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PULSE SHAPING PASSIVE FILTER DESIGN.

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Calculations are given for the construction of a two port network to slow down fast pulses.

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Introduction.

Sometimes there is the need for networks that take pulses of small rise time and produce output pulses of larger rise time. The rise time should be the same, within a few percent, independent of the input pulse rise time.[1]

Calculations are given for a passive filter of the following properties, a) its input impedance is resistive and independent of frequency, b) a step pulse of the form,

$$V_i(t) = a \quad , \; t \geq 0,$$
$$= 0 \quad , \; t < 0,$$

in its input produces a pulse at its output of the form,

$$V_o(t) = a \cdot (1 - e^{-b \cdot t}) \quad , \; t \geq 0$$
$$= 0 \quad , \; t < 0. \qquad (1)$$

In what follows all time functions will be considered to be zero for (time) $t < 0$.

Analysis.

The Laplace transform of the above input step function is given by,[2]

$$\mathcal{L}[V_i(t)] = \frac{a}{s}$$

The Laplace transform of the output voltage is,

$$\mathcal{L}[V_o(t)] = \frac{a}{s} - \frac{a}{s+b}$$

A two port network to have this response to the above input, should have a transfer function,

$$T(s) = \frac{E_o(s)}{E_i(s)} = \left(\frac{a}{s} - \frac{a}{s+b}\right) \Big/ \left(\frac{a}{s}\right), \text{ or } T(s) = \frac{b}{s+b} \quad . \qquad (2)$$

A) Symmetric network. Equal input and output impedances.

For a lattice network (Fig 1a) with the above $T(s)$ it is easy to proove[2,3] that,

$$T(s) = \frac{R - Z_A(s)}{R + Z_A(s)} \qquad (3)$$

and $\dfrac{E_i(s)}{I_i(s)} = R$ if

$$Z_A(s) \cdot Z_B(s) = R^2 \qquad (4),$$

R is ohmic resistance.

The network then if connected to a load R has an input impedance R independent of frequency. It is obvious that its output impedance is the same, R, if the circuit is fed by a source of impedance R.

From eqs (2) and (3) we get,

$$\frac{R - Z_A(s)}{R + Z_A(s)} = \frac{b}{s+b} .$$

From this one gets,

$$\frac{1}{Z_A(s)} = \frac{1}{\left(\frac{R}{2b}\right) \cdot s} + \frac{1}{R} \qquad (5)$$

By use of (4) and (5) one gets,

$$Z_B(s) = \frac{1}{\left(\frac{1}{2bR}\right) \cdot s} + R \qquad (6)$$

B) Asymmetric network.

For the asymmetric network of Fig. 2a it is easy to prove[2] that if,

$$Z_A(s) = R + \frac{R^2}{Z_B(s)} \quad , \text{ then } \quad T(s) = \frac{R}{R + Z_B(s)}$$

and $$\frac{E_i(s)}{I_i(s)} = R \ .$$

If again the transfer function is the same, $T(s) = \frac{b}{s+b}$

then, $\frac{b}{s+b} = \frac{R}{R + Z_B(s)}$ which gives,

$$Z_B(s) = \frac{R}{b} s \qquad (7)$$

$$Z_A(s) = R + \frac{1}{\left(\frac{1}{Rb}\right) \cdot s} \qquad (8)$$

Synthesis.

A) Asymmetric case.

The synthesis of the network is now trivial with the use of eqs (5) and (6).

Equation (5) suggests that Z_A is a parallel connected inductance

$$L = \frac{R}{2b} \qquad (9)$$

and ohmic resistance R, see Fig. 1b.

Equation (6) suggests that Z_B is a serially connected ohmic resistance R and a capacitance,

$$C = \frac{1}{2bR} \qquad (10).$$

See Fig.1b.

R, obviously, is the terminating resistance (and the network's characteristic impedance).

One may need to put a 1:1 pulse transformer at the input of this circuit to avoid braid effects from the grounding of the coaxial cable, if such a cable is used to feed the filter.

B) Asymmetric case.

