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STUDY OF IDENTICAL PARTICLE CORRELATIONS IN pp INTERACTIONS AT 405 GeV/c

A. M. COOPER, M. FUKAWA, S. KABE, H. KICHIMI, F. OCHIAI,
T. SATO, R. SUGAHARA, A. SUZUKI, Y. YOSHIMURA and K. TAKAHASHI
KEK, National Laboratory for High Energy Physics
Oho-machi, Tsukuba-gun, Ibaraki, 300-32, Japan
T. OKUSAWA, K. TANAHASHI, M. TERANAKA and O. KUSUMOTO
Osaka City University, Osaka, 558, Japan
T. KONISHI
Kinki University, Higashi Osaka, 577, Japan
and
O. MIYAMURA
Department of Applied Mathematics
Faculty of Engineering Science, Osaka University
Toyonaka, Osaka, 560, Japan

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Abstract

We first investigate identical pion correlations in pp interactions at 405 GeV/c in terms of the two pion invariant mass variable and the Kopylov-Podgoretsky-Cocconi formalism, and we obtain the radius of the pion source as $R = 1.50 \pm 0.25$ fm, and the coherence parameter of the source as $\lambda = 0.58 \pm 0.05$, for central region processes. We then present a new method of analysis in terms of the moments of the local number distribution for identical particles, and we find strong evidence for higher order correlations.

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1. INTRODUCTION

Since two-particle correlation data became available there have been numerous attempts to interpret it [1]. Most studies on rapidity correlations employ some form of cluster model [2]. It has also become evident that resonance formation has a dominant contribution in multiparticle production and gives a substantial effect in particle correlation [3, 4]. Detailed studies including the dependence of correlations on transverse variables revealed that the correlations of like and unlike charged pion pairs differ in some respects [5]. In particular when dealing with like bosons the effect of Bose Einstein statistics seems to be important [6].

Most studies of identical particle correlations at medium energies have used the formalism developed by Kopylov & Podgoretsky [7] and Cocconi [8] to measure the radius R, and lifetime τ of the hadron source. Despite the fact that this formalism has recently come under some criticism [9], it remains a useful way to parametrize the observed correlations so that one may compare results on different reactions at different energies.

Fowler and Weiner [10] have recently suggested that the pion source may be a mixture of chaotic (quantum statistical) and coherent (classical) fields. Thus we introduce into the Kopylov-Podgoretsky-Cocconi(K. P. C.) formalism a parameter λ which measures the mixture of coherent and incoherent field contributions in the source. If $\lambda = 1$, we have a completely incoherent source, and if $\lambda = 0$, we have a fully coherent source, in which case no higher-order interference (i.e. Bose Einstein (B.E.) correlation) would be observed. Values of λ may represent the first definite observation of classical (coherent) fields in the hadron source, which is of great interest at present in studies of the state of highly compressed quark matter in the framework of Q C D [10]. Fowler

and Weiner have pointed out that measurement of this parameter in different reactions and at different energies is important, since the mixture of coherent and incoherent fields in the source is expected to depend on the initial conditions.

The present work is concerned with the investigation of identical pion correlations in pp interactions at 405 GeV/c. We consider our data in terms of the two pion mass variable, in order to separate resonance and B.E. effects more clearly, and we then consider the application of the K. P. C. formalism including the coherence parameter λ.

We have also made some analysis of higher order correlation, which are of current theoretical interest [11]. The conventional investigations of 3 particle correlations in terms of the correlation functions $R(y_1, y_2, y_3)$ may not be sensitive to clustering in very small rapidity intervals, and previous investigation of correlations in terms of rapidity gap [12] has not considered the charge structure of the clusters. In the present study we use a new method of analysis of multiparticle correlations [13] from the point of view of Bose condensation, which makes local number fluctuations for identical particle groups very clear.

Section 2 deals with experimental procedure, and Section 3 presents the general features of the data in terms of the two pion invariant mass variable. Section 4 describes analysis in terms of the K. P. C. formalism and Section 5 presents the analysis of local number fluctuations. Section 6 is reserved for summary and conclusions.

2. EXPERIMENTAL PROCEDURE

The data used in this analysis were obtained from a 50k picture exposure of the Fermilab 30 inch hydrogen bubble chamber to a 405 GeV/c proton beam. Results are based on 3220 all pronged events. Two independent

scans were performed and a third check scan was made by physicists. The overall scanning efficiency was 98%. Events were track matched by physicists and then measured on conventional manual projectors. The measurements were processed using the TVGP programme and remeasured if necessary. Finally 92% of the secondary tracks measured were successfully reconstructed. Approximately 5% of scanned events were impossible to match due to secondary interactions or many overlapping tracks. Corrections due to this loss were made by comparison with the correct multiplicity distribution obtained in scanning. There is a relatively low efficiency for measurement of secondary tracks in the forward hemisphere. By using the complete forward -backward symmetry property of the $\pi^{\pm}$ c.m. rapidity distribution which is expected in pp interactions, correction factors were estimated for each track, in the forward hemisphere. It was checked that these weights did not cause any distortion of the double pion rapidity distribution. The average weight is less than 1.3. A check on ionization loss of low momentum secondaries (momentum less than 1.4 GeV/c) was also made by physicists, in order to discriminate positive pions from protons. All positive tracks with $x_F > 0.6$ (x_F = the Feynman variable) are taken to be protons, and all negative tracks to be pions. The multiplicity distribution for charged particles is shown in fig. 1. Further experimental details may be found elsewhere [4].

3. GENERAL FEATURES OF THE DATA

We first consider the two particle inclusive correlation function as a function of the invariant mass $M_{\pi\pi}$ of the two particle system, since one can more easily separate the contributions of resonances and the B.E. interference to correlations in terms of this variable. B.E.

effects are strongest when the 4-momenta of the two particles are nearly equal, and thus $q^2 = (p_1 - p_2)^2$ is small. Since the variable $M_{\pi\pi}$ is related to q by the equation $q^2 = 4m^2 - M_{\pi\pi}^2$ in the case of two pions, we expect B.E. effects to show up at threshold in the $M_{\pi\pi}$ variable, whereas resonances show up at the appropriate masses e.g. $M_{\pi\pi} = m_\rho$. The correlation function is defined as,

$$C_2^{\pm\mp}(M_{\pi\pi}) = \rho_2^{\pm\mp}(M_{\pi\pi}) - [\rho_1^{\pm} * \rho_1^{\mp}](M_{\pi\pi})$$

where, $\rho_2^{\pm\mp}(M_{\pi\pi})$ is the usual differential cross section $d\sigma/dM$, divided by the inelastic cross section σ_{inel}, and $[\rho_1 * \rho_1](M_{\pi\pi})$ is the product of single particle distributions at fixed pair mass,

$$[\rho_1 * \rho_1](M_{\pi\pi}) = \int d^3\vec{p}_1\, d^3\vec{p}_2\, \delta[(p_1 + p_2)^{1/2} - M_{\pi\pi}]\, \rho_1(p_1)\rho_1(p_2)$$

where, $\rho_1(p_1) = 1/\sigma_{inel} \times d\sigma/d^3\vec{p}$. Thus $[\rho_1 * \rho_1](M_{\pi\pi})$ represents the uncorrelated background. We have evaluated it by Monte Carlo techniques. Since the choice of the cross section σ as the inelastic cross section may be somewhat arbitrary, we have checked our results by normalizing the Monte Carlo generated shape of $[\rho_1 * \rho_1](M_{\pi\pi})$ to the data for $\rho_2(M_{\pi\pi})$, for values of $M_{\pi\pi}$ greater than 2.0 GeV, and the slight difference of normalization so introduced is included in our errors.

The results for $\pi^-\pi^-$ and $\pi^+\pi^-$ pion pairs are given in figs. 2a and 2b respectively. One can see that $C_2^{+-}(M_{\pi\pi})$ shows a broad threshold enhancement and evidence of ρ production. Analysis of ρ and f production in this data has already been published [14]. By contrast, $C_2^{--}(M_{\pi\pi})$ shows a much sharper threshold enhancement and no resonant structures may be fitted.

The data are compared with 205 GeV/c data [15] in fig. 3, in terms of the function $C_2^{\pm\mp}(M_{\pi\pi})/\ln s$ for each data sample. The shapes of

$C_2^{+-}(M_{\pi\pi})$ are similar, but the expectation that $C_2^{+-}/\ln s$ may scale [15] is not fulfilled. The dip at around 700 MeV in $C_2^{--}(M_{\pi\pi})$ for the 205 GeV/c data is not reproduced in our data, in fact the shapes of C_2^{+-}, and C_2^{--} for $M_{\pi\pi}$ greater than 500 MeV are similar. A quantitative comparison of the present data with data at 102 [16] and 205 GeV/c [15] is given in table 1 in terms of the Meuller moments $f_2^{\pm\pm}$, where $f_2^{\pm\pm} = \int C_2^{\pm\pm}(M_{\pi\pi})dM_{\pi\pi}$.

