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Determination of Quark-Nucleon Inclusive Cross Section from Leptoproduction of Hadrons in Nuclear Targets

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ABSTRACT

The A-dependence of hadron leptoproduction from nuclear targets is interpreted as measurement of inclusive inelastic and possibly also quasi-elastic quark-nucleon cross sections. The formulae which allow to extract these cross sections from the data are derived and discussed. Numerical estimates indicate that the measurement of total inelastic quark-nucleon cross section is perfectly feasible. Measurement of differential inclusive cross sections of the reaction quark + nucleon → quark + anything is possible as well but requires experiments with rather high statistics. Implications of our calculations to the process of heavy lepton pair production from nuclear targets are also briefly discussed.

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I. INTRODUCTION

It has been pointed out by many authors[F1] that high-energy experiments with nuclear targets can be useful in the investigation of elementary interactions at very short times. The essential idea in all these considerations is that the (multiple) collisions inside the nucleus can serve as detector of the short-living objects created in the first collision. In this paper we follow this argument and apply it to A-dependence of hadronic spectra produced in deep-inelastic scattering of leptons from nuclear targets, i.e. to the process

$$\ell + A \to \ell' + \text{anything} \quad . \tag{1.1}$$

We argue that such experiments can be interpreted as measurements of quark-nucleon cross sections.[4-6]

The idea is illustrated in Figure 1. The high-energy quark created by the incident lepton inside the nucleus travels through the nuclear matter and scatters from the nucleons. Its momentum distribution is affected by the presence of the nuclear matter and therefore depends on A and on the cross section for the process

$$\text{quark} + \text{nucleon} \to \text{quark} + \text{anything} \tag{1.2}$$

which we shall denote by

$$\sigma(\vec{p}, \vec{p}\,') \equiv \frac{E' d\sigma(\vec{p}, \vec{p}\,')}{d^3p'} \quad . \tag{1.3}$$

Here $\vec{p}$ and $\vec{p}\,'$ are initial and final momenta of the quark in quark-nucleon scattering. Momentum distribution of the quark is reflected in the distribution of final hadrons in the process (1.1) which thus also depends on A and on σ. Consequently, by studying A-dependence of the hadron spectrum one should be able to learn about σ and, hopefully, even measure it with a reasonable accuracy. The attractive feature of the process (1.1) is that it provides quite a good control of the initial conditions:[F2,4-7] the lepton scattered with large momentum transfer is known to produce predominantly a single quark with fairly well-defined momentum, determined basically by momentum transfer between initial and final lepton.

As is clear from Figure 1, however, the process leading to the observed final state is not a simple one, but involves, in general, multiple collisions in nuclear matter. Consequently, the dependence of the spectrum on σ is expected to be fairly complicated. But we ought to know it if we are to obtain useful information about σ from the process (1.1). The main purpose of this paper is just to study in some detail this relation between σ and A-dependence of the final hadron spectrum in order to determine the feasibility of the measurements of the quark-nucleon cross section (1.3).

The difficulties and complications are significantly reduced if one selects a convenient range of variables at which the reaction (1.1) (and thus also reaction (1.2)) is to be studied. First, by focusing attention on high-energy hadrons[4] (in the laboratory frame) one avoids possible contamination by hadronic cascading effects inside the nuclear matter. If we further require that the momentum of the final observed hadrons is greater than ½ of the momentum of the initial quark, only one parton with energy sufficient to create such hadrons can be present in the nucleus and we need not consider the possibility of "quark-gluon cascade"[8] in nuclear matter. We thus restrict our considerations to this kinematic region.[F3]

It is convenient to discuss separately the longitudinal and transverse momentum[F4] spectra of hadrons, since they reflect different aspects of the problem. The A-dependence of the longitudinal momentum distribution is sensitive to the total inelastic quark-nucleon cross section, because this cross section determines the amount of absorption of quarks in nuclear matter. The distribution of large transverse momentum hadrons, on the other hand, is sensitive to differential cross sections for deep-inelastic quark-nucleon scattering (1.3). We show how these two aspects of the hadron spectra complement each other and allow full determination of the cross section for the reaction (1.3).

Our study of the sensitiveness of hadronic spectra to the cross section (1.3), and numerical estimates of the expected particle yields lead us to the conclusion that the measurement of quark-nucleon cross sections is indeed possible. The measurement of the total inelastic cross section is actually fairly simple and can be done with existing experimental arrangements. The determination of differential cross sections at large transverse momenta requires experiments with rather good statistics, but is otherwise also straightforward. We are thus optimistic about the prospects of such measurements.

The plan of the paper is as follows. In Sections 2 and 3 we discuss the limiting case of very small quark-nucleon cross section, when single scattering of quarks dominates the reaction in the nucleus. Absorption and multiple quasielastic scattering of quarks in nuclear matter are discussed in Section 4. In Section 5 the general formula for hadron spectrum in terms of the quark-nucleon cross section (1.3) is written down. Numerical estimates of longitudinal and transverse momentum spectra are discussed in Sections 6 and 7. Our conclusions are listed in the last section, where we also comment on the relation of our considerations to the heavy lepton pair production in nuclei.

II. SINGLE SCATTERING OF A QUARK IN NUCLEAR MATTER

In this section we discuss a simple case when the quark-nucleon cross section is so small that probability of multiple scattering is negligible. In such a case one may consider only no-scattering and single-scattering contributions.

Consider the quark of momentum $\vec{P}$ created by a lepton at some point $(\vec{b}, z)$ inside the nucleus (the z-axis points along the direction of the virtual photon—see Figure 1). The probability that the quark scatters from a nucleon located at the point $(\vec{b}, z')$ is

$$\sigma(\vec{P}, \vec{p})\,\rho_A(\vec{b}, z')dz' \qquad \text{for } z' > z$$

and (2.1)

$$0 \qquad \text{for } z' < z$$

where $\rho_A(\vec{b}, z)$ is the nuclear density normalized to unity: $\int \rho_A(\vec{r})d^3r = 1$. Consequently, the probability for the quark to scatter from a nucleon located at any point (b, z') is

$$\sigma(\vec{P}, \vec{p})d_A(\vec{b}, z)$$

where

$$d_A(\vec{b}, z) \equiv \int_z^\infty \rho_A(\vec{b}, z')dz' \quad . \tag{2.2}$$

To calculate the total probability of scattering of the quark created by a lepton anywhere in the nucleus we observe that

(i) the probability to create the quark at a given position inside the nucleus is proportional to the nuclear density at this point. If the probability is counted per one lepton trigger, then it is just equal to $\rho_A(\vec{b}, z)d^2b dz$ (because $\rho_A(\vec{b}, z)$ is normalized to unity);

(ii) there are A - 1 nucleons on which the quark can scatter (one is used for creation of the quark).

Taking these remarks into account, one sees that the total probability of scattering is

$$\frac{EdN}{d^3p}\Big|_A = \sigma(\vec{P}, \vec{p})(A-1)\int d^2b \int_{-\infty}^{\infty} dz\, \rho_A(b, z) d_A(b, z) \quad . \tag{2.3}$$

The integral over z can be performed if one observes that $\rho_A(z) = -d_A'(z)$. We obtain

$$\int_{-\infty}^{\infty} dz\, \rho_A(\vec{b}, z) d_A(\vec{b}, z) = -\frac{1}{2}\int_{-\infty}^{\infty} dz \frac{d}{dz}[d_A(\vec{b}, z)]^2 = \frac{1}{2}[D_A(\vec{b})]^2 \tag{2.4}$$

where $D_A(\vec{b}) = d_A(\vec{b}, -\infty) = \int_{-\infty}^{\infty} \rho_A(\vec{b}, z)dz$. Substituting Eq. (2.4) into (2.3) we have

$$\frac{EdN}{d^3p}\Big|_A = \sigma(\vec{P}, \vec{p})\, \frac{A-1}{2} \int d^2b\, [D_A(b)]^2 \quad . \tag{2.5}$$

In the Figure 2 the coefficient

$$W_1 \equiv \frac{A-1}{2}\int d^2b [D_A(b)]^2 \tag{2.6}$$

is plotted versus A for different nuclei.[F5] One sees that W_1 does not follow the power behavior $A^{1/3}$ suggested by simple geometrical considerations. It increases much faster than $A^{1/3}$ for small and medium size nuclei, and even above $A \sim 150$ the increase is slightly faster than $A^{1/3}$.

The spectrum of the observed hadrons is composed of two parts, since they may arise from fragmentation of quarks which either did or did not scatter inside the nucleus. Again normalizing to one deep-inelastic trigger one obtains

$$\frac{E_h dN_2}{d^3p_h}\Big|_A = [1 - \sigma_T^* W_1]\; D_{p\to h}(\vec{P}, \vec{p}_h) + W_1 \sigma_h(\vec{P}, \vec{p}_h) \tag{2.7}$$

where

$$\sigma_h(\vec{P}, \vec{p}_h) \equiv \int \frac{d^3p}{E}\, \sigma(\vec{P}, \vec{p}) D_{p\to h}(\vec{p}, \vec{p}_h) \tag{2.8}$$

is the cross section for the process

$$\text{quark} + \text{nucleon} \to \text{hadron} + \text{anything} \tag{2.9}$$

and σ_T^* is the quark-nucleon cross section integrated over final quark momentum. $D_{p\to h}$ is the quark fragmentation function which is obtained from measurements in Hydrogen (as is seen from Eq. (2.7) by observing that $W_1 = 0$ for $A = 1$).

