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PENGUINS AND THE $\Delta I=1/2$ RULE †

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ABSTRACT

We discuss the role of "penguin" operators in the weak decays of light particles, including an extra operator, O_7, entering in two loops. The evidence for large penguin contributions is reviewed and found to be uncompelling.

INTRODUCTION

The connection between QCD and the $\Delta S=1$ nonleptonic weak decays was first explored in 1974,[1] and is conventionally discussed within the context of the Wilson expansion and the renormalization group. A "standard" operator basis may be summarized as follows:

$$(I=\tfrac{1}{2}) \quad O_1^q = \bar{s}\gamma_\mu q_L \bar{q}\gamma^\mu d_L - \bar{s}\gamma_\mu d_L \bar{q}\gamma^\mu q_L \,(q=u \text{ or } c) \qquad d_1^q=+4$$

$$(I=\tfrac{1}{2}) \quad O_2^q = \bar{s}\gamma_\mu q_L \bar{q}\gamma^\mu d_L + \bar{s}\gamma_\mu d_L \bar{q}\gamma^\mu q_L + 2\delta_{qu}(\bar{s}\gamma_\mu d_L \bar{d}\gamma^\mu d_L + \bar{s}\gamma_\mu d_L \bar{s}\gamma^\mu s_L) \qquad d_2^q=-2$$

$$(I=\tfrac{1}{2}) \quad O_3 = \bar{s}\gamma_\mu d_L \bar{u}\gamma^\mu u_L + \bar{s}\gamma_\mu u_L \bar{u}\gamma^\mu d_L + 2\bar{s}\gamma_\mu d_L \bar{d}\gamma^\mu d_L - 3\bar{s}\gamma_\mu d_L \bar{s}\gamma^\mu s_L \qquad d_3=-2$$

$$(I=\tfrac{3}{2}) \quad O_4 = \bar{s}\gamma_\mu d_L \bar{u}\gamma^\mu u_L + \bar{s}\gamma_\mu u_L \bar{u}\gamma^\mu d_L - \bar{s}\gamma_\mu d_L \bar{d}\gamma^\mu d_L \qquad d_4=-2 \tag{1}$$

with bare coefficients $c_1^u=-c_1^c=-1$, $c_2=1/5$, $c_3=2/15$, $c_4=2/3$, and the weak $\Delta S=1$ Hamiltonian is given by:

$$H_{wk} = \sqrt{2}\, G_F \sin\theta_c \cos\theta_c \sum_i \tilde{c}_i \theta_i . \tag{2}$$

Our problem is the exact calculation of the renormalized $\tilde{c}_i$ in QCD and, particularly the estimation of the operator matrix elements. In the absence of penguins we have $\tilde{c}_i = c_i(\bar{g}^2(m_W^2)/\bar{g}^2(\mu^2))^{-d_i/b_o}$ where d_i is the anomalous dimension ($\times\, 16\pi^2/g^2$) of Eq. (1) and $b_o=11 - 2/3\, n_f$.

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PENGUINS

In 1977 Shifman, Vainshtein and Zakharov[2] (SVZ) pointed out that the diagrams of Fig. 1 are potentially important as the GIM cancellation is only <u>logarithmic</u>. Direct computation yields the operator:

$$\frac{g}{16\pi^2}\int_0^1 dx\ 4(x-x^2)\log\left(\frac{q^2(x-x^2)-m_u^2}{q^2(x-x^2)-m_c^2}\right)\bar{s}\gamma_\mu \frac{\lambda^A}{2} d\left(D_\nu G^{AB\,\mu\nu B}\right) \ , \tag{3}$$

By use of the operator equation of motion[3]

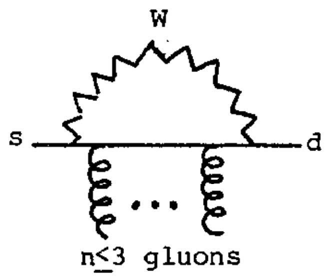

Fig. 1. Penguin Diagram.

