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Charmed Meson Decays and the Structure of the Charged Weak Current

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ABSTRACT

Decays of charmed mesons into two pseudoscalars are considered in the framework of weak interaction models with more than four quarks. Possible patterns of nonleptonic enhancement, and their experimental consequences are discussed.

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I. INTRODUCTION

Since the discovery[1] of the $\psi/J(3095)$ in 1974, the hope has persisted that the study of charmed particle decays would lead to new insights into the nature of the weak interactions.[2] A question of considerable importance is the possibility of a nonleptonic enhancement of the charm-changing weak interaction. The measurements of relative and absolute lifetimes of charmed particles and of branching ratios for semileptonic decays which are now becoming feasible open the way to detailed exploration of this topic. Of comparable importance is the structure of the charm-changing weak current itself. The dominant decays of charmed particles confirm the presence of the Cabibbo-favored charm to strange transition. Until recently, the only indication for Cabibbo-suppressed transitions came from the observation at approximately the expected rate of a valence component in neutrino-induced dilepton events.[3] The discovery of $\Upsilon(9460)$,[4] signalling a fifth quark, and of the fifth lepton $\tau(1782)$[5] indirectly suggests that the charm-changing current may be more complicated than the four-quark Glashow-Iliopoulos-Maiani[6] form. Some time ago it was pointed out by Donoghue and Wolfenstein[7] that branching ratios for Cabibbo-suppressed decays of charmed mesons may be quite sensitive to the presence of additional terms in the charm-changing current. Two Cabibbo-suppressed decays of the D^o-meson have now been detected.[8] In response, Suzuki[9,10] and Wang and Wilczek[11] have called attention to the implications of precise measurements of Cabibbo-suppressed decay rates for the structure of the weak current, in the particular context of weak interaction models with more than four quarks.

In this note two things are done. The issues of nonleptonic enhancement are reemphasized in anticipation of experimental information which will become available presently. Modifications to the charm-changing weak current which proceed from the existence of more than four quark flavors are explained, and some experimental consequences are derived. The analysis is based on the SU(3) symmetry approach which has proved so fruitful for the analysis of nonleptonic hyperon decays[12] and was applied to charmed meson decays in the four-quark model by many authors.[13,14,15,7] The treatment given here differs from the earlier work of Suzuki[9,10] and of Wang and Wilczek[11] in that attention is focussed upon the group theoretical structure of the weak Hamiltonian and upon the SU(3) representation of the final state. This permits a systematic discussion of nonleptonic enhancement and makes possible the straightforward imposition of additional symmetry requirements.

The plan of this article is as follows. In §II I review the representation structure of the weak interaction Hamiltonian in models that replicate the basic elements of the Weinberg[16]-Salam[17]-GIM[6] scheme, including the six-quark generalization due to Kobayashi and Maskawa[18] which is now in vogue. Section III is devoted to the presentation and discussion of amplitudes for charmed meson decays. Theoretical preconceptions for patterns of nonleptonic enhancement are recalled, and prospects for experimental tests are described. I then turn to the Cabibbo-suppressed decays and their implications for quark mixing. A number of sum rules and inequalities are derived. Some of these results may be sharpened by imposing a nonet scheme upon the final state, or a restricted form of SU(4) symmetry upon the weak Hamiltonian. In a closing §IV I summarize the opportunities presented by the vigorous experimental study of charmed meson decays.

II. REPRESENTATION CONTENT OF THE WEAK-INTERACTION HAMILTONIAN

In preparation for the calculations to follow, let us review the group structure of the hadronic weak currents. The conventions follow those of Einhorn and Quigg.[15] Consider models in which there are n generations of left-handed color-triplet quark doublets

$$\begin{pmatrix} q_{1+} \\ q_{1-} \end{pmatrix}_L , \begin{pmatrix} q_{2+} \\ q_{2-} \end{pmatrix}_L , \cdots \begin{pmatrix} q_{n+} \\ q_{n-} \end{pmatrix}_L \tag{2.1}$$

and in which flavor-changing neutral currents are eliminated by a generalization of the Glashow-Iliopoulos-Maiani mechanism.[6] The 2n quark fields may be represented as a composite spinor

$$\phi^\alpha = \begin{bmatrix} q_{1+} \\ q_{2+} \\ \vdots \\ q_{n+} \\ q_{1-} \\ q_{2-} \\ \vdots \\ q_{n-} \end{bmatrix} , \tag{2.2}$$

in which color indices have been suppressed. The space-time structure of the current, assumed to be of a V - A form, will not concern us. It is therefore convenient to adopt an abbreviated notation in which, for example, $\bar{u}d$ represents $\bar{u}\gamma^\mu(1-\gamma_5)d$. The charged weak current is then compactly written as

$$J = \bar{\phi}\,\mathcal{O}\,\phi \qquad , \tag{2.3}$$

where $\mathcal{O}$ is the $(2n) \times (2n)$ matrix

$$\mathcal{O} = \begin{pmatrix} 0 & V \\ 0 & 0 \end{pmatrix} \qquad , \tag{2.4}$$

