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Penguins in $\Delta S=1$ Nonleptonic Weak Decays

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Abstract

We discuss the use of the operator product expansion in computing nonleptonic weak decays. The estimation of "penguin" contributions is improved by a careful treatment of the u-c cancellation. Using the vacuum insertion technique and equations of motion to estimate the operator matrix element we find the penguin contribution is only $\frac{1}{10}$ of the experimentally observed $\Delta I=\frac{1}{2}$ amplitude in kaon or hyperon decays.

Ref: 75/79

Much of the recent work on $\Delta S=1$ nonleptonic weak decays[1,2,3] has used the operator product technique to sum radiative corrections to the basic Born graph. The resultant effective $\Delta S=1$ Lagrangian has the form[2]

$$\mathcal{L}_{eff} = \sqrt{2}\, G \sin\theta_c \cos\theta_c \sum_i c_i(M_W^2, m^2, g, \mu^2)\, O^i_{\mu^2}(0) \tag{1}$$

where the coefficient functions may depend on the W boson mass, M_W, and any other mass scales m together with the strong coupling constant g and the operator renormalisation point μ^2.

The operators 0^i are ordered according to their dimension and those with dimension ≤ 6 pick up large logarithmic corrections $\propto \log(\frac{M_W^2}{\mu^2})$ which may be summed by means of the renormalisation group. Operators of dimension > 6 arise, for example, from a diagram like that in Fig. 1(a). To $0(\frac{m^2}{M_W^2})$ the dominant part of this graph comes from the part $\propto \frac{1}{M_W^2}$ in the expansion of the W boson propagator $\frac{1}{(k^2+M_W^2)}$, the residual graph, Fig. 1(b), being convergent. If, as in Fig. 1(b), this graph only involves light quarks the scale of loop momentum is small and this part is normally identified as part of the matrix element of dimension ≤ 6 operators. Unfortunately this is not wholly correct as there are contributions of this type which are not corrected by $\ln(\frac{M_W^2}{\mu^2})$ terms and thus may not be included in the dimension 6 operator contribution. They genuinely correspond to higher dimension operators. It is hoped their contribution will not be significant (they contribute both to $\Delta I=\frac{1}{2}$ and $\Delta I=\frac{3}{2}$ transitions). One possibility is they have small matrix elements in the valence approximation. If the residual graph involves a heavy quark, mass m_{q_h}, in the loop, it will generate a coefficient $\propto (m_{q_h})^{-\ell}$ for a dimension $(6+2\ell)$ operator.

Thus it is usual only to include dimension ≤ 6 operators in eq.(1) to obtain the leading contribution to $\Delta S=1$ nonleptonic decays. Operators of dimension ≤ 4 are not present after renormalisation.

A list of candidate dimension 6 operators $\Delta I=\frac{1}{2}$ is given in ref. (3) and these may be reduced by use of equations of motion to the following set [2]

$$O_1 = \bar{s}_L\gamma_\mu d_L \bar{u}_L\gamma^\mu u_L - \bar{s}_L\gamma_\mu u_L \bar{u}_L\gamma^\mu d_L \; - \; (u \to c)$$

$$\begin{aligned} O_2 = \; & \bar{s}_L\gamma_\mu d_L \bar{u}_L\gamma^\mu u_L + \bar{s}_L\gamma_\mu u_L \bar{u}_L\gamma^\mu d_L + 2\,\bar{s}_L\gamma_\mu d_L \bar{d}_L\gamma^\mu d_L \\ & + 2\,\bar{s}_L\gamma_\mu d_L \bar{s}_L\gamma^\mu s_L - 5\,\bar{s}_L\gamma_\mu d_L \bar{c}_L\gamma^\mu c_L - 5\,\bar{s}_L\gamma_\mu c_L \bar{c}_L\gamma^\mu d_L \end{aligned}$$

$$O_3 = \bar{s}_L\gamma_\mu d_L \bar{u}_L\gamma^\mu u_L - \bar{s}_L\gamma_\mu u_L \bar{u}_L\gamma^\mu d_L \; + \; (u \to c)$$