The difference between C_2^{+-} and C_2^{--} is better illustrated in terms of the function

$$R(M_{\pi\pi}) = C_2(M_{\pi\pi})/[\rho_1 * \rho_1](M_{\pi\pi}).$$

Despite the slightly differing single particle rapidity distributions of π^+ and π^-, the uncorrelated distributions $[\rho_1^+ * \rho_1^-](M)$ and $[\rho_1^- * \rho_1^-](M)$ are very similar, apart from normalization, so that considering R rather than C should minimize the differences due to normalization and emphasize the difference in $M_{\pi\pi}$ dependence. Table 2 presents values of R in 40 MeV bins from threshold to one pion mass above threshold, and also in the ρ mass region and the higher mass region, $M_{\pi\pi}$ = 1.0 to 1.4 GeV for comparison. We see that the threshold dependence of R^{--} is much steeper than that of R^{+-}, but that the values are similar in the higher mass region. We may interpret the difference at threshold as a manifestation of B.E. interference. However, to evaluate this effect more quantitatively, we must estimate the background to such interference, since there may be other origins of correlation in the $\pi^-\pi^-$ system. There have been numerous attempts to interpret the threshold enhancements observed in the 205 GeV/c data [15, 17]. Some structure near threshold is expected as a result of the decay of resonances of multiplicity $m \geq 2$. Considerations along these lines yield a similar shape for C_2^{+-} and C_2^{--} [17]. At low energy it has been observed [18] that the broad threshold enhancements

disappear if one considers semi-inclusive data, leaving only a narrow threshold enhancement for like pairs and resonant structure for unlike pairs, i.e. the broad enhancements are a feature of multiplicity mixing.

In order to probe this problem further we may consider $C_2^{\pm\pm}(M_{\pi\pi})$ more differentially, and it seems most useful to do this in terms of the variables relevant to the K. P. C. formulation. Thus we define,

$$q_o = |E_1 - E_2|, \text{ and } q_t = |\vec{q}_t|, \ \vec{q}_t = \vec{q}_s - \vec{n}\,(\vec{q}_s \cdot \vec{n}),$$
$$\text{where } \vec{q}_s = \vec{p}_1 - \vec{p}_2 \text{ and } \vec{n} = (\vec{p}_1 + \vec{p}_2)/|(\vec{p}_1 + \vec{p}_2)|.$$

$(\vec{p}, E)$ represents the 4-momentum of the two pions. For reasons outlined in the next section, we have restricted further analysis to the central region $|y_\pi^\pm| < 2.0$, where y_π^+ and y_π^- stand for the rapidity in the center of mass system of the π^+, and π^-, respectively.

In figs. 4a and 4b we present $C_2^{\pm\pm}(M_{\pi\pi})$ for the kinematic region $q_o < 0.25$ GeV, and in figs. 4c, and 4d, for the kinematic region $q_t < 0.4$ GeV/c. One may expect some peak at small values of $M_{\pi\pi}$ on purely kinematic grounds since one is restricting the difference in 4-momenta. However, the difference between C_2^{+-} and C_2^{--} at threshold is striking. In table 3 we give a quantitative estimate of this difference in terms of $R^{\pm\pm}$. In the same table we also compare R^{--} with R^{+-} in the kinematic regions, $q_o > 0.25$ GeV and $q_t > 0.40$ GeV/c. They are quite similar, even though the ρ and f are visible in $C_2^{+-}(M_{\pi\pi})$. Thus one sees that the strong differences occur in precisely the kinematic regions where B.E. effects are expected to be strongest. These observations support the view that one may use the $\pi^+\pi^-$ spectra as a background to the B.E. interference, if one restricts the kinematic region to that of small 4-momentum difference.

4. ANALYSIS IN TERMS OF THE K. P. C. FORMALISM

Kopylov [7] wrote the B.E. interference term for incoherent pion production from sources distributed over the surface of a circular emission volume of radius R, and source lifetime τ as,

$$I = I^2(Rq_t)/(1.0 + (\tau q_o)^2) \qquad \text{(A)}$$

where $I(x) = 2\,J_1(x)/x$ and J_1 is the first order Bessel function. It was assumed that Lambert's Law applies and that the condition $c\tau >> R$ holds. However in most analyses of this effect at medium energies [19], this condition does not hold, and the alternative interpretation of Cocconi seems to be more appropriate. Cocconi [8] derived the same B.E. interference term using a minimum of assumptions, the most important of which is the incoherence of the source, as

$$I = \exp(-(Rq_t/2))^2/(1.0 + (\tau q_o)^2) \qquad \text{(B)}$$

for the case of a Gaussian luminosity distribution at the surface of the emission volume. In this case $c\tau$ is interpreted as the depth of the layer from which two pions may be emitted in order to be observed simultaneously, thus $c\tau < 2R$ is required. In the present work we use formula (B), although use of (A) does not make any large difference to our results.

If the distributions in the variables q_o and q_t without interference are written as W_o, then the distributions including interference are written as

$$W = W_o(1.0 + \lambda I),$$

where the parameter λ has been introduced to take account of the possibility that the source may not be fully incoherent [10, 20]. The importance of

this parameter has been outlined in the section 1. We have restricted our analysis to the central region $|y| < 2.0$ since this cut, and a restriction to multiplicities $m \geq 6$, ensure that we have eliminated leading particle and diffractive effects, and the interpretation of our results is thus clarified as relevant to central hadron production. As expected, the value of λ decreases if non central (possibly coherent) processes are included. We estimate that an uncertainty of 0.05 is introduced into the value of λ by the arbitrary choice of the cut, for central region process.

One must next choose the non-interfering background W_o suitably. Most analyses have used the unlike pion spectra, and, as we saw in the last section, this may not be too bad an approximation, if one restricts the kinematic region to small values of q_o and q_t, such that identical particle interference is enhanced, and resonant effects in $\pi^+\pi^-$ are negligible. We compare this choice of background to a Monte Carlo generated simple Independent Emission Model, based on the single particle distributions of our data. Fig. 5 illustrates the data and these two choices of background for the variable q_o in the kinematic regions $q_t < 0.4$ GeV/c (fig. 5a) and $q_t > 0.4$ GeV/c (fig. 5b). Fig. 6 presents the corresponding plots for the variable q_t in the kinematic regions $q_o < 0.25$ GeV and $q_o > 0.25$ GeV. The normalization of the backgrounds is the same as for Fig. 4, such that there is no difference between $\pi^+\pi^-$ and $\pi^-\pi^-$ spectra for values of $M_{\pi\pi}$ above 2.0 GeV.

We see that the backgrounds are not very different for any kinematic region. We prefer the use of the $\pi^+\pi^-$ spectra since, as discussed in the previous section, there may be some correlation in the $\pi^-\pi^-$ system other than the B.E. effect which is better accounted for by the use of this background. We have checked the effect on our values of R and τ of

the use of these alternative choices of background, and included the difference in our systematic errors.

The more significant observation we can make by considering figs. 5 and 6 is that whereas the background shape follows the data closely for figs. 5b and 6b, this is far from the case for figs. 5a and 6a, and the discrepancies are at small value of q_o and q_t i.e. exactly in the kinematic region where the B.E. interference is strongest. Thus we fit the data of these plots, simultaneously, using formula (B). The fit to fig. 5a is made in the range q_o from 0.0 to 0.9 GeV, and it employs an average value of q_t in the range $q_t < 0.4$ GeV/c. The fit to fig. 6a is made in the range q_t from 0.0 to 1.05 GeV/c and it employs an average value of q_o in the range $q_o < 0.25$ GeV. The normalization of the fitted form is chosen such that the number of events (combinations) in the data, and under the fitted curve, is the same, i.e. W_o differs from the background illustrated in figs. 5 and 6 by a normalization factor. However the value of this factor is never far from 1.0.

The results for R, $c\tau$ and λ are

$R = 1.50 \pm .25$ fm, $c\tau = 1.20 \pm 0.30$ fm and $\lambda = 0.58 \pm 0.05$.

The χ^2/n.d.f. of the fit is 46/36. Compare χ^2/n.d.f. = 95/39 if $\lambda = 0$ (i.e. no interference). An attempt to fit figs. 5b, and 6b to the same form yields $\lambda = 0.03 \pm 0.10$ for χ^2/n.d.f. = 44/39. The results have been further checked by altering the kinematic regions of fitting and the cuts on q_o and q_t. The fit is illustrated in figs. 7a and 7b.

Our value of R is consistent with, although slightly higher than, the values quoted in other studies on pp interactions at 28 GeV [21], $R = 1.3 \pm 0.1$, and at the ISR [22], $R = 1.34 \pm 0.31$. However the study at 28 GeV did not include the parameter λ, and there is some indication that including λ tends to increase the values found for R [23].

Our value is in good agreement with the most recent high statistics data of the ABBCCLVW collaboration for medium energy πp, Kp and $\bar{p}p$ interactions [23]. The new higher value of R has been interpreted as representing the primeval interaction region plus the decay length of the many resonances first produced. We have evidence for strong resonance production in the central region of our interaction [14], so our value for R must also include their decay length. Thus there seems to be no significant change in the value of R over a wide energy range.

Our value for $c\tau$ is consistent with Cocconi's condition $c\tau < 2R$. It is closer to the value found at the ISR (1.38 ± 0.6 fm) than to the value found at 28 GeV (0.58 ± 0.2 fm). Thus there may be some increase in this parameter from medium to high energies.

