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RADIATIVE CORRECTIONS FOR THE EXPERIMENTER

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ABSTRACT

A number of calculations exist for radiative corrections to the cross-section for exclusive reactions, but they are generally applicable only for the specific experimental arrangement for which they were derived. We show how to use these existing calculations to derive the correction for any other experimental situation. The radiative correction to high energy backward πp elastic scattering is calculated as an example.

I. INTRODUCTION

The calculation of radiative corrections to experimental data is a problem often considered difficult by experimenters because the complete calculation is complicated and because it is very dependent on the details of the experimental arrangement and analysis. The experimenter usually finds that published calculations are not adequate for the specific experimental cuts used in the analysis of his experiment. The alternative of calculating the correction from first principles involves many difficulties including the proper cancellation of divergent integrals.

In this paper we consider the problem of radiative corrections to the cross-section for exclusive reactions; we show how the experimenter can easily adapt a radiative correction (RC) which has already been calculated for a given set of experimental cuts to the cuts used in his experiment.

This paper specifically results from the calculation of the RC to pion-proton backward elastic scattering[1]. The complete calculation of the RC for this reaction was first made by Sogard[2] who published results for many elastic hadron scattering processes and showed that RC's can be significant for hadronic reactions, especially at energies of 100 GeV or greater. Additional work by E. Borie[3] was based on Sogard's formulae. Sogard's results, however, were questioned by Ginzburg, Kotkin and Serbo[4] (GKS). Using the results of Gribov[5], they showed that because the behavior of hadrons when interacting with massive or high transverse momentum photons is not point-like, the final correction should be considerably smaller than Sogard's correction. We have based our calculation of the RC for our own experiment on the GKS results.

In this article we do not use the "peaking approximation"[6], a simplification in which the angular distribution of the radiated photon is ignored, because it is often inadequate for muonic and hadronic scattering processes.

Section II is an introduction to radiative corrections explaining the basic concepts and definitions. While it refers to the radiative correction to a hadronic cross-section, the statements are quite general and can be made about the cross-section due to any type of interaction. Section III gives the formula for the experimenter's calculation, and in the appendix the formula is applied to the specific problem of backward pion-proton scattering.

II. GENERAL STATEMENTS ON RADIATIVE CORRECTIONS

We are interested in measuring a hadronic cross-section σ_H which corresponds to an integral involving the T-matrix element of the diagram in Fig. 1.

The cross-section experimentally measured includes not only the diagram of Fig. 1, the reaction to be studied, but also radiative diagrams in which a real photon is emitted and radiative diagrams in which a virtual photon is transferred. The lowest order diagrams of these types are shown in Fig. 2 and Fig. 3 respectively. If we assume we know the T-matrix element of Fig. 1 then it is possible to calculate the diagrams of Fig. 2 and Fig. 3 using the rules of quantum electrodynamics (QED).[3,4]

If we neglect the changes in the kinematics of the hadronic reaction due to the radiation, the integrals associated with the radiative diagrams will be proportional to the integral for the diagram of Fig. 1. Therefore, it is standard procedure[6-8] in this field to introduce the radiative correction, δ, which is defined by the equation

$$\sigma_1 = (1 + \delta) \quad \sigma_H \tag{1}$$

where σ_1 is the cross-section including radiative terms up to first-order in the fine structure constant α, and σ_H is the cross-section for the reaction of Fig. 1 alone. (We use the more common sign convention for δ in which δ is in practice generally less than zero.)

δ is the sum of two terms

$$\delta = \delta_R + \delta_V \tag{2}$$

where δ_R refers to the calculation of the diagrams of Fig. 2 and δ_V refers to the calculation of the interference term between the diagrams of Fig. 1 and those of Fig. 3. (The term from Fig. 3 alone is of second-order in α, and not included in the calculation of δ.) The integral which determines δ_R is positive (its integrand is always positive) and diverges logarithmically (the infrared divergence) as the photon momentum approaches zero. This integral is a function of the experimental cuts (see section III). The integral which determines δ_V is negative and similarly diverges. It is independent of the experimental situation since the diagrams of Fig. 3 lead to the same final state as that of Fig. 1. The two divergences cancel when the sum δ is calculated. In most practical cases $|\delta_V|$ is larger than $|\delta_R|$, making δ negative.