For the asymmetric case the synthesis is again easy from eqs (7) and (8). Eq. (7) suggests an inductance,

$$L = \frac{R}{b} \tag{11}$$

see Fig. 2b, and eq. (8) a capacitance,

$$C = \frac{1}{Rb} \tag{12}$$

and an ohmic resistance R in series, see fig. 2b.

More general case input voltage.

Let us consider now an input of the more general form,

$$V_i(t) = A(1 - e^{-D \cdot t}) \tag{13}$$

Its Laplace transform is,[2]

$$E_i(s) = A \cdot \left(\frac{1}{s} - \frac{1}{s+D}\right)$$

For the $T(s)$ found above, one gets,

$$E_o(s) = \frac{A \cdot b}{s+b} \cdot \left(\frac{1}{s} - \frac{1}{s+D}\right).$$

This can be written as,

$$E_0(s) = A\cdot\left(\frac{1}{s} - \frac{D}{D-b}\cdot\frac{1}{s+b} + \frac{b}{D-b}\cdot\frac{1}{s+D}\right).$$

The inverse Laplace transform of the last expression gives,

$$V_0(t) = A\cdot\left(1 - \frac{D\cdot e^{-b\cdot t} - b\cdot e^{-D\cdot t}}{D-b}\right) \quad (14).$$

It is easy to find from eq. 14 what happens when $D \to \infty$ (step function result), or when $D \to 0$ (zero input).

When $D \to b$, by using de L' Hospital's rule, one finds,

$$V_0(t) = A\cdot[1 - (1+b\cdot t)\cdot e^{-b\cdot t}] \quad (15).$$

Numerical estimates.

Let the rise time of the transmitted pulse (for the step pulse case) be, τ=10 nsec (defined as the time from 10% to 90% voltage).

Then $b = \frac{\ln 9}{10}$ nsec^{-1} = 0.2197 nsec^{-1}.

A) Symmetric case.

From eqs (9) and (10) we find, if R=50Ω,

L=0.1138 μH and C=45.51 pF.

B) Asymmetric case.

From eqs (11) and (12) we find, L=0.2276 μH and C=91.03 pF.

Fig. 3 shows the input and output voltages for various inputs (circuits of Figs 1b and 2b). Fig. 4 is a plot of output rise time τ_0 versus the corresponding input rise time τ_i, τ_0 was calculated graphically.

References.

1) W. Selove, private communication. W. Selove suggested the use of a pulse shaping filter for scintillator calorimeter pulses.

2) Charles M. Close, The Analysis of Linear Circuits, pages 324-325, Harcourt, Brace & World, publishers, Chicago-San Francisco-Atlanta (1966).

3) Anatol I. Zverev, Handbook of Filter Synthesis, page 6, John Wiley & Sons publishers, NY-London-Sydney (1967).

Figure captions.

Fig. 1. a) A lattice network.

b) A lattice filter with characteristics as described in the text.

Fig.2. a) An asymmetric network.

b) An asymmetric filter with characteristics as described in the text.

Fig. 3. a) to d). Input ($V_i(t)$) and output ($V_o(t)$) voltages of the filters of Figs 1b and 2b. The general input form is, $1- e^{-Dt}$. The general output form is given by eq. 14.

Fig. 4. Output rise time (defined as the time between 10% to 90% voltage) versus input rise time, for the filters of Figs 1b and 2b as calculated in the text. The output rise time was calculated graphically.