and V is the $n \times n$ unitary quark mixing matrix. Denoting the Hermitian conjugate field $\phi^{\alpha +}$ by ϕ_α, we may write the current as

$$J = \phi_\alpha \,\mathcal{O}_{\alpha\beta}\,\phi^\beta \qquad , \tag{2.5}$$

which is a linear combination of states that transform under SU(2n) as the direct product $\underline{(2n)}^* \otimes \underline{(2n)} = \underline{1} \oplus \underline{(4n^2 - 1)}$. Because the matrix $\mathcal{O}$ is traceless, the singlet representation does not appear in (2.5), which is to say that the weak current transforms like a member of the adjoint $\underline{(4n^2 - 1)}$ representation of SU(2n).

The form (2.4) for $\mathcal{O}$ has other interesting consequences. The contribution to the weak neutral current proportional to the commutator $[J, J^+]$ has the form

$$J_o \propto \bar{\phi}\left[\mathcal{O}, \mathcal{O}^+\right]\phi = \bar{\phi}\begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}\phi \qquad , \tag{2.6}$$

which ensures that the neutral current is flavor conserving.[19] In addition, it follows from the tracelessness of $\mathcal{O}$ and the fact that $\{\mathcal{O}, \mathcal{O}^+\} = I$ that the adjoint representation does not occur in the charged-current Hamiltonian

$$\mathcal{H}_W = \tfrac{1}{2}\{J, J^+\} \qquad . \tag{2.7}$$

This is shown explicitly for the two-generation case in the Appendix to Ref. 15 and holds in general for SU(2n).

To discuss the decays of charmed particles it is only necessary to consider the transformation properties of the weak Hamiltonian under the group SU(4) relating the c, u, d, and s quarks. Under the assumptions of this Section, the result

$$\mathscr{H}_W \simeq \underline{20} \oplus \underline{84} \quad , \tag{2.8}$$

familiar from the four-quark case, persists. To be more specific, let us represent the four quark fields of interest as

$$\psi^\alpha = \begin{bmatrix} \psi^0 \\ \psi^1 \\ \psi^2 \\ \psi^3 \end{bmatrix} = \begin{bmatrix} c \\ u \\ d \\ s \end{bmatrix} \quad , \tag{2.9}$$

and write the charged weak current as

$$J = \bar{u}(dV_{11} + sV_{12}) + \bar{c}(sV_{22} + dV_{21}) \quad . \tag{2.10}$$

In the conventional GIM four-quark model, $V_{11} = V_{22} = \cos\theta_C$ and $V_{12} = -V_{21} = \sin\theta_C$, where θ_C is the Cabibbo angle. In the six-quark generalization due to Kobayashi and Maskawa,[18]

$$V_{11} = \cos\theta_1 \quad ,$$

$$V_{12} = \sin\theta_1 \cos\theta_3 \quad ,$$

$$V_{21} = -\sin\theta_1 \cos\theta_2 \quad ,$$

$$V_{22} = \cos\theta_1 \cos\theta_2 \cos\theta_3 - \sin\theta_2 \sin\theta_3 e^{i\delta} \quad . \tag{2.11}$$

Hereafter it will be convenient to neglect the possibility of CP-violation and to regard V_{22} as real.

The $\Delta C = 1$ part of the Hamiltonian is

$$\begin{aligned} \mathcal{H}(\Delta C = 1) = & \{ \bar{c}s, \bar{d}u \} V_{11} V_{22} + \{ \bar{c}s, \bar{s}u \} V_{12} V_{22} \\ & + \{ \bar{c}d, \bar{d}u \} V_{11} V_{21} + \{ \bar{c}d, \bar{s}u \} V_{12} V_{21} \quad , \end{aligned} \tag{2.12}$$

which may be written as

$$\begin{aligned} \mathcal{H}(\Delta C = 1) = & T^2_{31} V_{11} V_{22} + (T^3_{31} - T^2_{21})\Sigma \\ & + (T^3_{13} + T^2_{12})\Delta + T^3_{21} V_{12} V_{21} \quad . \end{aligned} \tag{2.13}$$

Here the tensor

$$T^k_{ij} \equiv \{ \psi^k \psi_i, \psi^o \psi_j \} - \tfrac{1}{3} \delta^k_i \{ \psi^\ell \psi_\ell , \psi^o \psi_j \} \tag{2.14}$$

(latin indices run over 1, 2, 3) and the definitions

$$\Sigma \equiv \tfrac{1}{2}[V_{12}V_{22} - V_{11}V_{21}] \tag{2.15}$$

and

$$\Delta \equiv \tfrac{1}{2}[V_{12}V_{22} + V_{11}V_{21}] \tag{2.16}$$

have been introduced. If the two-by-two matrix V is unitary, meaning that u, d, s, c are decoupled from heavier quarks, Δ vanishes identically.