We have discussed so far how δ is determined by calculating the contribution of the radiative diagrams only up to the first order in α. Using the results of Yennie, Frautschi and Suura[9], we can include the effect of higher order terms without the need for additional calculations. They have shown that the infrared divergences cancel in the higher order terms also and that if $|\delta|$ is small compared to one (as it usually is), we can take into account the higher order radiative diagrams by replacing Eq. 1 with the formula,

$$\sigma_M = e^{\delta}\, \sigma_H$$

where σ_M is the experimentally measured cross-section. Therefore, the hadronic cross-section of interest is

$$\sigma_H = e^{-\delta}\, \sigma_M \tag{3}$$

III. CALCULATION OF RADIATIVE CORRECTIONS AS A FUNCTION OF THE EXPERIMENTAL CUTS

The experimental cuts can be represented by a surface in three-dimensional photon-momentum space enclosing zero photon momentum. This is the surface of the largest momentum a single emitted photon can have such that the outgoing particles are still accepted by the experimental cuts. The integral which determines δ_R is over the region of all photon momenta within this surface.

Given a calculation in which a radiative correction δ_0 is evaluated for a given set of experimental cuts, an experimenter can calculate the difference between δ_1, the correction for his cuts, and δ_0 by evaluating the integral for δ_R over the region, A, of photon momentum space between the two corresponding surfaces.

This procedure allows the experimenter to bypass the problems of cancelling the divergences and of calculating the δ_V integral which, since it is not limited by the experimental cuts, necessarily includes the large photon momentum region where many simplifying approximations break down. Furthermore, if the region, A, is limited to sufficiently small (see next paragraph) photon momenta, the experimenter's integral becomes quite simple.

Specifically, let us assume we have a scattering process $1 + 2 \rightarrow 3 + 4$ with four-momenta p_1 through p_4 and charges z_1 through z_4. We then have:

$$D \equiv \delta_1 - \delta_0 = \int_\lambda \frac{d^3\vec{k}}{\omega} \frac{\alpha}{4\pi^2} \chi^2 \qquad (4)$$

where ω is the magnitude of the photon momentum vector $\vec{k}$, α is the fine structure constant, and

$$\chi^2 = -\left(z_3 \frac{p_3}{p_3 \cdot k} - z_1 \frac{p_1}{p_1 \cdot k} + z_4 \frac{p_4}{p_4 \cdot k} - z_2 \frac{p_2}{p_2 \cdot k} \right)^2 \quad (5)$$

Equation (4) is derived from Tsai's[8] approximations to QED and two additional approximations. Tsai's approximations are made by neglecting the spin term[7] and internal radiative diagrams[7], and by assuming that the photons are sufficiently soft ($2k.p_i << s$ for $i=1-4$ where $s=(p_1+p_2)^2$).

In addition we assume that the changes in the kinematics (the momenta p_i) due to the radiation do not cause large fractional changes in χ^2 or in the hadronic cross-section. This approximation greatly reduces the complexity of the relevant equations[10] and is generally valid when the photon is soft (as defined above) since the changes in the p_i are slight.[11]

Finally, we treat hadrons as point particles. This approximation is considered valid by GKS[9] when the photon momentum transverse to the interacting particles is less than the mass of the pion.[12]

Equation (4) can be simplified by using spherical coordinates for the photon momentum vector (ω, θ and ϕ). Expressed in these variables, χ^2 factors into:

$$\chi^2 = (1/\omega^2) \quad R(\theta,\phi) \tag{6}$$

where $R(\theta,\phi)$ is the dependence of χ^2 on the angle variables and can be physically interpreted as giving the angular dependence of soft photons emitted in the interaction. After evaluating the integral in eq. (4) over ω, we are left with our final result:

$$D = \frac{\alpha}{4\pi^2}\int_0^{2\pi}\int_{-1}^{1} \ln \frac{\omega_1(\theta,\phi)}{\omega_0(\theta,\phi)} \, R(\theta,\phi) \quad d(\cos\theta)d\phi \tag{7}$$

where ω_0 and ω_1 are, for a given photon direction, the photon momentum limits for the existing calculation of δ and for the experiment, respectively.