Our value of λ is somewhat higher than the values given for πp, and Kp reactions at medium energy (we discount comparison with the ISR value since no restriction was made to the central region in their case and consequent inclusion of coherent processes could lead to an underestimate of the hadron source size and the value of λ). The approach of Giovanni and Veneziano [20] would predict that pp interactions are essentially the same as πp and Kp reactions, from the point of view of the dual sheet picture. However the approach of Fowler and Weiner [9], predicts that the value of λ depends on the initial conditions i.e. on the reaction and the c.m. energy. Using this model we may obtain the ratio $\frac{\langle n\rangle_{coherent}}{\langle n\rangle_{random}} \approx 2$ from our value of λ, which implies that about 2/3 of pions are still emitted coherently. The rise of λ with energy implies a corresponding decrease of this ratio, which is consistent with the observed increase in resonance production [3, 13, 14] and the fireball picture of Hagedorn [24] and Frautschi [25], which includes resonance production in a statistical bootstrap approach. It would be very interesting to see the results of

similar analysis of identical particle correlations, including the λ parameter, for other reactions in the 100 GeV/c energy range.

5. LOCAL NUMBER FLUCTUATIONS AND HIGHER ORDER CORRELATIONS OF IDENTICAL PARTICLES

While there is considerable evidence for two particle correlations, the real existence of higher order correlations has not yet been established [26, 27]. Observation of three negative pion correlations in 200 GeV/c $\pi^- p$ data has been explained as a reflection of two particle correlations [12, 28]. In this section we present a new method of analysis in terms of local number fluctuations [13], which makes interpretation of the observed fluctuations in terms of higher order correlations very clear. The idea of a local number distribution is similar to the usual multiplicity distribution except that it is defined in terms of a restricted domain of phase space, which we will lable $\underline{\delta}$. This window $\underline{\delta}$ is specified both in rapidity and azimuthal angle ϕ_o with respect to the beam direction.

We consider the normalized moments

$$C_m(\underline{\delta}) = M_m(\underline{\delta})/M_1(\underline{\delta})^m$$

where $M_m(\underline{\delta})$ are the local factorial moments

$$M_m(\underline{\delta}) = \sum_{n=m}^{\infty} \frac{n!}{(n-m)!} P_n(\underline{\delta})$$

and $P_n(\underline{\delta})$ is the local multiplicity distribution

$$P_n(\underline{\delta}) = \frac{\text{Number of events which have n } \begin{matrix}\text{positive}\\ \text{negative}\end{matrix} \text{ pions in } \underline{\delta}}{\text{Total number of inelastic events}}$$

There are two problems involved in making the calculation of $C_m(\underline{\delta})$. The first is that for high m, these quantities are extremely sensitive to the normalization of $M_1(\underline{\delta})$. The second is that track loss in the forward direction for high multiplicity events reduces the value of $C_m(\underline{\delta})$. We do not apply any rapidity dependent weight to account for this loss in the present section, instead the following alternative procedure is used to account for these problems. We evaluate the quantity $M_1(\underline{\delta})$ semi-inclusively for all multiplicities $4 < N < 24$. If the data is free from experimental bias we should obtain

$$M_1^{(N)}(\underline{\delta}) = \int_{\underline{\delta}} \frac{d\sigma_N}{d\vec{k}} \, d\vec{k}/\sigma_{inel}.$$

This is true within experimental errors, for low multiplicities, but

$$M_1^{(N)ob}(\underline{\delta}) < \int_{\underline{\delta}} \frac{d\sigma_N}{d\vec{k}} \, d\vec{k}/\sigma_{inel},$$

for $N > 16$ due to track loss. We thus introduce a multiplicity dependent weight w_N, where

$$w_N = \int_{\underline{\delta}} \frac{d\sigma_N}{d\vec{k}} \, d\vec{k}/\sigma_{inel} \times M_1^{(N)ob}(\underline{\delta}) \text{ for } N > 16; \; w_N = 1.0 \text{ for } 16 > N > 4$$

to recover the equality. Then the higher moments are calculated by using this weight w_N and $M_m(\underline{\delta})$ is obtained by summing over multiplicities.

In order to assess the significance of the values of $C_m(\underline{\delta})$ which we obtain from the data, we also calculate $C_m^{IEM}(\underline{\delta})$ in a simple Independent Emission Model for which

$$P_n(\underline{\delta}) = \sum_{N=n}^{\infty} \left(\frac{\sigma_N}{\sigma_{inel}}\right)_N C_n \left(\frac{1}{N\sigma_N} \int_{\underline{\delta}} \frac{d\sigma_N}{d\vec{k}} d\vec{k}\right)^n \left(\frac{1}{N\sigma_N} \int_{U-\underline{\delta}} \frac{d\sigma_N}{d\vec{k}} d\vec{k}\right)^{N-n}$$

where U is total phase space, so that $U - \underline{\delta}$ is phase space not in the window.

In practice, since we use the single particle semi-inclusive distributions of our data to make this calculation, we must again correct for forward loss, by applying the weight w_N before summing over multiplicities. The calculation is performed by Monte Carlo techniques. Positive and negative single particle distributions are considered separately, but since the window is restricted to the central region there is no significant difference in the values of $C_m(\underline{\delta})$ obtained. We present the average result.

Fig. 8 illustrates $C_m^{IEM}(\underline{\delta})$ for the case when $\underline{\delta}$ is restricted $|y| \leq a$ and $0 \leq \phi_o \leq \phi_{MAX}$, for (i) $a = 1.0$, $\phi_{MAX} = 2\pi$, and (ii) $a = 0.25$, $\phi_{MAX} = \pi$. In the former case the calculation is in agreement with a calculation made directly from the total multiplicity distribution in which case the window $\underline{\delta}$ spans all phase space. This is a good consistency check on our procedure. The fact that $C_m^{IEM}(\underline{\delta})$ for the more restricted window size lies above $C_m^{IEM}(\underline{\delta})$ for the less restricted window size may be a simple reflection of the central production of high multiplicity events.

In evaluating $C_m(\underline{\delta})$ from the data we have considered $m = 2, 3, 4, 5$ for window sizes $a = 0.25, 0.5, 1.0$ and $\phi_{MAX} = \pi, 2\pi$. In the case of the smaller window sizes we may increase statistics, by sliding a window of width 0.5 (1.0) from $y = -1.0$ to 1.0, since there is no significant y dependence of the results in the central region. Similarly to the case for the independent emission model, there is no systematic difference in the results for π^+ and π^-. We thus present the combined results in order to increase statistics.

In Fig. 8 we present the result for $a = 0.25$, $\phi_{MAX} = \pi$ for comparsion with the IEM. The discrepancy is striking, however we prefer to present the results in terms of the ratio $C_m(\underline{\delta})/C_m^{IEM}(\underline{\delta})$, such that the sensitive dependence on $M_1(\underline{\delta})$ is removed. As a self consistency check on our procedure, we have also evaluated $C_m(\underline{\delta})$ and $C_m^{IEM}(\underline{\delta})$ with no weighting procedure. Although the values of C_m are different, (since $M_1(\underline{\delta})$ is different) the ratio C_m/C_m^{IEM} is consistent with present results, for all m and $\underline{\delta}$.

In Table 4, we give the values of these ratios. We see that for large window $C_m(\underline{\delta}) \approx C_m^{IEM}(\underline{\delta}) \approx C_m$ (total phase space). However for small window $a = 0.25$, $C_m(\underline{\delta})/C_m^{IEM}(\underline{\delta}) > 1$, and this ratio increases with m.

In order to perform correlation analysis on the observed local number fluctuation, the expansion of moments by factorial cumulants is used. According to Mueller [29], each moment is expressed by

$$C_m(\underline{\delta}) = \left(\frac{\partial}{\partial z}\right)^m \exp\left[\sum_{\ell=2}^{\infty} \frac{z^\ell}{\ell!} F_\ell(\underline{\delta}) + z\right]\Big|_{z=0}$$

where $F_\ell(\underline{\delta})$ is the ℓ-th order normalized local factorial cumulant which represents the ℓ-th order correlation.

If there is only two particle correlation (i.e. $F_\ell(\underline{\delta}) = 0$ for $\ell \geq 3$), moments are simply given by

$$\begin{aligned} C_2(\underline{\delta}) &= 1 + F_2(\underline{\delta}) \\ C_3(\underline{\delta}) &= 1 + 3F_2(\underline{\delta}) \\ C_4(\underline{\delta}) &= 1 + 6F_2(\underline{\delta}) + 3(F_2(\underline{\delta}))^2 \\ C_5(\underline{\delta}) &= 1 + 10F_2(\underline{\delta}) + 15(F_2(\underline{\delta}))^2 \\ C_6(\underline{\delta}) &= 1 + 15F_2(\underline{\delta}) + 45(F_2(\underline{\delta}))^2 + 15(F_2(\underline{\delta}))^3. \end{aligned} \tag{5.2}$$

These relations are shown in fig. 9 in terms of $F_2(\underline{\delta})$ by dotted

lines and the observed values of the moments C_m/C_m^{IEM} are shown superimposed on the appropriate line with hatched error bars. The values of C_m/C_m^{IEM} are used rather than $C_m(\delta)$ in order to illustrate nontrivial local fluctuations. This represents a conservative estimate, since C_m^{IEM} itself increases with m. If there are no higher order correlations ($\ell \geq 3$), these observed values should be consistent with a single value of $F_2(\delta)$ for each window size(δ), i.e. the points would lie on a line parallel to the ordinate. For the larger window size the data are consistent with this assumption. Systematic deviations, however, are appreciable for the smaller window, a < 0.25. Therefore, in this case *it seems impossible to explain the observed large local fluctuations* without the contribution from higher order correlations above $\ell \geq 3$.

Such evidence has not been noticed previously [12, 26, 28], since no comparable analysis considering identical particle distributions in such restricted regions of phase space in both rapidity and azimuthal angle, has been made.

