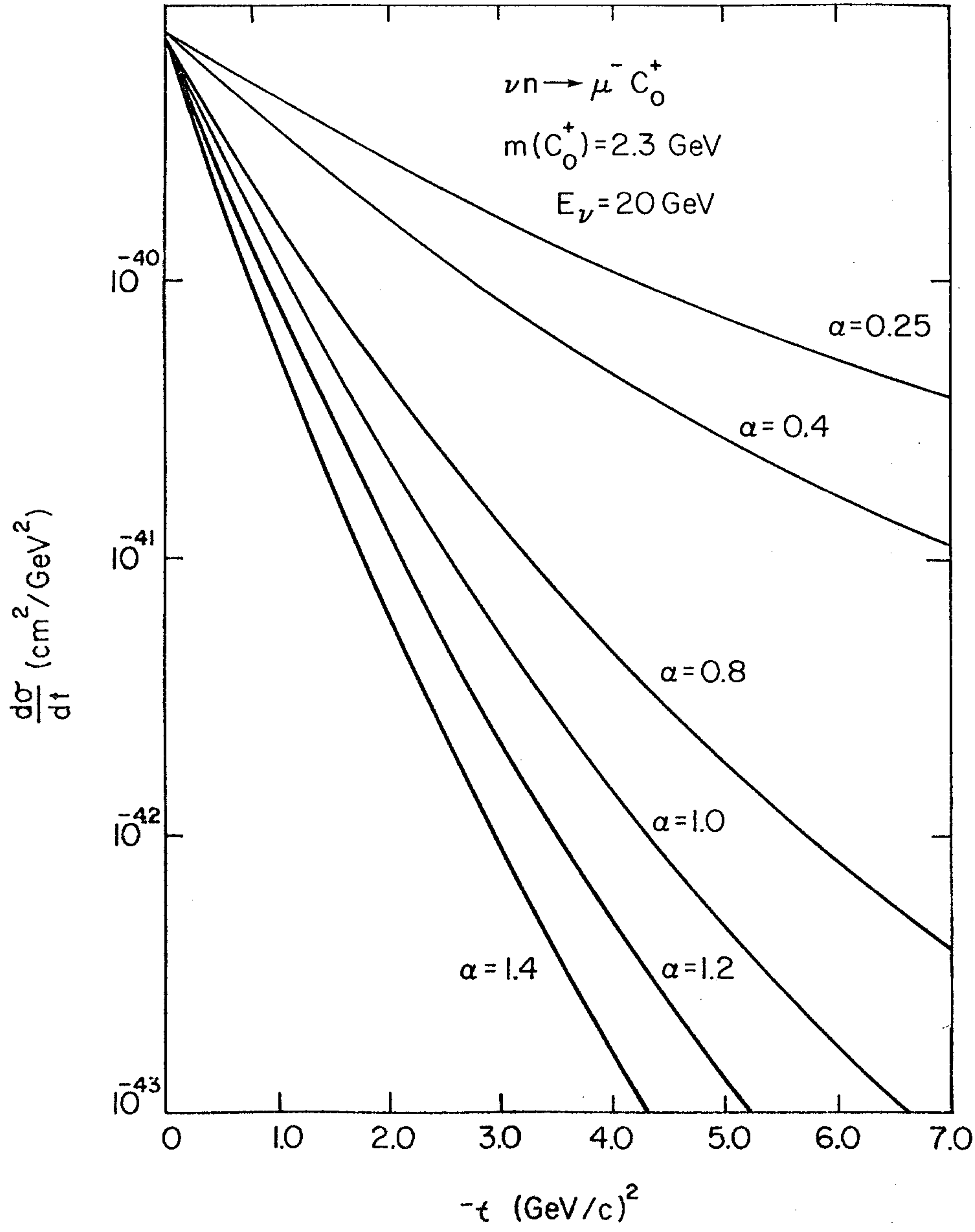
$\Gamma(\frac{1}{2}^{+} \rightarrow \frac{3}{2}^{+} \ell\nu)$
$\alpha = 1.2\,(\mathrm{GeV/c})^{-2}$
C_1^0
C_1^+
C_1^{++}
S^0
S^+
T^0
10^{14}
10^{13}
10^{12}
10^{11}
10^{10}
sec^{-1}
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
GeV