Equations (2.5) and (2.7) were derived by neglecting all multiple scattering effects so they are expected to be valid only if the quark-nucleon cross section is rather small. We discuss later corrections which may arise from multiple scattering phenomena. However, it should be emphasized that, since we do not know how large the quark-nucleon cross section is, it is not excluded that Eq. (2.7) shall describe correctly the A-dependence of the data. In such a case, the physical interpretation of the experiments is greatly simplified and the measurement of $\sigma(\vec{P}, \vec{p})$ or rather of $\sigma_h(\vec{P}, \vec{p}_h)$ should be relatively easy. We thus feel that the first step in data analysis should be comparison of the observed A-dependence of the spectra with the simple and well-defined formula (2.7).

In the next section we present the estimates of the A-dependence of hadron spectrum following from Eq. (2.7), assuming a simplified form of the quark-nucleon cross section.

III. NUMERICAL ESTIMATES OF THE SINGLE-SCATTERING FORMULA

In order to illustrate the nuclear effects which follow from the first order formula (2.7) it is necessary to make definite assumptions about the magnitude and shape of the quark-nucleon cross section. We shall be primarily interested in the region $p_\perp \lesssim 2$ GeV where little is known about this quantity, so any reliable prediction is difficult. It is nevertheless interesting to obtain even a crude estimate which may indicate what kind of effects one may expect and how difficult they will be to detect. In our calculations we adopted for $\sigma(P, \vec{p})$ a simplified factorized form

$$\sigma = \kappa \exp(4x_F)(1 - x_F) \frac{1}{(p_\perp^2/M^2 + 1)^4} \tag{3.1}$$

where $M = 1$ GeV, $\kappa = 1.5$ mb/GeV^2; x_F is the scaled Feynman momentum of the quark. Equation (3.1) was chosen for its simplicity[F6] and because its $p_\perp$-dependence approximates (up to factor $\backsim 2$) the QCD-suggested[10] quark nucleon cross section for 1 GeV $< p_\perp <$ 2 GeV and $.5 < x_F < .9$ at initial quark momentum of 100 GeV. The over-all normalization is essentially arbitrary and was chosen to yield the integrated cross-section $.55 < x_F < 1$ of $\simeq 4$ mb which is just a first guess (this underestimates the cross section from Ref. 10 extrapolated to the region of $p_\perp < 2$ GeV). For different values of quark-nucleon cross section our estimates should be scaled accordingly. We present only results for the spectrum integrated over x_F ($x_F > .55$). They are insensitive to the actual form of x_F dependence in (3.4). In the actual calculations we convoluted the cross section (3.1) with the distribution $\exp\{-6p_\perp^2\}$ to take approximately into account the internal motion of the quarks in the target.

For the quark fragmentation function we have also chosen the factorized form

$$D_{q\to h}(P;\vec{p}) = 11z \exp[-5z] \quad \exp\{-7.6\sqrt{p_\perp^2 + .26}\}/.011 \tag{3.2}$$

where $z = p_{\parallel}/P$. The form and the parameters of $p_\perp$ dependence were taken from Ref. 11. The z dependence was chosen to describe approximately the data of Ref. 11 for $.5 < z < 1$.

The A-dependence of the charge hadron spectra calculated from the formula (2.7) using the assumptions (3.1) and (3.2) is shown in Figure 3. The transverse momentum distribution of hadrons is plotted for Hydrogen and three nuclei: Al, Cu and U. One sees that the nuclear effects start to show up for transverse momenta above 1 GeV and are quite pronounced already at $p_\perp \backsim 1.6$ GeV. Unfortunately, the rates expected in this region are rather low and thus shall not be easy to measure.

Below 1 GeV the absorptive effects (as expressed crudely by $-\sigma_T^* W_1$ in the first term of Eq. (2.7)) take over and the hadron yields are actually smaller for heavy nuclei than for hydrogen. This important effect shall be discussed more carefully in Sections 4 and 6.

As we see from Fig. 3 the nuclear corrections are not very large. This is partly because the quark-nucleon cross section is not very large at high transverse momenta and because those quarks which scattered inelastically are slowed down and are not very effective if fragmenting into high energy hadrons. One loses a factor of about 4 by this last effect. We should stress again, however, that all our estimates are based on the simple forms (3.1) and (3.2) and thus serious uncertainties are involved.

In conclusion we feel that the experiment which has enough statistics to cover the region of $p_\perp$ above 1.5 GeV may be able to see substantial nuclear effects. In the following sections we shall discuss in more detail how one can extract the quark-nucleon cross section from such measurements.

IV. ABSORPTION AND MULTIPLE QUASI-ELASTIC SCATTERING OF QUARKS IN NUCLEAR MATTER

Equation (2.7) discussed in the previous two sections can be useful for phenomenological description of the data only if the quark-nucleon cross section (1.3) is small enough to allow termination of the general multiple scattering series on first two terms. This requirement is probably satisfied by large-angle cross section, as evidenced by the data on high $p_\perp$ particle production in hadron-hadron experiments. It is not excluded, however, that quarks can also scatter quasi-elastically–and with high probability–at small transverse momenta. When many of such small "kicks" are accumulated in a heavy nucleus, they may produce a substantial effect and thus exclude Eq. (2.7) as a practical tool for data analysis.

Another possibly important effect, which should be calculated more carefully than it was done in Eq. (2.7), is the absorption of the high-energy quarks in the nucleus by inelastic scattering. This absorption arises from simple conservation of probability: the quarks which scattered inelastically inside the nucleus should be subtracted from the number originally created by leptons. The effect of absorption can be quite substantial because it depends on the total quark-nucleon inelastic cross section, which may be a large number.

It is thus desirable to develop a scheme in which these effects of absorption and of quasi-elastic scattering of quarks are summed up so they need not be evaluated by perturbation series. We present such a scheme in this and in the next section. Our argument is an extension of standard multiple-scattering techniques[12-14] to the particular problem we consider. Many elements of the calculation are similar to those employed in Refs. 14-16.

To illustrate the method, we first consider a simplest case when only quasi-elastic scattering and absorption of quarks takes place and show how to sum up their effects to all orders.

Let us first define what we mean by quasi-elastic scattering of high-energy quarks. We define it as a process in which the longitudinal momentum of the quark remains (approximately) unchanged, so that we can write

$$\sigma(\vec{p}; \vec{p}') = \sigma_{\perp}(\vec{p}_{\perp} - \vec{p}_{\perp}')E'\delta(p_{\|} - p_{\|}') \quad . \tag{4.1}$$

The dependence of $\sigma_{\perp}$ on $(\vec{p}_{\perp} - \vec{p}_{\perp}')$ is not restricted, also the magnitude of the cross section is arbitrary.

We now calculate the momentum distribution of the quarks which underwent any number (including zero) of such quasi-elastic collisions. To this end, we first write down the formula for the final distribution of quarks which were created at some point $(\vec{b}, z)$ inside the nucleus:

$$\frac{dN^{(0)}(\vec{b}, z)}{d^2p_{\perp}}\Big|_A = \sum_{\nu=0}^{A-1} H_{\nu}(A; \vec{b}, z; \tilde{\sigma}_{\perp}(0))\phi_{\nu}(\vec{p}\ ) \tag{4.2}$$

where H_{ν} is the probability that exactly ν quasi-elastic collisions (and no other collisions) have taken place:

$$H_\nu = \binom{A-1}{\nu} [\tilde{\sigma}_\perp(0) d_A(\vec{b}, z)]^\nu [1 - \sigma_T d_A(\vec{b}, z)]^{A-1-\nu} \quad (4.3)$$

with $\tilde{\sigma}_\perp(0)$ being the integrated cross section for quasi-elastic collisions on a single nucleon

$$\tilde{\sigma}_\perp(0) = \int d^2p_\perp \sigma_\perp(\vec{p}_\perp) \quad . \quad (4.4)$$

σ_T is the total quark-nucleon cross section. It takes into account absorption of quarks. ϕ_ν is convolution of ν quark-nucleon quasi-elastic cross sections

$$[\tilde{\sigma}_\perp(0)]^\nu \phi_\nu(\vec{p}_\nu) = \int \sigma_\perp(\vec{p}_{1\perp}) d^2p_{1\perp} \sigma_\perp(\vec{p}_{1\perp} - \vec{p}_{2\perp}) d^2p_{2\perp} + \dots d^2p_{\nu-1\perp} \sigma_\perp(\vec{p}_{\nu-1\perp} - \vec{p}_\perp) \quad . \quad (4.5)$$

In order to evaluate the formula (4.2) we use the standard procedure of Fourier transforming the convolution (4.5). We obtain

$$\tilde{\phi}_\nu(\vec{\beta}) = [\tilde{\sigma}_\perp(\vec{\beta}) / \tilde{\sigma}_\perp(0)]^\nu \quad (4.6)$$

where the tilde denotes the Fourier-Bessel transform

$$\tilde{\sigma}_\perp(\vec{\beta}) = \int d^2q \sigma_\perp(\vec{q}) e^{i\vec{\beta}\vec{q}} \quad (4.7)$$

and similarly for $\tilde{\phi}_\nu(\vec{\beta})$. $\phi_\nu(\vec{p}_\perp)$ is then given by the inverse transform

$$\phi_\nu(\vec{p}_\perp) = \frac{1}{(2\pi)^2} \int d^2\beta e^{-i\vec{p}\,\vec{\beta}} \tilde{\phi}_\nu(\vec{\beta}) \quad . \quad (4.8)$$