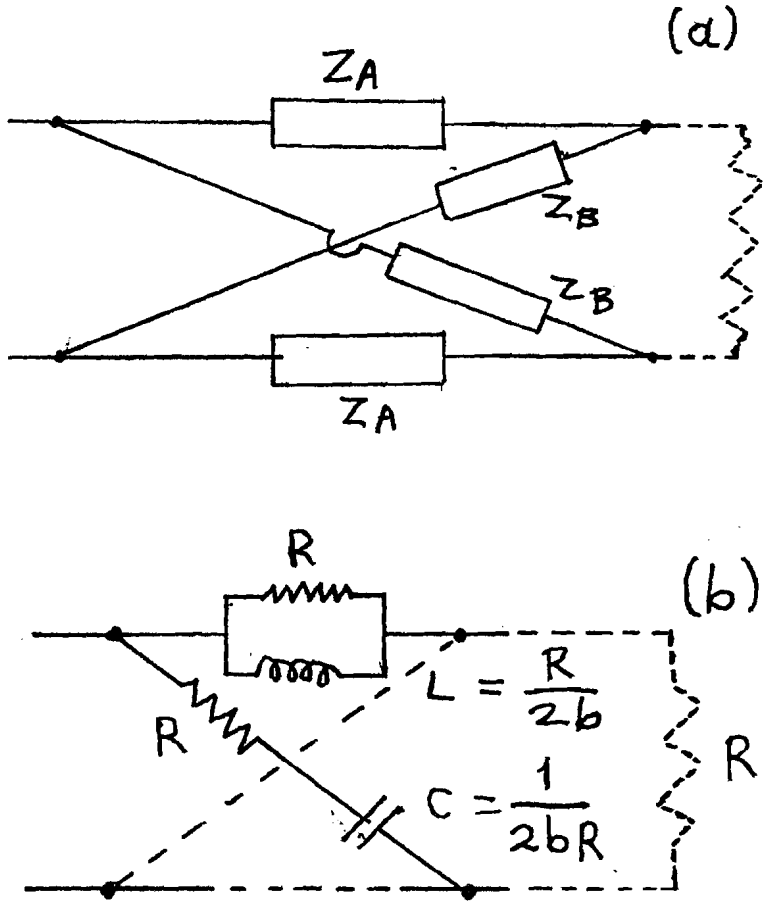

Fig 1

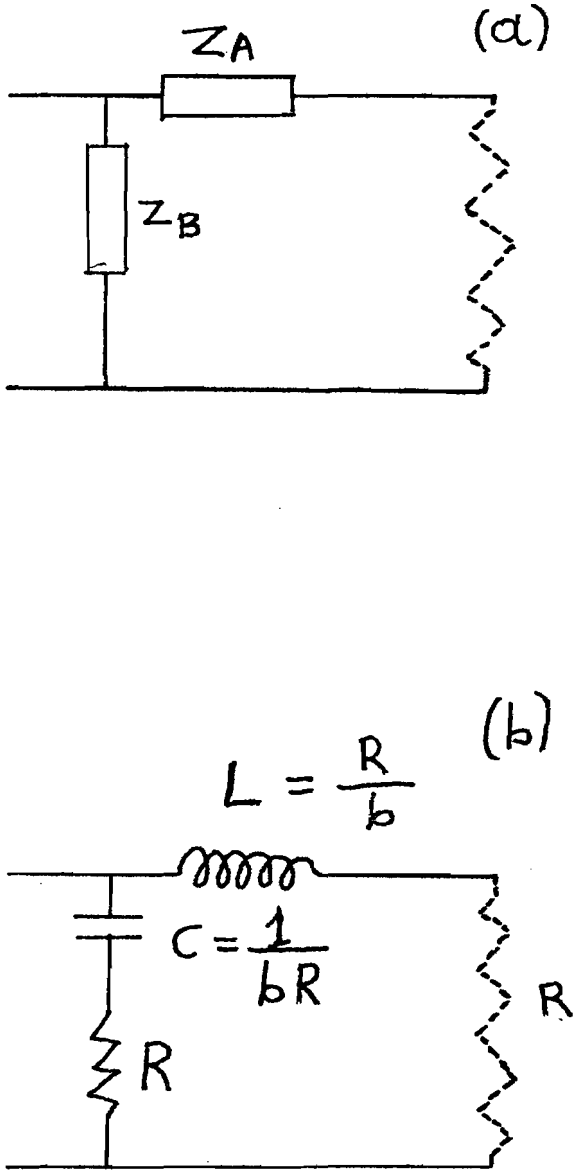

Fig. 2

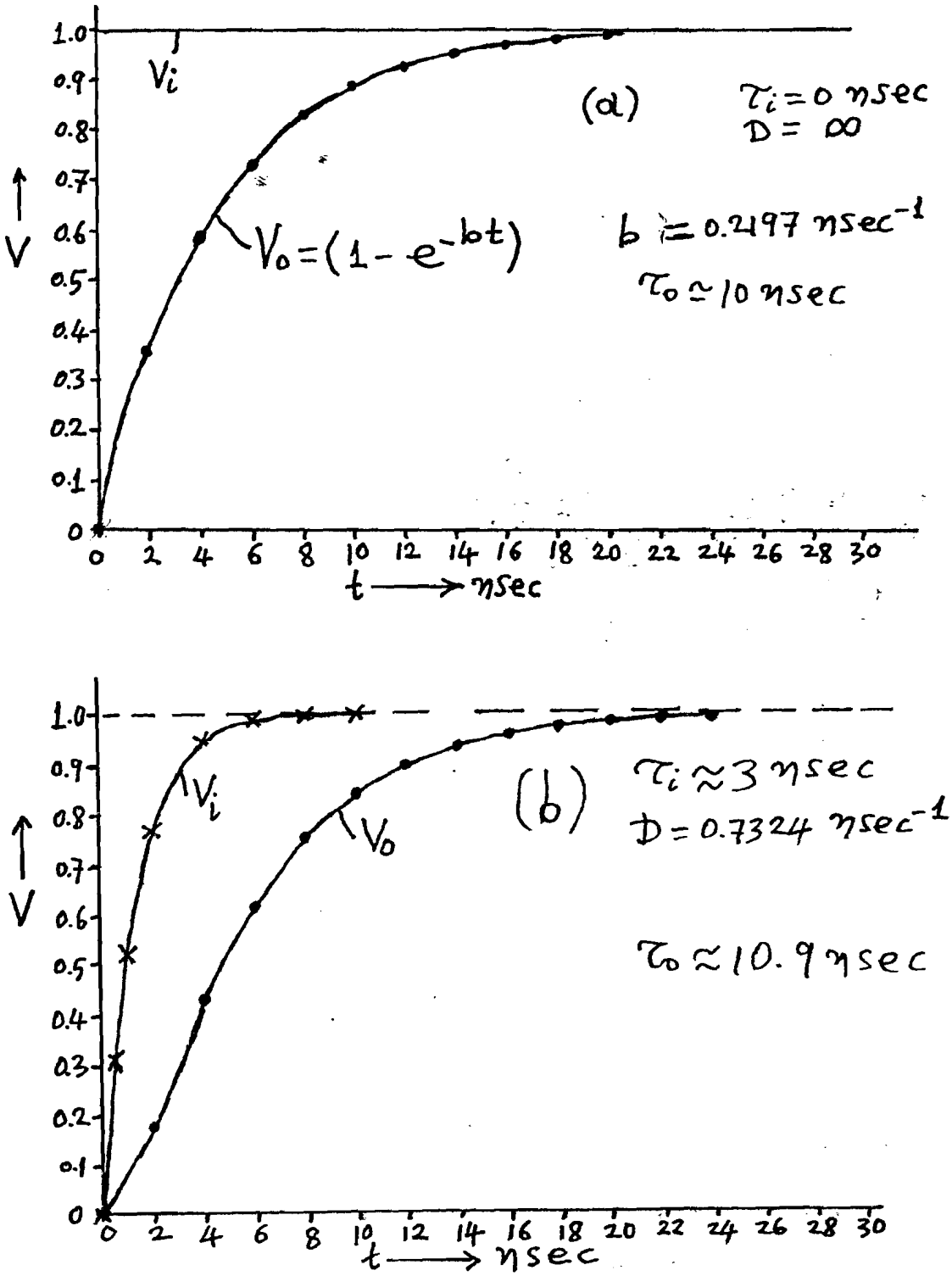
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
V
V_i
$V_o = (1 - e^{-bt})$
(a)
$\tau_i = 0$ nsec
$D = \infty$
$b = 0.2197$ nsec^{-1}
$\tau_o \simeq 10$ nsec
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30
t ⟶ nsec
V_i
V_o
(b)
$\tau_i \approx 3$ nsec
$D = 0.7324$ nsec^{-1}
$\tau_o \approx 10.9$ nsec

