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BOUND STATES OF HEAVY QUARKS AND ANTIQUARKS

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ABSTRACT

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1. Introduction

In the study of the strong interaction, several distance regimes may be distinguished according to the phenomena that occur and the tools–both experimental and theoretical–with which we explore them. Such a separation, in which the boundaries are necessarily vague, is shown in Fig. 1.

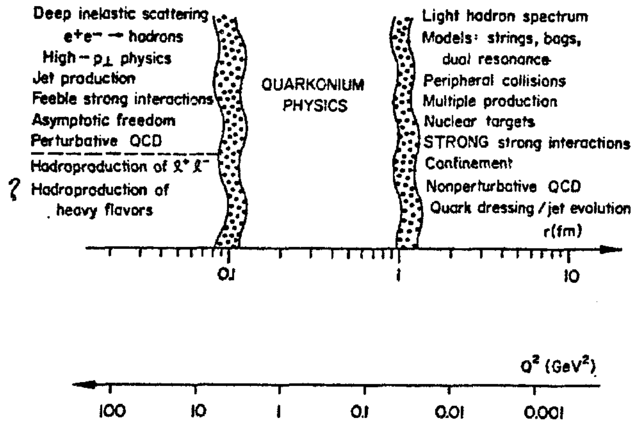


Fig. 1. A (one-dimensional) map of the strong interactions.

At long distances, exceeding about 1 fm, we encounter the traditional concerns of strong interaction physics: the spectroscopy and peripheral interactions of light hadrons. This is the realm of formidable strong interactions or, in the parlance of quarks and gluons, the scale on which confinement takes place and partons evolve into the hadrons we observe. If Quantum Chromodynamics[1] is the correct theory of strong interactions, then the development of new (nonperturbative) calculational techniques is essential to a full understanding of this regime. Progress reports are given by Mandelstam[2] and Polyakov[3] in these proceedings.

At very short distances, below about 0.1 fm, the so-called deep scattering phenomena[4] occur. Among these are deep-inelastic lepton-nucleon scattering and hadron production in electron-positron annihilations. According to the notion of asymptotic freedom,[5] the strong interactions become feeble at short distances, so that theoretical analysis can be based on the perturbative application of QCD.[6] Still other phenomena, including the hadronic production of massive lepton pairs or of heavy quark flavors, for which a short distance expansion is unjustified, may yield to perturbative analysis.

Between the rock of short-distance phenomena and the hard place of soft physics lies the domain of quarkonium states. At these intermediate distances–short enough that the strong interactions are weak but long enough that the forces cannot reliably be calculated–the theoretical treatment of choice is nonrelativistic quantum mechanics. The application of nonrelativistic techniques to quarkonium is the principal topic of this report.

The boundaries between the short, intermediate, and long-distance regions are indistinct and disputable. An obvious ultimate goal is to unify all three regimes, but this is not yet within our capabilities. For now, the interplay between zones is important, at least in shaping prejudices and developing intuition. This will be illustrated below.

2. Historical Remarks

Around the time of the November Revolution, Appelquist and Politzer[7] were led by the ideas of asymptotic freedom to suggest that quarks and antiquarks of sufficient mass would be bound in nonrelativistic motion in a Coulomb potential arising from one gluon exchange. What they envisaged is not quite what Nature has presented us, at least until now. As we shall see, the ψ and Υ families may fruitfully be described in terms of nonrelativistic level schemes, but their properties are not those of Coulomb bound states.

Following the discovery[8] of the $J/\psi(3095)$, many authors have explored the consequences of specific potential models. Among the early workers, it is appropriate to recognize the Cornell group[9] who developed the charmonium model in depth and successfully[10] predicted the ^3P_J psion levels, the E1 transitions, and the location of the 3^3D_1 level $\psi(3767)$.

Over the past five years, the picture of quarkonium levels as nonrelativistic bound states has provided much constructive guidance for experiment, and has made possible many useful inferences from experiment. Theoretical efforts may be divided among three categories. The earliest technique to be employed, which still remains important, is the adjustment of explicit potentials to reproduce known data and to make predictions for future experiments to confront. More recently, the full array of tools of nonrelativistic quantum mechanics has been brought to bear on the quarkonium problem. Both of these approaches will be discussed in this report. A third category, mentioned here only in passing, consists of attempts to derive the static interquark interaction from QCD.

3. Data on the ψ and Υ Families

Current experimental knowledge[11-14] of the charmonium system is summarized in Fig. 2a. Data from the Crystal Ball experiment[13] at SPEAR appear to rule out (decisively!) the previous candidates for pseudoscalar states X(2830),[15] χ(3454),[16] and X(3600).[17] The difficulties these candidates posed to the simple charmonium model have been reviewed extensively.[18] In their place we now have the strong suggestion of a new state U(2976), seen as an enhancement in the inclusive spectrum of photons from the decay

$$\psi' \to \gamma + \text{anything} \quad . \qquad (1)$$

For the other known states, there are only minor changes in masses and branching ratios to be noted. A template for the spectrum, in the form of a nonrelativistic level scheme, appears in Fig. 2(b).

