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Isospin-violating Mixing in Meson Nonets†

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ABSTRACT

Segregation into ideally mixed nonets results when the OZI-violating interaction which would mix $u\bar{u}$, $d\bar{d}$, and $s\bar{s}$ mesons into isospin and SU(3) eigenstates is much weaker than the $s\bar{s}$ - $d\bar{d}$ mass difference. We show that the $d\bar{d}$ - $u\bar{u}$ mass difference can begin to induce a similar segregation into $d\bar{d}$ and $u\bar{u}$ mesons which leads to large isospin violations. An experimental example of such large isospin breaking ($\sim$30%) which we predict has probably already been seen in $f \to K\bar{K}$.

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Isospin violations in hadron physics are generally assumed to be small . In this paper we show **that** such effects can be appreciably enhanced in meson nonets with nearly degenerate isoscalar and isovector states. One consequence is a predicted thirty percent isospin violation in $f \rightarrow K\bar{K}$ decays already seen[1] in the experimental data, thereby resolving an apparent conflict between two sets of experiments.

Meson nonets contain three neutral non-strange states $u\bar{u}$, $d\bar{d}$, and $s\bar{s}$ which are degenerate in the nonet symmetry limit and which split and mix into the observed mesons. Nonet symmetry is broken by 1) quark mass differences which keep $u\bar{u}$, $d\bar{d}$, and $s\bar{s}$ as eigenstates but split their masses, 2) electromagnetic interactions which (apart from small annihilation effects) have the same qualitative effects, and 3) strong annihilation interactions which would in the absence of the first two effects mix these states to give a symmetry basis which includes isospin and SU(3). These strong interactions, however, violate the OZI[2] rule and should therefore be smaller in some sense than ordinary strong interactions. Thus they do not necessarily overwhelm the other effects. The physical meson states are determined by the competition between these various interactions and are obtained by diagonalizing a three by three mass matrix.

In most meson nonets the OZI-violating strong interaction is small compared to the $s\bar{s}$ - $d\bar{d}$ mass difference but large compared to the $d\bar{d}$ - $u\bar{u}$ mass difference. This leads to the "ideal mixing" pattern with the eigenstates $M_1 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$ and $M_0 = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$, the combinations of $u\bar{u}$ and $d\bar{d}$ which are isospin eigenstates, and $M_s = s\bar{s}$ [3,4]. The splitting between M_0 and M_1 then gives a measure of the OZI-violating strong interaction. The one exceptional case is the pseudoscalars where the OZI violation is large and M_s is mixed roughly 50-50 with M_0 [3].

We consider here the isospin mixing between isoscalar and isovector states resulting from the mass difference between the $d\bar{d}$ and $u\bar{u}$ states and point out some new manifestations of this mixing. This mass difference is roughly the same for all nonets, being of the order of 10 MeV with the $u\bar{u}$ state lighter than the $d\bar{d}$. There are variations between nonets because of flavour-dependent strong and electromagnetic interactions which depend upon the spins and orbital angular momenta of the states. We use values for this mass difference obtained from a recent analysis of hadron masses[5], but our qualitative conclusions are insensitive to the exact values. Because this mass difference is still relatively small compared with the OZI-violating mass shifts, the mixing can be given to a good approximation by treating the $d\bar{d}$ - $u\bar{u}$ mass difference as a perturbation and using the experimental values of mass differences to estimate the OZI-violating strong interaction. The result is always to change the mixture to give the lighter state more $u\bar{u}$ than $d\bar{d}$ and vice versa for the heavier state. This immediately determines the <u>sign</u> of the isospin breaking in the coupling to kaon pairs mentioned above: the lighter state couples more strongly to K^+K^- than to $K^0\bar{K}^0$ and vice versa for the heavier state.

The nature of the OZI-violating strong interaction which dominates the $d\bar{d}$ - $u\bar{u}$ mass difference is very different in the different nonets. The vector nonet, where the mixing has been extensively investigated by ρ-ω interference experiments, is anomalous and should not be used to guide the intuition for other cases. Because all the PP decay channels excepting $\pi\pi$

are closed, the $\pi\pi$ decay channel dominates the OZI violation and the imaginary part of the mass difference is much bigger than the real part; i.e., $\frac{1}{2}(\Gamma_\rho-\Gamma_\omega) >> m_\omega-m_\rho$. The narrowness of the ω, its position on the tail of the ρ, and the imaginary mixing which brings the peak of the forbidden $\omega\to\pi\pi$ amplitude into phase with the tail of the allowed $\rho\to\pi\pi$ amplitude, maximize the interference. These well known properties are essentially absent in the other nonets, where the OZI-violation tends to be real and the mixing can be detected by looking for a difference between the magnitudes of the $u\bar{u}$ and $d\bar{d}$ components of the wave function.