$$\begin{aligned} O_4 = \; & \bar{s}_L\gamma_\mu d_L \bar{u}_L\gamma^\mu u_L + \bar{s}_L\gamma_\mu u_L \bar{u}_L\gamma^\mu d_L + 2\,\bar{s}_L\gamma_\mu d_L \bar{d}_L\gamma^\mu d_L \\ & + 2\,\bar{s}_L\gamma_\mu d_L \bar{s}_L\gamma^\mu s_L + 2\,\bar{s}_L\gamma_\mu d_L \bar{c}_L\gamma^\mu c_L + 2\,\bar{s}_L\gamma_\mu c_L \bar{c}_L\gamma^\mu d_L \end{aligned}$$

$$\begin{aligned} O_5 = \; & \bar{s}_L\gamma_\mu \lambda^a d_L \left(\bar{u}_R\gamma^\mu \lambda^a u_R + \bar{d}_R\gamma^\mu \lambda^a d_R + \bar{s}_R\gamma^\mu \lambda^a s_R \right. \\ & \left. + \bar{c}_R\gamma^\mu \lambda^a c_R\right) \end{aligned}$$

$$O_6 = \bar{s}_L\gamma_\mu d_L \left(\bar{u}_R\gamma^\mu u_R + \bar{d}_R\gamma^\mu d_R + \bar{s}_R\gamma^\mu s_R + \bar{c}_R\gamma^\mu c_R\right)$$

$$O_7^{\pm} = m_s\,\bar{s}_R\sigma_{\mu\nu}\frac{\lambda_a}{2} d_L G^{\mu\nu,a} \pm m_d\,\bar{s}_R\sigma_{\mu\nu}\frac{\lambda_a}{2} d_R G^{\mu\nu,a} \tag{2}$$

Here λ^A are SU(3) colour matrices and $G^{\mu\nu,A}$ is the gluon field tensor.

In the standard model only the operators O_1 and O_2 arise through the W boson Born graph. This gives (1,2,3) $C_1 = -1$, $C_2 = \frac{1}{5}$. In first order in the strong coupling constant g_s the operators O_5 and O_6, the so-called penguin operators, arise through the graph of Fig.2.

Note that this graph gives the operator $s\gamma_\mu \lambda^a d D_\nu G^a_{\mu\nu}$ which is related by an equation of motion to O_5. The fact that the momentum flowing through the gluon g can be soft does not affect this equation of motion [4]. Also note that the contribution of Fig.2 is $\propto \ln(\frac{m_c^2}{m_u^2})$ and vanishes if $m_u = m_c$ due to the cancellation of the u and c contributions in a theory with the conventional GIM currents. Higher order graphs such as those in Fig.3,4 contribute terms involving large logarithms and these we will sum using renormalisation group techniques.

In order g_s, $O_7^{\pm}$ does not occur. Its leading contribution comes in order g_s^3, and may be expected to be small (2). We discuss this operator elsewhere (10). The anomalous dimension matrix for the set of operators $\{O_1, O_2, O_3, O_4, O_5, O_6\}$ is

$$\gamma(g,\mu^2) = \left(\frac{-2g^2}{16\pi^2}\right)\rho$$

where

$$\rho = \begin{bmatrix} 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{32}{9} & \frac{14}{9} & \frac{16}{9} & 0 \\ 0 & 0 & \frac{2}{9} & \frac{-25}{9} & \frac{-8}{9} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{-7}{6} & \frac{17}{3} & \frac{3}{2} \\ 0 & 0 & 0 & 0 & \frac{16}{3} & 0 \end{bmatrix} \qquad (3)$$

The solution of the renormalization group equations for the coefficient gives

$$\underset{\sim}{C}(M_W^2, m^2, g, \mu^2) = \mathcal{G}\exp\left\{-\int_{\mu_0^2}^{\mu_1^2} \gamma(\bar{g}(\mu'^2), \mu'^2)\,\frac{d\mu'^2}{\mu'^2}\right\}$$
$$\underset{\sim}{C}\left(\mu^2, \mu^2, \bar{g}\left(\frac{\mu_1^2}{\mu_0^2}\right), \mu^2\right) \quad (4)$$

where $\underset{\sim}{C}$ (M_W^2, m^2, g, μ^2) is the column vector of coefficient functions $C_2(M_W^2, m^2, g, \mu^2)$.

