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A Natural Composite Model for Quarks and Leptons

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ABSTRACT

It is shown that the radial excitation model of quark/lepton flavor naturally possesses a unitary charged current matrix and thus ensures the absence of the flavor changing neutral currents by means of the GIM mechanism. The non-vanishing overlap of the quark wave functions gives rise to the mixing angles. The CP violating phases are small.

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I. INTRODUCTION

It is conceivable that the key for resolving the "generation problem" of the quarks and leptons will lie in their composite structure. An especially attractive possibility is that flavor is dynamically generated, that is that different generations correspond to certain excitation levels of a composite system. In a recent paper[1] the radial excitation model of flavor has been proposed and studied in greater detail. It has been shown that the model is compatible with the standard QED results in the sense that the contribution, to typical QED processes (like Compton effect on electrons) from the coupling of electrons to excited states are many orders of magnitude smaller than the standard QED results. In particular, it was argued that for bound states of sufficiently small size one obtains arbitrarily small anomalous magnetic moments of the composite leptons. Subsequently, similar results have been obtained by Shaw, Silverman and Slansky[2] and, more detailed, by Brodsky and Drell.[2]

In this paper I show that the model leads to a unitary charged current matrix[3] and thus has naturally no flavor changing neutral currents and provides an explanation of the origin of the mixing angles. Moreover, since the charged current matrix turns out to be approximately real, the Kobayashi-Maskawa CP violating phases[4] are small.

The paper is organized in the following way. The model is described in Section II. A computation of typical charged and neutral current matrix elements is presented in Section III. The results are summarized and commented on in Section IV.

II. THE MODEL

According to the model, the fermions of the first generation (ν_e, e, u, d) are the $1S_{1/2}$ states of some (unspecified) composite systems, those of the second generation (ν_μ, μ, c, s) the $2S_{1/2}$ states of the same systems and so on. The universality is automatic in this scheme since, e.g. electron and muon are by construction two different energy levels of the same quantum system.

This scheme is essentially nonrelativistic and it is appropriate to ask whether it can be based on a set of non self-contradicting assumptions, and if so, what are the assumptions involved.

The fact that the leptons and the quarks appear to be pointlike at momentum transfer much larger than their masses suggests that there must be a large mass scale involved. This can be either the characteristic mass scale Λ in case of a QCD-like theory with light subquarks, or the typical constituent mass m_c in models with heavy constituents. This also seems to be needed in order for the anomalous magnetic moment of leptons to be small: as pointed out in Refs. 1,2 a composite fermion of size a and mass m has an anomalous magnetic moment of the order

$$\Delta(g-2) = O(ma) \quad .$$

The size a of the bound state is $O(\Lambda^{-1})$ and $O(m_c^{-1})$ in the two kinds of models mentioned above. Brodsky and Drell[2] obtain an additional suppression factor of m_f/m_b in the case of a bound state of a fermion of mass m_f and a much heavier boson of mass m_b.

In this paper we consider only the models with two-body bound states and require the constituents (a fermion and a scalar) to be massive.

In principle it is possible to construct light and nonrelativistic bound states of heavy constituents. The kinetic, potential and total bound-state energies are related by means of the virial theorem (see e.g. 5) and it is possible to choose potential so as to keep the kinetic energy small. In particular any such potential must possess an intrinsic length scale. Notorious example is provided by the square well: the depth and the width of the well, and thus the potential and the kinetic energy can be varied independently. Such potentials do not entail the L degeneracy and thus do not necessarily produce the undesired low-lying P states. A phenomenological lower bound for the mass of a P state lepton

$$M > 90 \text{ GeV}$$

has been derived in Ref. 1 from the present bound for the decay $\mu \to e\gamma\gamma$. This would mean that there are at least three radially excited levels (generations) below the lowest P state. Greenberg and Sucher[6] point out that in the case of power-like potentials,

$$V(r) \propto r^{\alpha} \quad ,$$

as α changes from -1 (Coulomb) towards -2 more and more S states sink below the lowest P state. In order to have n radial excitations below the lowest orbital excitation one must have

$$\alpha < -\frac{2n-3}{n-1} \quad .$$

For $n = 3$ generations $\alpha < -3/2$.