In general, the product $\mathscr{H}(\Delta C = 1)$ of charm-changing and charm-conserving currents transforms under SU(3) as

$$[\underline{3}^*] \otimes [\underline{8}] = [\underline{3}^*] \oplus [\underline{6}] \oplus [\underline{15}_M^*] \quad . \tag{2.17}$$

The $[\underline{3}^*]$ and $[\underline{15}_M^*]$ are contained in the 84-dimensional representation of SU(4); the $[\underline{6}]$ lies in the SU(4) $\underline{20}$. The states of these representations may be defined in the T_{ij}^{k} basis as

$$[\underline{3}^*]_i \equiv T_{ij}^{j}$$

$$[\underline{6}]^{k\ell} \equiv \varepsilon^{\ell ij} T_{ij}^{k} + \varepsilon^{kij} T_{ij}^{\ell}$$

$$[\underline{15}_M^*]_{ij}^{k} \equiv T_{ij}^{k} + T_{ji}^{k} - \tfrac{1}{4}\delta_i^k T_{j\ell}^{\ell} - \tfrac{1}{4}\delta_j^k T_{i\ell}^{\ell} \quad . \tag{2.18}$$

Consequently, $\mathscr{H}(\Delta C = 1)$ may be decomposed as

$$\mathscr{H}(\Delta C = 1) = (\tfrac{1}{2}[\underline{15}_M^*]^2_{31} + \tfrac{1}{4}[\underline{6}]^{22})V_{11}V_{22}$$

$$+ \tfrac{1}{2}([\underline{15}_M^*]^3_{31} - [\underline{15}_M^*]^2_{21} + [\underline{6}]^{23})\Sigma + (-\tfrac{1}{2}[\underline{15}_M^*]^1_{11} + \tfrac{3}{4}[\underline{3}^*]_1)\Delta$$

$$+ (\tfrac{1}{2}[\underline{15}_M^*]^3_{21} - \tfrac{1}{4}[\underline{6}]^{33})V_{12}V_{21} \quad . \qquad (2.19)$$

The term proportional to Σ is antisymmetric in the interchange $d\bar{d} \leftrightarrow s\bar{s}$, and so transforms as a U-spin vector, as do the terms multiplying $V_{11}V_{22}$ and $V_{12}V_{21}$. The term proportional to Δ is symmetric under $d\bar{d} \leftrightarrow s\bar{s}$, and thus transforms as a scalar under U-spin. The U-spin symmetry suffices to derive many interesting sum rules, just as the observation that the Cabibbo-favored $[\underline{6}]^{22}$ piece is a V-spin ($u \leftrightarrow s$) singlet leads to strong selection rules.[20] The U-spin and V-spin analyses are particularly potent for multibody decays, for which the tensor method becomes cumbersome. In the case of the four-quark model, for which charmed meson decays have been discussed by many authors, $V_{11}V_{22} = \cos^2\theta_C$, $\Sigma = \sin\theta_C \cos\theta_C$, $\Delta = 0$, and $V_{12}V_{21} = -\sin^2\theta_C$. There is then no contribution from the $[\underline{3}^*]$ representation.

Let us now consider the decays of the SU(3) triplet of C = 1 pseudoscalar mesons D^o, D^+, F^+. To evaluate the matrix element $\langle P_c|\mathscr{H}|\mathscr{P}\mathscr{P}\rangle$, where P_c is a charmed pseudoscalar and $|\mathscr{P}\mathscr{P}\rangle$ denotes a charmless two-pseudoscalar final state, note that Bose symmetry requires the final state to be symmetric in SU(3) indices. Therefore only the following (complex) matrix elements occur:[F1]

$\langle P_c|[\underline{6}]|[\underline{8}]\rangle$, denoted S

$\langle P_c|[\underline{15}_M^*]|[\underline{8}]\rangle$, denoted E

$$<P_c|[\underline{15}^*_M]|[\underline{27}]>, \text{ denoted T}$$

$$<P_c|[\underline{3}^*]|[\underline{8}]>, \quad \text{denoted F}$$

$$<P_c|[\underline{3}^*]|[\underline{1}]>, \quad \text{denoted G} \quad .$$

Assuming the validity of SU(3) symmetry, there are five independent matrix elements. Under stronger assumptions, the number can be reduced. Imposing a nonet scheme on the $\mathscr{P}\mathscr{P}$ final states connects F and G. The matrix elements E and F correspond to transitions to octet final states mediated by the same SU(4) representation. They are therefore related if SU(4) is a useful symmetry of the Hamiltonian.

In the next Section, decay amplitudes are expressed in terms of the five matrix elements S, E, T, F, G. The signals for and implications of nonleptonic enhancement will then be discussed, as well as the effects of deviations from the four-quark GIM scheme.