Fig. 3 (a, b)

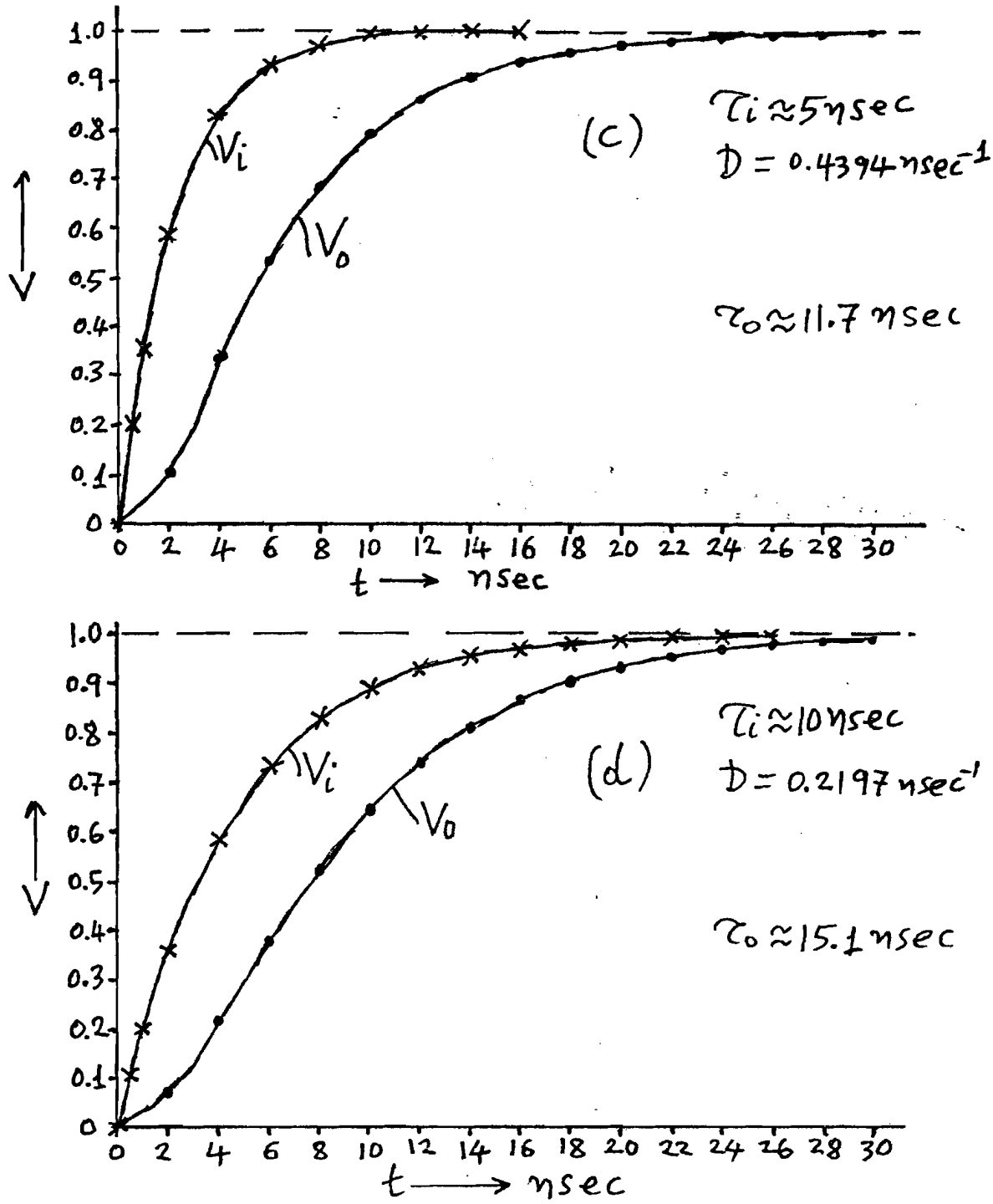
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
V
V_i
V_o
(c)
$\tau_i \approx 5$ nsec
$D = 0.4394$ nsec^{-1}
$\tau_o \approx 11.7$ nsec
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30
t ⟶ nsec
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
V
V_i
V_o
(d)
$\tau_i \approx 10$ nsec
$D = 0.2197$ nsec^{-1}
$\tau_o \approx 15.1$ nsec
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30
t ⟶ nsec