$\mathcal{G}$ means the exponential is ordered, and $\bar{g}(Q^2)$ is the usual running coupling constant defined by

$$Q^2 \frac{\partial \bar{g}(Q^2)}{\partial Q^2} = -\left(11 - \frac{2}{3} n_f\right) \frac{g^3}{16\pi^2} + \ldots \equiv -b_{n_f} \frac{g^2}{16\pi^2} + \ldots \qquad (5)$$
$$\bar{g}(\mu^2) = g.$$

n_f is the number of quark flavours.

Equation (4) sums the leading logs of the form $\log\left(\frac{\mu_1^2}{\mu_o^2}\right)$. The identification of $\frac{\mu_1^2}{\mu_o^2}$ requires some care.

The leading log corrections to O_1 and O_2 arise from graphs of the form shown in Fig.3 and explicit calculation shows the argument $\left(\frac{\mu_1^2}{\mu_0^2}\right)$

is $\approx (\frac{M_W^2}{\mu^2})$. The inclusion of "penguin" diagrams involves graphs of the form shown in Fig.4. Explicitly evaluating the contributions to O_5 and O_6 we find terms involving $(\log \frac{M_W^2}{m_{q_2}^2})^2$ and $(\log \frac{m_{q_1}^2}{m_{q_2}^2})^2$, but no mixed terms of the form $\log M_W^2 \log m_{q_1}^2$ that would evidence an enhancement like $(\log \frac{M_W^2}{m_c^2})^2 - (\log \frac{M_W^2}{m_u^2})^2$ after the G.I.M. cancellations.

This persists in higher order as may be seen from the general form of the Feynman integral expressed in terms of Feynman parameters.[5] Logarithms come from minimising the denominator function D which has the form

$$D = \left(\alpha^W M_W^2 + \sum_i \alpha_i^q m_q^2\right) C(\alpha) + F(\mu^2, \alpha) \tag{6}$$

where α_W and α_i^q are the Feynman parameters associated with the W boson propagator and the quark propagator respectively. The dependence of D on m_q will be negligible unless $\alpha^W \lesssim \frac{m_q^2}{M_W^2}$.

Integrating with respect to α^W in this region gives logarithmic terms

$$\ln\left[M_W^2 C'' + \sum_i \alpha_i^q m_q^2 C' + \sum_i \alpha_i^q m_q^2 C'' + F''(\mu^2) + F'(\mu^2)\right] \tag{7}$$

and

$$\ln\left[\sum_i \alpha_i^q m_q^2 C'' + F'[\mu^2]\right] \tag{8}$$

together with the new denominator

$$D' = M_W^2 C'' + \sum_i \alpha_i^q m_q^2 C' + F'' \tag{9}$$

Here we have written

$$C(\alpha) = \alpha^W C' + C''$$
$$F(\alpha) = \alpha^W F' + F'' \tag{10}$$

Further logs of M_W^2 come from the region $C'' \lesssim \frac{m_c^2}{M_W^2}$. The first log term in eq. (7) will give either $\ln^2 M_W^2$ or $\ln^2 m_c^2$ terms. The second log term in eq. (8) gives only $\ln F'(\mu^2) \ln M_W^2$. Clearly this argument may be repeated for further zeros of C''.

Thus the log summation appropriate to diagrams as in Fig.4 has the argument $(\frac{\mu_1^2}{\mu_0^2})$ approximately given by $(\frac{m_{q_1}^2}{m_{q_2}^2})$. All other terms cancel between the u and c contributions. This gives zero mixing between $\bar{d}\gamma_\mu u_L \bar{u}\gamma^\mu s_L$ and 0_5, 0_6 since $m_{q_1}^2 \approx m_{q_2}^2$. For the mixing between $\bar{d}\gamma_\mu c_L \bar{c}\gamma^\mu s_L$ and 0_5, 0_6 we choose $\frac{\mu_1^2}{\mu_0^2} = \frac{1}{a} \frac{m_c^2}{\mu^2}$ where the constant of proportionality $\frac{1}{a}$ is chosen to take account of the fact that the argument of the logs is not quite $\frac{m_c^2}{\mu^2}$. For example Fig.2 gives

$$\int_0^1 dx \, (x-x^2)^2 \ln \left\{ \frac{m_c^2 + \mu^2 x(1-x)}{m_u^2 + \mu^2 x(1-x)} \right\}$$

having a factor $a \lesssim \frac{1}{4}$. In higher order this will not be the same factor but we expect some similar suppression of μ^2 relative to μ_c^2.