III. AMPLITUDES FOR CHARMED MESON DECAY

The amplitudes for the decay of D^o, D^+, and F^+ into two pseudoscalar mesons are collected in Table I. Here $\eta(549)$ is regarded as the eighth member of the pseudoscalar octet, and $\eta'(958)$ (denoted X to avoid confusion) is treated as an SU(3) singlet state which completes the pseudoscalar nonet. Attention is restricted to these two-body decays because the relatively small number of independent matrix elements offers the hope of making definite inferences from experiment.[F2]

Let us first review some characteristics of the Cabibbo-favored decays. It is a familiar fact that the piece of the Hamiltonian which transforms as an SU(4) $\underline{20}$ contains the octet which is known to dominate charm conserving weak decays, i.e. the decays of kaons and hyperons. On the other hand, the $\underline{84}$ part of the Hamiltonian contains in its $\Delta C = 0$ sector the [$\underline{27}$] which is suppressed in kaon and hyperon decays. It was therefore natural to speculate that the appropriate generalization of octet dominance should be $\underline{20}$-dominance.[13,14,15,20] The origin of nonleptonic enhancement is incompletely understood, but it is believed to arise from the effects of strong interactions at short distances,[21] which may be less pronounced for the decays of heavy quarks (such as the charmed quark) than for the light quarks.[22] Consequently the sextet portion of the charm-changing Hamiltonian (which is contained in the $\underline{20}$) is expected to be enhanced relative to the triplet and pentadecimet (which lie in the $\underline{84}$), but perhaps by less than the order-of-magnitude enhancement of the octet over the [$\underline{27}$] in the $\Delta C = 0$ sector.

What are the consequences of an enhancement of the charm-changing nonleptonic decays? Most basic is an increase in the nonleptonic decay rate compared to the expectations of unversality. This is reflected in a semileptonic branching ratio $\Gamma(\text{charm} \to \text{hadrons} + e\nu)/\Gamma(\text{charm} \to \text{all})$ which is less than the 20%

predicted by quark counting. Indeed, the observed branching ratio for semileptonic decays of D mesons[23] (unselected by charge) is (9 ± 1)%, indicating a modest degree of enhancement. The SU(3) analysis of charmed meson decays shows that the situation may be somewhat subtle. The two-body decay amplitude initiated by the sextet component of the Hamiltonian, which is the candidate for enhancement, contributes only to the decays of D^0 and F^+, not to decays of D^+. This raises the possibility that D^0 and F^+ may have enhanced, Cabibbo-favored two-body decays whereas D^+ may have none. In this event, the reduced decay rate[F3] $\tilde{\Gamma}(D^+ \to \overline{K^0}\pi^+)$ would be substantially smaller than either $\tilde{\Gamma}(D^0 \to K^-\pi^+)$ or $\tilde{\Gamma}(F^+ \to K^+\overline{K^0})$. An alternative hypothesis, to be kept in mind as the data are accumulated, would be that strong interaction enhancement effects are not as vigorous in the exotic final states of D^+ decay as in the strongly resonant final states of D^0 and F^+ decay.

Pais and Treiman[24] have observed that because of the expected equality of the semileptonic decay rates for D^0 and D^+, the ratio of semileptonic branching ratios gives the ratio of lifetimes:

$$\frac{\tau(D^+)}{\tau(D^0)} = \frac{\Gamma(D^+ \to \text{hadrons} + e\nu)}{\Gamma(D^+ \to \text{all})} \Bigg/ \frac{\Gamma(D^0 \to \text{hadrons} + e\nu)}{\Gamma(D^0 \to \text{all})} \quad . \tag{3.1}$$

The nonleptonic enhancement and final-state representation considerations given above (and expounded at greater length elsewhere[15,20]) raise the possibility that $\tau(D^+)$ significantly exceeds $\tau(D^0)$. The ratio of semileptonic branching ratios may soon be accessible in the study of the reactions

$$e^+e^- \to \psi(3772) \to D^+D^- \text{ or } D^0\overline{D^0} \quad , \tag{3.2}$$

where a nonleptonic decay tags an event as a charged D or neutral D event. Absolute measurements of lifetimes in the expected neighborhood of 10^{-12} - 10^{-14}s. may be forthcoming from neutrino events in emulsions, and may be achieved in bubble chambers or high resolution detectors.[25] The lifetime ratio $\tau(D^+)/\tau(D^o)$ is a crucial parameter of the charm-changing weak interaction.

Measurement of the rates for the Cabibbo-favored decays listed in Table I will permit the determination of the quantities $|S|^2 + |E|^2$, $|T|^2$, $\mathrm{Re}(S^*E)$, $\mathrm{Re}(S^*T)$, and $\mathrm{Re}(E^*T)$. Except in special circumstances the separation of $|S|^2$ and $|E|^2$ cannot be made unambiguously. However, the observation that $\tau(D^+) \gg \tau(D^o)$, indicating that two-body decay rates are significantly smaller for D^+ than for D^o, would be strongly suggestive of sextet dominance which would imply $|S|^2 \gg |E|^2$.