Another reason for requiring a potential with an intrinsic scale is the disparity between the mass splitting among the generations (O(1 GeV)) and the inverse radius of the leptons (> 100 GeV). For a scale invariant potential (Coulomb case), by a dimensional argument

$$\text{level splitting} = \mathcal{O}(\text{size}^{-1}) \quad ,$$

and in order to avoid that we need another dimensional parameter in the potential.

Although apparently possible, the potential picture may seem somewhat unnatural: it would involve some "fine tuning" to precisely match the values of various independent parameters so as to obtain the bound states which are essentially massless on the scale of the binding energies. However, as a potential approximation to a field theory which does that in a natural way (i.e. by means of a symmetry) it may be correct. What I have in mind here could be illustrated by the Nambu-Jona-Lasinio model where the massive nucleons bind in a massless pion. This happens naturally--the chiral symmetry is spontaneously broken and the pion is the Goldstone boson. However, not knowing that, we could achieve the same result in a potential model in which various parameters--the depth and the width, spin-spin force etc.--would have to be adjusted and the model would look unnatural. A similar model based on supersymmetry where a massive fermion and a massive scalar bind in a massless fermion is presently under investigation.

Summarizing, the radial excitation model (1) of the quark/lepton generations involves the following set of assumptions:

(1) The quarks and the leptons are two-body bound states of a massive scalar-fermion pair.

(2) The underlying dynamics allows the non-relativistic description in terms of a potential.

(3) The potential is characterized by an intrinsic length scale such that the bound states are of very small size ($<<$ mass $^{-1}$) thereby having (i) small anomalous magnetic moment, (ii) orbital excitations more massive than a certain number of S states, and (iii) the level spacing smaller than the inverse size of the bound state.

Since we wish to consider the general case of N generations, we shall adopt the following notation: the quarks with the charge 2/3 will be denoted by α_k, where $k = 1, 2, \ldots N$ is the generation number, while the quarks with the charge -1/3 are denoted by κ_k. The model can be schematically described by the following table:

State:	$1S_{1/2}$	$2S_{1/2}$	$3S_{1/2}$	...	$NS_{1/2}$	
$Q = +\frac{2}{3}$	$u\ (= \alpha_1)$	$c\ (= \alpha_2)$	$t\ (= \alpha_3)$	...	α_N	(1)
$Q = -\frac{1}{3}$	$d\ (= \kappa_1)$	$s\ (= \kappa_2)$	$b\ (= \kappa_3)$	...	κ_N	

Table 1: The quark generations as radial excitation levels.

and similarly for leptons. N can be infinite (confining composite models) or finite and followed by the continuum. The wave function of each quark (lepton) can be written as a product $\psi_r\psi_i$ of the radial and the internal wave functions. The internal wave functions are <u>identical</u> horizontally and <u>orthogonal</u> vertically, while the radial wave functions are <u>orthogonal</u> horizontally, but only <u>approximately</u> (due to the breaking of the isospin symmetry) <u>identical</u> vertically, i.e.

$$u_i = c_i = \ldots = \alpha_{Ni} \equiv \alpha \qquad ,$$

$$d_i = s_i = \ldots = \kappa_{Ni} \equiv \kappa \qquad ,$$

$$< \alpha, \kappa > = 0 \qquad ,$$

$$<\alpha_{kr}, \alpha_{\ell r}> = \int \alpha^*_{kr}\alpha_{\ell r} r^2 dr = \delta_{k\ell} \qquad ,$$

$$<\kappa_{kr}, \kappa_{\ell r}> = \int \kappa^*_{kr}\kappa_{\ell r} r^2 dr = \delta_{k\ell} \qquad ,$$

