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INSTANTON INDUCED INTERACTIONS

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Abstract

Quarks interact under the influence of instantons. These interactions affect the wavefunctions of pions and nucleons and produce differences in the transverse momentum distributions of the up and down quarks within a nucleon.

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Quantum chromodynamics may provide a viable description of the quark substructure of hadronic states. At short distances this theory is amenable to perturbative analysis, with results which accord well with a number of high energy scattering experiments. At long distances, however, non-perturbative effects are probably important, and the details of how quarks are bound into hadronic states remain unclear. This binding poses two difficult questions: how to generate the spectrum of the nonrelativistic quark model and how to incorporate in these states the constraints of current algebra and chiral symmetry.

A solution to this second problem may be provided[1,2] by the instanton configurations[3] of quantum chromodynamics. If instantons generate dynamical masses for N species of massless fermions, then the same interactions must produce[2] $N^2 - 1$ massless pseudoscalar bound states of quarks and anti-quarks as Nambu-Goldstone bosons for the chiral symmetry SU(N) X SU(N). If, moreover, this spontaneous symmetry breakdown occurs at distances smaller than typical hadronic radii, it should be possible to treat instanton interactions independently of details of the mechanism which confines quarks to the interior of hadrons.

In this letter we argue that the instanton scale is indeed smaller than the confinement scale. It follows that hadronic wavefunctions should have momentum components of order μ, the inverse of the dominant instanton size, a number which is significantly larger than 200 MeV, the inverse of the confinement length. Experiments[4] which probe the transverse momentum distribution of quarks within pions and nucleons already indicate the presence[5] of momentum components on the order of 800 - 1000 MeV. Our model helps explain this result and predicts that there should be large differences in the transverse momentum distributions of the up and down quarks within a nucleon.

The instanton-induced quantum tunnelling amplitude involves the running coupling g in the following combination:[6,7]

$$\gamma = \left[\frac{8\pi^2}{g^2(\lambda\rho)}\right]^6 \exp\left[\frac{-8\pi^2}{g^2(\lambda\rho)}\right]\left[1 + O(g^2)\right], \qquad (1)$$

Here ρ denotes the instanton size and λ is a constant which is indeterminate at the one loop level (i.e. with the consideration of only the lowest order quantum fluctuations about the instanton field). Since γ is maximal for $g^2(\lambda\rho)/8\pi^2 = 1/6$, this factor selects a certain range of instanton sizes dependent upon the parameter λ. The approximate value of this parameter can be estimated in an analysis of the dynamical mass which instantons are presumed to generate. For the purpose of that analysis we will choose a specific form for the running coupling,

$$\frac{8\pi^2}{g^2(\rho)} = \frac{33-2N}{6} \log\left(1 + \frac{1}{\Lambda^2\rho^2}\right), \qquad (2)$$

which extrapolates from asymptotic freedom at small ρ to a linear potential at large ρ. The scale factor Λ is determined from analyses of scaling violations in deep inelastic scattering and electron-positron annihilation to have the approximate value[8], $\Lambda \simeq 500$ MeV.

The dynamical quark mass m(p) is described by an integral equation which can be solved by numerical or approximate analytical techniques. The result is

$$m(p) \sim \int d\rho \; \rho^{2N-4} \; \gamma(\lambda\rho) \; f^2\left(\frac{\rho p}{2}\right), \tag{3}$$

where $f(\rho p/2)$ describes[9] the wavefunction of the lowest eigenmode of a quark field in the presence of an instanton of size ρ. A plot of the mass function m(p) is given in Fig. 1 for the case N = 3. The derivation of Eq. (3) assumes that $\rho\, m(p) \ll 1$ over the range of ρ selected by the factor $\gamma(\lambda\rho)$. The approximate value of λ can be specified as follows. If our quantum amplitudes are defined[5] by Pauli-Villars regularization, then the long distance mass m(0) will have a numerical value

$$m(0) \simeq 3.6 \; \lambda^{10} \; \Lambda \, . \tag{4}$$

Thus, if this number is to be of the order of 300 MeV, we must have $\lambda \simeq 0.8$. A different regularization scheme would change the numerical coefficient in Eq. (4). Thus, for example, in the dimensional regularization scheme[6] (still using $\Lambda = 500$ MeV) we would obtain an estimate $\lambda \simeq 1.5$. In either case the strong λ dependence of Eq. (4)

requires that λ have a value of order unity.