Now let us consider the Cabibbo-suppressed decays listed in Table I. The contributions proportional to Σ are present within the four-quark GIM model. Because the Cabibbo-suppressed transitions $D^+ \to \pi^+\eta$, π^+X, $K^+\overline{K^o}$ lead to nonexotic final states and may be mediated by the enhanced term in the Hamiltonian, they may occur at larger rates compared to the Cabibbo-favored decay $D^+ \to \overline{K^o}\pi^+$ than a casual estimate would suggest. The contributions proportional to Δ signal the presence in the weak current of couplings to additional quarks. In the absence of these new couplings several equalities are predicted, among them:

$$\tilde{\Gamma}(D^o \to K^+K^-) = \tilde{\Gamma}(D^o \to \pi^+\pi^-) = \tilde{\Gamma}(D^o \to K^-\pi^+) \times \tan^2\theta_C \quad , \quad (3.3)$$

$$\tilde{\Gamma}(D^o \to K^o\overline{K^o}) = 0 = \tilde{\Gamma}(D^o \to XX) \quad , \quad (3.4)$$

$$\tilde{\Gamma}(D^o \to \pi^o\pi^o) = \tilde{\Gamma}(D^o \to \eta\eta) = \frac{3}{2}\tilde{\Gamma}(D^o \to \pi^o\eta) = \tilde{\Gamma}(D^o \to \overline{K^o}\pi^o) \times \tan^2\theta_C \quad , \quad (3.5)$$

$$\tilde{\Gamma}(D^o \to \eta X) = 3\tilde{\Gamma}(D^o \to \pi^o X) = \frac{3}{2}\tilde{\Gamma}(D^o \to \overline{K^o} X) \times \tan^2\theta_C \quad , \tag{3.6}$$

$$\tilde{\Gamma}(D^+ \to K^+\overline{K^o}) = \tilde{\Gamma}(F^+ \to K^o\pi^+) = \frac{3}{2}\tilde{\Gamma}(F^+ \to \pi^+\eta) \times \tan^2\theta_C \quad . \tag{3.7}$$

The equalities may all be broken in the presence of couplings to additional quarks.

How important are the deviations from equality likely to be? Without making additional assumptions (which will be done below) one cannot predict the relative importance of the new reduced matrix elements F and G. However, some recently completed analyses of the quark mixing matrix lead to estimates for the parameters Σ, Δ, etc. Two cases considered by Shrock, Treiman, and Wang[26] will serve as representative examples. The resulting parameters are shown in Table II. They suggest that

$$|\Delta|/|\Sigma| \lesssim 1/15 \quad , \tag{3.8}$$

but this ratio is probably uncertain by a factor of two.

Measurement of the ratio

$$\frac{\tilde{\Gamma}(D^+ \to \pi^+\pi^o)}{\tilde{\Gamma}(D^+ \to \overline{K^o}\pi^+)} = \frac{(\Sigma - \Delta)^2}{2(V_{11}V_{22})^2} = \frac{1}{2}\left(\frac{V_{21}}{V_{22}}\right)^2 \tag{3.9}$$

would provide direct information on elements of the quark mixing matrix which are now poorly determined.[11] The ratio expected on the basis of Table II is approximately 3%.

The amplitudes for D^o decay into $K^-\pi^+$, K^+K^-, and $\pi^+\pi^-$ lead to two potentially useful triangle inequalities. These amplitudes satisfy the sum rule

$$\mathscr{A}(D^0 \to K^+K^-) - \mathscr{A}(D^0 \to \pi^+\pi^-) = \frac{2\,\Sigma\,\mathscr{A}(D^0 \to K^-\pi^+)}{V_{11}V_{22}} \tag{3.10}$$

which implies that

$$\frac{\left|\sqrt{\tilde{\Gamma}(D^0 \to \pi^+\pi^-)} - \sqrt{\tilde{\Gamma}(D^0 \to K^+K^-)}\right|}{\sqrt{\tilde{\Gamma}(D^0 \to K^-\pi^+)}} \le \left|\frac{2\Sigma}{V_{11}V_{22}}\right| \le \frac{\sqrt{\tilde{\Gamma}(D^0 \to \pi^+\pi^-)} + \sqrt{\tilde{\Gamma}(D^0 \to K^+K^-)}}{\sqrt{\tilde{\Gamma}(D^0 \to K^-\pi^+)}} \; . \tag{3.11}$$

A recent experimental report[8] gives the branching ratios

$$\left.\begin{aligned} \frac{\Gamma(D^0 \to K^+K^-)}{\Gamma(D^0 \to K^-\pi^+)} &= (11.3 \pm 3.0)\% \;, \\ \frac{\Gamma(D^0 \to \pi^+\pi^-)}{\Gamma(D^0 \to K^-\pi^+)} &= (3.3 \pm 1.5)\% \;, \end{aligned}\right\} \tag{3.12}$$

which lead to the bounds

$$(0.20 \pm 0.06) \le \left|\frac{2\Sigma}{V_{11}V_{22}}\right| \le (0.54 \pm 0.06) \; . \tag{3.13}$$