The use of equation (7), in conjunction with existing calculations of the radiative correction parameter (such as refs. 2-4) enables an experimenter to calculate the radiative corrections for his own experiment.

The application of equation (7) to the calculation of the radiative correction for backward pion-proton elastic scattering is given in the appendix.

IV ACKNOWLEDGEMENT

I wish to thank Michael Sogard for explaining radiative corrections to me when I was beginning this project and Roy Rubinstein for many useful comments on this article.

APPENDIX
RADIATIVE CORRECTIONS FOR BACKWARD $\pi^{\pm}p$ ELASTIC SCATTERING

We studied $\pi^{\pm}p$ backward (near 180° in the cms) elastic scattering for incident energies of 30 to 90 Gev[1]. In our experiment we measured the momenta of both outgoing tracks and extracted an elastic signal in the analysis using four cuts on the data. For a given scattering angle of the outgoing backward pion, we made cuts on the momentum of the pion, the momentum of the proton and finally on the horizontal and vertical projected angle of the proton. These cuts can be defined by limits on the parameters a, b, Δp_x, and Δp_y respectively.

$$a = (|\vec{p}_3| - |\vec{p}_{3el}|)/|\vec{p}_{3el}|$$

$$b = (|\vec{p}_4| - |\vec{p}_{4el}|)/|\vec{p}_{4el}|$$

$$\Delta p_x = p_{x3} + p_{x4} ; \qquad \Delta p_y = p_{y3} + p_{y4}$$

where the x and y subscripts refer to the two components of the momentum transverse to the incident beam direction.

We determined $\omega_1(\theta, \phi)$, the maximum photon energy corresponding to the above cuts, with a computer program which calculated the kinematics of the radiative reaction. We similarly calculated $\omega_0(\theta, \phi)$ for the cuts defined both in refs. 2 and 4 and using equations (5) and (6), we evaluated $R(\theta, \phi)$. The three functions were obtained only for πp scattering at exactly 180° because the variation in the radiative correction with the scattering angle for our range of angles is negligible. R and ω_o are both independent of ϕ because of the azimuthal symmetry of our reaction. Therefore, in order to make $\omega_1(\theta, \phi)$ azimuthally symmetric, we approximated our cuts on Δp_x and Δp_y, which were equal, using a cut on $\Delta p_\perp$(where $\Delta p_\perp = (\Delta p_x{}^2 + \Delta p_y{}^2)^{1/2}$) equal to 1.1 times the cut on Δp_x.

After integrating equation (7) over ϕ, we found:

$$D = \frac{\alpha}{2\pi} \int_{-1}^{+1} \ln(\omega_1/\omega_0)\, R\, d(\cos\theta) \qquad (8)$$

This one dimensional integral was calculated numerically on the Fermilab CDC 6600 computer. The function R, ω_1 and ω_0 were all calculated in the laboratory system reference frame but the final result, D, is independent of the Lorentz frame chosen. We show the functions R, ω_1, and ω_0 as calculated in the lab frame for an incident pion beam energy of 50 GeV in figures 4 and 5. We checked that the photon momenta in our region, A, (between ω_0 of Ginzberg and ω_1 in Fig. 5) all satisfy the limits described in Section III.

The integral of eq. 8 must be evaluated with special care when $\cos\theta$ approaches 1 due to the large variations of R in that region and because R, evaluated using eq. (5), becomes the difference of two very large but nearly equal numbers.

As we stated in the introduction, we used the GKS calculation of δ_0 for the correction in $\pi^- p$ backward scattering instead of the earlier results of Sogard.[2] GKS claimed a correction approximately 25 percent less than that of Sogard. They did not give an estimate of accuracy for their method for all their results but stated that for 100 GeV $\pi^- p$ backward scattering, their value (-0.35) for δ was within .05 of being the correct value.

However, consistently applying their correction at different energies should lead to considerably smaller errors in the cross-sections of one momentum relative to another. We estimate, therefore, that these relative errors of the cross-sections in our energy range (30-90 GeV) are less than 1% although the absolute radiatively corrected cross-section may have an error up to 5% due to the uncertainty in the radiative correction. The corrections for our experimental cuts are shown in Table I.

The radiative corrections for π^+p backward scattering using Sogard's calculation are quite small (2 to 3%). We have used this calculation for δ_0 as it is sufficiently accurate for our purpose.