The momentum $\mu = \rho^{-1}$ which maximizes the function γ of Eqs. (1) and (2) has the value

$$\mu = \left[e^{4/3} - 1\right]^{1/2} \lambda\Lambda \simeq 835 \text{ MeV} \tag{5}$$

for $\lambda = 1.0$ and $\Lambda = 500$ MeV. Note that this estimate is large compared to 200 MeV, meaning that the dominant instantons are significantly smaller than the confinement length. Although the magnitude of the product $\lambda\Lambda$ is somewhat uncertain, we believe that this qualitative conclusion will survive any juggling of the parameters in Eq. (5). In physical terms, the parameter μ sets the scale of variation of the mass function m(p) (see Fig. 1). The same scale characterizes the interactions which instantons induce between quarks and antiquarks. These interactions[10] can be expressed[2] in terms of a Bethe-Salpeter kernel,

$$K(p_2, q_2; p_1 q_1) \sim \int d\rho\, \rho^{2N-3} \gamma(\lambda\rho)\, \delta^4(p_2 + q_2 - p_1 - q_1)$$

$$\times f\left(\frac{\rho p_2}{2}\right) f\left(\frac{\rho q_2}{2}\right) f\left(\frac{\rho p_1}{2}\right) f\left(\frac{\rho q_1}{2}\right) A_2 A_1, \tag{6}$$

where A_1 and A_2 denote color, flavor and spin dependent factors formed from the incoming and outgoing particles, respectively. The relative strengths of the kernel for different states of color, flavor and spin are indicated in Table 1. A strong attractive interaction occurs[2] in the pseudoscalar, color singlet, flavor adjoint channel and generates the Nambu-Goldstone bosons expected to follow from the spontaneous breaking of chiral SU(N) X SU(N) symmetry. The corresponding Bethe-Salpeter wavefunction for a pion of zero four-momentum is

$$\Pi_i(p) \sim \frac{m(p)\,\gamma_5\,\lambda_i}{p^2+m(p)^2} \tag{7}$$

Note that the small momentum (or long distance) behavior of this wavefunction is governed by the long distance mass m(0). The large momentum behavior of $\Pi_i(p)$ is determined by the large momentum behavior of m(p) and involves the scale μ as indicated in Fig. 1.

Crossing to quark-quark channels, we can recast the results of Table I into the form of Table II. The same approximations[2] which produced the massless bound state of Eq. (7) lead now to a (massive) diquark state. The diquark is spinless and transforms as an anti-triplet of color and flavor. Like the pseudoscalar mesons this diquark includes in its wavefunction momentum components of characteristic size μ. The quantum numbers of the diquark allow it to reside in the nucleon but not in the Δ resonance. This fact underscores the role of instanton interactions in breaking the SU(6) symmetry of the naive quark model. (Similarly, we note that in Table I there are entries corresponding to pseudoscalar mesons but none for any vector meson states.)

If now we construct nucleons as bound states of diquarks and quarks, an interesting physical picture results. Since the confinement length is larger than typical instanton sizes, the quark-diquark wavefunction should involve typically smaller momentum components than the wavefunction for quarks bound within the diquark. This implies that the average transverse momentum of a down quark within a proton will be typically larger than that of an up quark. Quantitative measurements of these distributions can be obtained by studying[4] the production of $\mu^+\mu^-$ pairs in the bombardment of protons with $\pi^\pm$ beams. Neglecting entirely the momentum arising from the quark-diquark wavefunction, we obtain a prediction[11,12]

$$\langle p_\perp^2\rangle_{\pi^+} - \langle p_\perp^2\rangle_{\pi^-} \simeq \frac{1}{2}\mu^2 \qquad (8)$$

for the difference in mean-square transverse momenta of the produced muon pairs. To test this prediction one must measure the difference $\langle p_\perp^2\rangle_{\pi^+} - \langle p_\perp^2\rangle_{\pi^-}$ and use Eq. (8) to deduce from it a value for μ. If this number is significantly larger than 200 MeV, then we would interpret the $\mu^+\mu^-$ production data as an experimental observation of instantons, and would identify the experimentally determined μ^{-1} as the dominant instanton size.