Substituting Eqs. (4.6) and (4.8) into Eq. (4.2) it is possible to carry out the summation over ν and one obtains

$$\frac{dN^{(0)}(\vec{b}, z)}{d^2p_\perp}\Big|_A = \frac{1}{(2\pi)^2}\int d^2\beta\, e^{-i\vec{p}_\perp\vec{\beta}}\,[1-(\sigma_T-\tilde{\sigma}_\perp(\vec{\beta}))d(\vec{b}, z)]^{A-1} \quad . \qquad (4.9)$$

The last step is to integrate over nuclear volume in order to obtain the distribution for the quarks created anywhere inside the nucleus. We obtain

$$\frac{dN^{(0)}}{d^2p_\perp}\Big|_A = \int d^2b\int_{-\infty}^{\infty} dz\,\rho(\vec{b}, z)\frac{dN_o(\vec{b}, z)}{d^2p_\perp}\Big|_A \quad . \qquad (4.10)$$

The integral over z can be evaluated using the formula

$$\int_{-\infty}^{\infty}[1-\lambda d(b, z)]^{A-1}\rho(b, z)dz = \frac{1-[1-\lambda D(b)]^A}{\lambda A} \qquad (4.11)$$

and we finally have

$$\frac{dN^{(0)}}{d^2p_\perp}\Big|_A = \frac{1}{(2\pi)^2}\int d^2\beta\, e^{-i\vec{p}_\perp\vec{\beta}}\,W_A^{(0)}(\beta) \qquad (4.12)$$

where

$$W_A^{(0)}(\beta) = \int d^2b\,\frac{1-\{1-[\sigma_T-\tilde{\sigma}_\perp(\beta)]D(b)\}^A}{A[\sigma_T-\tilde{\sigma}_\perp(\vec{\beta})]} \quad . \qquad (4.13)$$

Equation (4.12) gives transverse momentum distribution of the quarks which underwent any number of quasi-elastic collisions inside the nucleus (according to our definition of quasi-elastic collisions (Eq. (4.1) the longitudinal momentum of these quarks did not change). The distribution of hadrons is obtained by folding in the quark fragmentation function:

$$\frac{E_h dN_h^{(0)}}{d^3p_h}\Big|_A = \int d^2p_\perp\frac{dN^{(0)}}{d^2p_\perp}\Big|_A D_{q\to h}(P, \vec{p}_\perp; \vec{p}_h) \quad . \qquad (4.14)$$

V. INELASTIC SCATTERING OF QUARKS IN NUCLEAR MATTER

As we have seen in the previous section, the crucial step which allows to sum up all quasi-elastic collisions was factorization of the Fourier transform of the convolution ϕ_ν , as shown in Eq. (4.6). Equation (4.6) is a consequence of the assumption that the quasi-elastic scattering conserves the longitudinal momentum of the quark. In deep-inelastic ("hard") quark scattering the longitudinal momentum is allowed to change during the collision so Eq. (4.6) is not valid. Instead we obtain multiple integrals over longitudinal momenta of all intermediate states. Consequently, the explicit summation cannot be performed without additional assumptions. Fortunately, as we have already discussed in the previous section, the hard scattering cross section is relatively small and thus the perturbative methods may be available.

In this section we show how such a perturbative expansion in powers of hard scattering cross section can be constructed. We derive explicitly the first order term (the zeroth order term is given by Eqs. (4.12) and (4.13)), and write down the formula for higher orders. Similarly as in the previous section, the quasi-elastic scattering and absorption of quarks will be summed to all orders.

To this end we write down the quark nucleon inclusive cross section as a sum of quasi-elastic and inelastic cross sections:

$$\sigma(\vec{p}, \vec{p}\,') = \sigma_\perp(\vec{p}_\perp - \vec{p}_\perp{}')E\delta(p_{||} - p_{||}{}') + \sigma_{in}(\vec{p}, \vec{p}\,') \quad . \tag{5.1}$$

In the inelastic ("hard") collisions described by $\sigma_{in}(\vec{p}, \vec{p}\,')$ the final longitudinal momentum of the quark is smaller than the initial one.

Consider the quark created by lepton at point $(\vec{b}, z)$ which scatters inelastically at the nucleon located at the point $(\vec{b}, z')$, $z' > z$. Before and after hard scattering the quark can scatter quasi-elastically or be absorbed in nuclear matter by other processes. The distribution of the quark momentum is then given by

$$\frac{EdN^{(1)}(\vec{b}; z, z')}{d^2p}\Big|_A = \sum_{\nu=0}^{A-2} \sum_{\substack{\nu_1=0 \\ \nu_1+\nu_2=\nu}}^{\nu} H_{\nu_1\nu_2} \phi_{\nu_1\nu_2}(\vec{P}; \vec{p}) \tag{5.2}$$

where $H_{\nu_1\nu_2}$ is the probability that exactly ν_1 quasi-elastic collisions have taken place before and ν_2 after the inelastic one

$$H_{\nu_1\nu_2} = \binom{A-2}{\nu}\binom{\nu}{\nu_1} [\tilde{\sigma}_\perp^{(1)}(0)d_A(b; z, z')]^{\nu_1} [\tilde{\sigma}_\perp^{(2)}(0)d_A(b; z')]^{\nu_2} [1 - \sigma_T^{(1)}d_A(\vec{b}; z, z') - \sigma_T^{(2)}d(\vec{b}; z')]^{A-2-\nu} \quad . \tag{5.3}$$

Here $\tilde{\sigma}_\perp^{(1)}(0)$ and $\sigma_T^{(1)}$ [$\tilde{\sigma}_\perp^{(2)}(0)$, $\sigma_T^{(2)}$] are quasi-elastic and total quark-nucleon cross-sections before [after] the quasi-elastic scattering took place and

$$d_A(\vec{b}; z, z') = \int_z^{z'} \rho(\vec{b}, z'')dz'' \quad , \tag{5.4}$$

so that $d_A(\vec{b}, z') = d_A(\vec{b}; z', \infty)$. $\phi_{\nu_1\nu_2}(\vec{P}; \vec{p})$ is the convolution of one inelastic and $\nu_1 + \nu_2$ quasi-elastic scatterings

$$[\tilde{\sigma}_\perp^{(1)}(0)]^{\nu_1} [\vec{\sigma}_\perp^{(2)}(0)]^{\nu_2} \phi_{\nu_1\nu_2}(\vec{P}; \vec{p}) = \int \sigma_\perp^{(1)}(\vec{p}_{1\perp})d^2p_{1\perp} \dots d^2p_{\nu_{1\perp}} \sigma_{in}(P; p_{\|}; \vec{p}_{\nu_{1\perp}} - \vec{q}_1) \, d^2q_1\sigma_\perp^{(2)}(q_1 - q_2) \dots d^2q_{\nu_2}\sigma_\perp^{(2)}(q_{\nu_2} - \vec{p}) \quad . \tag{5.5}$$

Taking Fourier-Bessel transform of $\phi_{\nu_1\nu_2}$ it is possible to perform explicitly summations in Eq. (5.2). The integration over dzdz' can also be done using methods similar as in previous section.[14,16] The result is

$$\frac{EdN^{(1)}}{d^3p} = \frac{1}{(2\pi)^2}\int d^2\beta\, e^{-i\vec{\beta}\vec{p}}\tilde{\sigma}_{in}(P;\, p_{\|},\, \vec{\beta})W_A^{(1)}(\vec{\beta}) \tag{5.6}$$

where

$$W_A^{(1)}(\vec{\beta}) = \frac{F(\lambda_1)}{\lambda_2 - \lambda_1} + \frac{F(\lambda_2)}{\lambda_1 - \lambda_2} \tag{5.7}$$

with

$$F(\lambda) = \int \frac{1 - [\,1 - \lambda D(b)\,]^A}{\lambda A}\, d^2b \tag{5.8}$$

and

$$\lambda_i = \lambda_i(\vec{\beta}) = \sigma_T^{(i)} - \tilde{\sigma}_{\perp}^{(i)}(\vec{\beta}) \quad . \tag{5.9}$$

To complete the formula for the quark spectrum we have to add contribution from quarks which scattered only quasi-elastically (Eq. 4.12). Using similar arguments, one can show that the general expansion in powers of σ_{in} can be written as

$$\frac{EdN}{d^3p}\Big|_A = \sum_{k=0}^{A-1}\frac{EdN^{(k)}}{d^3p} \tag{5.10}$$

with

$$\frac{EdN^{(k)}}{d^3p}\Big|_A = \frac{1}{(2\pi)^2}\int d^2\beta e^{-i\vec{\beta}\vec{p}}\int \tilde{\sigma}_{in}(P;p_1,\vec{\beta})\,\frac{dp_{\parallel 1}}{E_1}\,\tilde{\sigma}_{in}(P_{\parallel 1};P_{\parallel 2},\vec{\beta})\,\ldots$$

$$\ldots\frac{dp_{\parallel k-1}}{E_{k-1}}\,\tilde{\sigma}_{in}(P_{\parallel k-1},p_{\parallel},\vec{\beta})W_A^{(k)}(\vec{\beta}) \tag{5.11}$$

where $W_A^{(k)}(\vec{\beta})$ is given by the formula

$$W_A^{(k)}(\vec{\beta}) = \frac{1}{k!}\sum_{i=1}^{k+1}\frac{F(\lambda_i)}{\prod_{j\neq i}(\lambda_j-\lambda_i)} \quad . \tag{5.12}$$

The hadron spectrum is obtained by folding in the quark fragmentation function, similarly as in Eq. (2.8):

$$\frac{E_h dN_h}{d^3p_h}\Big|_A = \int \frac{EdN}{d^3p}\Big|_A D_{q\to h}(\vec{P},\vec{p}_h)\,\frac{d^3p}{E} \quad . \tag{5.13}$$

In a special case when total and quasi-elastic cross sections do not depend on quark momentum, we have $\lambda_1 = \lambda_2 = \ldots \equiv \lambda$. The Eq. (5.12) reduces then to

$$W_A^{(k)}(\vec{\beta}) = \frac{(-1)^k}{k!}\,\frac{d^k}{d\lambda^k}F(\lambda) \quad . \tag{5.14}$$

Formulae (5.10)-(5.14) give hadron spectrum in terms of nuclear parameter and quark-nucleon cross-sections. They are generalizations of the formulae from Ref. 14, and can be used to extract the quark-nucleon cross-sections from the data. In the next two sections we discuss A-dependence of longitudinal and transverse momentum spectra and indicate how they can be possibly used to this purpose.