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[10] See Ref. 2 equations 2.2 to 2.4

[11] Using our method with this approximation for the specific case of backward πp elastic scattering, we obtained good agreement with Borie's[3] parameter C which gives the variation of the radiative correction with an experimental cut. Borie's results, based on the equations of Sogard[2], do not make this approximation so the use of this approximation in our case is justified.

[12] The GKS[4] disagreement with Sogard arises in the evaluation of δ_V where part of the integration necessarily deals with massive or high transverse momentum photons and this approximation can not be used.

TABLE I

THE RADIATIVE CORRECTION FOR $\pi^- p$ BACKWARD SCATTERING

Energy (GeV)	Analysis Cuts			(a)			(c)
	$\|a\|$	$\|b\|$	$\Delta p_\perp$ (MeV)	δ_o	D	δ	$e^{-\delta_1}$
30	15.%	2.5%	55.	−.266	.139	−.127	1.14
50	15 %	2.0%	44	−.323	.158	−.165	1.18
50 (b)	10	2.0	44	−.323	.157	−.166	1.18
50 (b)	20	2.0	44	−.323	.159	−.164	1.18
50 (b)	15	4.0	44	−.323	.169	−.154	1.17
50 (b)	15	2.0	88	−.323	.202	−.121	1.13
50 (b)	15	4.0	88	−.323	.221	−.102	1.11
70	15	2.0	66	−.362	.216	−.146	1.16
90	15	2.0	77	−.394	.251	−.143	1.15

(a) δ_o is calculated using Ref. 4 with ω_M = 40 MeV.

(b) These entries show the effect of varying the cuts on the result.

(c) This is equal to σ_R/σ_M

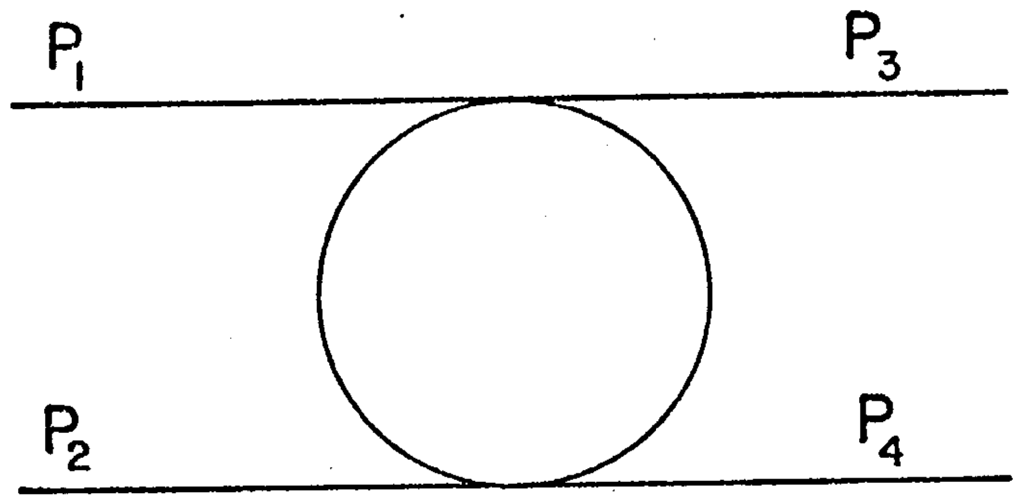

Fig. 1. Feynman diagram for the reaction the experimenter wants to study.