Figure 10



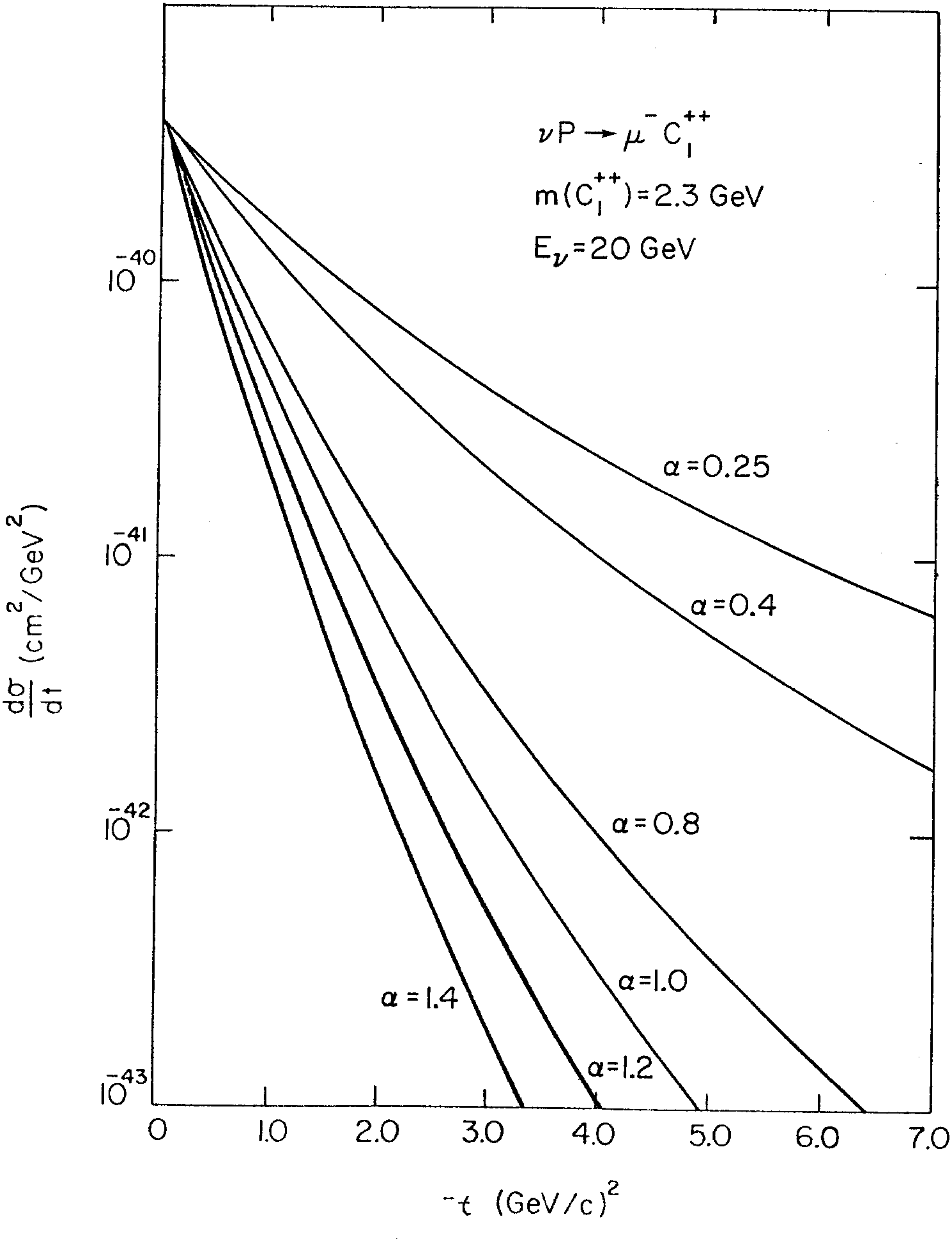
$\Gamma(\frac{1}{2}^+ \to \frac{3}{2}^+ \ell\nu)$
$\alpha = 1.4\,(\mathrm{GeV}/c)^{-2}$
C_1^0
C_1^+
C_1^{++}
S^0
S^+
T^0
10^{14}
10^{13}
10^{12}
10^{11}
10^{10}
$\sec^{-1}$
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
GeV