VI. A-DEPENDENCE OF THE LONGITUDINAL MOMENTUM SPECTRA

By integrating Eq. (5.13) over transverse momentum one obtains a rather transparent formula for the longitudinal momentum distribution of hadrons[F7]

$$\frac{E_h dN}{dp_{h\parallel}}\Big|_A = W_A^{(0)}(\beta = 0)\bar{D}_{q\to h}(P;\, p_{h\parallel}) + W_A^{(1)}(\beta = 0)\int \frac{dp_\parallel{}'}{E'}\,\bar{\sigma}_{in}(P;\, p_\parallel{}')\bar{D}_{q\to h}(p'_\parallel;\, p_{h\parallel}) + \ldots \quad (6.1)$$

where bar denotes integral over $d^2p_\perp$:

$$\bar{D}_{q\to h}(P;\, p_\parallel) = \int d^2p_\perp D_{q\to h}(P;\, p_\parallel, \vec{p}_\perp) \quad (6.2)$$

$$\bar{\sigma}_{in}(P;\, p_\parallel) = \int d^2p_\perp \sigma_{in}(P;\, p_\parallel, \vec{p}_\perp) \quad . \quad (6.3)$$

The interesting feature of Eq. (6.1) is that all A-dependence of the spectrum is contained in the coefficients $W_A^{(0)}(\beta = 0)$, $W_A^{(1)}(\beta = 0)$, ..., the other quantities being independent of A. Now, the point is that the coefficients $W_A^{(k)}(\beta = 0)$ are entirely determined by the nuclear density and the quantity

$$\sigma_a = \sigma_T - \tilde{\sigma}_\perp(0) \quad (6.4)$$

which is the total cross section for inelastic quark-nucleon scattering. By studying the A-dependence of the longitudinal momentum spectrum of produced hadrons it is thus possible to measure σ_a.

In Figure 4 the coefficients

$$W_A^{(0)}(\beta = 0) = \frac{1}{A\sigma_a}\int d^2b\,\{1 - [\,1 - \sigma_a D(b)\,]^A\} = \frac{\sigma_{qA}}{A\sigma_a} \quad (6.5)$$

where σ_{qA} is the total quark-nucleus inelastic scattering cross section, and

$$W_A^{(1)}(\beta = 0) = \frac{\sigma_{qA}}{A\sigma_a^2} - \frac{1}{\sigma_a} \int D(b)[1 - \lambda D(b)]^{A-1} d^2b \qquad (6.6)$$

are plotted versus A for $\sigma_a = 2, 5, 10$ and 15 mb. (For $\sigma_a = 0$ the values are given by $W^o = 1$ and W_1 shown in Figure 2.)

For the ratio R_A of hadron yields from a heavy nucleus and Hydrogen Eq. (6.1) implies

$$R_A = W_A^{(0)}(\beta = 0) + h(P; p_{h||})W_A^{(1)}(\beta = 0) + \dots \qquad (6.7)$$

where $h(P; p_{h||})$ is the A-independent function of P and $p_{h||}$. In Figure 5 the A-dependence of R_A for $x_h \equiv p_{h||}/P \geq .55$ is plotted versus A. It was calculated using Eqs. (3.1) and (3.2). One sees that the dependence of R_A on σ_a is quite significant and thus with good data one should be indeed able to determine σ_a. For comparison also the contribution from the first term in Eq. (6.7) is plotted in Figure 5. It is seen that the second term is quite small and thus does not affect very significantly determination of σ_a. It follows from this observation that we do not expect very much longitudinal momentum dependence of R_A for large x_h. If x_h dependence is observed, it is a manifestation of the presence of the second term in Eq. (6.7).

The data of Ref. 17 are also plotted in Figure 5. These are rather low energy data and it is not clear if they can be interpreted according to the ideas of this paper. However, if we do attempt such an interpretation, the data indicate $\sigma_a \simeq 15$ mb. This seems to be a rather large number. A possible explanation[17] may be that at these low energies the contribution from hadrons produced directly by leptons cannot be neglected. Any such contribution has a tendency to increase σ_a because hadron-hadron inelastic cross sections are large.

In Ref. 18 the zeroth order formula $R_A = W_A^{(0)}(\beta = 0)$ was compared with the data of Ref. 17 using $\sigma_a = 11$ mb, as suggested by additive quark model of low-$p_\perp$ hadronic interactions. It is by no means obvious[3-6] that the cross section of a point-like quark created by leptons in a deep-inelastic collision should have anything to do with the cross section of the constituent quarks which are relevant in low $p_\perp$ hadronic interactions. Nevertheless, the search for such a possible connection is certainly very interesting. It is clear from Eq. (6.7) and from Fig. 5 that the high-energy deep-inelastic lepton experiments on nuclear targets can provide the answer to this problem.

VII. A-DEPENDENCE OF THE TRANSVERSE MOMENTUM SPECTRUM

A-dependence of the transverse momentum spectrum derived from Eq. (5.13) is a function of

$$\lambda \equiv \sigma_T - \tilde{\sigma}_\perp(\beta) = \sigma_a + \tilde{\sigma}_\perp(0) - \tilde{\sigma}_\perp(\vec{\beta}) \tag{7.1}$$

and of $\sigma_{in}(\vec{P}, \vec{p}')$. To estimate what effects are expected, we have to choose, in addition to $\sigma_{in}(\vec{p}; \vec{p}')$ and $D_{q\to h}(\vec{P}; \vec{p}')$ (we take them as given by Eqs. (3.1) and (3.2)) also $\tilde{\sigma}_\perp(\vec{\beta})$. For simplicity we assumed $\tilde{\sigma}_\perp(\vec{\beta})$ to be a Gaussian

$$\tilde{\sigma}_\perp(\vec{\beta}) = \tilde{\sigma}_\perp(0)\exp\{-\beta^2/4\gamma^2\} \tag{7.2}$$

with $\gamma^2 = 6\ \mathrm{GeV}^{-2}$. This corresponds to $p_\perp$ distribution of quasi-elastic scattering $\sigma_\perp(p_\perp) \sim \exp\{-\gamma^2 p_\perp^2\}$.

The values of σ_a and of total quasi-elastic cross section $\tilde{\sigma}_\perp(0)$ were varied to see the sensitivity of the results to them.

In Figure 6 the effects of quark absorption on A-dependence of the transverse momentum spectrum is illustrated. The $p_\perp$ spectrum for Uranium target is plotted for different values of σ_a and for the quasi-elastic cross section $\tilde{\sigma}_\perp(0)$ equal to zero. It is seen that the absorption decreases the hadron yield almost uniformly in $p_\perp$. The effects are significant but not very dramatic and they can be easily taken into account since σ_a is independently determined from longitudinal momentum spectrum, as described in the previous section.

The last point we investigated was the effect of quasi-elastic scattering $\sigma_\perp(\vec{p}_\perp)$. Although we do not expect the contribution from $\sigma_\perp(\vec{p}_\perp)$ to be very large, it may influence the determination of σ_{in} and therefore requires careful examination.

Let us first describe how one can test the presence of the effects of quasi-elastic quark-nucleon scattering in the data. To this end, let us observe that if $\sigma_\perp(\vec{p}_\perp) = 0$, the formula (5.10) gives <u>one parameter</u> description[F7] of the data and thus should not be difficult to verify. Indeed, if $\tilde{\sigma}_\perp(\vec{\beta}) = 0$ we have $\sigma_a = \sigma_T$ and thus

$$\frac{E_h dN}{d^3 p_h}\Big|_A = \frac{\sigma_{qA}}{A\sigma_a}\frac{E_h dN}{d^3 p_h}\Big|_H + W_A^{(1)}(\beta = 0)\sigma_h(P,\vec{p}_h) \tag{7.3}$$

where σ_h is given by Eq. (2.8) with $\sigma \to \sigma_{in}$. Since σ_a can be measured from the analysis of the longitudinal momentum spectrum, the only unknown in Eq. (7.3) is the cross section $\sigma_h(P;\vec{p}_h)$. In Figure 7 we plot the difference $\frac{dN}{dp_\perp^2}\Big|_A - \frac{\sigma_{qA}}{A\sigma_a}\frac{dN}{dp_\perp^2}\Big|_H$ versus $W_A^{(1)}(\beta=0)$ for σ_a=10 mb, x_F>.55 and different values of $\tilde{\sigma}_{\perp}(0)$. When $\tilde{\sigma}_{\perp}(0) = 0$ this difference is a linear function of $W_A^{(1)}(\beta = 0)$ and the slope measures $\sigma_h(P; \vec{p}_h)$. One sees from Figure 7 that already for $\tilde{\sigma}_{\perp}(0) = 2.5$ mb the deviation from linearity is quite visible (particularly if we take into account that all curves must pass through the origin). Thus we feel that it should be possible to identify the presence of $\sigma_{\perp}(\vec{p}_{\perp})$.