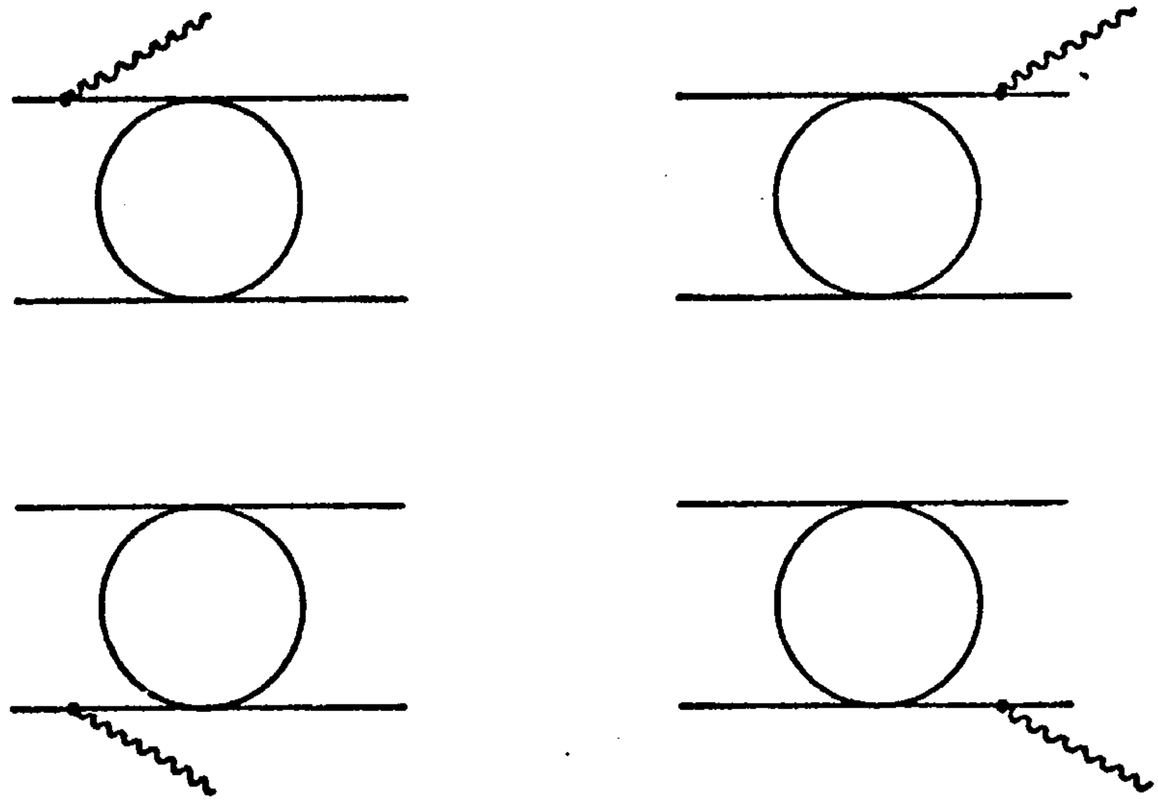

Fig. 2. Feynman diagram associated with the emission of a real photon.

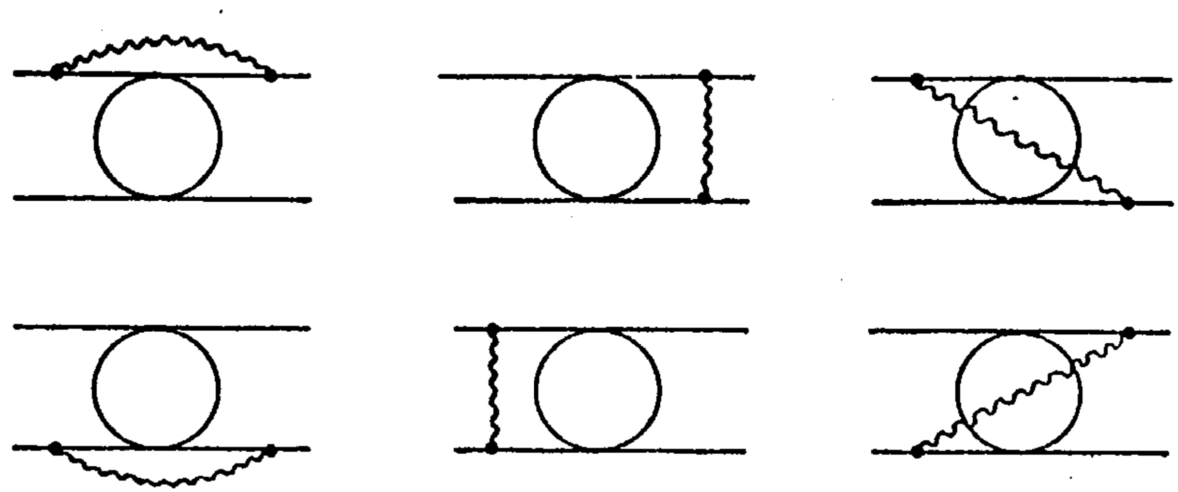

Fig. 3. Feynman diagrams associated with the exchange of a virtual photon.