The existence of such large fluctuations in the local number distribution for identical particles is consistent with the "pi-ball" effect discussed by Michael and Webber [11]. More detailed theoretical consideration of our results in terms of the cooperative existence of higher order correlations due to Bose symmetry may be found in a separate publication [13].

6. SUMMARY AND CONCLUSION

We have investigated identical particle correlations in multiparticle production in pp interactions at 405 GeV/c.

In two particle correlation analysis we have first considered the two pion invariant mass variable, and we have observed a significant

difference between $\pi^-\pi^-$ and $\pi^+\pi^-$ correlations, at threshold. We have parametrized this difference in terms of the Kopylov-Podgoretsky-Cocconi *formalism including the parameter* λ, which accounts for the possibility of coherent field contributions in the pion source. We obtain the values of the radius R and lifetime τ of the source as

$$R = 1.50 \pm 0.25 \text{ fm and } c\tau \approx 1.2 \pm 0.3 \text{ fm}$$

and the coherence parameter λ is obtained as

$$\lambda \approx 0.58 \pm 0.05$$

We have also investigated the possible existence of higher order correlations among identical particles considering the moments of the local number distribution, and we conclude that the observed fluctuations cannot be explained by two particle correlation alone, and the real existence of higher order correlations is required.

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References

[1] J. Whitmore, Phys. Reports 27C (1976) 187

[2] E. L. Berger, Phys. Letters 49B (1974) 459; J. Ranft and G. Ranft, Nucl. Phys. 97B (1975) 297 and references therein

[3] R. E. Diebold, Repporteur's Talk at the XIX High Energy Physics Conference, Tokyo (1978); W. Kittel, Invited report at the XIII-th International Symp. on Multiparticle Dynamics, Kayserberg (1977)

[4] H. Kichimi et al., KEK-Preprint-78-10, to be published in Phys. Rev. D; Nuovo Cim. Letters 24 (1979) 129

[5] C. M. Bromberg et al., Phys. Rev. D9 (1974) 1864 and Phys. Rev. D10 (1974) 3100; M. Pratap et al., Phys. Rev. Letters 33 (1974) 797; B. Y. Oh et al., Nucl. Phys. 86B (L975) 201; N. N. Biswas et al., Phys. Rev. Letters 35 (1975) 1059, Phys. Rev. Letters 37 (1976) 175

[6] W. J. Knox, Phys. Rev. D10 (1974) 65; M. Chaichian et al., Nucl. Phys. 92B (1975) 445; I. Montvay and H. Satz, Nuovo Cim. 39A (1977) 425

[7] G. I. Kopylov and M. I. Podgoretsky, Sov. J. of Nuc. Phys. 19 (1974) 215 and references therein; G. I. Kopylov, Phys. Letters 50B (1974) 472

[8] G. Cocconi, Phys. Letters 49B (1974) 459

[9] P. Grassberger, Nuc. Phys. 120B (1977) 231

[10] G. N. Fowler and R. M. Weiner LA-UR-77-2567 (1977)

[11] C. Michael and D. Webber, University of Liverpool LTH-41 (1978); E. H. de Groot and H. Satz, Nuc. Phys. 130B (1977) 257

[12] J. W. Lamsa et al., Nuc. Phys. 135B (1978) 258

[13] A. M. Cooper et al., to be published

[14] A. Suzuki et al., Nuovo Cim. Letters 24 (1979) 449

[15] E. L. Berger et al., Phys. Rev. D15 (1977) 206

[16] C. M. Bromberg et al., Phys. Rev. Letters 31 (1973) 1563

[17] G. H. Thomas, Phys. Rev. D15 (1977) 2636; J. Engels and K. Schilling, CERN TH 2401 (1977)

[18] D. Kriegel and R. Naunhauer, PHE 78-9 (1978)

[19] M. Deutschmann et al., Nucl. Phys. B103 (1976) 198; F. Grard et al., Nucl. Phys. B102 (1976) 221; E. Calligarich et al., Nuovo Cim. Letters 16 (1976) 129; N. Angelov et al., Dubna P1-10502 (1977); E. De Wolf, ITHE 76-11; C. Angelini et al., Nuovo Cim. Letters 19 (1977) 279, V. Simak, Proc. of Int. Symp. on $N\bar{N}$ interactions, Stockholm 1976, p319, R. Ledunicky, Proc. of Int. Symp. on $N\bar{N}$ interactions, Stockholm 1976, p447; G. Boreani, Nuovo Cim. Letters 36A (1976) 245

[20] A. Giovanni and G. Veneziano, Nucl. Phys. 130B (1977) 61

[21] C. Ezell et al., Phys. Rev. Letters 38 (1977) 873

[22] D. Drijard et al., CERN/EP/PHYS 78-13

[23] M. Deutschmann et al., CERN/EP/78-1

[24] R. Hagedorn, Nuovo Cim. Suppl. 3 (1965) 147

[25] S. Frautschi, Phys. Rev. D3 (1971) 2821

[26] T. Kafka et al., Phys. Rev. D15 (1977) 1261

[27] M. Biyajima and O. Miyamura, Phys. Letters 77B (1978) 425

[28] V. P. Kenney et al., Tokyo Conference #496

[29] A. H. Meuller, Phys. Rev. D9 (1971) 150

Table Captions

1) The Meuller moments $f_2^{\pm -}$ for data at different laboratory incident momenta, P_{lab}.

2) The functions $R^{\pm -}(M_{\pi\pi})$ for various ranges of $M_{\pi\pi}$.

3) The functions $R^{\pm -}(M_{\pi\pi})$ for various ranges of $M_{\pi\pi}$, in the kinematic regions $q_o \lesssim 0.25$ GeV, $q_t \lesssim 0.40$ GeV/c, when single particle rapidity is restricted to the central region $|y| < 2.0$.

4) Values of $\frac{C_m(\underline{\delta})}{C_m^{IEM}(\underline{\delta})}$ for m = 2, 3, 4, 5 in various window sizes.

Figure Captions

1) The multiplicity distribution.

2) The functions $\rho_2(M_{\pi\pi})$, $[\rho_1 * \rho_1](M_{\pi\pi})$ and $C_2(M_{\pi\pi})$ for (a) $\pi^-\pi^-$ pairs (b) $\pi^-\pi^+$ pairs, multiplied by the inelastic cross section.

3) The functions $C_2^{\pm -}(M_{\pi\pi})/\ln s$ for 200 GeV pp data[15] (full lines) and the present data (dashed lines).

4) The functions $C_2(M_{\pi\pi})$ for (a) $\pi^-\pi^-$, $q_o < 0.25$ GeV (b) $\pi^+\pi^-$, $q_o < 0.25$ GeV (c) $\pi^-\pi^-$, $q_t < 0.4$ GeV/c (d) $\pi^+\pi^-$, $q_t < 0.4$ GeV/c, multiplied by the inelastic cross section. In all figs. the single particle rapidity distribution is restricted to the central region $|y| < 2.0$.

5) The q_o distribution for $\pi^-\pi^-$ pairs (a) for $q_t < 0.40$ GeV/c (b) for $q_t > 0.40$ GeV/c. The dot-dashed line represents Monte Carlo background, and the dashed line represents the $\pi^+\pi^-$ background.

6) The q_t distribution for $\pi^-\pi^-$ pairs (a) for $q_o < 0.25$ GeV (b) for $q_o > 0.25$ GeV. The dot-dashed line represents Monte Carlo background, and the dashed line represents the $\pi^+\pi^-$ background.

7) The K. P. C. fit illustrated on (a) q_t, for $q_o < 0.25$ GeV (b) q_o, for $q_t < 0.40$ GeV/c.

8) The functions $C_m(\underline{\delta})$ for A) the data, in window size a = 0.25, $\phi_{MAX} = \pi$, B) the Independent Emission model, in window size a = 0.25, $\phi_{MAX} = \pi$, C) the Independent Emission model, in window size a = 1.0, $\phi_{MAX} = 2\pi$.

9) The calculated functions $C_m(\underline{\delta})/F_2(\underline{\delta})$ for m = 2, 3, 4, 5 shown as dashed lines, with the values of $C_m(\underline{\delta})/C_m^{IEM}(\underline{\delta})$ of the data superimposed as open circles with hatched error bars, for (a) a = 0.25, $\phi_{MAX} = 2\pi$, (b) a = 0.25, $\phi_{MAX} = \pi$, (c) a = 0.5, $\phi_{MAX} = 2\pi$, (d) a = 0.5, $\phi_{MAX} = \pi$.