We estimate the mixing by taking phenomenological values of the quark mass differences m_s-m_d and m_d-m_u from an analysis of baryon masses[5)], combining them with experimental masses in each nonet to get the diagonal matrix elements of the mixing matrix in the quark basis ($u\bar{u}$,$d\bar{d}$,$s\bar{s}$), and calculating the mixing from these diagonal elements and the experimental masses. Two kinds of flavour dependent contributions appear in the diagonal elements, one arising from the differences in the quark charges,the other from the differences in the quark masses. The charge-dependent contribution, which is mainly due to Coulomb and magnetic hyperfine interactions, is given directly by the experimental mass difference within the isovector state (or by theoretical estimates in cases like the tensor nonet where the experimental splitting is not available). The mass-dependent contribution includes the mass difference itself and mass-dependent strong interactions like the colour hyperfine, annihilation, and zero-point energies. The value of this contribution is estimated from the mass differences between the M_1, $d\bar{s}$, and M_s states. Since the masses of these three states approximately satisfy an equal spacing rule, they can be fit by an expansion in the quark mass difference stopping with a small quadratic term. For the mass difference between $d\bar{d}$ and $u\bar{u}$ a linear expansion will suffice so that we arrive at the result

$$M(d\bar{d})-M(u\bar{u})= \{4M(d\bar{s})-3m_1-M(s\bar{s})\}\left(\frac{m_d-m_u}{m_s-m_d}\right) + \frac{2}{3}\{M(u\bar{d})-m_1\} \tag{1}$$

where m_1 denotes the mass of the neutral isovector state M_1.

The values of the meson masses on the right of eq.(1) are taken directly from experiment for all nonets except the pseudoscalars, since the deviation from ideal mixing is small in these cases. For the pseudoscalars, a model for η-η' mixing is required, the details of which are given in ref.(5). Our results for the mixing angles are given in Table I, based on the values m_d-m_u = 6 MeV and m_s-m_d= 220 MeV from ref.(5). Eq.(1) shows that our results depend only on the ratio of these masses; note that a value of 180 MeV for m_s-m_d suggested by the Λ-N mass difference[6)] would raise our mixing estimates by about 20% but would give the same results if m_d-m_u were reduced to 5 MeV. The first term in eq.(1) is seen from the Table to give the dominant contribution to $M(d\bar{d})-M(u\bar{u})$, showing that these isospin violations are dominated by the effects of the quark mass difference and not by explicit electromagnetic effects. The matrix elements m_{10} and $m_{10'}$ which appear in the Table are the off-diagonal matrix elements of the mass matrix in the basis of isospin eigenstates. In the ideally mixed nonets M_s is unmixed with M_0 so that its matrix element $m_{10'}$ with M_1 vanishes and m_{10} is just half the mass difference from eq.(1). Deviations from ideal mixing are considered only for the pseudoscalars where we take

$$\eta = 0.75\, M_0 - 0.65\, M_s + \ldots \tag{2a}$$

$$\eta' = 0.45\, M_0 + 0.62\, M_s + \ldots \tag{2b}$$

from ref.(4); in this case m_{10} and $m_{10'}$ denote the mixing matrix elements between the π^0 and the η and η' respectively. The corresponding mixing angles in the various nonets α=P,V,T,3 are denoted by χ_α and χ'_α (where χ'_α differs from zero only for the pseudoscalars) with the sign convention exemplified by $\pi^0 = \pi_1 - \chi_P\, \eta - \chi'_P\, \eta'$, etc. The uncertainty in the V, T, and 3 mixing angles is mainly due to the parameter choice mentioned above and snould be relatively small; the P mixing angles are more model dependent and should be treated with some caution.

We now discuss some possible experimental manifestations of the isospin mixing of isoscalar and isovector meson states. In doing so we will frequently invoke an SU(3) invariant OZI rule. Whenever possible we have explicitly checked this rule against experiment and found that it is well satisfied; in particular, we have found that to a good approximation strong $s\bar{s}$ creation occurs with the same probability as $u\bar{u}$ or $d\bar{d}$ creation once angular momentum barrier and phase space factors are taken into account. For convenience of discussion we have classified the various effects we will mention into four sections:

1. <u>Isospin-forbidden transitions</u>. The $\omega \to 2\pi$ decay is the classic example of this type. Our approach provides a value for χ_V consistent in magnitude and phase with $(0.48 \pm 0.14)e^{i(87\pm 10)^\circ}$, the measured[7,8] value.