The quark-diquark structure proposed here provides a straightforward explanation for aspects of the nucleon that have been previously linked with SU(6) symmetry breaking effects. The diquark wavefunction (cf. Eq. (7))

falls off rapidly at large momenta ($p \gg$ [illegible]). Thus in deep inelastic scattering experiments near $x = 1$, scattering should occur dominantly from the odd quark in the nucleon. This provides that $F^n_2(x)/F^p_2(x) \to 1/4$ as $x \to 1$. Furthermore, since the diquark is spinless there is a suggestion in this picture of large polarization asymmetries as $x \to 1$. Such asymmetries do indeed seem to be a feature of the most recent data in this area[13].

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Footnotes and References

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7. The factor g^{-12} is appropriate for the color gauge group SU(3).

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10. Qualitative features of these interactions have been discussed by D. Horn and S. Yankielowicz, Phys. Letters 76B, 343 (1978).

11. A qualitatively similar, but quantitatively smaller, prediction was obtained by R. D. Carlitz, S. D. Ellis and R. Savit, Phys. Letters 68B, 443 (1977). The larger value of the present result can be attributed directly to the relatively large size of the parameter μ, which sets the momentum scale of all instanton effects.

12. Perturbative effects neglected in this paper should lead to an increase of $\langle p_\perp^2\rangle_{\pi^\pm}$ with the energy of the incident pions. These effects will not, however, alter our expectation of differences in $\langle p_\perp^2\rangle_{\pi^\pm}$.

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Table Captions

Table I. Relative strength of the Bethe-Salpeter kernel for quark-antiquark channels. Underlined numbers label representations of the SU(3) color and SU(N) flavor groups. Spin indices of the two particles are coupled via the indicated Dirac matrix.

Table II. Relative strength of the Bethe-Salpeter kernel for quark-quark (diquark) channels. The symbol A denotes anti-symmetrization of the flavor labels of the two quarks, corresponding to the representations $\underset{\sim}{2}$, $\underset{\sim}{\overline{3}}$ and $\underset{\sim}{6}$ for N = 2, 3 and 4, respectively.

Figure Caption

Fig. 1. The dynamical mass function. The dominant instanton size is of order μ^{-1}.

Table I

Color	Flavor	Spin	Strength
$\underset{\sim}{1}$	$\underset{\sim}{1}$	$\frac{i}{2}\gamma_5$	$1-N$
$\underset{\sim}{1}$	$\underset{\sim}{N^2-1}$	$\frac{i}{2}\gamma_5$	1
$\underset{\sim}{1}$	$\underset{\sim}{1}$	$\frac{1}{2}$	$N-1$
$\underset{\sim}{1}$	$\underset{\sim}{N^2-1}$	$\frac{1}{2}$	-1
$\underset{\sim}{8}$	$\underset{\sim}{1}$	$\frac{i}{2}\gamma_5$	$(1-N)/16$
$\underset{\sim}{8}$	$\underset{\sim}{N^2-1}$	$\frac{i}{2}\gamma_5$	$1/16$
$\underset{\sim}{8}$	$\underset{\sim}{1}$	$\frac{1}{2}$	$(N-1)/16$
$\underset{\sim}{8}$	$\underset{\sim}{N^2-1}$	$\frac{1}{2}$	$-1/16$
$\underset{\sim}{8}$	$\underset{\sim}{1}$	$\frac{\sqrt{2}}{4}\sigma_{\mu\nu}$	$-3(1-N)/16$
$\underset{\sim}{8}$	$\underset{\sim}{N^2-1}$	$\frac{\sqrt{2}}{4}\sigma_{\mu\nu}$	$-3/16$

