



## Why Most Flavor Dependence Predictions for Nonleptonic Charm Decays are Wrong

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### ABSTRACT

Conclusions about weak interactions from nonleptonic  $D^0$  decays are completely changed by strong final-state interactions. The suppression of  $D^0 \rightarrow \bar{K}^0 \pi^0$  found in some models is completely reversed to  $\Gamma(D^0 \rightarrow \bar{K}^0) = 8\Gamma(D^0 \rightarrow K^- \pi^+)$  by introducing experimentally measured  $K\pi$  scattering phase shifts in the exotic  $I=3/2$  and non-exotic  $I=1/2$  amplitudes. The  $K^+K^-/\pi^+\pi^-$  ratio is very sensitive to meson resonances expected at the  $D$  mass. Predictions less sensitive to final state interactions are discussed.



Recent treatments of nonleptonic decays of charmed mesons<sup>1</sup> consider very specific and detailed properties of the weak interactions but completely ignore strong interaction effects that can completely swamp the effects under consideration.<sup>2,3</sup> The D meson mass is sufficiently close to the resonance region in hadron-hadron scattering to produce large flavor-dependent effects in final state interactions which can completely destroy predictions of flavor dependence in the weak decays. The purpose of this letter is to point out the existence of these effects explicitly and to give some examples of predictions which may be less sensitive to strong interactions.

A simple example of how strong interactions can completely change flavor dependence predictions is given by the  $K\pi$  decays of the  $D^0$ . Some treatments suggest that the  $\bar{K}^0\pi^0$  decay mode is strongly suppressed<sup>3,4</sup> relative to  $K^-\pi^+$ . However, both the  $\bar{K}^0\pi^0$  and  $K^-\pi^+$  states are linear combinations of isospin eigenstates with  $I=1/2$  and  $I=3/2$ . To see effects of strong interactions, the decay amplitudes should be expressed in terms of these isospin amplitudes. Suppression of the  $\bar{K}^0\pi^0$  mode implies that the two amplitudes nearly cancel in the  $\bar{K}^0\pi^0$  mode and add constructively in the  $K^-\pi^+$  mode. This cancellation is changed by final state interactions which shift the relative phases.

This phenomenon is seen quantitatively in a simple model with all final state interactions parametrized by introducing phase shift factors,  $e^{i\delta_1}$  and  $e^{i\delta_3}$  and writing the  $D^0$  decay amplitudes

$$A(D^0 \rightarrow K^- \pi^+) = \sqrt{(1/3)} A_3 e^{i\delta_3} - \sqrt{(2/3)} A_1 e^{i\delta_1} \quad (1a)$$

$$A(D^0 \rightarrow \bar{K}^0 \pi^0) = \sqrt{(2/3)} A_3 e^{i\delta_3} - \sqrt{(1/3)} A_1 e^{i\delta_1} \quad (1b)$$

where  $A_1$  and  $A_3$  denote the  $I=1/2$  and  $I=3/2$  amplitudes when the final state interactions are neglected. The effect of final state interactions on models predicting the suppression of the neutral state (1b) is tested by assuming a complete suppression in the absence of final state interactions. Then

$$A_1 = -\sqrt{2} A_3 \quad (2a)$$

and

$$\Gamma(D^0 \rightarrow \bar{K}^0 \pi^0) = \frac{8\Gamma(D^0 \rightarrow K^- \pi^+)}{\{3 \cot [(\delta_3 - \delta_1)/2] + 1\}^2} \quad (2b)$$

The neutral decay is seen to be suppressed only if  $\delta_3 \delta_1$ . But the  $I=3/2$  channel is exotic and has no resonances; the  $I=1/2$  channel is not exotic and has many  $K^*$  resonances. The  $D$  mass is sufficiently close to the resonance region so that the two  $K\pi$  phase shifts should be affected very differently by nearby resonances. This is shown dramatically in a recent partial wave analysis of elastic  $K\pi$  scattering.<sup>5</sup> The  $I=1/2$  s-wave shows a resonance with a mass of 1.4 to 1.45 GeV and a width of 200-300 MeV, giving an s-wave phase at 1.85 GeV varying between  $110^\circ$  and  $160^\circ$  for different solutions to the analysis. The  $I=3/2$  s-wave shows no resonances and a smooth phase variation well described by an effective range fit with a value around  $-25^\circ$  to  $-30^\circ$  at 1.85 GeV. For  $\delta_3 - \delta_1 = -180^\circ$  the suppression is completely reversed,  $\Gamma(D^0 \rightarrow \bar{K}^0 \pi^0) = 8\Gamma(D^0 \rightarrow K^- \pi^+)$ , and other predictions<sup>3,4</sup> are drastically modified.