Since only $\bar{d}\gamma_\mu c_L \bar{c}\gamma^\mu s_L$ mixes to 0_5 and 0_6 the relevant initial coefficients necessary to calculate the penguin contribution are those corresponding to this operator: $C_3 = -\frac{1}{2}$, $C_4 = \frac{1}{+}$.

Then we find from eq.(4)

$$\begin{aligned}
c_1 &= -x_1^{4} \\
c_2 &= \frac{1}{5} x_1^{-2} \\
c_5 &= 10^{-2}\left(2.76\, x_2^{3.51} + 3.18\, x_2^{-3.03} - 4.89\, x_2^{7.05} - 1.04\, x_2^{-1.09}\right) \\
c_6 &= 10^{-3}\left(4.19\, x_2^{3.51} - 5.60\, x_2^{-3.03} - 3.70\, x_2^{7.05} + 5.12\, x_2^{-1.09}\right)
\end{aligned}$$

where, for $n_f = 4$

$$x_1 = \left(\frac{\bar{g}^2(m_c^2)}{\bar{g}^2(M_W^2)}\right)^{1/b_4} \left(\frac{\bar{g}^2(\mu^2)}{\bar{g}^2(M_W^2)}\right)^{1/b_3} \qquad (10)$$

$$x_2 = \left(\frac{\bar{g}^2(\mu^2)}{\bar{g}^2(a m_c^2)}\right)^{1/b_3}$$

In table 1 we tabulate these coefficients for two choices of $g^2(\mu^2)/4\pi$, and two choices of a.

Before we can compare our effective Lagrangian with experiment it is necessary to estimate matrix elements of the contributing operators. One technique that has been widely used[6,2] is to factorise the operators, or operators related by Fierz transformations, in all possible ways. Thus, for example,

$$\langle \pi^+\pi^- | O_5 | K_S^0 \rangle = -\frac{32\sqrt{2}}{9} \left\{ \langle \pi^+ | \bar{u}_R d_L | 0 \rangle \langle \pi^- | \bar{s}_L u_R | K^0 \rangle + \langle \pi^+\pi^- | \bar{d}_R d_L | 0 \rangle \langle 0 | \bar{s}_L d_R | K^0 \rangle \right\} \tag{11}$$

The remaining matrix elements are estimated via equations of motion e.g.

$$\begin{aligned} \langle \pi^+ | \bar{u}_R d_L | 0 \rangle &= -\frac{i}{(m_u + m_d)} \langle \pi^+ | \partial_\mu \bar{u} \gamma_\mu \gamma_5 d | 0 \rangle \\ &= -\frac{i f_\pi m_\pi^2}{2(m_u + m_d)} \end{aligned} \tag{12}$$

In writing eq. (11) and eq. (12) no mention has been made of the operator normalisation μ^2 - yet this is important.

O_5, for example, is not invariant under a change in μ^2 and this is reflected in eq. (12) by the fact that the quark masses used should be those relevant to the scale μ^2. What are reasonable values for these masses? Current algebra gives a value for $\frac{m_{u,d}}{m_s} \sim \frac{1}{20}$ [8]. This ratio is approximately valid for μ^2 much greater than symmetry breaking effects. The absolute value of m_s is normally taken from mass differences between members h of a unitary multiplet using the relation[8]