If such effects of $\sigma_{\perp}(\vec{p}_{\perp})$ are indeed present in the data then the analysis becomes more complicated, because it is necessary to fit data with the full formula (5.11). We would like to emphasize, however, that this procedure is still rather straightforward.

We close this section with the following remarks. In all our considerations we ignored the process

$$\text{quark} + \text{nucleon} \rightarrow \text{gluon} + \text{anything} \tag{7.4}$$

which also can contribute to the production of hadrons at large transverse momenta. Indeed, it was observed in Ref. 8 that this and other gluon interactions are crucial in understanding hadron production from nuclei. However, as we already emphasized in the introduction, by restricting our kinematic limit to $x_h > .55$ we largely eliminate the contribution from the process (7.4). There are two reasons for that: (i) the gluons from the process (7.4) tend to be less energetic than quarks from process (1.2); (ii) in the region of large z the gluon fragmentation function is believed to be much smaller than the quark decay function.[10] Our estimate is that gluons do not contribute more than 5% of hadrons in this kinematic region. This contribution can be reduced further by imposing even stronger cut on hadron momenta.

Finally, let us add that the formulae (5.10) and (5.11) are perfectly suited to describe also the more general case when both quarks and gluons contribute to hadron production. Their practical use is restricted, however, by the limited information we have about the gluon fragmentation function.

VIII. CONCLUSIONS

We have investigated the possibilities of measuring the high-energy inclusive quark-nucleon cross section by studying the A-dependence of hadron spectra produced in deep-inelastic leptoproduction from nuclear targets. Our conclusions can be summarized as follows:

(i) The spectra of fast hadrons are particularly useful in determining quark-nucleon cross sections[4] because they are not influenced by cascading effects inside the nuclear matter (both hadronic and quark-gluon cascade effects are minimized by this selection).

(ii) The study of A-dependence of the longitudinal momentum spectra of fast hadrons should provide an easy and fairly good determination of total inelastic quark-nucleon cross section σ_a. The study of energy (ν) and Q^2 dependence of this cross section might prove very interesting and give information on the relation between point-like quarks produced in deep-inelastic lepton scattering and constituent quarks which are relevant in low-$p_\perp$ hadronic interactions.

(iii) The determination of the quark-nucleon inclusive cross section at large $p_\perp$ of the scattered quark is also possible but requires high statistics experiments because the expected hadron production rates in this region are rather low. However, if these low rates can be measured, our discussion provides a well-defined and relatively simple procedure for extracting the quark-nucleon cross section from the data.

(iv) The quasi-elastic quark-nucleon cross section $\sigma_\perp(\vec{p}_\perp)$ is probably most difficult to measure. Its estimate rests on a complicated fitting procedure which may be quite sensitive to the particular assumptions one makes on shape of $\sigma_\perp(\vec{p}_\perp)$. It is difficult to say anything more precise before actual data are available.

Let us close the paper with a few remarks.

(a) Most of our results can be extended to the process of heavy lepton pair production from nuclear targets. To see this let us observe that if one imposes the condition[6]

$$t \equiv \frac{E_{\ell\bar{\ell}}}{M^2} < 1 \text{ fermi}$$

where $E_{\ell\bar{\ell}}$ is the energy of the pair in the laboratory and M is the mass of the pair, the life-time t of the heavy vector meson in the laboratory is so short that it has no chance to interact in the nucleus before disintegrating into a pair of leptons. Consequently, the A-dependence of the process can be influenced only by interaction of projectile quarks inside the target nucleus before the Drell-Yan process has taken place. Thus we have the situation which is just inverse to the one depicted in Fig. 1 where the quark interacts after the lepton-nucleon collision. Obviously, all our analysis holds, but now instead of convolution with quark fragmentation function $D_{q\to h}$ it is necessary to consider rather convolution with the projectile structure function $G_{h\to q}$.

Thus by invoking the results of our discussion of leptoproduction we see that A-dependence of the $\ell\bar{\ell}$ mass spectrum provides information on inelastic quark-nucleon scattering cross section σ_a, whereas the A-dependence of the $\vec{p}_\perp$ distribution of the pairs measures the differential inclusive cross section in the process (1.3). We feel that the precise measurements of these phenomena are of importance.

(b) It seems particularly interesting to compare the quark-nucleon cross sections obtained from leptoproduction data and from production of heavy Drell-Yan pairs. In the first case one measures the cross section of a "quasi-free" quark which travels alone through nuclear matter. In the second case one measures the cross section of a quark surrounded by other quarks and gluons forming together a color singlet state. If "color screening" effects[19] are important, it may well be that these cross sections are substantially different.

(c) We assumed in this paper that high-energy quarks behave as "normal" particles with well-defined properties which do not change during the time needed for passing through the nucleus. It was also assumed that fragmentation of quarks into hadrons is independent of secondary interactions in the target. We feel that this is a reasonable "conservative" starting point. It is possible, however, that these assumptions are violated. For example, some properties of the quarks may be time-dependent due to radiative corrections from gluon emission and exchange. We also do not know the effects of confinement forces. The issue then is what is the time scale of such secondary effects and how it depends on energy of the quark. If this time scale is Lorentz-dilated and thus long enough for high energy quarks, our analysis is expected to be applicable. The comparison with data should thus give information on this important problem.

ACKNOWLEDGMENTS

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FOOTNOTES

F1 It is impossible to quote all references. A sample of reviews is given as Refs. 1-6.

F2 This is to be contrasted with large p_T hadronic processes in nuclei where the initial state contains a not-too-well-known mixture of quarks, antiquarks and gluons with a rather broad distribution of momenta.

F3 This condition restricts the kinematic region in which the cross section (1.3) can be measured. The discussion of lower x_F region would thus be also very interesting. It seems, however, more complicated.[8]

F4 All directions are defined with respect to the momentum of the virtual photon.

F5 Following Ref. 9, we used the Saxon-Woods nuclear density $\rho(\vec{r}) = \rho_o/[1 + \exp\{(r-R)/a\}]$ with $R = (.978 + .0206A^{1/3})A^{1/3}$ and $a = .54$.

F6 The purpose of this paper is to present a <u>method</u> of extracting the quark-nucleon cross-sections from the data and <u>not</u> to make detailed predictions of the spectra. The estimates we give here are to be considered only as an illustration and not as predictions of a theory.

F7 To simplify our semi-quantitative discussion, we assume from now on that the total and quasi-elastic quark-nucleon cross-sections σ_T and $\tilde{\sigma}(\vec{p})$ do not depend on energy of the quark and thus $W_A^{(k)}(\vec{\beta})$ are given by Eq. (5.14). We assume also that the general series (5.10) can be terminated after the first two terms.

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FIGURE CAPTIONS

Fig. 1: Final state interaction of quark in the nucleus.

Fig. 2: Coefficient $W_1(A) = \frac{1}{2}(A - 1) \int d^2 b D_A^2(b)$ plotted versus nuclear number of the target.

Fig. 3: A-dependence of the $p_\perp$ spectrum following from the single-scattering formula (2.7).

Fig. 4: Parameters $W_A^{(0)}(\beta = 0)$ and $W_A^{(1)}(\beta = 0)$ plotted versus nuclear number of the target for different values of the total quark-nucleon inelastic cross section σ_a.

Fig. 5: A-dependence of the ratio R_A of hadronic yields from nuclei and hydrogen for different values of total quark-nucleon inelastic cross section σ_a. The full lines are results from Eq. (6.7). The dotted lines are results of the zero order formula $R_A = \sigma_{qA}/A\sigma_a$.

Fig. 6: Transverse momentum dependence of hadrons produced from U target for different values of σ_a.

Fig. 7: Test of the presence of the quasi-elastic quark-nucleon cross section $\sigma_\perp(\vec{p}_\perp)$, as described in the text.