Even above the resonance region there is a considerable energy range in the Regge region where the phases of exotic and nonexotic amplitudes are known to be very different. Only at high energies where the Pomeron completely dominates the scattering and gives an almost pure imaginary phase can the difference between  $\delta_3$  and  $\delta_1$  be neglected. Thus the experimental fact that the  $\bar{K}^0 \pi^0$  and  $K^- \pi^+$  decays are of the same order of magnitude is simply explained by hadronic final state interactions. Any attempts to explain the data only by weak interactions or gluon exchange diagrams without

considering hadronic final state interactions and isospin factors are open to serious criticism. The difference between exotic and nonexotic channels is defined by hadron flavor exchange processes and these essential physical features cannot be omitted from any realistic treatment.

Complete descriptions of nonleptonic decay must include such final state interactions in full dynamical calculations which are not possible at present. Approximate estimates of final state interactions are obtainable from phenomenological models together with hadron scattering data and constraints from analyticity and unitarity.<sup>6</sup> Alternatively, effects of final state interactions are avoided in flavor symmetry predictions using groups which are approximately symmetries of strong interactions and automatically take into account all final state interactions invariant under this approximate symmetry.

One popular procedure has been to neglect the mass differences among the light (u,d,s) quarks and to assume that  $SU(3)_{uds}$  flavor symmetry is a good symmetry broken only by  $H_{\text{weak}}$ , while rejecting all higher symmetries as being badly broken by masses.<sup>1,7,8</sup> In this way a number of independent  $SU(3)$  amplitudes are defined, which are taken as free parameters in fitting the data. The disadvantage of the  $SU(3)_{uds}$  approach is that a large number of different amplitudes contribute to any given process, and  $SU(3)$  breaking is not easily incorporated. It is therefore

difficult to interpret any disagreement in fits to data.

Symmetry does not generally lead to simple predictions because there are too many different independent invariant amplitudes. If the initial state transforms under the symmetry like a member of an  $i$ -dimensional representation of the group and  $H_{\text{weak}}$  transforms like a member of an  $h$ -dimensional representation, the number of independent amplitudes is equal to the number of irreducible representations appearing in the product  $i \times h$  which are also allowed for the particular final states considered. In the  $SU(3)_{\text{uds}}$  treatments, the charmed mesons transform like a 3 and  $H_{\text{weak}}$  has three pieces which transform respectively like a  $3^*$ , a 6 and a  $15^*$ , giving independent amplitudes corresponding to all representations appearing in the products

$$\begin{aligned} 3 \otimes 3^* &= 1 \oplus 8 \\ 3 \otimes 6 &= 8 \oplus 10 \\ 3 \otimes 15^* &= 8 \oplus 10^* \oplus 27 . \end{aligned}$$

These define seven independent amplitudes for a particular type of final state, unless some are excluded by the allowed final state couplings. For the particular case of decays into two octet pseudoscalar mesons, Bose statistics of a  $0^+$  state excludes the antisymmetric 10 and  $10^*$  representations

and uniquely chooses the D coupling for the octet so that only five independent amplitudes remain. But five is still an unwieldy number for extracting the physics of symmetry breaking when the predictions are violated by experiment.

An alternative approach is to use other subgroups of the maximum flavor symmetry, chosen to give simple predictions. The effects of symmetry breaking can then be considered for each individual case, and the simplicity of the predictions makes the underlying physics more transparent. The product  $i \times h$  simplifies when the initial state transforms like a singlet under the symmetry group and the relevant terms in  $H_{\text{weak}}$  are classified in only one irreducible representation. In this case the final state transforms like these relevant terms in  $H_{\text{weak}}$  and gives only a single amplitude. Simple predictions are also obtainable from discrete transformations like Weyl reflections which transform one decay process into another and can give simple equalities.<sup>9</sup> The relevant reflections correspond to a simple interchange of two quark flavors and are useful if the active terms in  $H_{\text{weak}}$  behave simply under this interchange.