As Suzuki[10] has stressed, this expression may be used together with experimental information on V_{11} and V_{12} to derive bounds on V_{21} in terms of Re V_{22}. Taking $V_{12}/V_{11} = 0.23$ as given in Table II, one has

$$(0.20 \pm 0.06) \le \left|0.23 - V_{21}/V_{22}\right| \le (0.54 \pm 0.06) \;, \tag{3.14}$$

which complements other information[26,27] on the quark mixing matrix. As the data on charmed meson decays improve, such bounds may prove increasingly restrictive. They have the virtue of remaining valid for any number of new quark generations, within the general assumptions set out in §II. For the moment it will suffice to remark that the parameters listed in Table II are consistent with the bounds (3.13):

$$\frac{2\Sigma}{V_{11}V_{22}} = \begin{cases} 0.48 & \text{(case a)} \\ 0.44 & \text{(case b)} \end{cases} \quad . \tag{3.15}$$

It is appealing to reduce the number of independent amplitudes by imposing additional symmetry requirements beyond SU(3)-invariance. In the general analysis reported above, the amplitudes for the transitions $< P_c | [\underline{3}^*] | [\underline{8}] >$ and $< P_c | [\underline{3}^*] | [\underline{1}] >$ were regarded as independent. However, it is rather natural to impose a nonet symmetry which expresses the amplitudes for both these transitions in terms of a single amplitude for $< P_c | [\underline{3}^*] | [\underline{9}] >$. With the conventions I have adopted, this is equivalent to the choice

$$G = F \quad . \tag{3.16}$$

An interesting consequence of this choice is that the amplitude for the decay $D^o \to K^o\overline{K^o}$ can be expressed entirely in terms of amplitudes which may be known from Cabibbo-favored decays as

$$\mathscr{A}(D^o \to K^o\overline{K^o}) = -\frac{1}{2}(T - 2E)\Delta \quad . \tag{3.17}$$

The singlet amplitude G is itself directly measurable in the decay $D^o \to XX$, once the factor Δ is known.

A further reduction in the number of independent amplitudes may be had by exploiting the full SU(4) symmetry of the weak Hamiltonian. The transitions $\langle P_c | [\underline{15}^*_M] | [\underline{8}] \rangle$ and $\langle P_c | [\underline{3}^*] | [\underline{8}] \rangle$ can both be expressed in terms of a single amplitude for $\langle P_c | \underline{84} | [\underline{8}] \rangle$. This implies the requirement

$$F = E \quad \text{(SU(4) symmetry)} \quad . \tag{3.18}$$

In fact, the requirement that the two $|\Delta C| = 1$ components of the $\underline{84}$ contribute with equal strength does not entail the full SU(4) symmetry, which would also relate the $|\Delta C| = 1$ and $\Delta C = 0$ components. Although SU(4) may be badly broken (as it surely is by quark masses), the condition (3.18) may hold to a good approximation. In spite of this distinction, it will be convenient to refer to (3.18) as a consequence of SU(4) symmetry. If (3.18) is satisfied, the relation (3.6) among decays leading to pure octet final states is only trivially modified from its four-quark form; it becomes

$$\tilde{\Gamma}(D^o \to \eta X) = 3\tilde{\Gamma}(D^o \to \pi^o X) = \frac{3}{2}\tilde{\Gamma}(D^o \to \overline{K^o} X)\left(\frac{\Sigma}{V_{11}V_{22}}\right)^2 \quad . \tag{3.19}$$

With the two additional symmetry properties (the nonet scheme and SU(4) invariance) the amplitudes for Cabibbo-suppressed charm decay can be expressed entirely in terms of the three independent amplitudes that determine the Cabibbo-favored decays. As an illustration, consider a second triangle inequality which can be derived from the sum $\mathscr{A}(D^o \to K^+K^-) + \mathscr{A}(D^o \to \pi^+\pi^-)$. Using the general expressions given in Table I one finds

$$\frac{\left|\sqrt{\tilde{\Gamma}(D^0 \to \pi^+\pi^-)} - \sqrt{\tilde{\Gamma}(D^0 \to K^+K^-)}\right|}{\sqrt{\tilde{\Gamma}(D^0 \to K^-\pi^+)}} \leq \frac{|(3T + 2G - E + F)\Delta|}{|2T + E - S|\,|V_{11}V_{22}|}$$

$$\leq \frac{\sqrt{\tilde{\Gamma}(D^0 \to \pi^+\pi^-)} + \sqrt{\tilde{\Gamma}(D^0 \to K^+K^-)}}{\sqrt{\tilde{\Gamma}(D^0 \to K^-\pi^+)}} , \quad (3.20)$$

or, inserting the experimental values,

$$(0.20 \pm 0.06) \leq \frac{|(3T + 2G - E + F)|\,|\Delta|}{|2T + E - S|\,|V_{11}V_{22}|} \leq (0.54 \pm 0.06) \quad . \quad (3.21)$$