but, in general,

$$\langle \alpha_{kr}, \kappa_{\ell r} \rangle \neq \delta_{k\ell} \quad .$$

In order to make the subsequent discussion more transparent, consider a "model" containing 4 fermions U, D, E, N and a scalar ϕ. Let the fermions be grouped into left-handed doublets and right-handed singlets of the conventional SU(2) × U(1)

$$\begin{pmatrix} U \\ D \end{pmatrix}_L \quad \begin{pmatrix} E \\ N \end{pmatrix}_L \qquad U_R \quad D_R \quad E_R \quad N_R$$

with the quantum number assignments as for u, d, e, ν. Let the field ϕ bind with the fermions to produce a set of light states--quarks and leptons--$(U\phi)$, $(D\phi)$, $(E\phi)$ and $(N\phi)$ but otherwise play no role in the electroweak interactions. There is nothing in this scheme that makes it unique or even necessary; it is only convenient for the purpose of illustration.

The subquark currents are then $\bar{U}\gamma_\mu U$, $\bar{D}\gamma_\mu D$, $\bar{E}\gamma_\mu E$ (the electromagnetic ones), $\bar{U}\gamma_\mu(1 + \gamma_5)D$, $\bar{E}\gamma_\mu(1 + \gamma_5)N$ (the charged weak ones) etc.

III. THE CURRENT MATRIX ELEMENTS

Consider first the case N = 2 and neglect the contribution of the continuum. The quark charged current $\bar{u}\gamma_\mu(1 + \gamma_5)d$ can be written as the subquark charged current $\bar{U}\gamma_\mu(1 + \gamma_5)D$ multiplied by the inelastic form factor, i.e.

$$\bar{u}\gamma_\mu(1 + \gamma_5)d = \bar{U}\gamma_\mu(1 + \gamma_5)Df_{ud}(q^2) \quad ,$$

where, in Born approximation

$$f_{ud}(q^2) = \int u_r^{\ *} d_r e^{-i\vec{q}\cdot\vec{r}}\, r^2 dr$$

as illustrated in Fig. 1. Similarly, the charged current form factor between u and s quarks is

$$f_{us}(q^2) = \int u_r^{\ *} s_r e^{-i\vec{q}\cdot\vec{r}}\, r^2 dr \quad .$$

For the momentum transfers much smaller than the inverse size of the quarks $e^{-i\vec{q}\cdot\vec{r}} \approx 1$ in whole integration region and the inelastic form factors are essentially overlap integrals of the quark wave functions. By using the completeness of the radial wave functions (neglecting the continuum) one can show that $|f_{ud}(0)|^2 + |f_{us}(0)|^2 = 1$. Therefore we can write

$$\cos\theta_C = f_{ud}(0) = \langle u_r d_r \rangle \approx \langle c_r, s_r \rangle$$

$$\sin\theta_C = f_{us}(0) = \langle u_r, s_r \rangle \approx -\langle c_r, d_r \rangle \quad , \qquad (2)$$

where approximate equality indicates neglecting the "leak" of unitarity into the continuum.

The neutral current matrix elements, however, vanish between different flavors by orthogonality of the wave functions, e.g.

$$f_{uc}(0) = \langle u_r, c_r \rangle = 0 \quad .$$

The generalized formulation in case of N generations can be given as follows. At momentum transfers much smaller than the inverse size of quarks the composite quarks can be treated as elementary fields. Their charged currents are determined by the matrix

$$C_+(q^2)\big|_{q^2 \ll R^{-2}} \simeq C_+(0) = \begin{pmatrix} \langle u_r, d_r \rangle & \langle u_r, s_r \rangle & \cdots & \langle u_r, \kappa_{Nr} \rangle \\ \langle c_r, d_r \rangle & \langle c_r, s_r \rangle & \cdots & \langle c_r, \kappa_{Nr} \rangle \\ \vdots & \vdots & & \vdots \\ \langle \alpha_{Nr}, d_r \rangle & \langle \alpha_{Nr}, s_r \rangle & \cdots & \langle \alpha_{Nr}, \kappa_{Nr} \rangle \end{pmatrix} \quad . \quad (3)$$