Figure II (a)



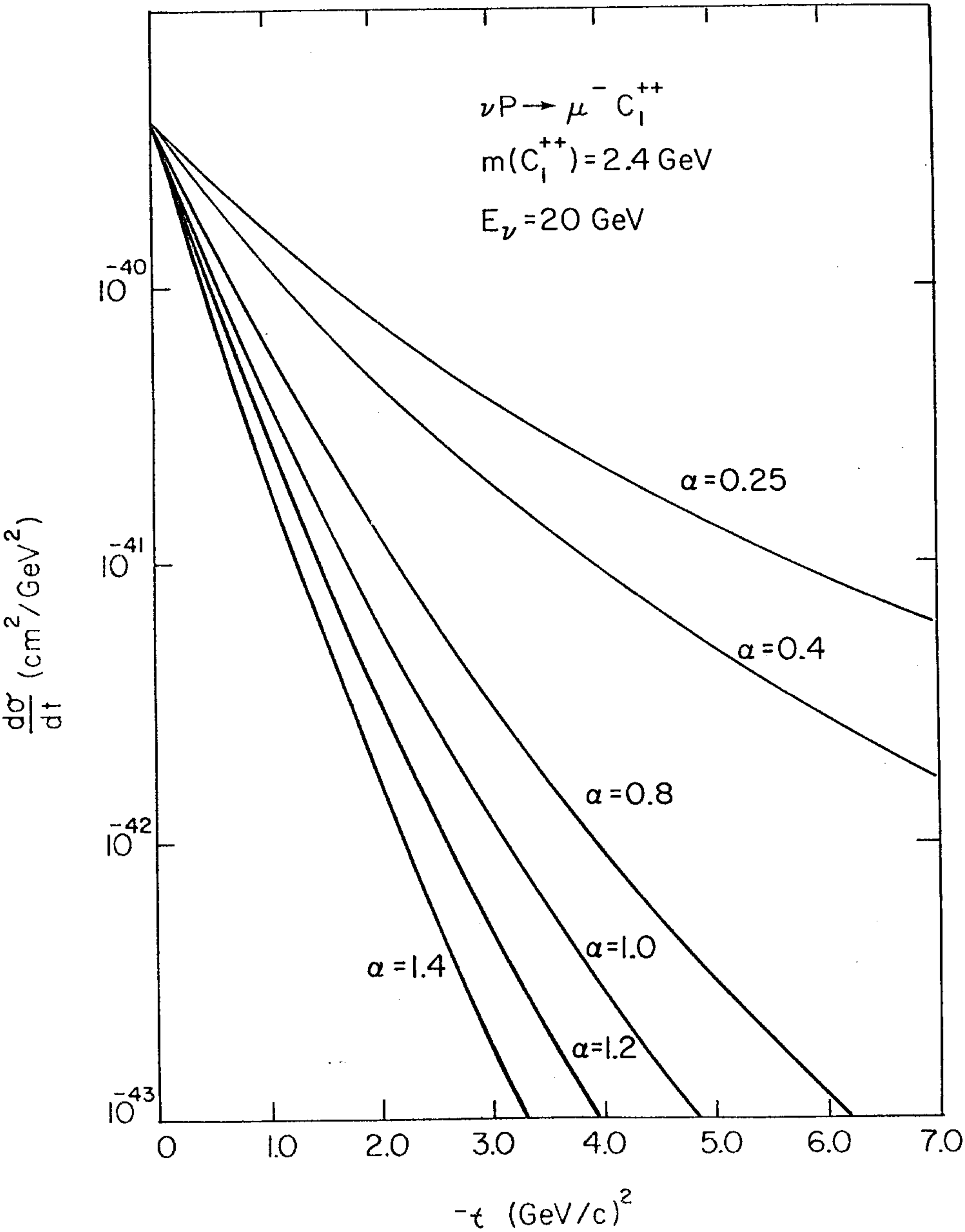
$\nu n \rightarrow \mu^- C_0^+$
$m(C_0^+) = 2.3$ GeV
$E_\nu = 20$ GeV
$\alpha = 0.25$
$\alpha = 0.4$
$\alpha = 0.8$
$\alpha = 1.0$
$\alpha = 1.2$
$\alpha = 1.4$
10^{-40}
10^{-41}
10^{-42}
10^{-43}
$\frac{d\sigma}{dt}$ (cm^2/GeV^2)
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
$-t$ (GeV/c)^2