$$\left(D_\mu G^{\mu\nu}\right)^A = g\sum_q \bar{q}\gamma^\nu \frac{\lambda^A}{2} q \tag{4}$$

we have:

$$= \frac{g^2}{16\pi^2}\ \frac{2}{3}\left(\log\frac{m_c^2}{\mu^2}\right)\bar{s}\gamma_\mu \frac{\lambda^A}{2}\ d\left(\sum_q \bar{q}\gamma\ \frac{g^A}{2}\ q\right) \ . \tag{5}$$

By color + Dirac Fierz rearrangement Eq. (5) can be rewritten in terms the preceding O_1^7, O_2^9 and the new ops:

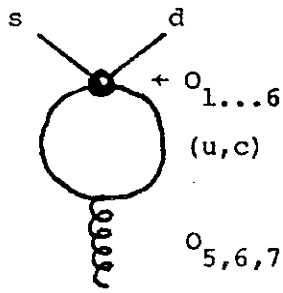

Fig. 2. Operator Mixing Diagram.

$$O_5 = (\bar{s}\gamma_\mu \frac{\lambda^A}{2} d_L)(\bar{u}\gamma^\mu \frac{\lambda^A}{2} u_R + \ldots + \bar{c}\gamma^\mu \frac{\lambda^A}{2} c_R)$$

$$O_6 = \bar{s}\gamma_\mu d_L(\bar{u}\gamma^\mu u_R + \ldots + \bar{c}\gamma^\mu c_R) \ . \tag{6}$$

In two loops we further encounter[4]

$$O_7^{(-)} = m_s \bar{s}_R \sigma_{\mu\nu} \frac{\lambda^A}{2} d_L G^{\mu\nu A} - m_d \bar{s}\sigma_{\mu\nu} \frac{\lambda^A}{2} d_R G^{\mu\nu A} \ , \tag{7}$$

with coefficient: $3/8\ g^3/(16\pi^2)^2\ G_F \cos\theta_c \sin\theta_c\ (\log\ m_c^2/\mu^2)$. No further d=6 operators can occur (5).

The operators $O_1^u \ldots O_6$ mix into O_5, O_8 and O_7 through the diagram of Fig. (2) (O_7 requires two loops). An anzätz for treating this expanded operator mixing problem, with inherent subtleties due to the GIM cancellation, is discussed in Ref. (2). The mixing generated by

Fig. (2) leads to mixed leading logs like $(\log M_w^2)^p(\log m_c^2)^q$. But is this the exact log structure of the actual Feynman diagrams? In Ref.(6) this is answered in the negative for two reasons: (a) Large $(\log m_c^2)^{p+q}$ are present which are leading and not properly summed by this ansatz (b) the $(\log M_w^2)^p(\log m_c^2)^q$ were found to cancel in some diagrams in which they should be present. To get an idea as to the potential size of these effects we compare in Table I the results for $\tilde{c}_1,\ldots\tilde{c}_7$ computed by the anzatz of Ref. (2) and by an anzatz in Ref. (6) in which only $(\log m_c^2)^{p+q}$ terms are summed for $\tilde{c}_5$, $\tilde{c}_6$, $\tilde{c}_7$.

Table I. Comparison of operator coefficients (Λ=0.5 GeV, μ=1, , m_c=2)

$\tilde{c}_1^u$	$\tilde{c}_2^u$	$\tilde{c}_4$	$\tilde{c}_5$	$\tilde{c}_6$	$\bar{g}(\mu^2)\tilde{c}_7$	
-2.5	0.10	0.41	-0.05	-0.01	---	SVZ
-2.4	0.13	0.41	-0.032	-0.006	-0.003	Ref. (4,6)

AMPLITUDES

We give a parameterization for the $\Delta I=1/2$, $K_s\to\pi^+\pi^-$ decay in the valence quark approximation[2] (VQA), ignoring effects of $O_7^{(-)}$ which we expect to be small[4] (the MIT bag model (7) is expected to give roughly similar results):

$$\frac{A_{\text{theory}}}{A_{\text{expt}}} = (-0.087)\tilde{c}_1^u + (\tilde{c}_5+\frac{3}{16}\tilde{c}_6)\frac{(-0.292)m_\pi^2}{(m_u+m_d)(m_s-m_d)} \quad . \tag{8}$$

Note the appearance of the renormalization group noninvariant ratio of physical masses to quark masses $(m_\pi^2/(m_u+m_d)(m_s-m_d))$ associated with penguin terms. What quark masses are to be used here?