The simplest example is the isospin group, for which symmetry breaking effects are completely negligible in strong interactions. The Cabibbo favored component of the charm-changing part of  $H_{\text{weak}}$  transform like the charged components of an isovector. The  $F^+$  meson is a singlet under isospin. Thus only isovector final states are allowed for

Cabibbo-favored  $F^+$  decays and a well known result is obtained from the observation that the spin zero  $\pi^+\pi^0$  state has isospin 2,

$$\Gamma(F^+ \rightarrow \pi^+\pi^0) = 0, \quad (3)$$

This selection rule follows only from the isospin transformation properties of  $H_{\text{weak}}$  and the isospin invariance of the strong interactions and should be unaffected by SU(3) symmetry breaking, strong final-state interactions or an increase in the number of quark flavors. It is therefore no surprise that this selection rule holds in an SU(3) treatment even when the number of flavors are increased from four to six.<sup>2</sup>

The  $D^+$  and  $D^0$  mesons constitute an isospin doublet. With an isovector  $H_{\text{weak}}$ , two values of isospin are allowed for the final state,  $I=1/2$  and  $3/2$ . Thus isospin gives no simple predictions for D decays; the best obtainable is a triangular inequality relating the amplitudes for  $D^0 \rightarrow K^-\pi^+$ ,  $\sqrt{2}(D^0 \rightarrow \bar{K}^0\pi^0)$  and  $D^+ \rightarrow \bar{K}^0\pi^+$ . Again it is no surprise that this inequality holds in SU(3) treatments independent of the number of quark flavors.<sup>2</sup> The contrast between this complicated inequality and the selection rule (3) shows the advantage of singlet initial states. A doublet initial state gives two independent amplitudes which are already too

many. Triangular inequalities are not very satisfactory tests for models, in comparison with equalities or selection rules.

The U spin group gives simple predictions for decays of the  $D^0$  meson which transforms like a singlet.<sup>9</sup> U spin is particularly useful in the four quark model, where the charm-changing part of  $H_{\text{weak}}$  transforms like a pure U spin vector, and the final state in  $D^0$  decays is pure U vector. The U spin analog of the selection rule (3) is

$$\Gamma(D^0 \rightarrow K^0 \bar{K}^0) = 0 \quad (4a)$$

Another well known U spin prediction is

$$\Gamma(D^0 \rightarrow K^+ K^-) = \Gamma(D^0 \rightarrow \pi^+ \pi^-) . \quad (4b)$$

However, these relations (4) are not as solid as the isospin selection rule (3) for two reasons.

1. U spin symmetry breaking cannot be neglected to the same degree as isospin.

2.  $H_{\text{weak}}$  transforms like a pure U vector only in the four quark model. With more than four quarks, a U-spin scalar component also appears.

The effects of U spin symmetry breaking are similar to those already discussed<sup>10</sup> for the analogous electromagnetic U spin predictions,

$$\sigma(e^+e^- \rightarrow \gamma + K^0 \bar{K}^0) = 0 \quad (5a)$$

$$\sigma(e^+e^- \rightarrow K^+ K^-) = \sigma(e^+e^- \rightarrow \pi^+ \pi^-) . \quad (5b)$$

1. Violation by SU(3) breaking of equalities or cancellations between pairs of diagrams. The  $K^0 \bar{K}^0$  state contains two quark-antiquark pairs, one  $s\bar{s}$  and one  $d\bar{d}$ . In the dominant diagrams contributing to both forbidden reactions (4a) and (5a) one pair is created in a hard electroweak vertex and the other in a soft strong vertex. There are two diagrams in which the roles of the  $s\bar{s}$  and  $d\bar{d}$  pairs are reversed. In the U spin or SU(3) limit, these two diagrams exactly cancel. The symmetry is broken by the s-d mass difference, which can destroy this cancellation. It is reasonable to assume that the hard electroweak vertices are pointlike and are unaffected by the s-d mass difference. But if it is easier to create nonstrange quark pairs out of the vacuum than strange pairs in strong processes, then the diagram in which the  $s\bar{s}$  pair is created strongly will not cancel the other diagram and the selection rule will fail. Whether this U spin breaking is significant at this mass is still an open question, with arguments presented on both sides.<sup>7,11,12</sup>