By using the completeness of the radial wave functions one can show that the matrix C_+ is unitary. Since the neutral current is generated by the commutator of the charged currents, and hence proportional to $C_+C_+^\dagger$, this ensures the absence of flavor changing neutral currents.[3] Moreover, since the radial wave functions can be chosen real, the matrix $C_+(0)$ is orthogonal and thus does not contain the CP violating phases[4] for any number of flavors. For $q^2 \neq 0$ the elements of C_+ become complex, however the imaginary parts are probably bound to remain small as long as $q^2 \ll R^{-2}$.

The physical picture which emerges from the foregoing is very simple: the charged weak current couples to the quark/lepton <u>constituents</u>--actually there is only one charged current in the quark sector, $J_+ = \bar{U}D$. However, the mass eigenstates, at least at low energies, are the bound states--quarks and leptons--and their charged currents are modified by the overlap integrals of their spatial wave functions (Fig. 1):

$$\bar{\alpha}_\ell \gamma_\mu (1+\gamma_5)\kappa_k = \bar{U}\gamma_\mu(1+\gamma_5)D\langle\alpha_{\ell r},\kappa_{kr}\rangle = \bar{U}\gamma_\mu(1+\gamma_5)D(C_+)_{\ell k} \quad .$$

It is interesting to ask under what conditions the charged current matrix (2) is $\mathbb{1}$, i.e. there is no mixing. There are two possibilities:

(i) $C_+ = \mathbb{1}$ in the limit of exact isospin symmetry: then $u_r = d_r$, $c_r = s_r, \ldots, \alpha_{Nr} = \kappa_{Nr}$ and $m_u = m_d, m_c = m_b, \ldots, m_{\alpha N} = m_{\kappa N}$ and $C_+ = \mathbb{1}$ follows immediately. Thus the origin of the mixing angles in this model lies in the breaking of isospin symmetry.

(ii) The second case of no mixing, which may be approximately realized in the lepton sector is that of degeneracy of several members of a charge multiplet. Let e.g. first k neutrinos be degenerate, i.e. $m_{\nu_e} = m_{\nu_\mu} = \ldots = m_{\nu_k}$, $k \le N$, and neglect the contribution of higher generations and/or of the continuum. The relevant part of the lepton charged current matrix can be written as

$$C_+(0) = \begin{pmatrix} e_r \\ \mu_r \\ \vdots \\ L_{kr} \end{pmatrix} \widehat{\nu_{er}\nu_{\mu r}\cdots\nu_{kr}} \quad . \tag{4}$$

However, since the wave functions $\nu_{er},\ldots,\nu_{kr}$ are degenerate, one can choose an arbitrary linear combination of them, which amounts to multiplying (4) on right by a unitary $k \times k$ matrix U. Choosing

$$U = \begin{pmatrix} \nu_{er} \\ \nu_{\mu r} \\ \vdots \\ \nu_{kr} \end{pmatrix} \widehat{e_r \mu_r \ldots L_{kr}} \,,$$

C_+ can always be made $\mathbb{1}$.

In both the cases the "generation number" is an additive conserved quantum number.

IV. CONCLUSION

We have shown that the radial excitation model of fermion generations implies absence of flavor changing neutral currents and provides an explanation of the origin of the mixing angles. It does that "naturally," i.e. this is a result of the orthogonality and the completeness of the eigenstates of the "horizontal" Hamiltonian and not of a particular numerical value of a parameter in the model.