Fig 3 (c, d)

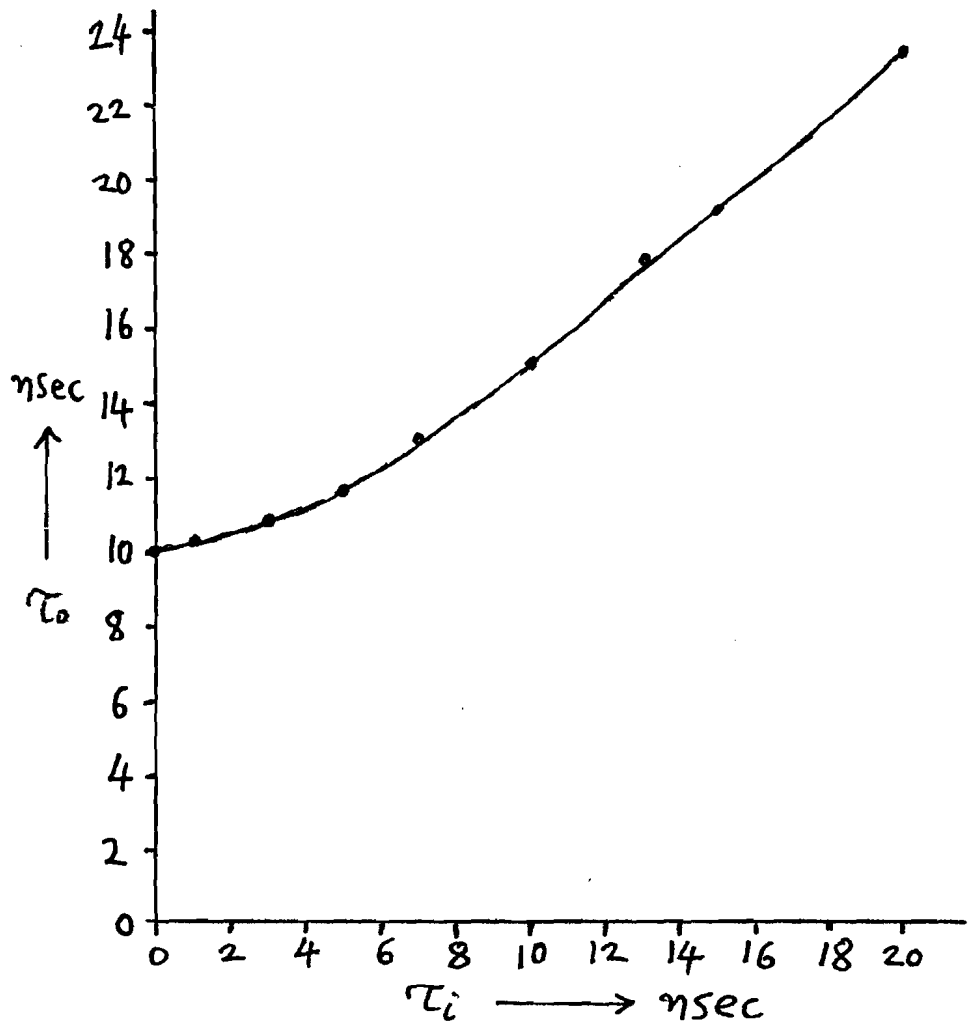
24
22
20
18
16
14
12
10
8
6
4
2
0
nsec
τ_o
0
2
4
6
8
10
12
14
16
18
20
τ_i → nsec

Fig. 4