Table 1

P_{lab} in GeV/c	f_2^{--}	f_2^{+-}
102	0.28 ± .07	-
205	0.8 ± 0.1	4.0 ± 0.2
405	1.8 ± 0.3	7.3 ± 0.2

Table 2

$M_{\pi\pi}$ in GeV	R^{--}	R^{+-}
0.28 - 0.32	1.06 ± 0.07	0.63 ± 0.04
0.32 - 0.36	0.58 ± 0.06	0.68 ± 0.04
0.36 - 0.40	0.55 ± 0.04	0.64 ± 0.03
0.68 - 0.84	0.33 ± 0.04	0.52 ± 0.04
1.00 - 1.40	0.26 ± 0.06	0.34 ± 0.04

Table 3

	q_o < 0.25 GeV		q_o > 0.25 GeV	
$M_{\pi\pi}$ in GeV	R^{--}	R^{+-}	R^{--}	R^{+-}
0.25 - 0.35	0.79 ± 0.06	0.30 ± 0.04	0.40 ± 0.09	0.28 ± 0.04
0.35 - 0.45	0.54 ± 0.04	0.29 ± 0.02	0.28 ± 0.04	0.30 ± 0.02
0.60 - 0.85	0.35 ± 0.04	0.22 ± 0.03	0.23 ± 0.03	0.17 ± 0.02
	q_t < 0.40 GeV/c		q_t > 0.40 GeV/c	
0.25 - 0.35	0.78 ± 0.06	0.26 ± 0.03	-	-
0.35 - 0.45	0.40 ± 0.04	0.30 ± 0.02	-	-
0.60 - 0.85	0.25 ± 0.04	0.14 ± 0.03	0.35 ± 0.03	0.28 ± 0.02

Table 4

Window Size	m =	2	3	4	5
$a = 0.25$, $\phi_{MAX} = 2\pi$	$\frac{C_m}{C_m^{IEM}}$	1.13 ± 0.10	1.6 ± 0.2	2.6 ± 0.3	4.2 ± 1.2
$a = 0.25$, $\phi_{MAX} = \pi$	$\frac{C_m}{C_m^{IEM}}$	1.17 ± 0.16	1.8 ± 0.3	5.0 ± 1.5	-
$a = 0.5$, $\phi_{MAX} = 2\pi$	$\frac{C_m}{C_m^{IEM}}$	1.05 ± 0.06	1.15 ± 0.15	1.5 ± 0.2	2.2 ± 0.7
$a = 0.5$, $\phi_{MAX} = \pi$	$\frac{C_m}{C_m^{IEM}}$	1.07 ± 0.08	1.3 ± 0.2	1.9 ± 0.4	3.1 ± 1.0
$a = 1.0$, $\phi_{MAX} = 2\pi$	$\frac{C_m}{C_m^{IEM}}$	1.01 ± 0.04	1.1 ± 0.1	1.2 ± 0.2	1.5 ± 0.4

Fig. 1

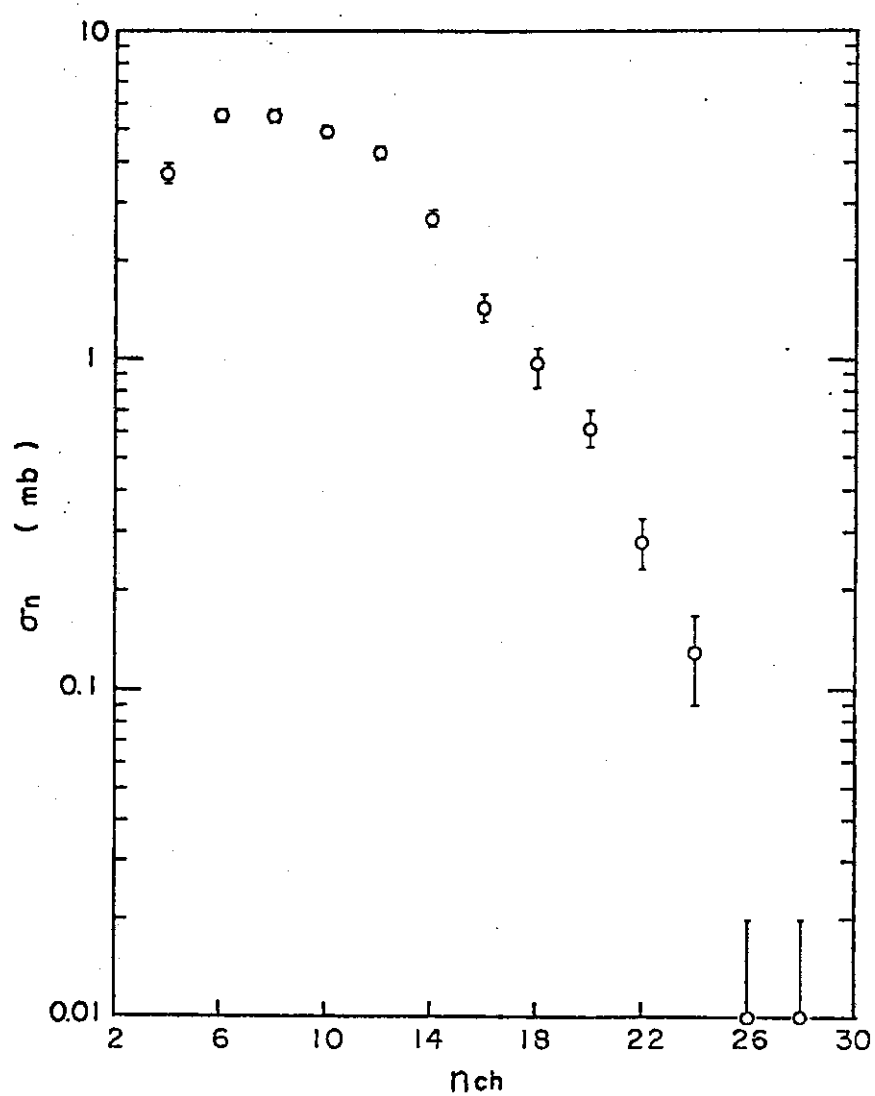


Fig. 2

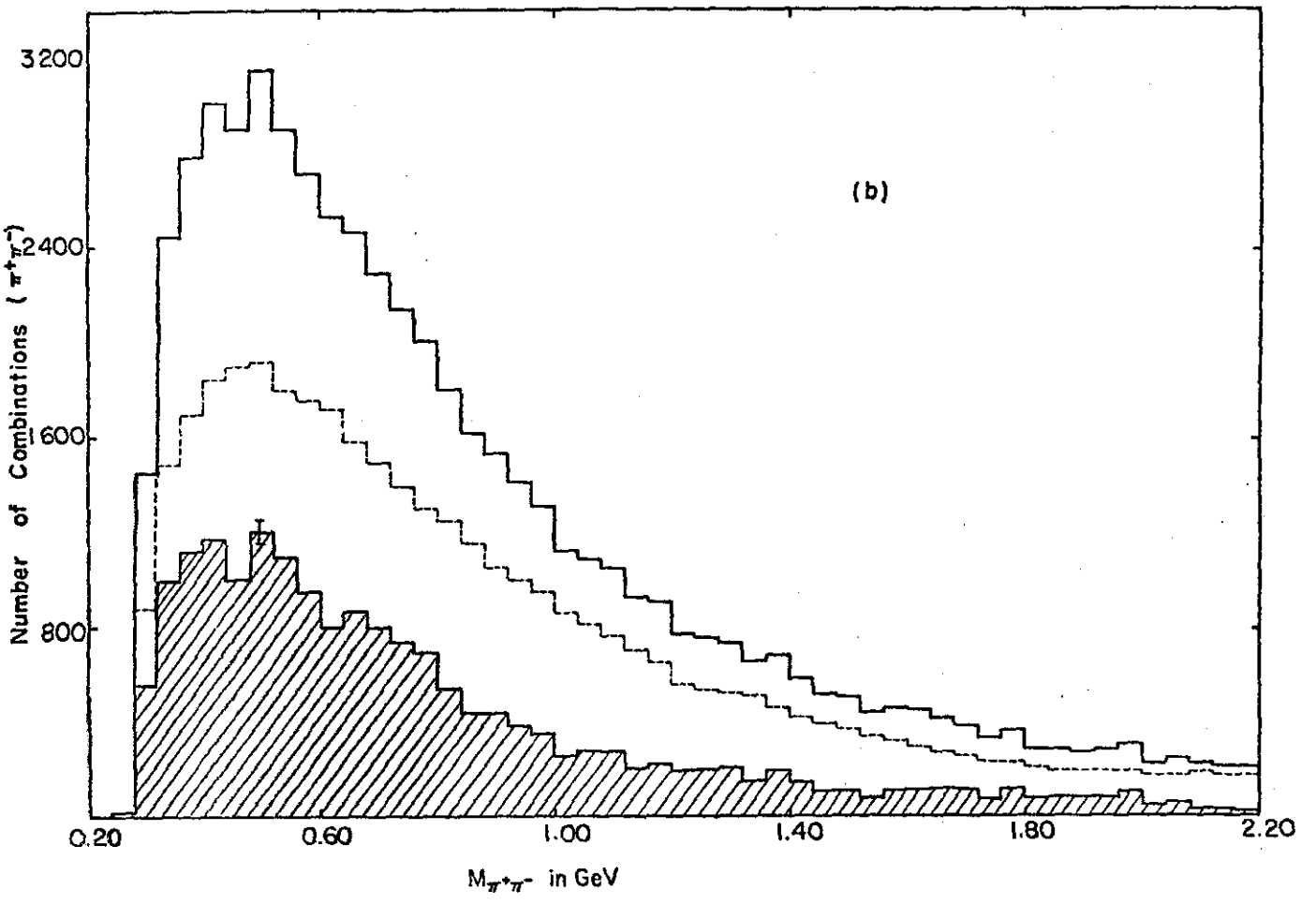

3200
2400
1600
800
Number of Combinations ($\pi^+\pi^-$)
(b)
0.20
0.60
1.00
1.40
1.80
2.20
$M_{\pi^+\pi^-}$ in GeV