In the form (3.21) this constraint is not particularly informative. However the additional symmetry requirements (3.16) and (3.18) simplify the bounded expression, which becomes

$$R \equiv \frac{|(3T + 2G - E + F)|\,|\Delta|}{|2T + E - S|\,|V_{11}V_{22}|} = \frac{|(3T + 2E)|\,|\Delta|}{|2T + E - S|\,|V_{11}V_{22}|} \quad . \quad (3.22)$$

Ultimately one may hope to use (3.21) to bound the poorly-known parameter Δ. For now, using the estimate $|\Delta|/|V_{11}V_{22}| \lesssim 0.02$ from Table II we may usefully conclude only that

$$\left|\frac{3T + 2E}{2T+E-S}\right| \gtrsim (10 \pm 3) \quad . \quad (3.23)$$

Current knowledge of charmed meson decay rates is far too fragmentary to elicit a definite response to this result. A large ratio would offhand be somewhat surprising, however.

Among the many other sum rules which can be constructed from Table II are two which relate the decays of D^+ and F^+:

$$-\mathscr{A}(D^+ \to \overline{K^o}K^+) + \mathscr{A}(F^+ \to K^o \pi^+) = +\sqrt{6}\mathscr{A}(F^+ \to \pi^+\eta)\Sigma/(V_{11}V_{22}) \quad , \quad (3.24)$$

and

$$\mathscr{A}(F^+ \to K^+X) - \mathscr{A}(D^+ \to \pi^+X) = 2\mathscr{A}(F^+ \to \pi^+X)\,\Sigma/(V_{11}V_{22}) \quad , \quad (3.25)$$

which lead to potentially useful triangle inequalities such as (3.13) and (3.21).

IV. CONCLUSIONS

Extensive experimental investigations of the decays of charmed mesons can be expected to lead to significant new insights into the nature of the weak interactions. In this paper I have elaborated some of the important issues which can be addressed by measurements of charmed particle lifetimes and of branching ratios for two-body decays. A systematic survey of the Cabibbo-favored decays of D^o, D^+ and F^+ will help to eliminate long-standing uncertainties concerning the patterns and origin of nonleptonic enhancement. Cabibbo-suppressed decays are sensitive to departures from the four-quark Glashow-Iliopoulos-Maiani current, which are to be expected in models incorporating more than four quark flavors. Within a broad class of such models, including the Kobayashi-Maskawa six-quark model, charmed meson decay rates may be used to derive new experimental bounds on elements of the quark mixing matrix. These in turn have implications for the weak couplings of heavier quarks (b, t, ...).

One consequence of the richer structure implied by the existence of more than four flavors is that the decay rates $\tilde{\Gamma}(D^o \to K^-K^+)$ and $\tilde{\Gamma}(D^o \to \pi^-\pi^+)$, which must be equal in the GIM scheme, are permitted to differ. The magnitude of the difference is not now predictable. Knowledge of both mixing angles and matrix elements of the weak Hamiltonian is too primitive to fix the expected rate. As data are accumulated on other decay modes, this unsatisfactory state of affairs will be ameliorated. Because the inequality of $D^0 \to K^+K^-$ and $\pi^+\pi^-$ decay rates is to be expected, given the apparent instability of the b-quark,[28] unconventional structures in the weak Hamiltonian[29] seem not to be compelled.

The SU(3) symmetry approach taken here is expected to provide a reliable framework for relating data on various decay modes of charmed mesons, and for perceiving the systematics of nonleptonic enhancement. If successive quark generations repeat the familiar pattern of left-handed doublets, the present analysis remains valid for any number of quark generations. Relative magnitudes of the five independent decay matrix elements are not determined by SU(3) invariance alone, although additional symmetry requirements can reduce the number of independent amplitudes. Whether specific dynamical mechanisms[30] succeed in explaining the relative importance of the amplitudes remains to be seen.