A new type of isospin-forbidden transition which may have interesting applications is the pionic decay of heavy quarkonium states via the isoscalar component of the π^0.[9)] One such decay is $\psi' \to \psi + \pi^0$, related to the decay $\psi' \to \psi + \eta$ already observed[8)] with an appreciable branching ratio. On the basis of the η decay mode we can estimate that the branching ratio for $\psi' \to \psi + \pi^0$ will be about 0.11%, near the present experimental limit[8)]. Other interesting decays of this type which may be useful in studying some of the more elusive of the charmonium states are $\psi' \to {}^1P_1 + \pi^0$, ${}^1P_1 \to \psi + \pi^0$, ${}^3P_2 \to \eta_c + \pi^0$, and ${}^3P_0 \to \eta_c + \pi^0$ all of which (in the absence of any point for comparison) we can only roughly estimate to have branching ratios between 10^{-3} and 10^{-4}. These decays may provide a way to find ${}^1P_1$.

We conclude this section by mentioning two further "old" quark systems with effects of this type. The forbidden $\Lambda\Lambda\pi^0$ coupling[10)] gets contributions from both Σ^0-Λ and π^0-η-η' mixing. Using the Σ^0-Λ mixing of ref.(5) gives $g_{\Lambda\Lambda\pi} \approx 0.04\, g_{\Lambda\Sigma\pi}$ in good agreement with experiment[11)]. Finally we mention the decays $\eta \to 3\pi$. The contribution to these decays from π^0-η-η' mixing has been discussed in the current quark picture with promising results[12)]. Our values for χ_P and χ_P' are similar and certainly seem capable of resolving the problems associated with these decays, but to relate our mixing angles to the observed rates requires a model for the allowed $P \to 3P$ processes.

2. Couplings to kaon pairs. The neutral isospin eigenstates couple equally to charged and neutral kaon pairs, since they are equal mixtures of $u\bar{u}$ and $d\bar{d}$ which couple to charged and neutral pairs respectively [†1]. The mixing which breaks the equality of the $u\bar{u}$ and $d\bar{d}$ components

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[†1] This result is more general than the quark line rules; see, e.g., ref.(13).

thus breaks the equal coupling to charged and neutral kaons to give:

$$|g_{LK^+K^-}|:|g_{M^-K^oK^-}|:|g_{LK^o\bar{K}^o}| = |g_{HK^o\bar{K}^o}|:|g_{M^-K^oK^-}|:|g_{HK^+K^-}| = 1+|\mathrm{Re}\chi|:1:1-|\mathrm{Re}\chi| \tag{3}$$

where L and H denote the lighter and heavier states in the nonstrange isoscalar-isovector doublet and M^- the negatively charged isovector state in a nonet. We have chosen this form to emphasize that the lighter state goes preferentially into the charged mode and the heavier into the neutral.

The most dramatic example of this effect is in the A_2 and f decays where (assuming $\Gamma \sim p_{cm}^5$ as appropriate to a D-wave) eq.(3) and the value of χ_T from Table I give

$$\Gamma(A_2 \to K^+K^-) = 0.86\ \Gamma(A_2 \to K^o\bar{K}^o) \tag{4a}$$

$$\Gamma(f \to K^+K^-) = 1.30\ \Gamma(f \to K^o\bar{K}^o) \tag{4b}$$

i.e., huge violations of the isospin predictions of unity (1.05 and 1.06, respectively, after correcting for p_{cm}^5). Surprisingly enough, these huge effects are not inconsistent with experiments; on the contrary, recent results give[1)]

$$\Gamma(f \to K^+K^-)_{exp} = (1.68 \pm 0.35)\ \Gamma(f \to K^o\bar{K}^o)_{exp} \tag{5}$$

These experiments have until now been interpreted as measurements of the <u>same</u> quantity, the $f \to K\bar{K}$ branching ratio, with an inconsistency attrtibuted to experimental systematics and with isospin invariance unquestioned. Our prediction (4b) resolves this inconsistency by explaining the difference as due to isospin violations. It would clearly be interesting to reduce the error in the ratio (5) somewhat to test our predictions more quantitatively; in the meantime we view this result as encouraging.

Precisely analogous effects are expected to occur in the 3^{--} states g and ω_3, though these two resonances are so close together in mass and width (just the circumstance which is responsible for mixing them so effectively) that they may be difficult to resolve in a $K\bar{K}$ experiment.