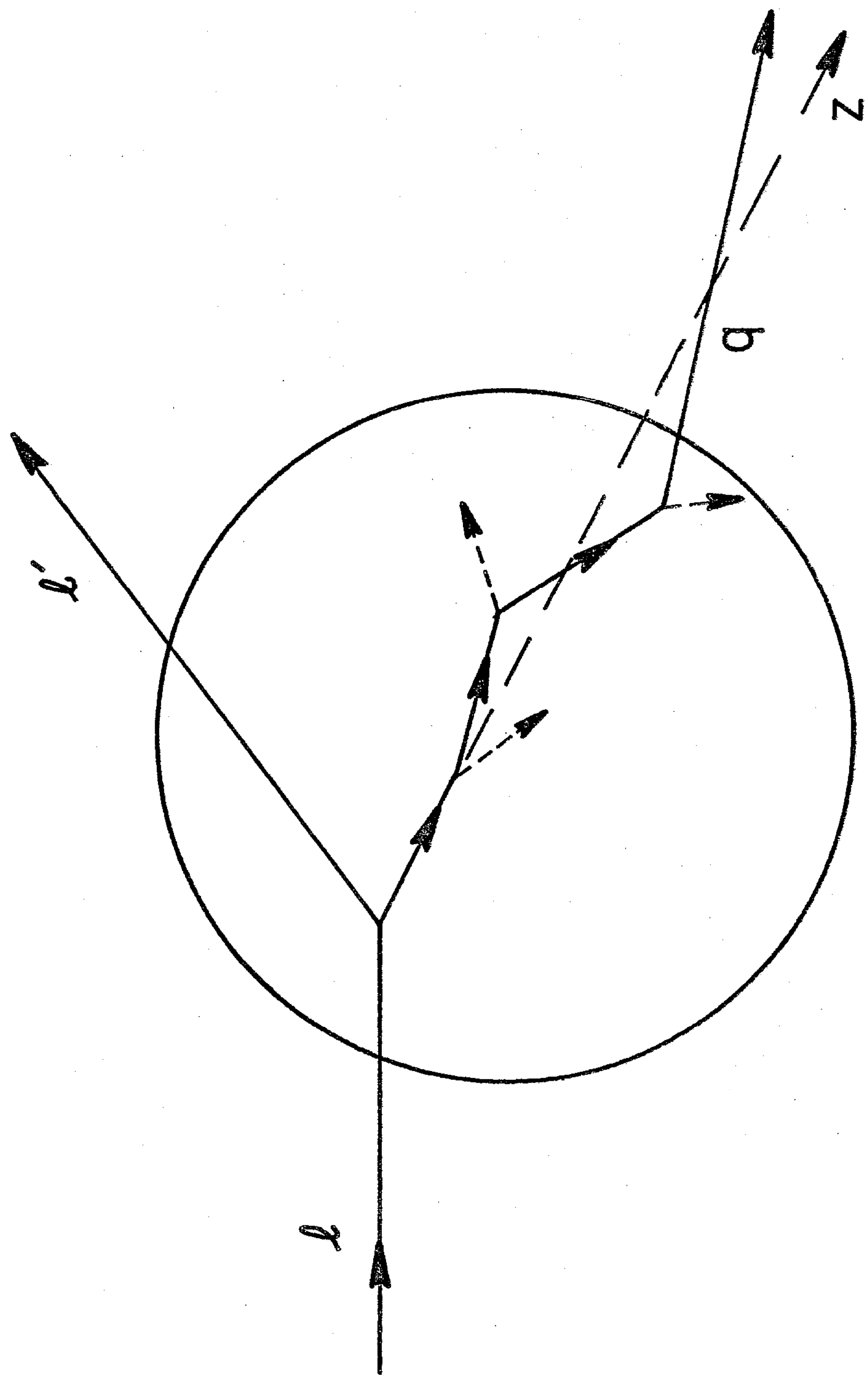


Fig. 1

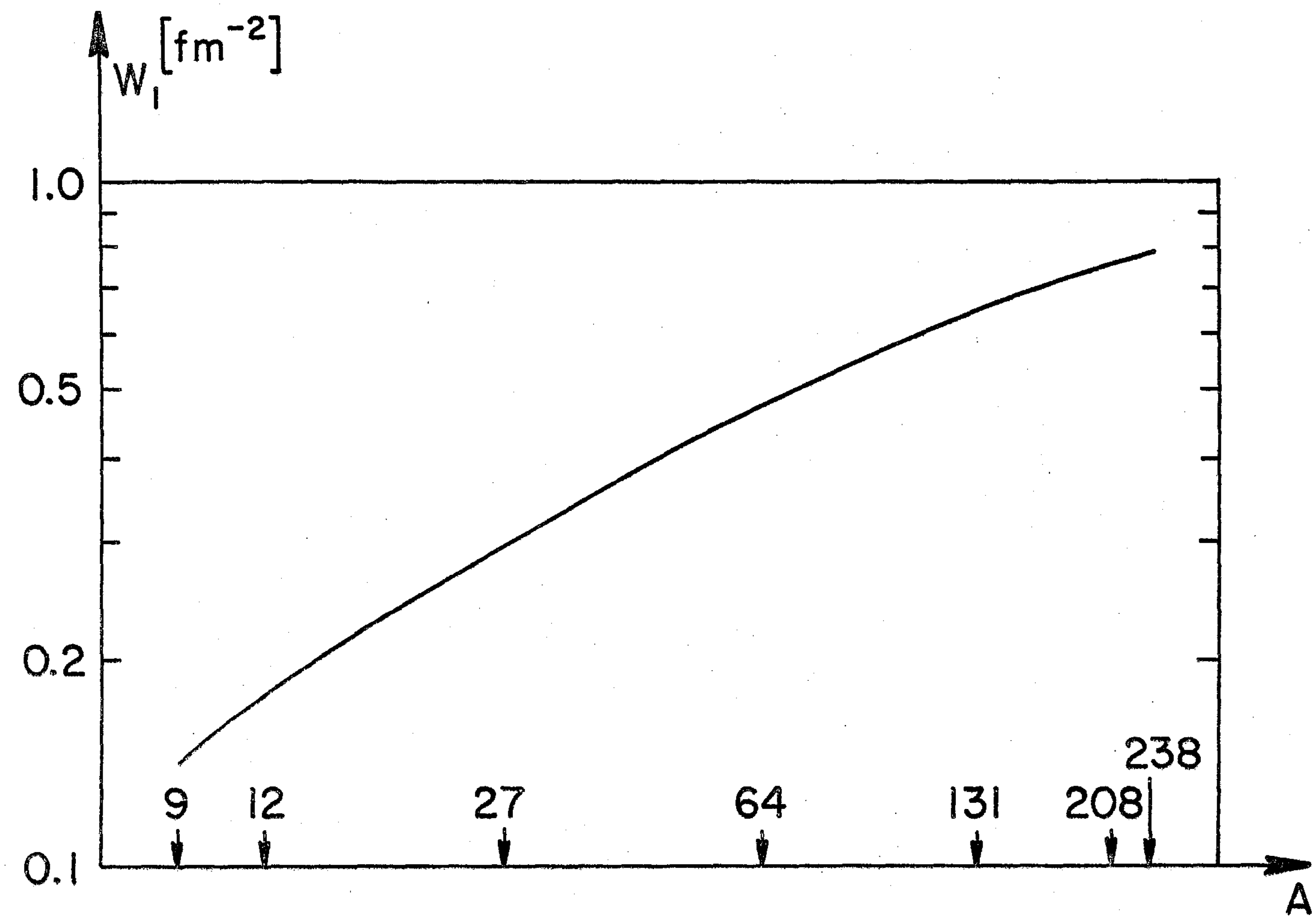


Fig. 2

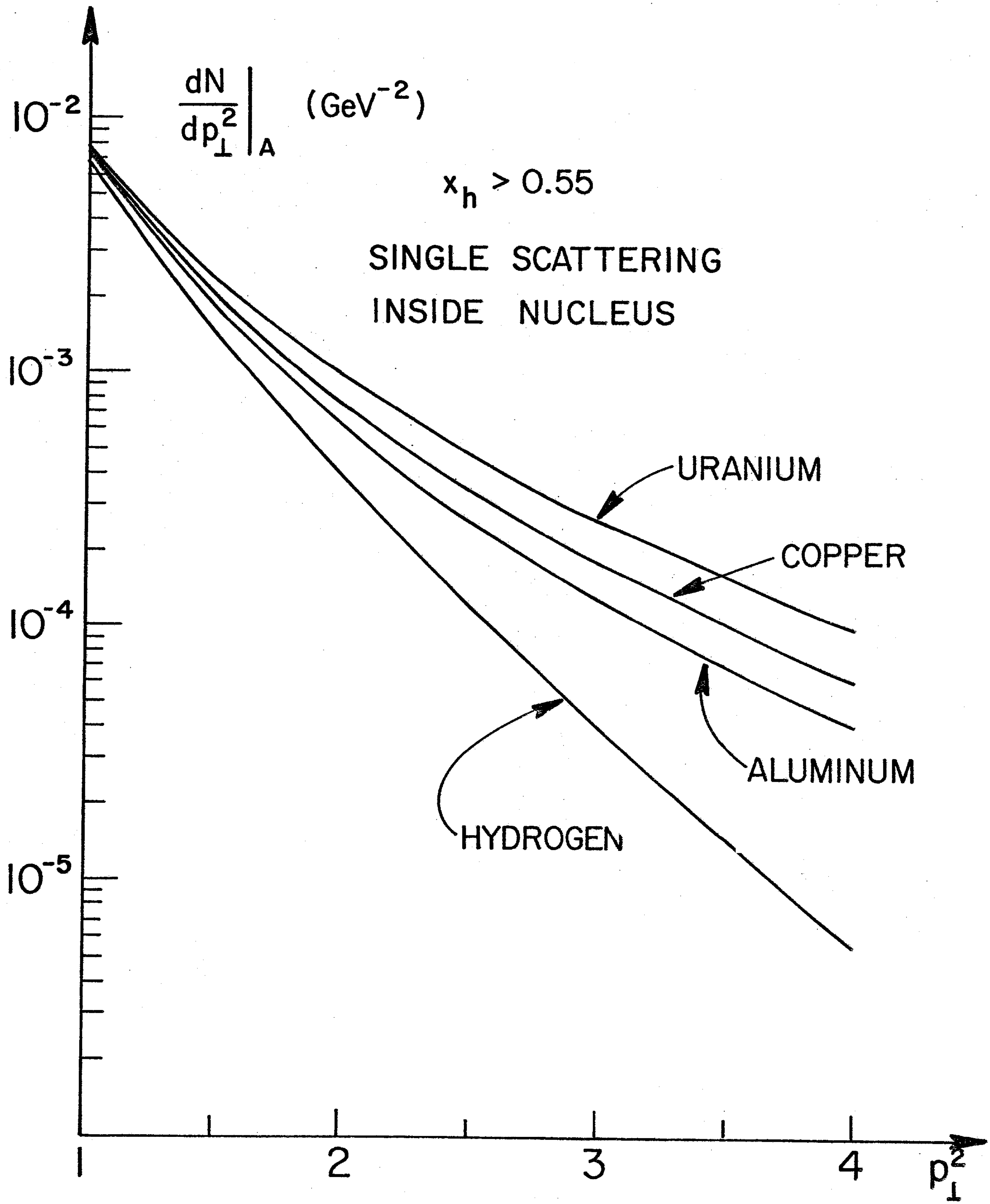
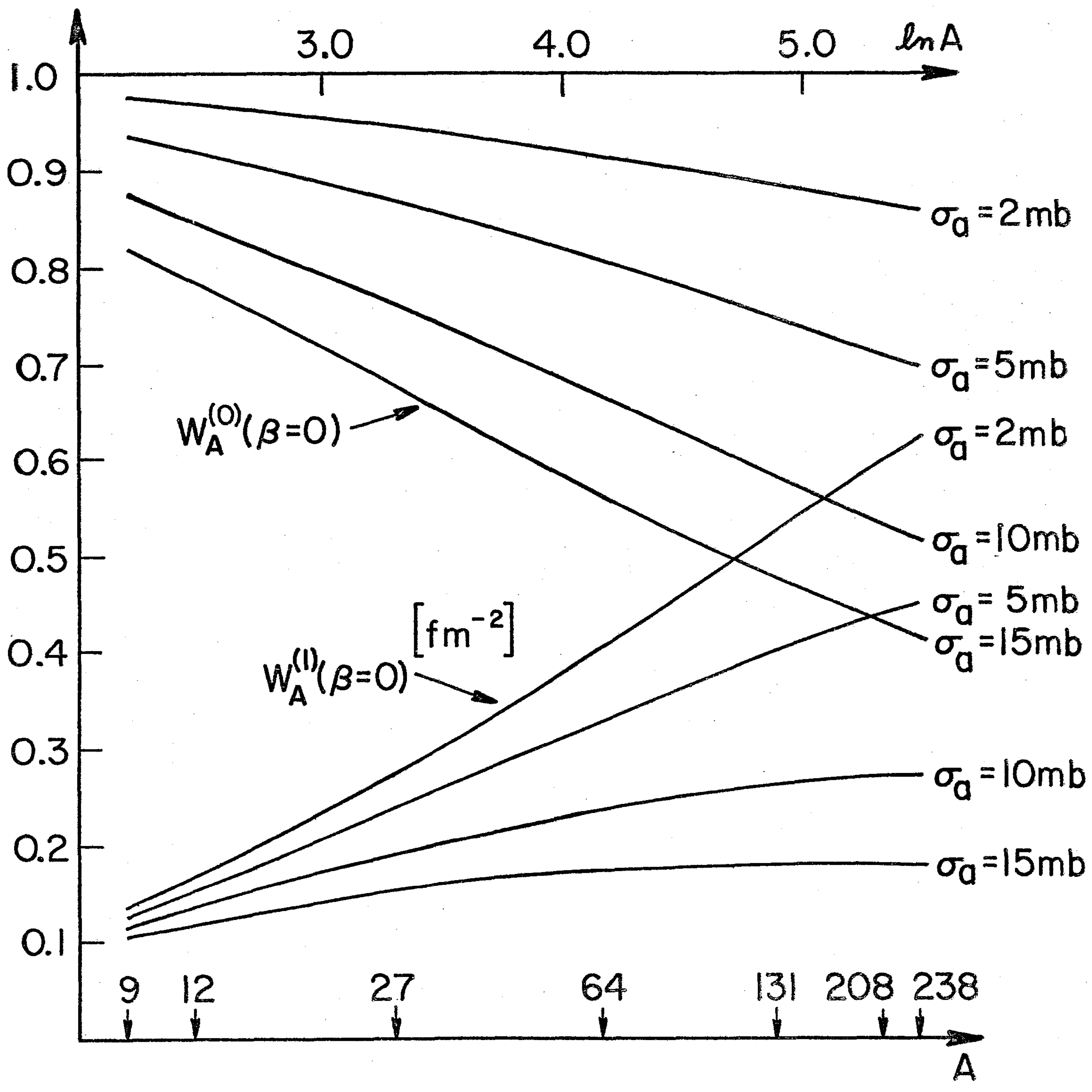