Figure II (b)



$\nu P \rightarrow \mu^- C_1^{++}$
$m(C_1^{++}) = 2.3$ GeV
$E_\nu = 20$ GeV
$\alpha = 0.25$
$\alpha = 0.4$
$\alpha = 0.8$
$\alpha = 1.0$
$\alpha = 1.2$
$\alpha = 1.4$
10^{-40}
10^{-41}
10^{-42}
10^{-43}
$\frac{d\sigma}{dt}$ (cm^2/GeV^2)
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
$-t$ (GeV/c)^2

Figure II (c)



$\nu P \rightarrow \mu^- C_1^{++}$
$m(C_1^{++}) = 2.4$ GeV
$E_\nu = 20$ GeV
$\alpha = 0.25$
$\alpha = 0.4$
$\alpha = 0.8$
$\alpha = 1.0$
$\alpha = 1.2$
$\alpha = 1.4$
10^{-40}
10^{-41}
10^{-42}
10^{-43}
$\frac{d\sigma}{dt}$ (cm^2/GeV^2)
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
$-t$ (GeV/c)^2

Figure 12 (a)

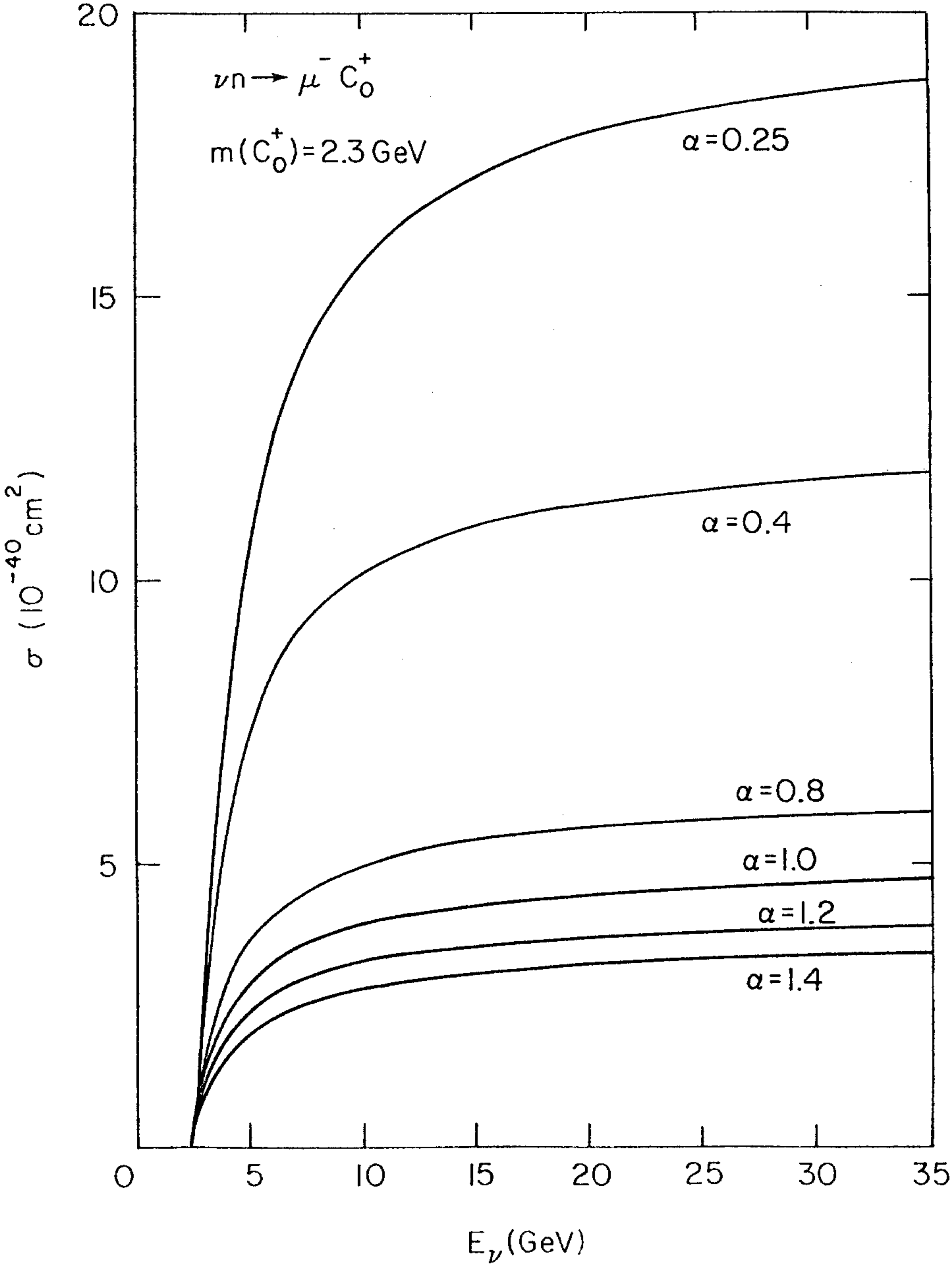