Similar effects also occur in the couplings of pions to kaon pairs. Here the mixing is smaller, but observable effects can still be found. Thus for the $K\pi$ decay modes of the vector K^* states we obtain

$$\Gamma(K_V^{*o}\to K^+\pi^-):2\Gamma(K_V^{*o}\to K^o\pi^o):2\Gamma(K_V^{*+}\to K^+\pi^o):\Gamma(K_V^{*+}\to K^o\pi^+) = 1:0.92:1.06:0.95 \qquad (6)$$

versus 1:0.99:0.99:0.95 using only p_{cm}^3. By contrast, the tensor states show essentially no effect and the analogous ratios are expected to be very near to the p_{cm}^5 results of 1:0.99:0.98:0.97. This small effect in the tensors is a consequence of the small coupling of the η relative to the π^o, which has also the more interesting consequence that

$$\Gamma(K_T^{*+}\to K^+\eta) = 1.30\ \Gamma(K_T^{*o}\to K^o\eta) \qquad (7)$$

in compensation.

The same couplings arise also in semileptonic kaon decays, where the $\Delta S=-1$ current couples to the $K\pi$ system like a K_V^* giving

$$\Gamma(K^+\to\pi^o e^+\nu_e) = \tfrac{1}{2}(.987)(.994)\{1-\chi_p(\alpha_p+\sqrt{2}\sigma\beta_p)-\chi_p'(\alpha_p'-\sqrt{2}\sigma\beta_p')\}^2\ \Gamma(K_L^o\to\pi^- e^+\nu_e) \qquad (8)$$

where α_p=0.75, β_p=0.65, α_p'=0.45, and β_p'=0.62 are from eq.(2), the first and second parenthetical terms are radiative[14] and phase space[15] corrections, and where σ=1 (0) if the $u\bar{s}\to s\bar{s}$ transition contributes fully (nothing) to the decay. If we assume our calculated value for χ_p'/χ_p and allow for the uncertainty in σ, then the measured value[8) †2] of this ratio indicates that

$$\chi_p' = -\ .030\pm.015 \qquad (9)$$

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†2 This "measured value" should be viewed with some caution as it is an average over data with different radiative cuts and corrections.

in agreement with our predicted value.

3. Couplings to the photon. The $u\bar{u}$ and $d\bar{d}$ states couple to the photon with a ratio of 2:-1, the ratio of the constituent charges. These two components interfere destructively in isoscalar photon (odd G) transitions and constructively in isovector photon (even G) transitions. Thus isoscalar-isovector mixing effects are enhanced for odd G transitions. For example, we predict that

$$\Gamma(\rho^0 \to \pi^0 \gamma) = 1.11\ \Gamma(\rho^+ \to \pi^+ \gamma) \tag{10a}$$

$$\Gamma(A_2^0 \to \rho^0 \gamma) = 1.32\ \Gamma(A_2^+ \to \rho^+ \gamma) \tag{10b}$$

Since χ_V is nearly imaginary, the dominant effects in (10) are due to χ_P and χ_T respectively.

4. Other processes. Finally, we mention a few other assorted effects: a) the $NN\eta$ couplings should violate isospin with $g^2_{pp\eta} \simeq 0.76 g^2_{nn\eta}$, b) $\bar{p}d$ annihilations at rest display[16] violations of isospin relations on the partition of energy between neutral and charged π's which have the sign and magnitude expected from our π-η-η' mixing, c) $\pi^- p \to \pi^0 n$ backward scattering near zeroes in the differential cross section is sensitive to isospin mixing in the π^0 and seems to show evidence for a quite substantial value for χ_P[17], and d) the strong η decay of the lowest-lying S-wave nucleonic resonance at 1535 MeV leads to the prediction[18] that $\Gamma(N^{*+} \to p\pi^0) \simeq 1.38 \Gamma(N^{*0} \to n\pi^0)$.

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Table I. Meson multiplet mass matrix parameters and mixing angles (all masses in MeV)

Nonet(J^{PC})	M(dd)-M(uu) first term eq.(1)	total	m_{10}	$m_{10'}$	χ	χ'
$P(0^{-+})$	25	27	-7.6	-2.2	-.02	-.003
$V(1^{--})$	6.6	6.2	-3.1	~ 0	$.04e^{i(100^{o})}$	~ 0
$T(2^{++})$	7.8	8.2	-4.1	~ 0	$.07e^{i(45^{o})}$	~ 0
$3(3^{--})$	4.8	5.2	-2.6	~ 0	large	~ 0