We expect that the usual PCAC masses $m_u:m_d:m_s$=5:10:150 MeV are appropriate at energy scales $\sim$ few GeV where PCAC sum rules saturate. For our problem we must evolve (hopefully) down to mass scales of order m_K, m_π where the quark masses increase. In the MIT bag, for example,[7] we have m_u=10, m_d=20, m_s=300 (MeV). With these masses and c_1^u=-2.5 and c_5 + 3/16 c_6 = -0.06 we find A_{thy}/A_{expt} = 0.26; the O_1^u contribution is 87% here where the penguin is only 13%. In Ref. (2) the (extremely optimistic) values $m_u=m_d$=5 MeV, m_s=150 are used with c_5 + 3/16 c_6 = -0.25 (five times greater than theory predicts) to obtain A_{thy}/A_{expt} = 1.13. In this estimate the "reliable" short distance calculation of c_5 and c_6 has been abandoned and the penguin contribution is greatly enhanced by the "unreliable" matrix element estimate. Since the matrix element of O_1^u is expected to be accurate only to within a factor of 2$\sim$3, we believe that the former estimate more accurately reflects the real situation than the latter. Here penguins account for $\sim$ 1/5 of the total amplitude.

If we simply fit hyperon s-waves by pure penguins in the VQA we deduce an effective c_5' and c_6' to compare with theory. With the bag model quark masses one obtains[6] $c'_{5,6}/\tilde{c}_{5,6} \sim 30$. With a pure O_1^u fit we obtain $c_1'/c_1 \sim 3$. Hence, one must conclude that the penguin coefficients are far too small to allow a large contribution to the observed amplitudes with reasonable PCAC quark masses.

The hyperon processes have been reexamined by Finjord & Gaillard[8] and do not agree with theory in any approximation. Also, the $\Delta I=3/2$ decays involve only O_4 and are overestimated by ~ 2 in theory.

Clearly, external input is needed to determine whether penguins are really important or not. If they have a substantial contribution here then they may be isolated in CP-violation[8] measurements of $|\varepsilon'/\varepsilon|$[9]. Also, such measurements as $\Omega^- \to \Xi^- \pi/\Omega^- \to \Xi^* \pi$[8] are sensitive to penguins. We expect penguin contributions to be $\sim$ 1/5 to 1/10 the contribution of other operators.[6]

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$$H_{wk} = \sqrt{2}\, G_F \sin\theta_c \cos\theta_c \sum_i \tilde{c}_i \theta_i . \tag{2}$$

$$\propto \frac{g}{16\pi^2} \int_0^1 dx \; 4(x-x^2) \log \frac{q^2(x-x^2)-m_u^2}{q^2(x-x^2)-m_c^2} \; \bar{s}\gamma_\mu \frac{\lambda^A}{2} d \,(D_\nu^{AB} G^{\mu\nu B}) , \tag{3}$$

$$D_\mu G^{\mu\nu\,A} = g \sum_q \bar{q}\gamma^\nu \frac{\lambda^A}{2} q \tag{4}$$

$$= \frac{g^2}{16\pi^2} \; \frac{2}{3} \log \frac{m_c^2}{\mu^2} \; \bar{s}\gamma_\mu \frac{\lambda^A}{2} \; d\Big(\sum_q \bar{q}\gamma \frac{g^A}{2} q\Big) . \tag{5}$$

$$\sigma_s = \Big(\bar{s}\gamma_\mu \frac{\lambda^A}{2} d_L\Big)\Big(\bar{u}\gamma^\mu \frac{\lambda^A}{2} u_R + \ldots + \bar{c}\gamma^\mu \frac{\lambda^A}{2} c_R\Big)$$

$$\sigma_6 = \bar{s}\gamma_\mu d_L (\bar{u}\gamma^\mu u_R + \ldots + \bar{c}\gamma^\mu c_R) . \tag{6}$$

$$\sigma_7^{(-)} = m_s \bar{s}_R \sigma_{\mu\nu} \frac{\lambda^A}{2} d_L G^{\mu\nu A} - m_d \bar{s} \sigma_{\mu\nu} \frac{\lambda^A}{2} d_R G^{\mu\nu A} , \tag{7}$$

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