Table II

Color	Flavor	Spin	Strength
$\underset{\sim}{\bar{3}}$	A	$\frac{i}{2}\gamma_5$	-1
$\underset{\sim}{\bar{3}}$	A	$\frac{1}{2}$	1
$\underset{\sim}{6}$	A	$\frac{\sqrt{2}}{4}\sigma_{\mu\nu}$	$-\frac{1}{2}$

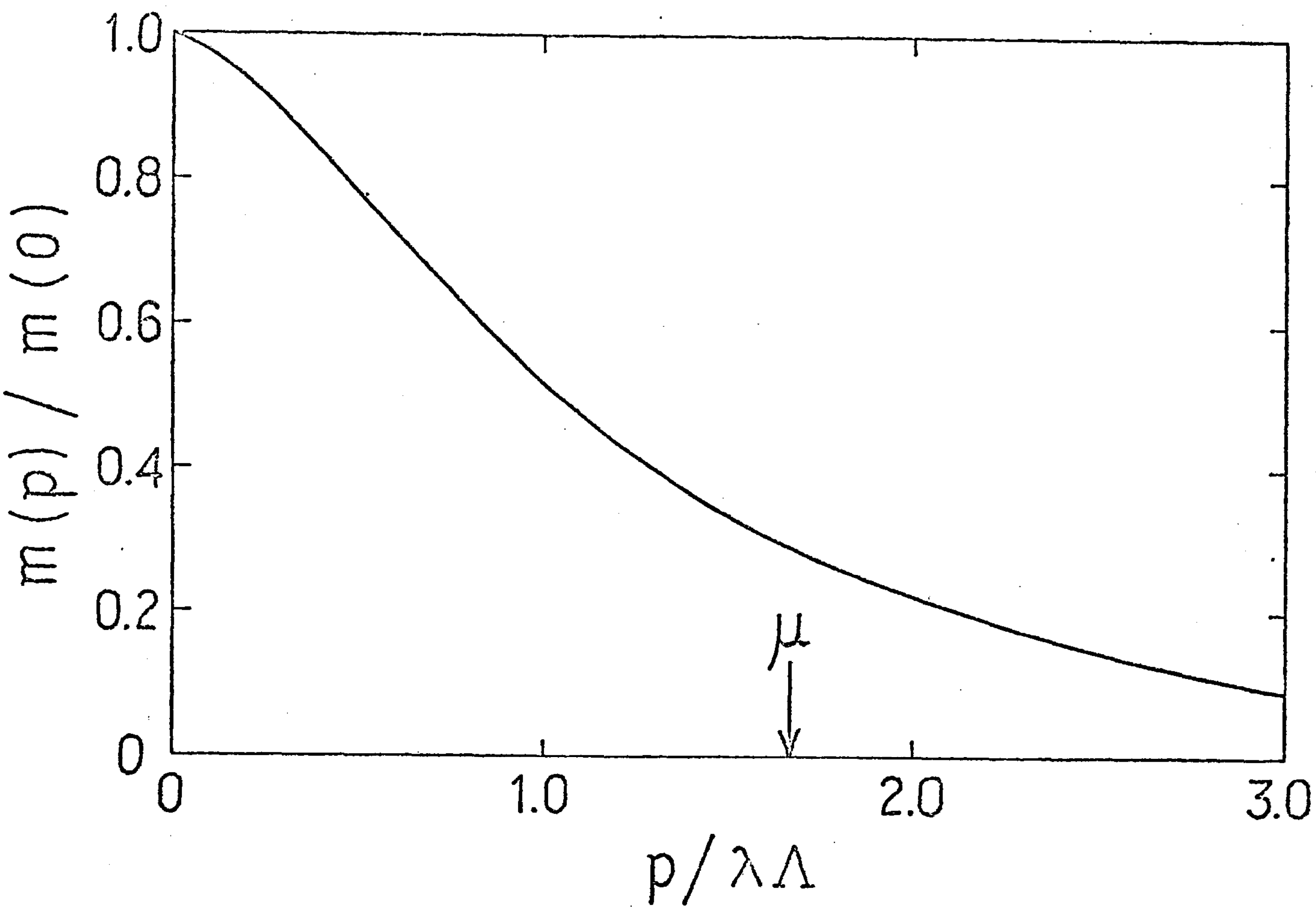

Fig. 1