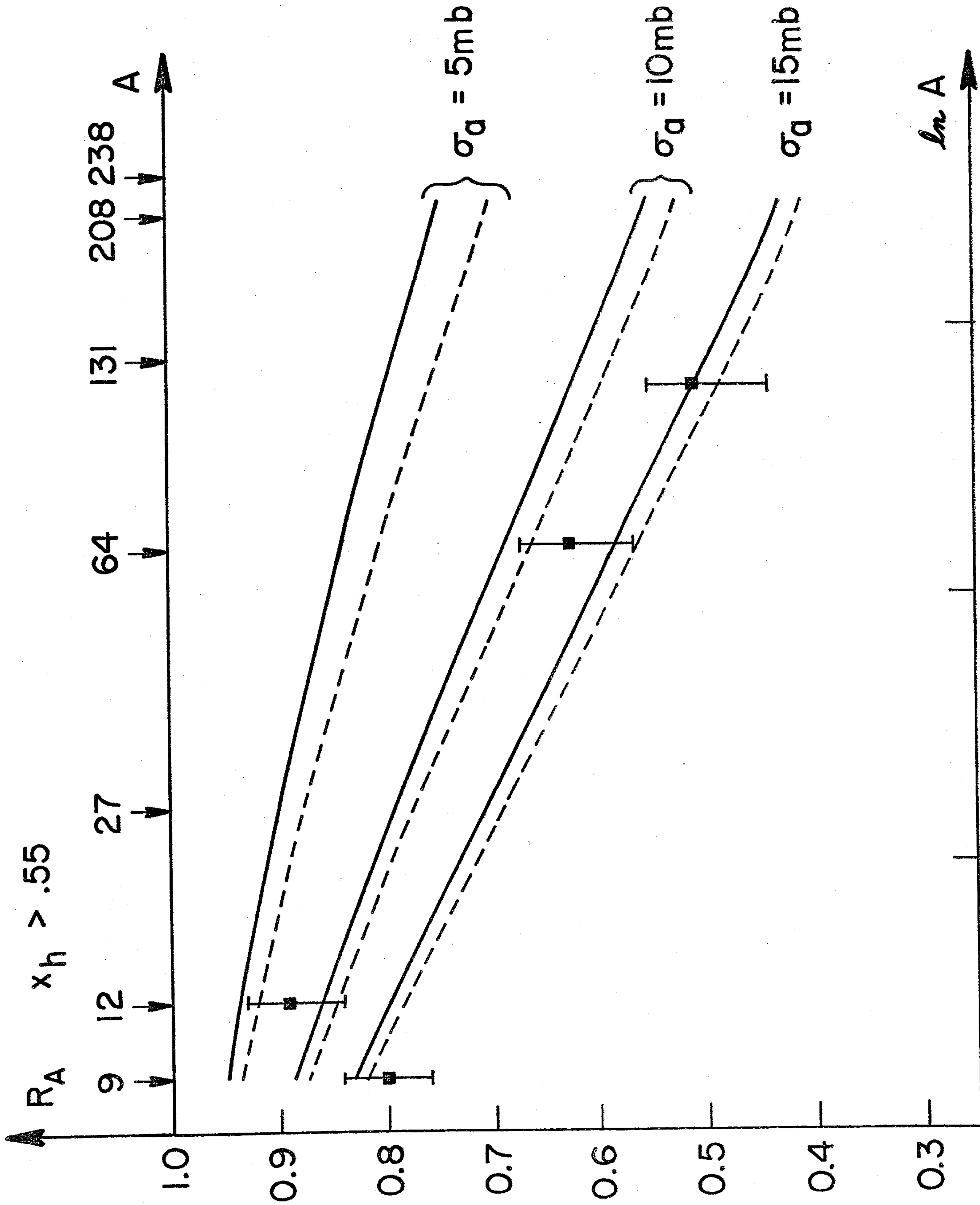

dN/dp⊥² |A (GeV⁻²)
x_h > 0.55
SINGLE SCATTERING
INSIDE NUCLEUS
URANIUM
COPPER
ALUMINUM
HYDROGEN
10⁻²
10⁻³
10⁻⁴
10⁻⁵
1
2
3
4
p⊥²