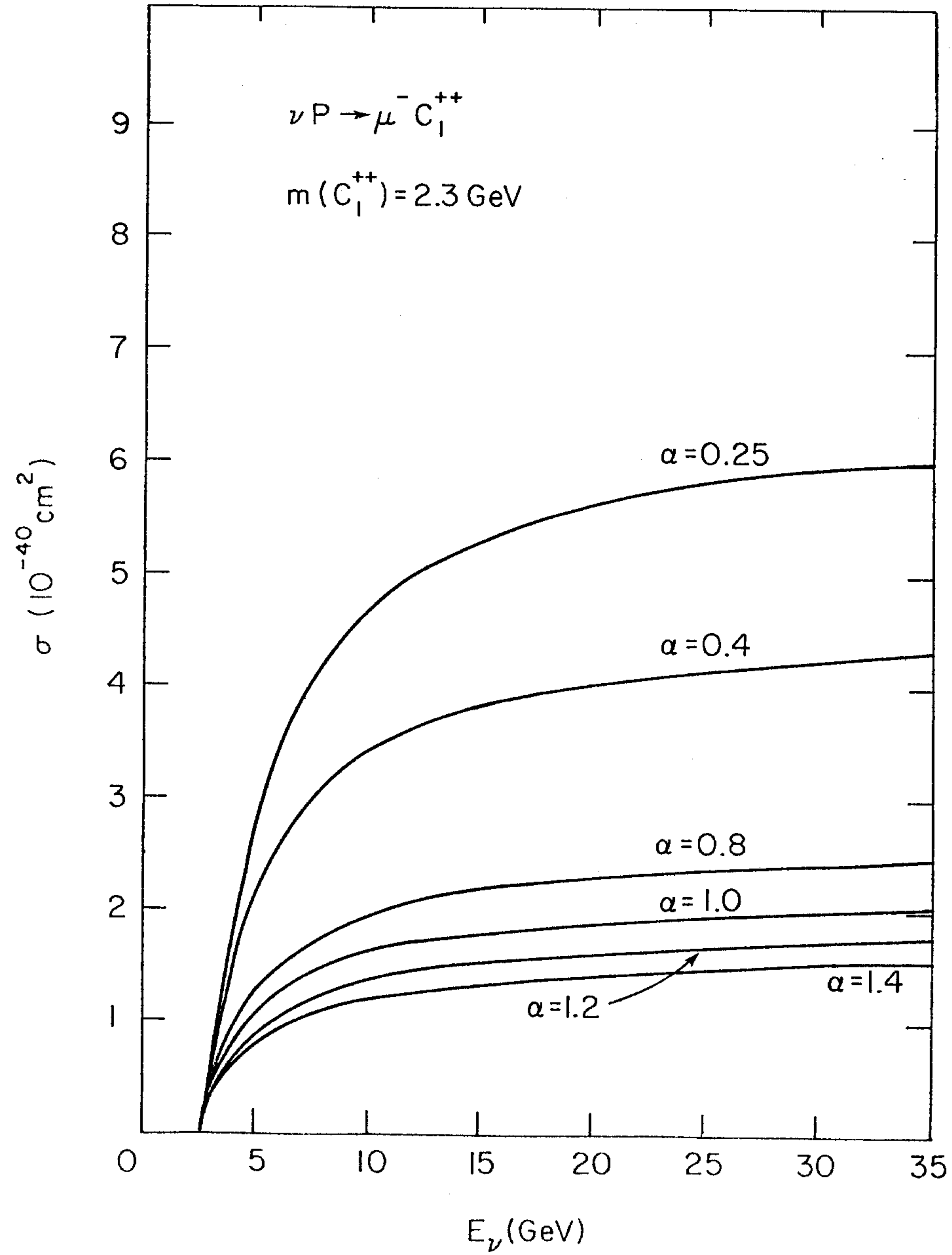
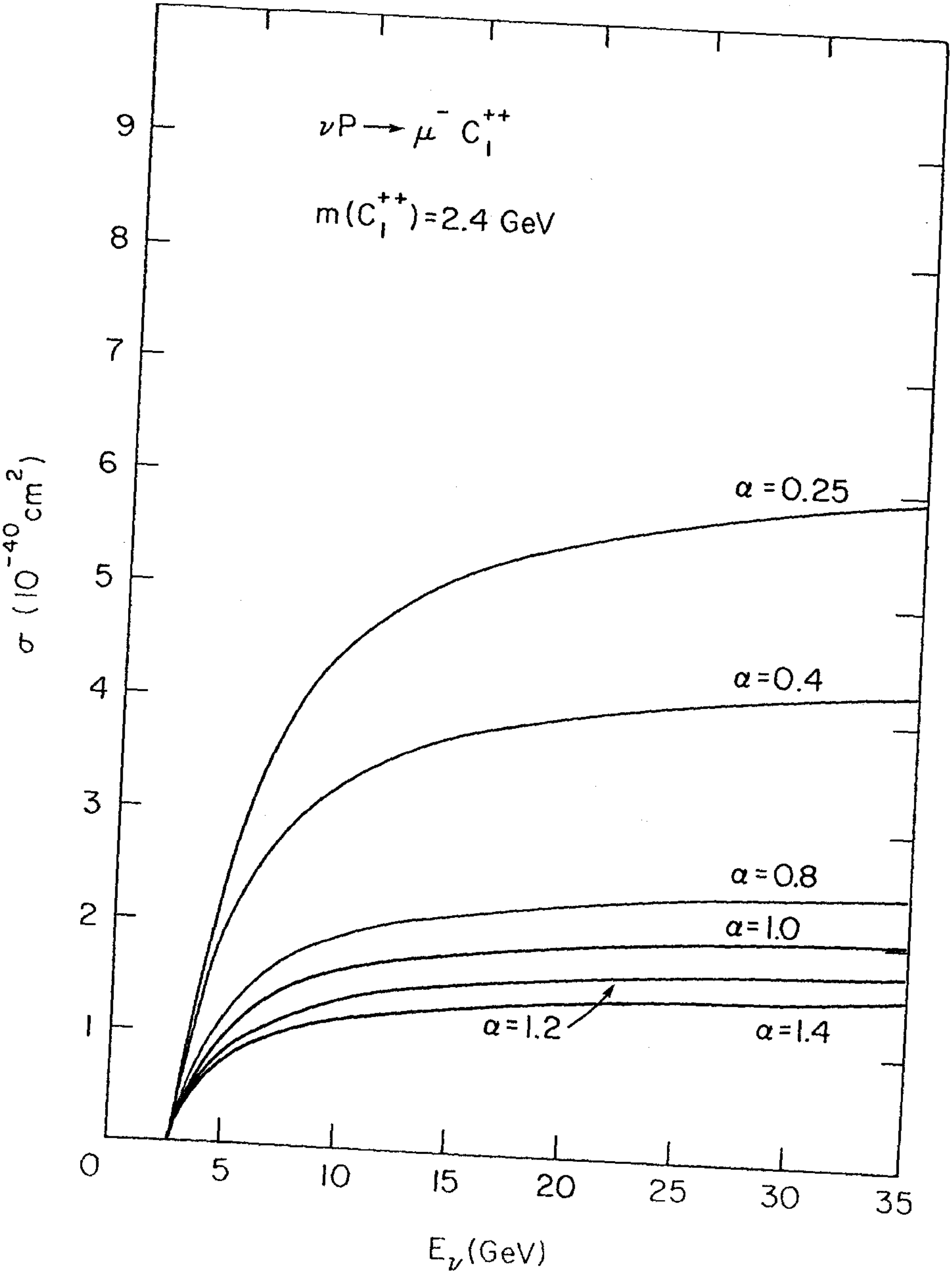

νn → μ⁻ C₀⁺
m(C₀⁺)=2.3 GeV
α=0.25
α=0.4
α=0.8
α=1.0
α=1.2
α=1.4
σ (10⁻⁴⁰cm²)
20
15
10
5
0
5
10
15
20
25
30
35
E_ν(GeV)

Figure 12 (b)



$\nu P \rightarrow \mu^- C_1^{++}$
$m(C_1^{++}) = 2.3$ GeV
$\alpha = 0.25$
$\alpha = 0.4$
$\alpha = 0.8$
$\alpha = 1.0$
$\alpha = 1.2$
$\alpha = 1.4$
σ $(10^{-40} cm^2)$
E_ν (GeV)
0
5
10
15
20
25
30
35
1
2
3
4
5
6
7
8
9

Figure 12 (c)



$\nu P \rightarrow \mu^- C_1^{++}$
$m(C_1^{++}) = 2.4$ GeV
$\alpha = 0.25$
$\alpha = 0.4$
$\alpha = 0.8$
$\alpha = 1.0$
$\alpha = 1.2$
$\alpha = 1.4$
σ (10^{-40} cm^2)
E_ν (GeV)
0
1
2
3
4
5
6
7
8
9
5
10
15
20
25
30
35