Fig. 2

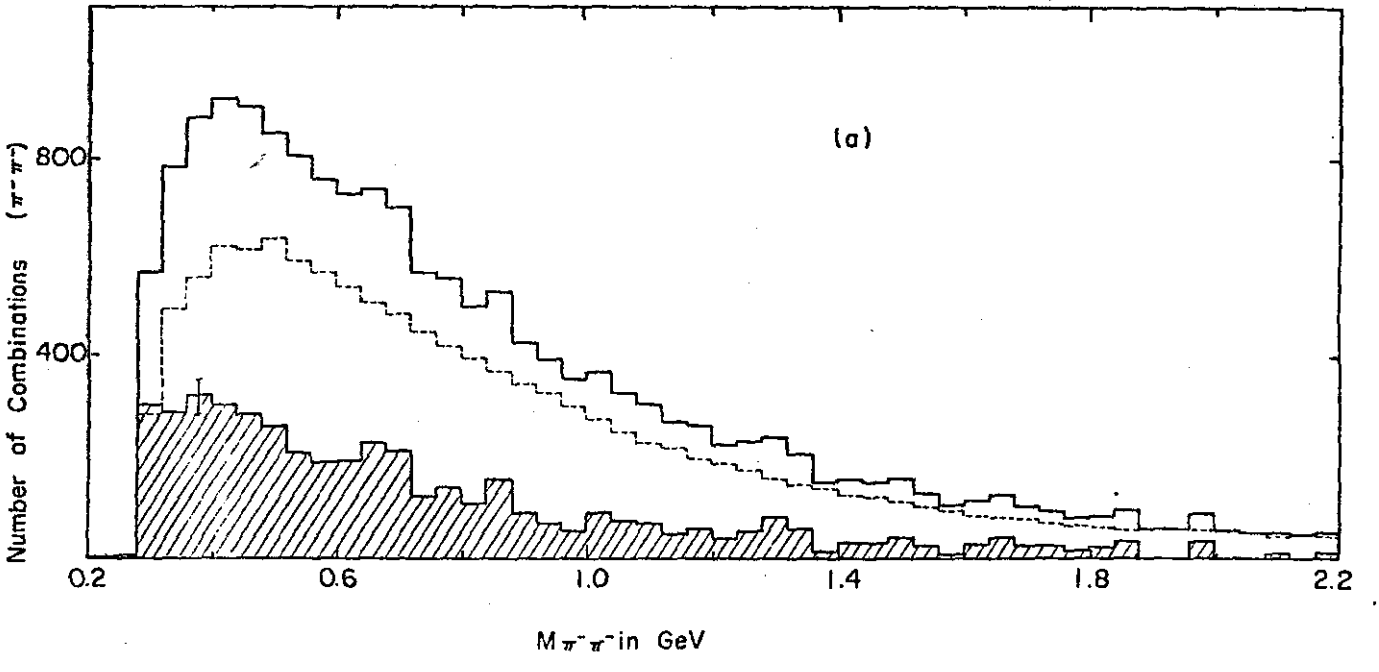

800
400
Number of Combinations ($\pi^-\pi^-$)
(a)
0.2
0.6
1.0
1.4
1.8
2.2
$M_{\pi^-\pi^-}$ in GeV

Fig. 4

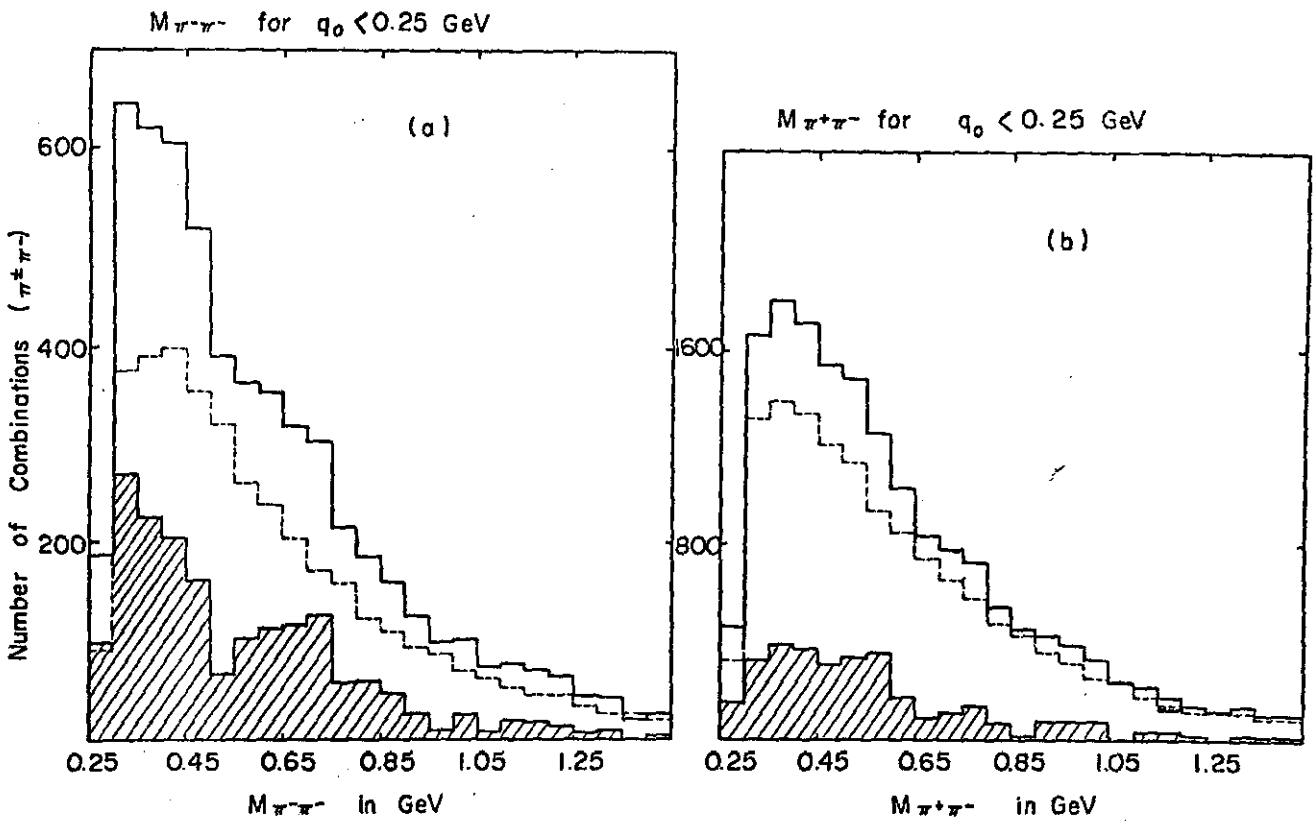

$M_{\pi^-\pi^-}$ for $q_0 < 0.25$ GeV
(a)
$M_{\pi^+\pi^-}$ for $q_0 < 0.25$ GeV
(b)
Number of Combinations ($\pi^\pm\pi^-$)
600
400
200
1600
800
0.25
0.45
0.65
0.85
1.05
1.25
$M_{\pi^-\pi^-}$ in GeV
$M_{\pi^+\pi^-}$ in GeV

Fig.3

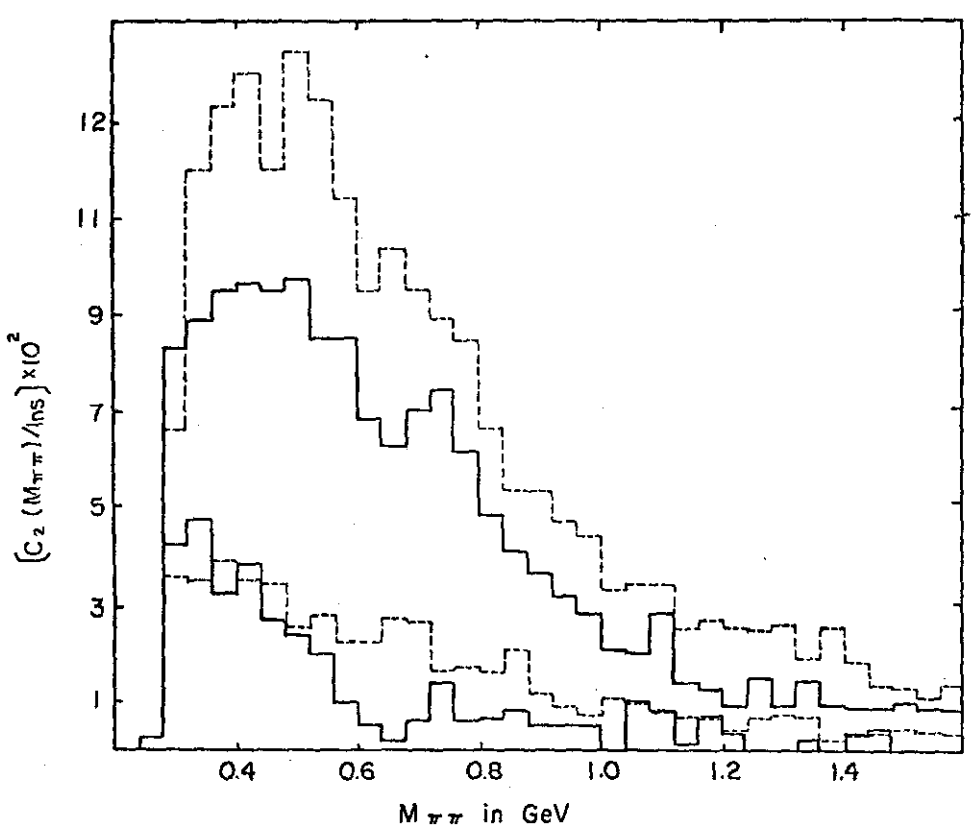

$[C_2(M_{\pi\pi})/I_{nS}] \times 10^2$
12
11
9
7
5
3
1
0.4
0.6
0.8
1.0
1.2
1.4
$M_{\pi\pi}$ in GeV