$$m_h = \text{const} + m_s(\mu^2) \langle \bar{s}s(\mu^2) \rangle_h \tag{11}$$

Obviously before $m_s(\mu^2)$ can have a meaning the relative normalisation of m_s and $\bar{s}s$ must be defined. If μ^2 is chosen to be of the same order as the mean momentum squared as that found in hadron h it should be reasonable then to use for $\langle\bar{s}s\rangle_h$ a bag model estimate. Such a calculation gives $\langle\bar{s}s\rangle_h = 0.48\, N_h$ where N_h is the number of strange valence quarks in h. Assuming the current algebra ratios work at the same scale gives[8] $m_s \approx 300$ MeV, $m_d \approx 15$ MeV, $m_u \approx 8$ MeV. With the values for C_5 and C_6 given in Table 1 (for $\mu^2 = (0.7\ \text{GeV})^2$) together with the operator matrix elements evaluated as above we find the operators 0_5 and 0_6 can account for only $(\frac{1}{24} - \frac{1}{18})$ to $(\frac{1}{9} - \frac{1}{6})$ of the $\Delta I=\frac{1}{2}$ amplitudes observed in kaon and hyperon decay (corresponding to $\frac{g^2(\mu^2)}{4\pi} = 1$ and 2.4 respectively). The contribution of the operators 0_1 and 0_2 is essentially unchanged from previous analyses and, in the factorisation approximation, contributes about $(\frac{1}{5})$ to $(\frac{3}{10})$ of the observed amplitudes in kaon decays. For the case $\frac{g^2}{4\pi}(\mu^2) = 2.4$ the perturbation expansion probably breaks down near μ^2. We include it as an upper estimate of the calculable contribution to 0_i.

It is amusing to ask what happens for a different choice of μ^2. $m(\mu^2)$ decreases as μ^2 increases approximately as[7]

$$m(\mu_1^2) = \left(\frac{\bar{g}^2(\mu_1^2)}{\bar{g}^2(\mu^2)}\right)^{12/25} m(\mu^2)$$

For $\mu_1^2 = 2\text{GeV}^2$ this increases the estimate of the matrix elements of 0_5 and 0_6 by a factor of 2.5 to 4. However calculation of $C_5(\mu^2)$ and $C_6(\mu^2)$ for this new subtraction point (cf Table 2) shows they are suppressed by factors of $\frac{1}{5}$ to $\frac{1}{7}$. The product C0 is reasonably constant (it would be exactly so if the factorisation technique were exact) and remains too small to explain the large $\Delta I=\frac{1}{2}$ amplitude.

Of course the factorisation technique for evaluating matrix elements is suspect and it is important to look for other methods. One recent approach uses PCAC plus the bag model[9]. It suffers from serious difficulties in continuing to the soft pion limit but the results are broadly in agreement with the factorisation results. We also note that with the normalisation of the s quark given above charm PCAC [8] gives a value for $m_c \approx 1.7$ GeV. This is in reasonable agreement with the determination of m_c from the ψ mass [7] assuming free field matrix elements for ψ operator matrix elements normalised at the scale m_c.

In conclusion we have re-examined the operator product analysis for $\Delta S=1$ nonleptonic weak decays. The operator ordering according to dimension is not justified by short distance arguments alone and requires assumptions about the relevant operator matrix elements. Evaluation of the contribution of the dimension 6 operators suggests that the penguin contribution is too small to account for the large $\Delta I=\frac{1}{2}$ enhancements found unless important contributions to the coefficient functions arise from the non-short distance parts of the integrand. In this case the contribution of other operators such as O_7 may be important.[10,11]

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$\frac{g^2(\mu^2)}{4\pi}$	(Equivalent Λ (Gev))	a	C_1	C_2	C_5	C_6
$\frac{1}{2}$	.25	1	-2.33	0.13	-0.033	-0.006
$\frac{1}{2}$	.25	$\frac{1}{4}$	-2.33	0.13	-0.052	-0.013
2.4	.5	1	-3.68	0.10	-0.089	-0.031
2.4	.5	$\frac{1}{4}$	-3.68	0.10	-0.139	-0.058

Table 1

Coefficients C_1, C_2, C_5, C_6 calculated for $\mu = .7$ Gev, $M_W = 100$ Gev and various values of the strong coupling constant and scale factor a.