If quarks and leptons are considered composite, present gauge theories have to be regarded as effective low-energy theories. In Ref. 1 it was suggested that such theories contain the renormalizable sector which is independent of a (size of the bound state) and thus survives as a $\to 0$ and a non-renormalizable sector containing terms proportional to a, i.e.

$$\mathscr{L}_{\text{total}} = \mathscr{L}_R + \mathscr{L}' + \ldots \tag{5}$$

where $\mathscr{L}_R = O(a^0)$, $\mathscr{L}' = O(a)$, etc. Due to very small size of leptons and quarks, $\mathscr{L}'$ is negligible at present energies. However (for dimensional reasons), it contains operators of higher dimensions (typical term being the Pauli term $a\bar{\psi}\sigma_{\mu\nu}\psi F^{\mu\nu}$) and therefore is expected to grow with energy. The nonrenormalizable couplings of this type introduce flavor changing neutral and electromagnetic currents of same strength as the muon number non-conservation and the anomalous magnetic moment of leptons.

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FIGURE CAPTION

Fig. 1: The charged current vertex of composite quarks.

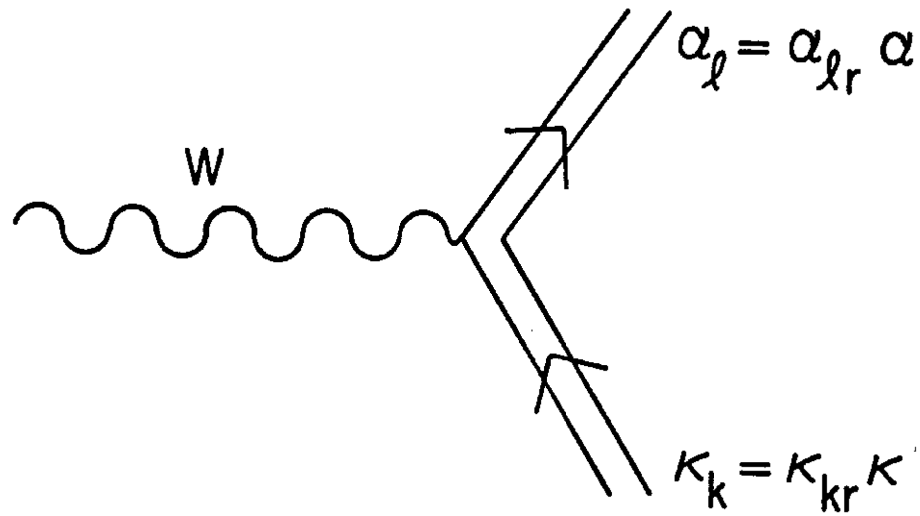

Fig. 1

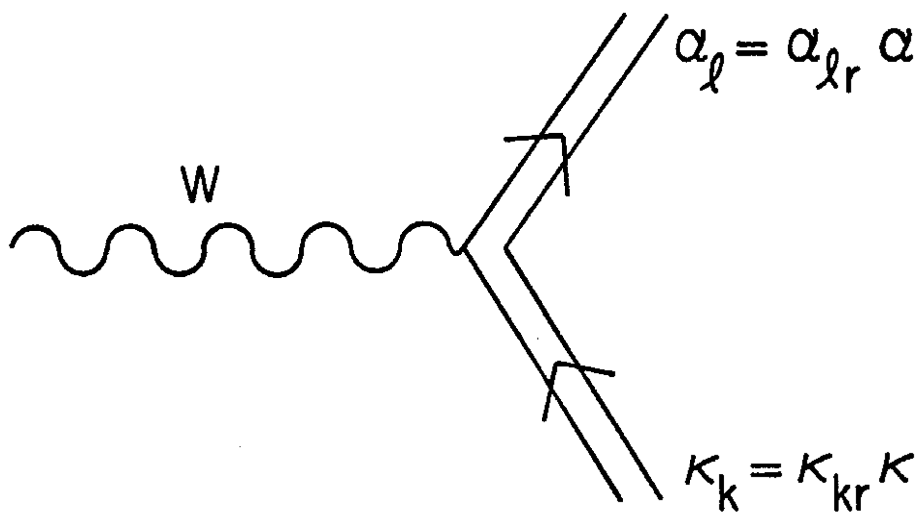

Fig. 1