Fig. 5

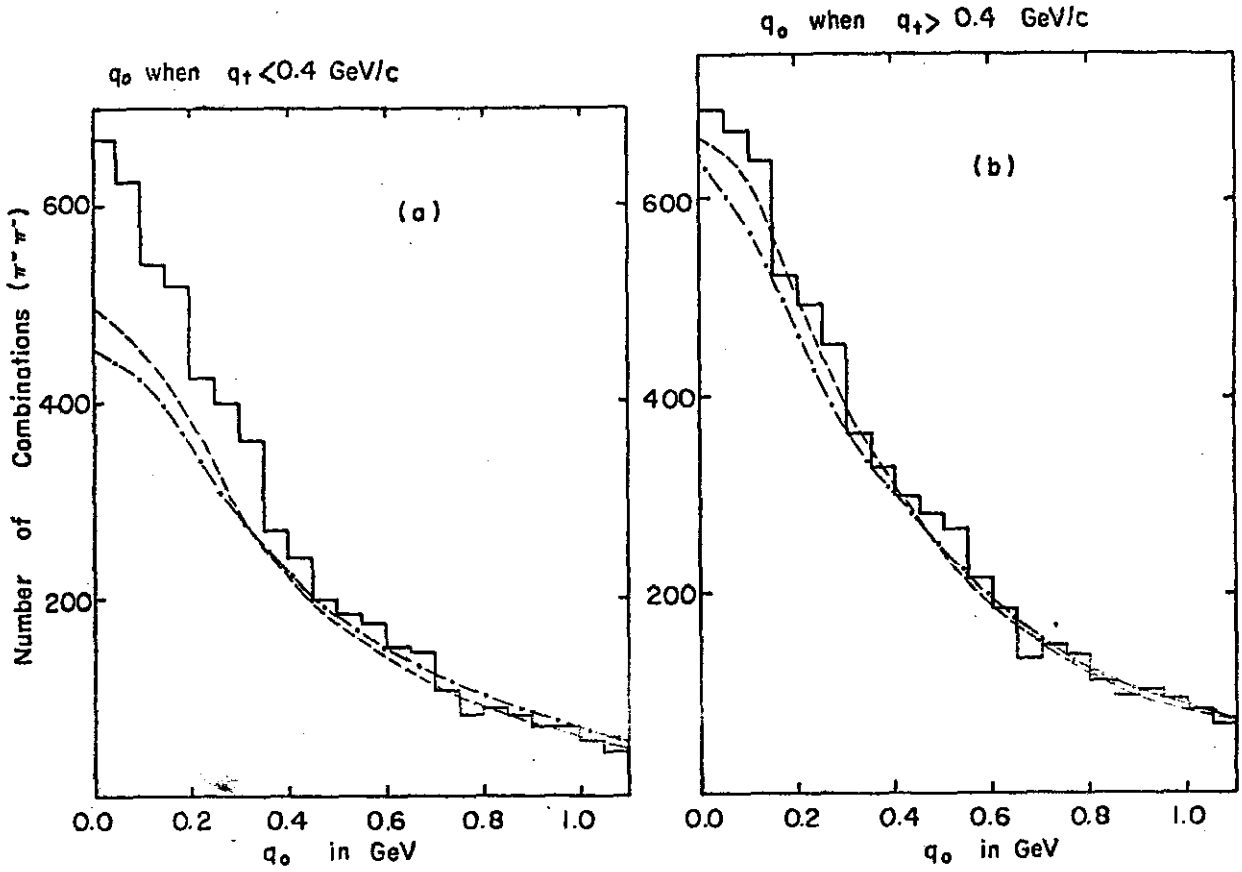

q_o when q_t <0.4 GeV/c
(a)
q_o when q_t> 0.4 GeV/c
(b)
Number of Combinations ($\pi^-\pi^-$)
600
400
200
0.0
0.2
0.4
0.6
0.8
1.0
q_o in GeV

Fig. 4

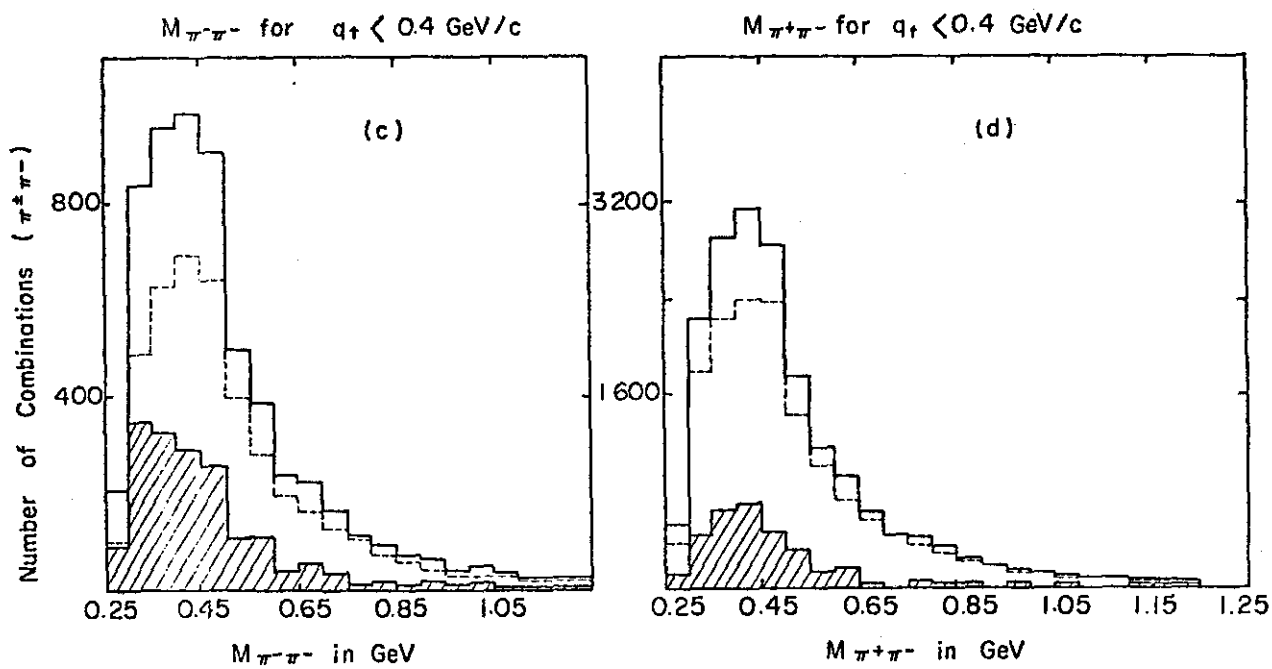

$M_{\pi^-\pi^-}$ for q_t < 0.4 GeV/c
(c)
$M_{\pi^+\pi^-}$ for q_t <0.4 GeV/c
(d)
Number of Combinations ($\pi^\pm\pi^-$)
800
400
3200
1600
0.25
0.45
0.65
0.85
1.05
1.15
1.25
$M_{\pi^-\pi^-}$ in GeV
$M_{\pi^+\pi^-}$ in GeV