The equalities (4b) and (5b) do not depend upon such cancellations but upon the equality of contributions from pairs of diagrams in which a  $d\bar{d}$  or  $s\bar{s}$  pair is created by the hard vertex, while the additional  $u\bar{u}$  pair is created in the same way in both cases either hard or soft. If these diagrams provide the major contribution, the equalities (4b) and (5b) would be less sensitive to symmetry breaking than the selection rules (4a) and (5a). The electromagnetic case also has dominant diagram in which the  $u\bar{u}$  pair is created by the photon and  $d\bar{d}$  and  $s\bar{s}$  pairs are created strongly. This is absent in the dominant contribution to the  $D^0$  decays (4b) since the  $u\bar{u}$  state is forbidden by U spin for a pure U spin vector state, and additional U spin breaking is required to obtain the diagram in the first place as in the case of the models with more than four quarks. Once such diagrams are introduced, the mass breaking must also be considered; but the mass breaking alone cannot introduce a violation by this mechanism.

2. SU(3) breaking in resonance mass spectra. The predictions (5) are clearly violated at the  $\phi$  mass, where the forbidden reaction (5a) is equal to the allowed production of charged kaon pairs, and there are no charged pion pairs.<sup>10</sup> In the SU(3) symmetry limit, the amplitude for the reaction (5a) via the  $\phi$  would be canceled by contributions from the  $\rho$  and  $\omega$ , and these would also restore a charged pion amplitude satisfying the equality (5b). But

because the vector nonet is not degenerate, the relations (5) are strongly violated.

A similar situation clearly obtains for the charmed meson decay predictions (4). If there are any scalar meson resonances at the  $D^0$  mass which are not in degenerate nonets, the predictions can be strongly violated.

The same approach used in Eqs. (1) can be applied to estimate the correction of the selection rule (4a) for final state interactions in the  $K\bar{K}$  system. Assuming isospin invariance we define phase shifts  $\delta_0$  and  $\delta_1$  for the final states of isospin zero and one respectively. Then the amplitudes for the  $K\bar{K}$  decays can be written

$$A(D \rightarrow K^+ K^-) = \sqrt{(1/2)} (A_0 e^{i\delta_0} + A_1 e^{i\delta_1}) \quad (6a)$$

$$A(D \rightarrow K^0 \bar{K}^0) = \sqrt{(1/2)} (A_0 e^{i\delta_0} - A_1 e^{i\delta_1}), \quad (6b)$$

where  $A_0$  and  $A_1$  are the isoscalar and isovector amplitudes when the final state interactions are neglected. The selection rule (4a) implies that  $A_0 = A_1$  in the U spin vector approximation. The correction to the selection rule (4a) due to final state interactions is

$$\Gamma(D \rightarrow K^0 \bar{K}^0) = \Gamma(D \rightarrow K^+ K^-) \tan^2 [(\delta_0 - \delta_1)/2] . \quad (6c)$$

In the SU(3) symmetry limit the isoscalar and isovector phase shifts are equal and the selection rule (4a) is recovered from (6c). However, known symmetry breaking in the structure of isoscalar and isovector  $K\bar{K}$  resonances suggests that  $\delta_0$  and  $\delta_1$  are not equal so close to the resonance region.

A scalar resonance denoted by  $\epsilon(1300)$  with a width of 200-400 MeV has been reported under the f meson. If this resonance and the  $K^*$  resonance at 1420 mentioned above are members of an SU(3) nonet, a similar scalar state coupled only to kaons can be expected under the f(1516). If this resonance has a large width, its tail could still be appreciable at the D mass and effect the decay to the  $K^+K^-$  final state with no effect on  $\pi^+\pi^-$ . A relatively small resonant amplitude interfering constructively with non-resonant background could explain effects of the order of the experimental discrepancies reported for the relation (4b).

Experimental information on s-wave  $\pi\pi$  and  $K\bar{K}$  scattering amplitudes at the D mass is necessary in order to either take these effects into account properly or to prove that they are negligible. Without such information it is very difficult to trust any calculation which attempts to explain the observed discrepancy between experiment and the prediction (4b). Until effects of this kind are properly investigated, any attempts to fit the data by introducing

new weak interaction contributions are unconvincing.