Fig. 3



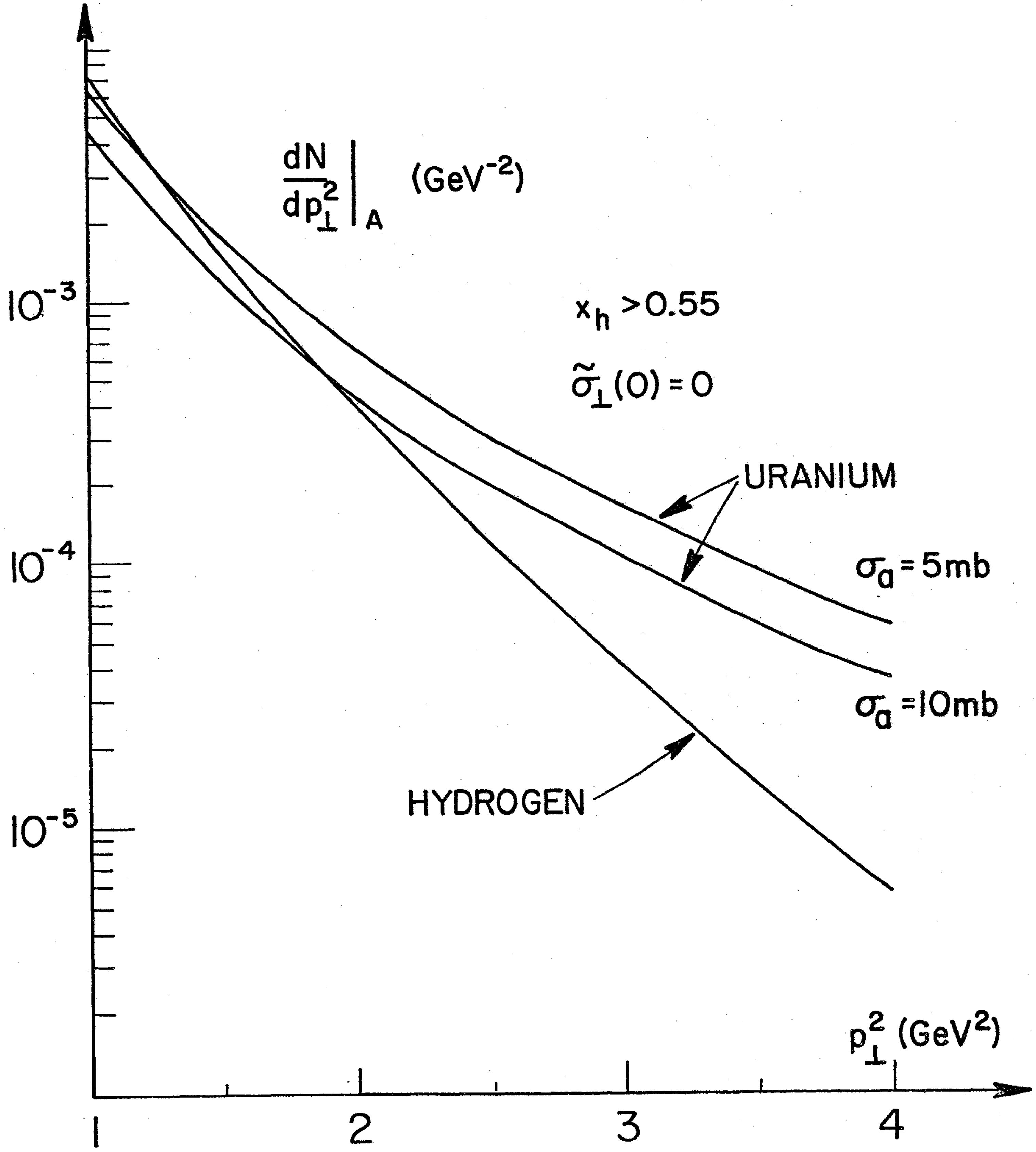
3.0
4.0
5.0
ℓn A
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
$\sigma_a = 2$mb
$\sigma_a = 5$mb
$\sigma_a = 2$mb
$\sigma_a = 10$mb
$\sigma_a = 5$mb
$\sigma_a = 15$mb
$\sigma_a = 10$mb
$\sigma_a = 15$mb
$W_A^{(0)}(\beta=0)$
$W_A^{(1)}(\beta=0)$ [fm^{-2}]
9
12
27
64
131
208
238
A

Fig. 4



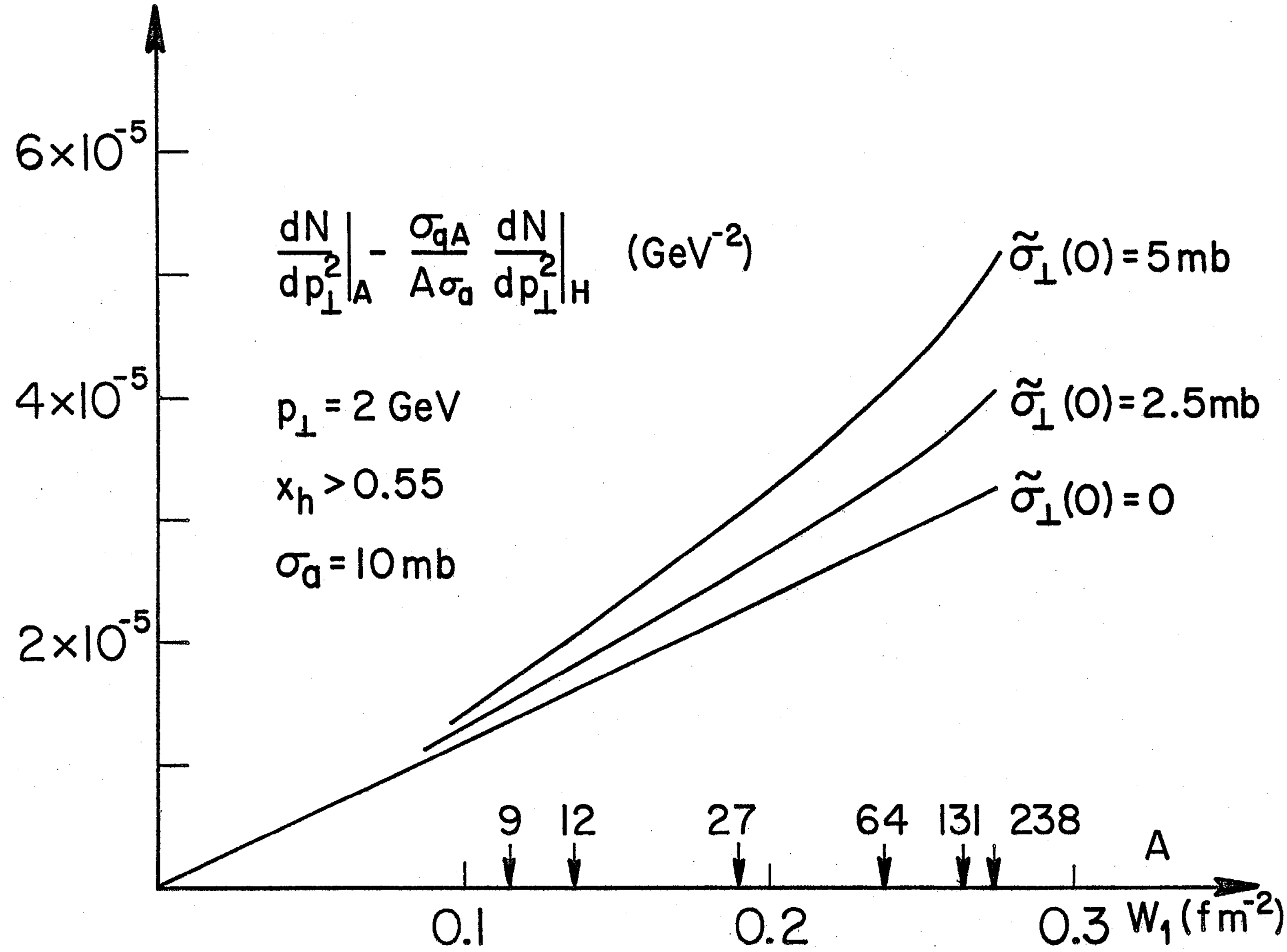
R_A
$x_h > .55$
A
9
12
27
64
131
208
238
ln A
2.0
3.0
4.0
5.0
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
σ_a = 5mb
σ_a = 10mb
σ_a = 15mb

Fig. 5



$\frac{dN}{dp_\perp^2}\Big|_A$ (GeV^{-2})
$x_h > 0.55$
$\tilde{\sigma}_\perp(0) = 0$
URANIUM
$\sigma_a = 5mb$
$\sigma_a = 10mb$
HYDROGEN
10^{-3}
10^{-4}
10^{-5}
$p_\perp^2$ (GeV^2)
1
2
3
4

Fig. 6



$\frac{dN}{dp_\perp^2}\Big|_A - \frac{\sigma_{qA}}{A\sigma_q}\frac{dN}{dp_\perp^2}\Big|_H$ (GeV^{-2})
6×10^{-5}
4×10^{-5}
2×10^{-5}
$p_\perp = 2$ GeV
$x_h > 0.55$
$\sigma_q = 10$ mb
$\tilde{\sigma}_\perp(0) = 5$ mb
$\tilde{\sigma}_\perp(0) = 2.5$ mb
$\tilde{\sigma}_\perp(0) = 0$
9
12
27
64
131
238
A
0.1
0.2
0.3
W_1 (fm^{-2})

Fig. 7