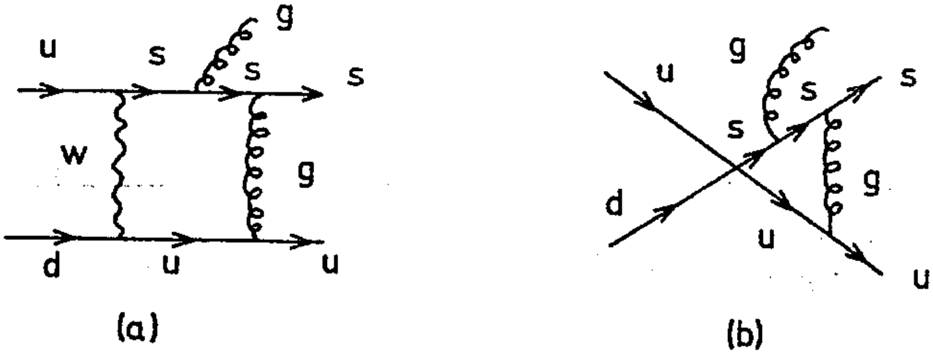

Fig. 1

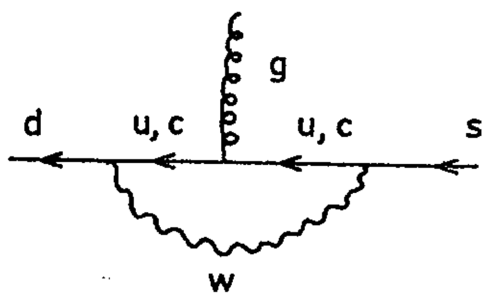

Fig. 2

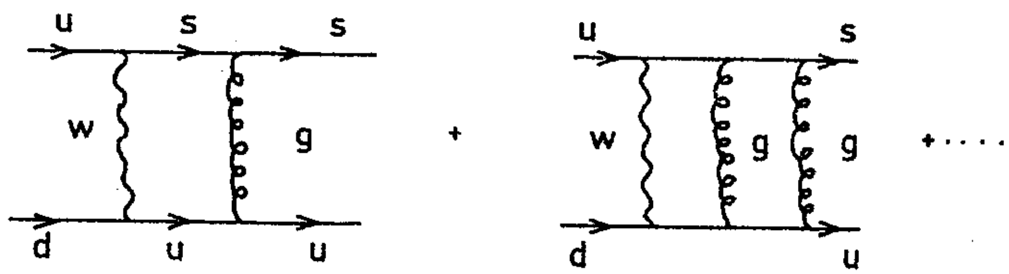

Fig. 3

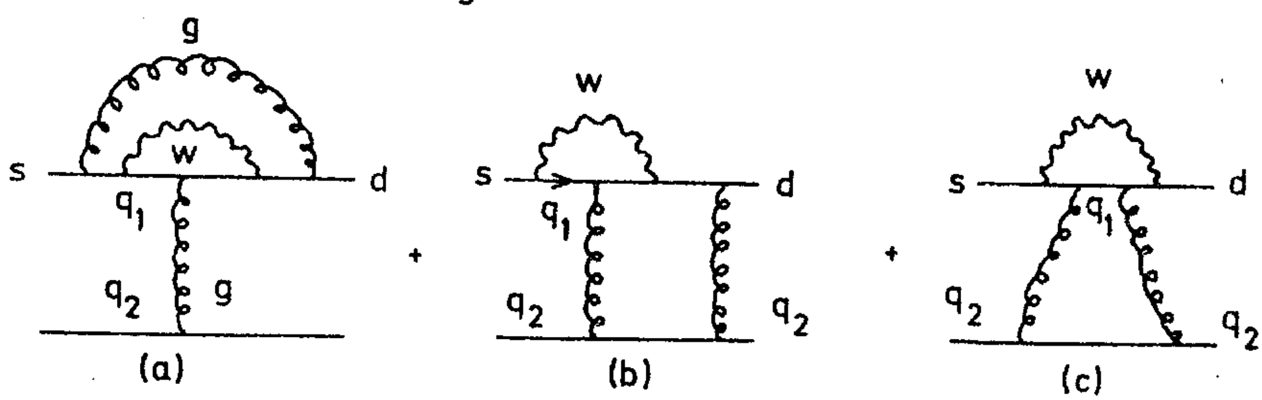

Fig. 4