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Table I. Amplitudes for Charmed Meson Decay*

<u>D^0 Decays</u>

$K^-\pi^+$	$2T+E-S$	$\times\ V_{11}V_{22}$
$\overline{K^0}\pi^0$	$(3T-E+S)/\sqrt{2}$	
$\overline{K^0}\eta$	$(3T-E+S)/\sqrt{6}$	
$\overline{K^0}X$	$2(E-S)/\sqrt{3}$	
K^+K^-	$(2T+E-S)\Sigma$	$+\frac{1}{2}(3T+2G+F-E)\Delta$
$\pi^+\pi^-$	$-(2T+E-S)\Sigma$	$+\frac{1}{2}(3T+2G+F-E)\Delta$
$\pi^0\pi^0$	$\frac{1}{2}(3T-E+S)\Sigma$	$+\frac{1}{4}(-7T+2G+F-E)\Delta$
$K^0\overline{K^0}$		$\frac{1}{2}(-T+2G-2F+2E)\Delta$
$\eta\eta$	$-\frac{1}{2}(3T-E+S)\Sigma$	$+\frac{1}{4}(-3T+2G-F+E)\Delta$
XX		$\frac{1}{2}G\,\Delta$
$\pi^0\eta$	$-\frac{(3T-E+S)\Sigma}{\sqrt{3}}$	$+\frac{1}{2}\frac{(-6T+3F-3E)\Delta}{\sqrt{3}}$
ηX	$\sqrt{2}(S-E)\Sigma$	$+\frac{(F-E)\Delta}{\sqrt{2}}$
$\pi^0 X$	$\sqrt{\frac{2}{3}}(E-S)\Sigma$	$-\sqrt{\frac{2}{3}}\,\frac{(3F-3E)\Delta}{2}$
$K^+\pi^-$	$(2T+E-S)$	$\times\ V_{12}V_{21}$
$K^0\pi^0$	$(3T-E+S)/\sqrt{2}$	
$K^0\eta$	$(3T-E+S)/\sqrt{6}$	
K^0X	$2(E-S)/\sqrt{3}$	

D^+ Decays

$\overline{K^o}\pi^+$	$5T \quad V_{11}V_{22}$	
$\pi^+\pi^o$	$(5T/\sqrt{2})\Sigma$	$-(5T/\sqrt{2})\Delta$
$\pi^+\eta$	$-\frac{(9T+2E+2S)\Sigma}{\sqrt{6}}$	$+\frac{(-3T+E+3F)\Delta}{\sqrt{6}}$
$K^+\overline{K^o}$	$(3T-E-S)\Sigma$	$+(T+\frac{E}{2}+\frac{3F}{2})\Delta$
π^+X	$-\frac{2}{\sqrt{3}}(E+S)\Sigma$	$+\frac{1}{\sqrt{3}}(E+3F)\Delta$
$K^o\pi^+$	$2T+E+S$	$\times V_{12}V_{21}$
$K^+\pi^o$	$(-3T+E+S)/\sqrt{2}$	
$K^+\eta$	$(3T-E-S)/\sqrt{6}$	
K^+X	$\frac{2}{\sqrt{3}}(E+S)$	

F^+ Decays

$\overline{K^o}K^+$	$(2T+E+S)$	$\times V_{11}V_{22}$
$\pi^+\eta$	$-\sqrt{\frac{2}{3}}(3T-E-S)$	
π^+X	$\frac{2}{\sqrt{3}}(E+S)$	
$K^o\pi^+$	$(-3T+E+S)\Sigma$	$+(T+\frac{E}{2}+\frac{3F}{2})\Delta$
$K^+\pi^o$	$\frac{(2T+E+S)\Sigma}{\sqrt{2}}$	$+\frac{(-4T+\frac{E}{2}+\frac{3F}{2})\Delta}{\sqrt{2}}$
$K^+\eta$	$-\frac{(12T+E+S)\Sigma}{\sqrt{6}}$	$-\frac{(6T+\frac{E}{2}+\frac{3F}{2})\Delta}{\sqrt{6}}$
K^+X	$\frac{2}{\sqrt{3}}(E+S)\Sigma$	$+\frac{1}{\sqrt{3}}(E+3F)\Delta$
K^oK^+	$5T \quad V_{12}V_{21}$	

*Bose statistics convention: For decays into pairs of identical particles, reduced rates are given by $2 \times |\text{Amplitude}|^2$

Table II. Parameters of the Charm-Changing Hamiltonian [from Ref. 26]

	Case a	Case b
V_{11}	.97	.97
V_{12}	.22	.22
V_{21}	- .22	- .20
V_{22}	$0.85-0.66\times 10^{-3}i$	$0.95-0.75\times 10^{-3}i$
$\lvert V_{11}V_{22}\rvert$	0.82	0.92
Σ	0.20	0.20
Δ	-0.013	0.008
$V_{12}V_{21}$	-0.05	-0.04

FOOTNOTES

[F1]Numerical factors have been absorbed into the definitions of S, E, T, F, G to simplify the entries in Table I. In labelling amplitudes by the SU(3) transformation properties of the Hamiltonian and the final state I follow the practice of refs. 15 and 7.

[F2]These results agree with those of refs. 9 and 11 for the cases considered there. They may also be recovered from the amplitudes presented in ref. 7.

[F3]The reduced two-body decay rate is defined as $\tilde{\Gamma}(A \to \alpha\beta) = (M_A^2/p_{\alpha\beta})\Gamma(A \to \alpha\beta)$, where M_A is the mass of the decaying particle and $p_{\alpha\beta}$ is the momentum of the products in the rest frame of A.

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