Predictions from U-spin properties of  $H_{\text{weak}}$  less sensitive to symmetry breaking may be obtained by using the charge conjugation invariance of strong interactions. The U-spin Weyl reflection which interchanges s and d flavors induces the transformations:

$$K^0 \leftrightarrow \bar{K}^0 \quad (7a)$$

$$K^+\pi^- \leftrightarrow K^-\pi^+ . \quad (7b)$$

A final state containing only  $K^0$  and  $\bar{K}^0$  mesons together with  $K^+\pi^-$  and  $K^-\pi^+$  pairs goes into its charge conjugate state under the U-spin reflection. Since the  $D^0$  goes into itself under any U spin transformation, the transformation (7) relates any  $D^0$  decay into these particles to a  $D^0$  decay into its charge conjugate state. For example, the assumption that  $H_{\text{weak}}$  transforms like a pure U spin vector which leads to the relations (4) also gives the relation:

$$\Gamma(D^0 \rightarrow K^0 K^-\pi^+) = \Gamma(D^0 \rightarrow \bar{K}^0 K^+\pi^-) . \quad (8)$$

Like the predictions (4), this prediction (8) no longer holds if there are more than four quarks, or if additional diagrams introduce a U spin scalar component into the effective  $H_{\text{weak}}$ . However, the kind of symmetry breaking

discussed in connection with Eq. (6) and in particular the effects of resonances should not affect the relation (8) since the resonance structure of the two final states must be identical. Thus a comparison of the experimental tests of the two predictions (6) and (8) should give an indication of whether the violation of (6) presently observed comes from an additional  $U=0$  component in  $H_{\text{weak}}$  or from  $U$ -spin violating final state interactions.

Additional predictions are obtainable from  $U$  spin reflections which relate Cabibbo allowed transitions to doubly unfavored transitions. The terms in  $H_{\text{weak}}$  which generate these transitions go into one another under  $U$  spin reflection. Consider the  $\Delta C=1$  part of  $H_{\text{weak}}$  in the notation of Quigg<sup>1</sup>

$$\begin{aligned} \mathcal{H}(\Delta C = 1) = & \{\bar{c}s, \bar{d}u\}V_{11}V_{22} + \{\bar{c}s, \bar{s}u\}V_{12}V_{22} \\ & + \{\bar{c}d, \bar{d}u\}V_{11}V_{21} + \{\bar{c}d, \bar{s}u\}V_{12}V_{21} , \end{aligned} \quad (9)$$

The first and fourth terms of (9) are seen to be two components of the same  $U$  spin vector which go into one another under the  $U$ -spin Weyl reflection, except for the difference in the coefficients. These terms describe Cabibbo favored and doubly unfavored transitions respectively. Thus any pair of favored and doubly unfavored transitions which go into one another under the  $U$  spin

reflection must satisfy the relation,

$$\frac{\Gamma(D^0 \rightarrow f)}{\Gamma(D^0 \rightarrow \tilde{f})} = \frac{\Gamma(D^+ \rightarrow f')}{\Gamma(D^+ \rightarrow \tilde{f}')} = \frac{\Gamma(F^+ \rightarrow f'')}{\Gamma(D^+ \rightarrow \tilde{f}'')} = \left[ \frac{V_{12}V_{21}}{V_{11}V_{22}} \right]^2, \quad (10)$$

where  $f$ ,  $f'$  or  $f''$  denotes any Cabibbo favored final state for the decay considered, and  $\tilde{f}$  denotes the doubly unfavored state obtained from  $f$  by a U spin reflection.

For the case where the final states contain only the mesons  $K^0$  and  $\bar{K}^0$  and the meson pairs  $K^+\pi^-$  and  $K^-\pi^+$ , relations are obtained between final states which are charge conjugates and where effects of final state interactions can be expected to be much smaller. For the case of final states of two pseudoscalar mesons, the  $\pi^0$  and  $\eta$  also appear in simple U-spin equalities because the contribution from the  $U=1$  mixture to  $\pi^0$  and  $\eta$  vanishes as a result of a selection rule related to the selection rule (4a) by a U spin rotation. Thus

$$\frac{\Gamma(D^0 \rightarrow K^+\pi^-)}{\Gamma(D^0 \rightarrow K^-\pi^+)} = \frac{\Gamma(D^0 \rightarrow K^0\pi^0)}{\Gamma(D^0 \rightarrow \bar{K}^0\pi^0)} = \frac{\Gamma(D^0 \rightarrow K^0\eta)}{\Gamma(D^0 \rightarrow \bar{K}^0\eta)} = \left[ \frac{V_{12}V_{21}}{V_{11}V_{22}} \right]^2. \quad (11)$$

These predictions are unaffected by a treatment of final

state interactions like Eq. (1a) if the phase shifts are invariant under charge conjugation.

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## FOOTNOTES AND REFERENCES

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