Fig .7

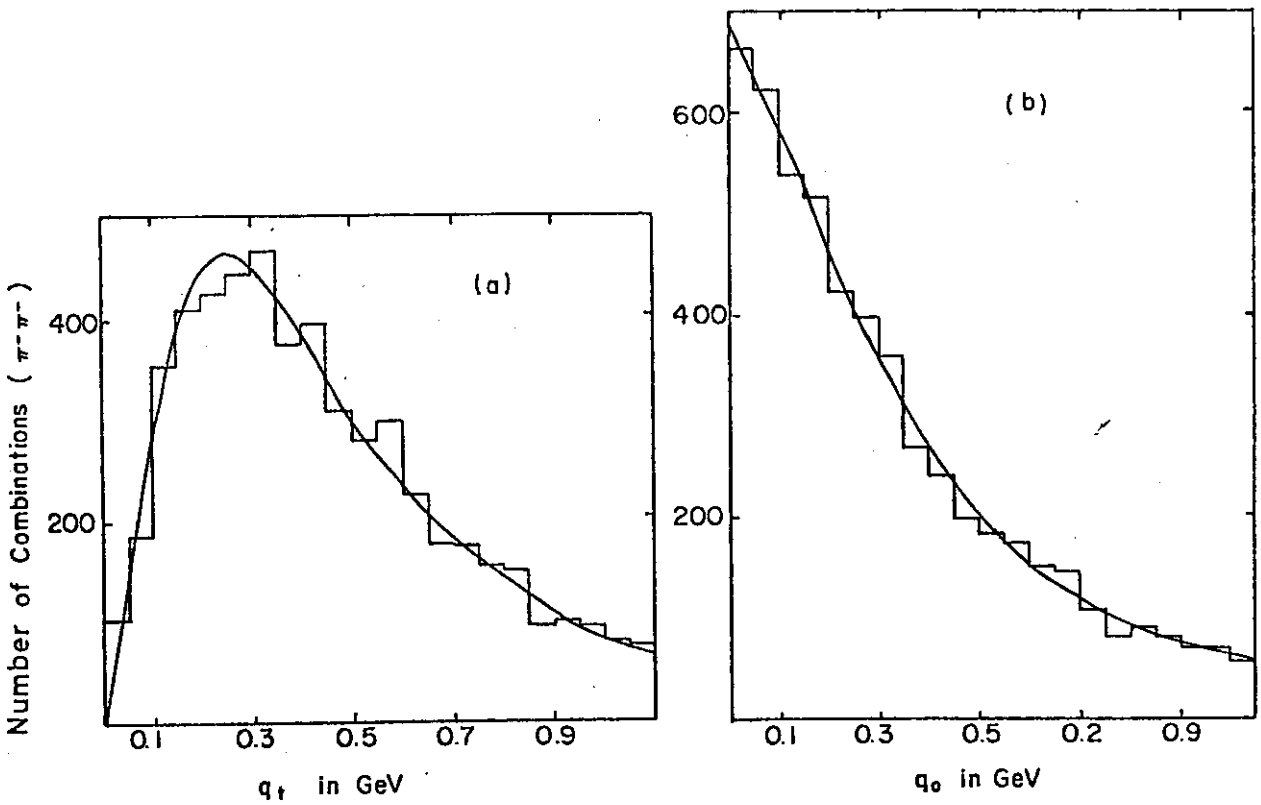
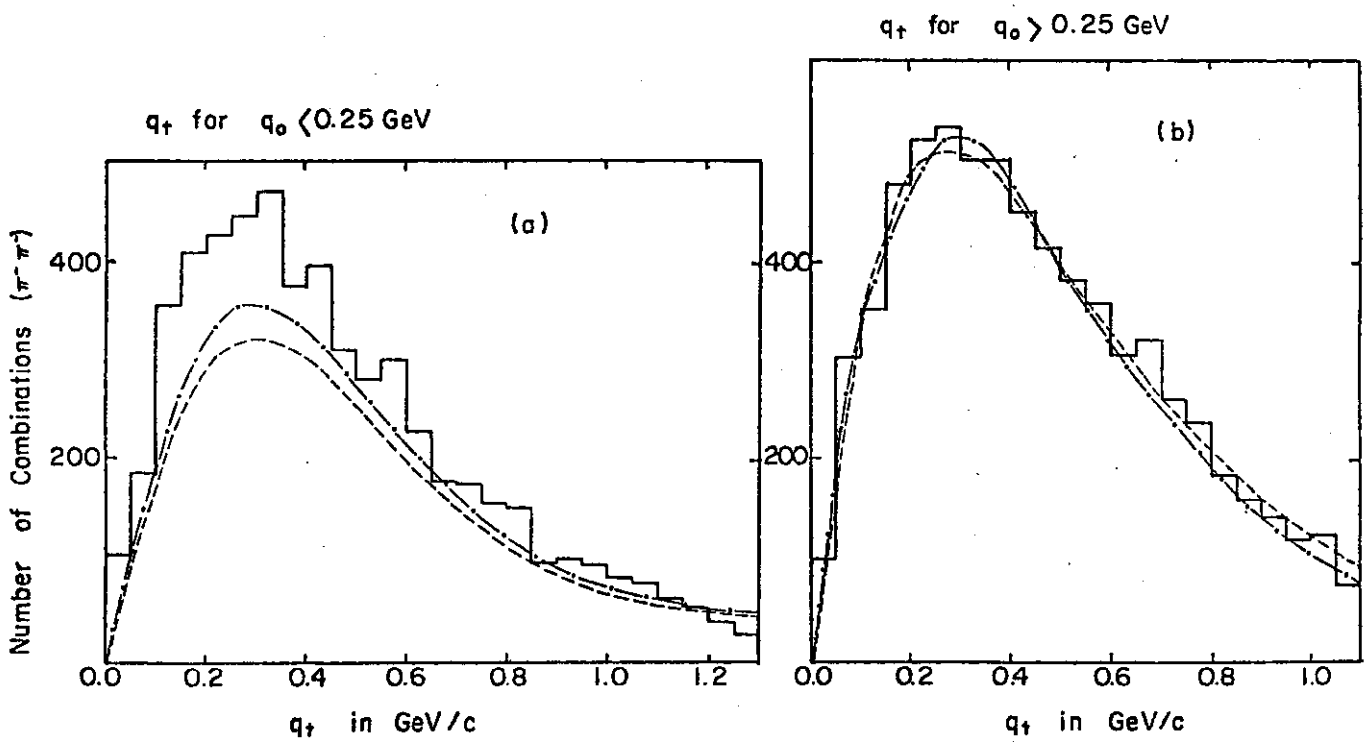

(a)
(b)
Number of Combinations ($\pi^-\pi^-$)
400
200
600
400
200
0.1
0.3
0.5
0.7
0.9
0.1
0.3
0.5
0.2
0.9
q_t in GeV
q_o in GeV

Fig. 6


q_t for q_o <0.25 GeV
q_t for q_o > 0.25 GeV
(a)
(b)
Number of Combinations ($\pi^-\pi^-$)
400
200
400
200
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0.0
0.2
0.4
0.6
0.8
1.0
q_t in GeV/c
q_t in GeV/c

Fig·9

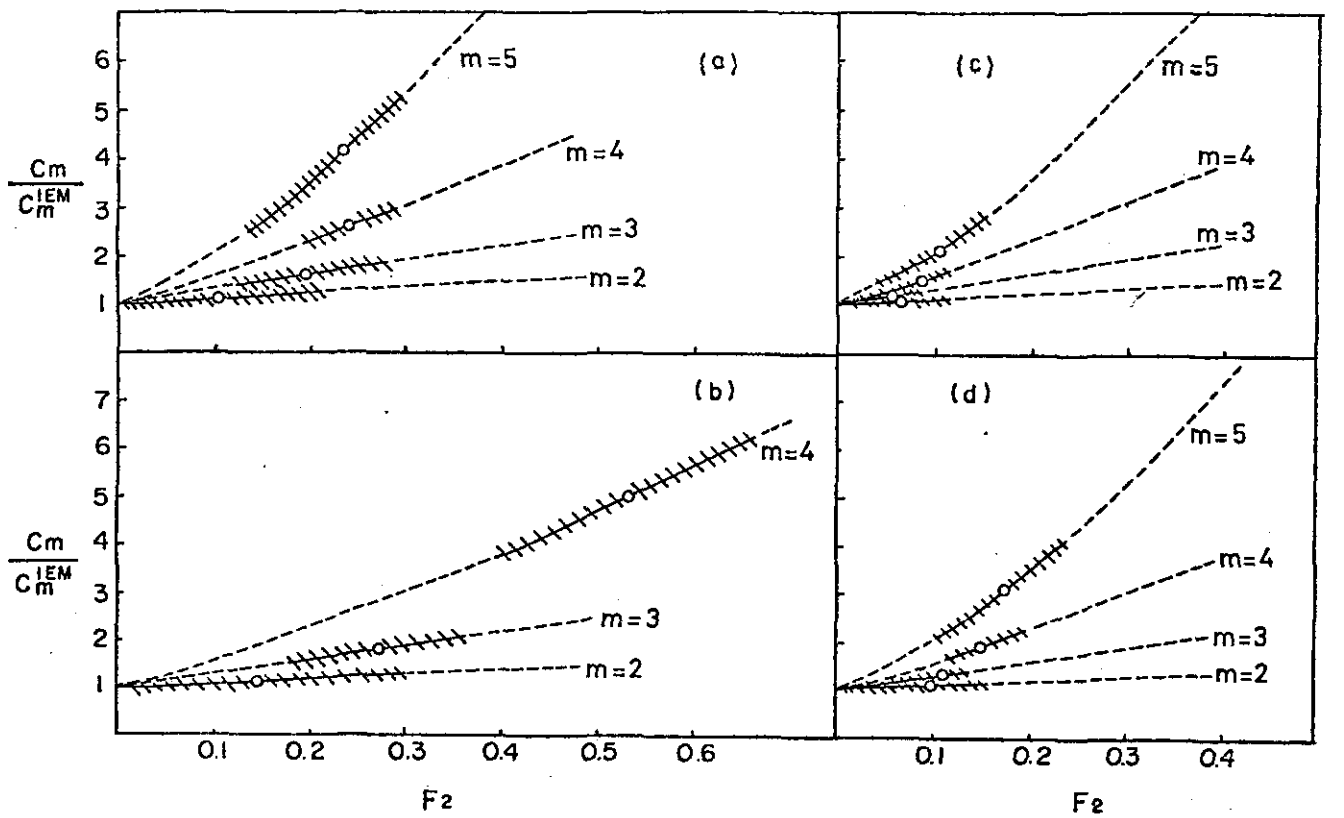
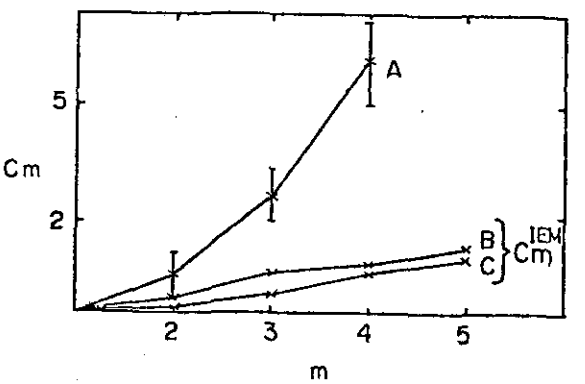



Fig.8