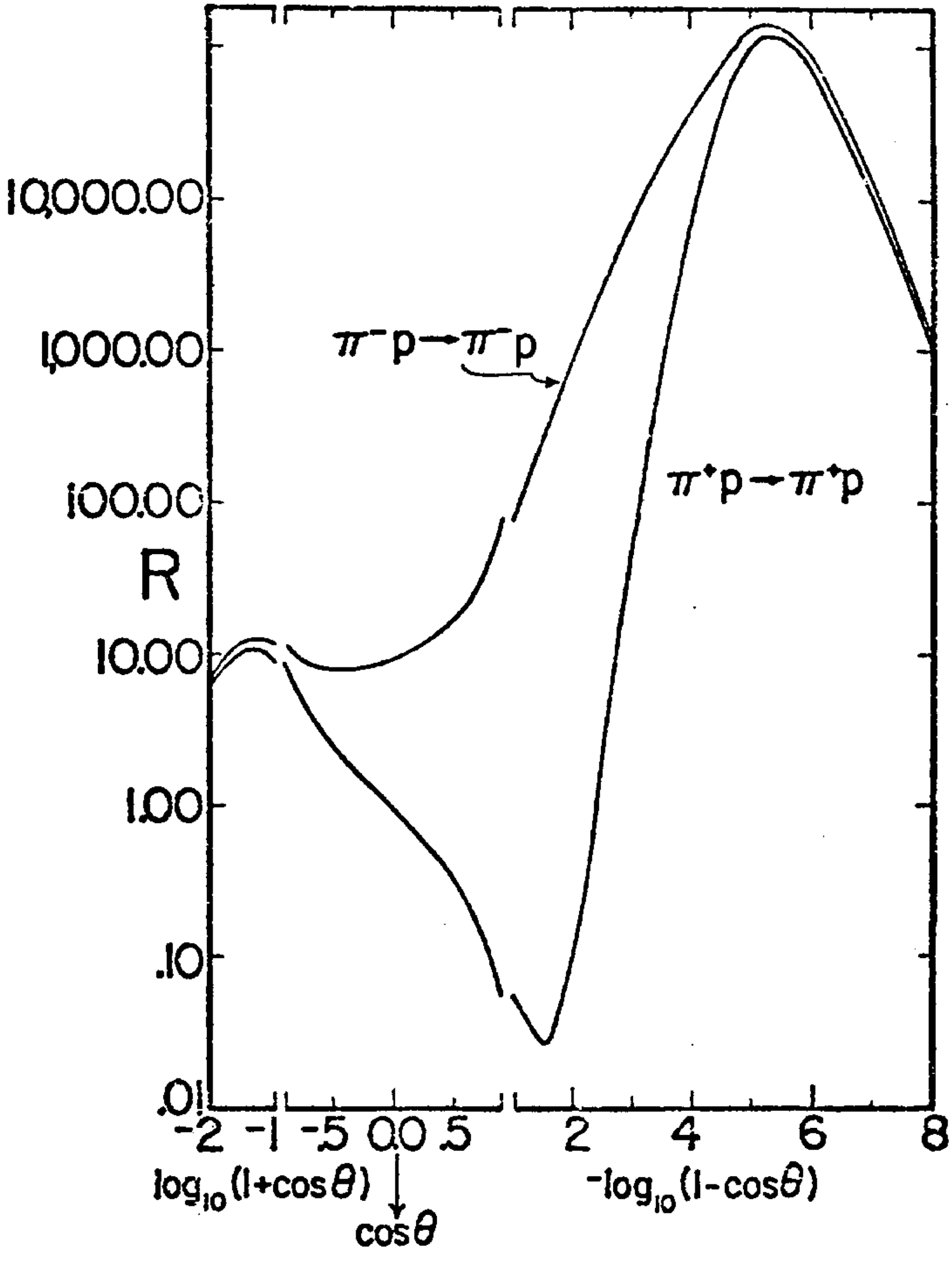

Fig. 4. $R(\cos\theta) = \omega^2\chi^2$, as a function of $\cos\theta$ for backward elastic scattering of a 50-GeV incident pion. θ is the polar angle of emission of the photon in the laboratory frame.

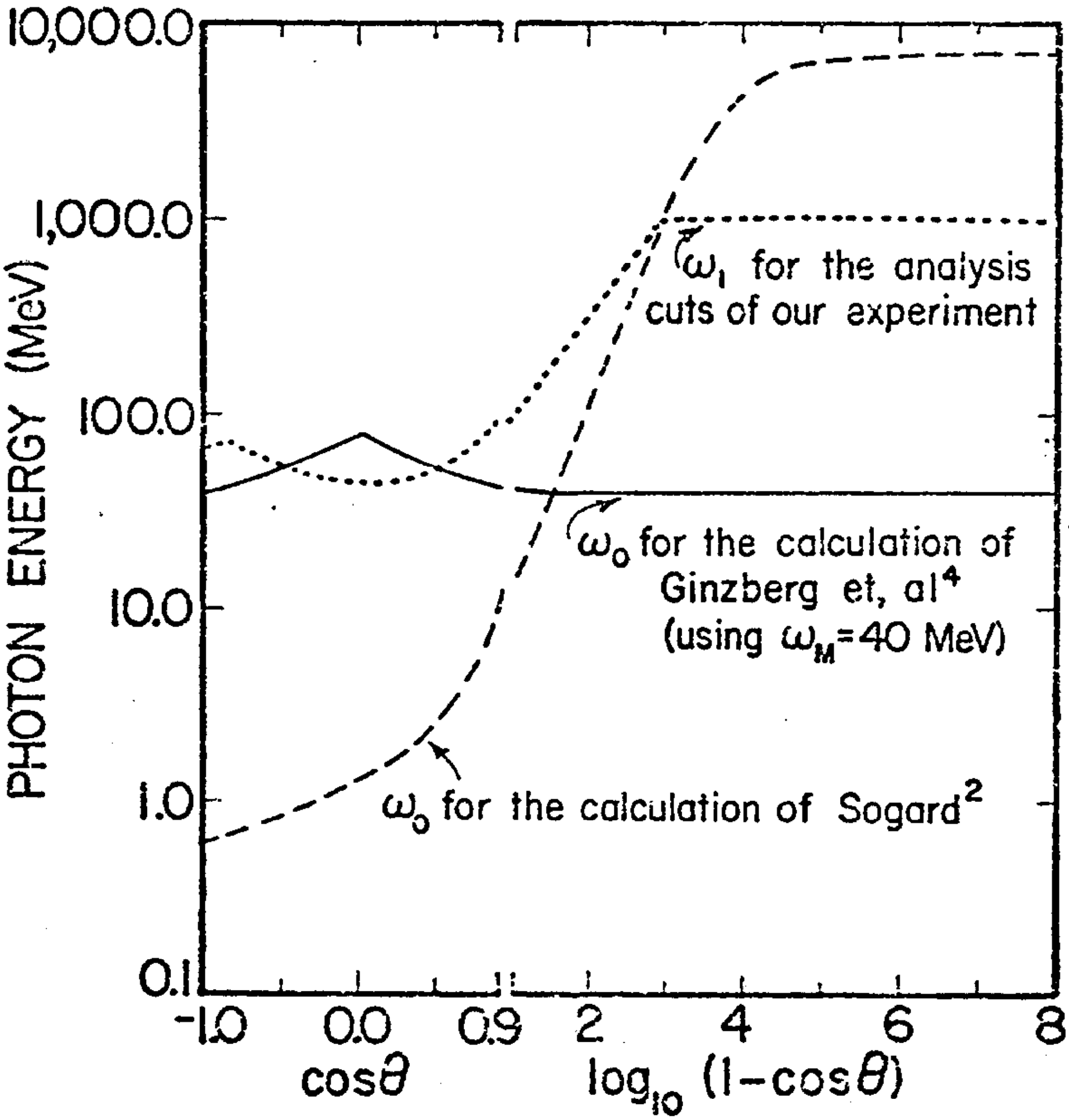

Fig. 5. The photon energy limits ω_0 and ω_1 as a function of the angle of the photon in the Lab and for an incident pion energy of 50 GeV. ω_m is a parameter used in Ref. 4.