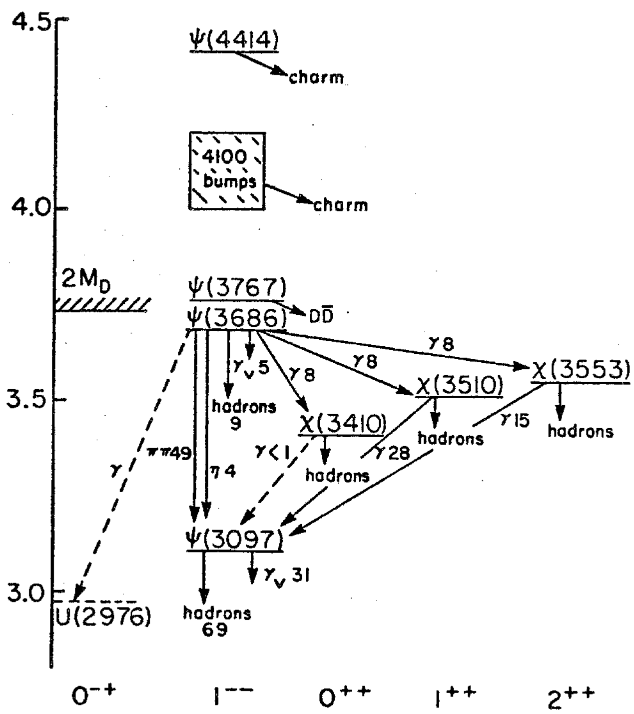
The established levels[19-22] of the upsilon family are indicated in Fig. 3. Only vector states have been observed until now, but a rich spectrum of levels is expected on the basis of potential models. Prospects for upsilon spectroscopy will be taken up briefly in §9.

4. Scaling Laws and Effective Potentials

One may find in the literature a large number of potentials with parameters adjusted to reproduce the data on ψ and Υ levels.[23] To indicate that a consistent description of the experimental information is possible (at least in broad terms), I will briefly describe an alternative program for determining properties of the potential. Consider an effective-power-law potential of the form

$$V(r) = \lambda r^{\nu} \quad , \qquad (2)$$

where the exponent ν is to be determined from the data. Such a form is appealing because many exact or approximate but sharp statements can be derived about bound-state properties.[24] Among these are scaling properties of observables as functions of the constituent (quark) mass or the principal



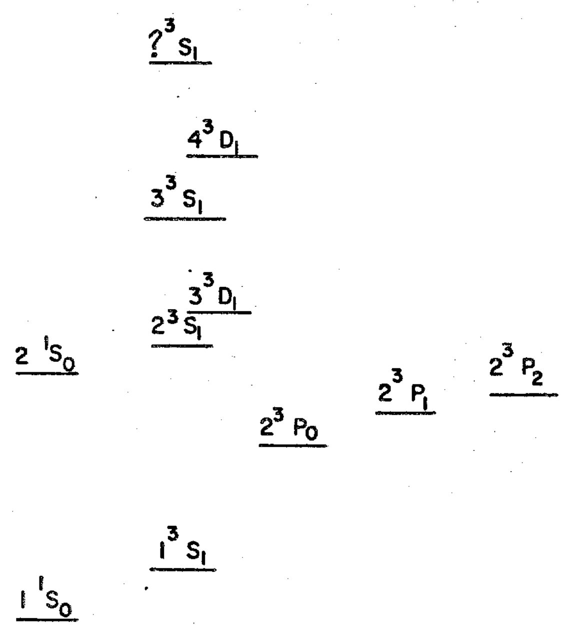


Fig. 2(a). The spectrum of charmonium ($c\bar{c}$). Branching fractions (in percent) are shown for the important classes of decays. Charm threshold is indicated at twice the D meson mass. (b) Spectroscopic notation for the levels of charmonium. The identification of $^1S_0$ levels is speculative.

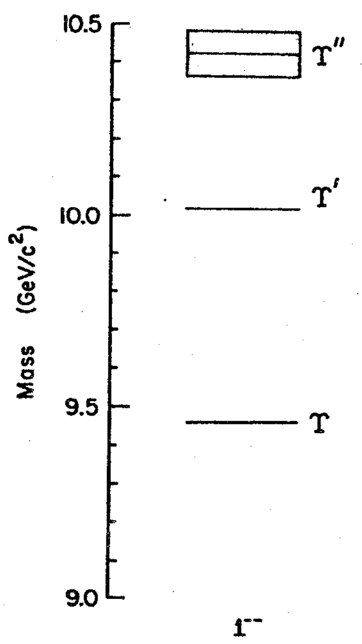


Fig. 3. The observed levels of the Υ family.

of the energy scale is arbitrary, any power-law potential (i.e., $-2 < \nu < \infty$) can accommodate the positions of the 1^3S_1 and 2^3S_1 levels for a single quarkonium family. The relative splitting of the 1^3S_1, 2^3S_1, and 3^3S_1 levels is, however, peculiar to a specific potential, as shown in Fig. 4(a). The data on $^3S_1$ levels in the ψ and Υ families, summarized in Fig. 4(b), are consistent with equal to a small positive power, for both families: $\nu(\psi) = 0.20 \pm 0.06$ and $\nu(\Upsilon) = 0.33 \pm 0.23$.

The 2S-2P splitting is also an identifying feature of a power-law potential, as shown in Fig. 5. The most familiar case is the 2S-2P degeneracy of Coulomb bound states. The data on the spin-triplet ψ states[25] again indicate an effective power $\nu(\psi) \approx 0.15$ which is close to zero, and slightly positive.

In addition to information on the level spacings, we have available experimental data on the square of the Schrödinger wavefunction at the origin, as inferred from the nonrelativistic connection[26]

$$|\Psi(0)|^2 = M_V^2 \Gamma(V \to e^+e^-)/16\pi\alpha^2 e_Q^2 \qquad (3)$$

quantum number. If the potential at very short distances indeed is Coulombic, the observables should indicate $\nu \approx -1$ in extremely massive quarkonium families.

Let us now examine a few characteristics of bound states in a power-law potential. Because the choice of a zero

between leptonic decay width and wavefunction of a vector state.[27] Here M_V is the vector meson mass and e_Q is the quark charge in units of the proton charge. The relationship between the potential and the ratio of $|\Psi(0)|^2$ for the 2^3S_1 and 1^3S_1 levels is depicted in Fig. 6. The data are again

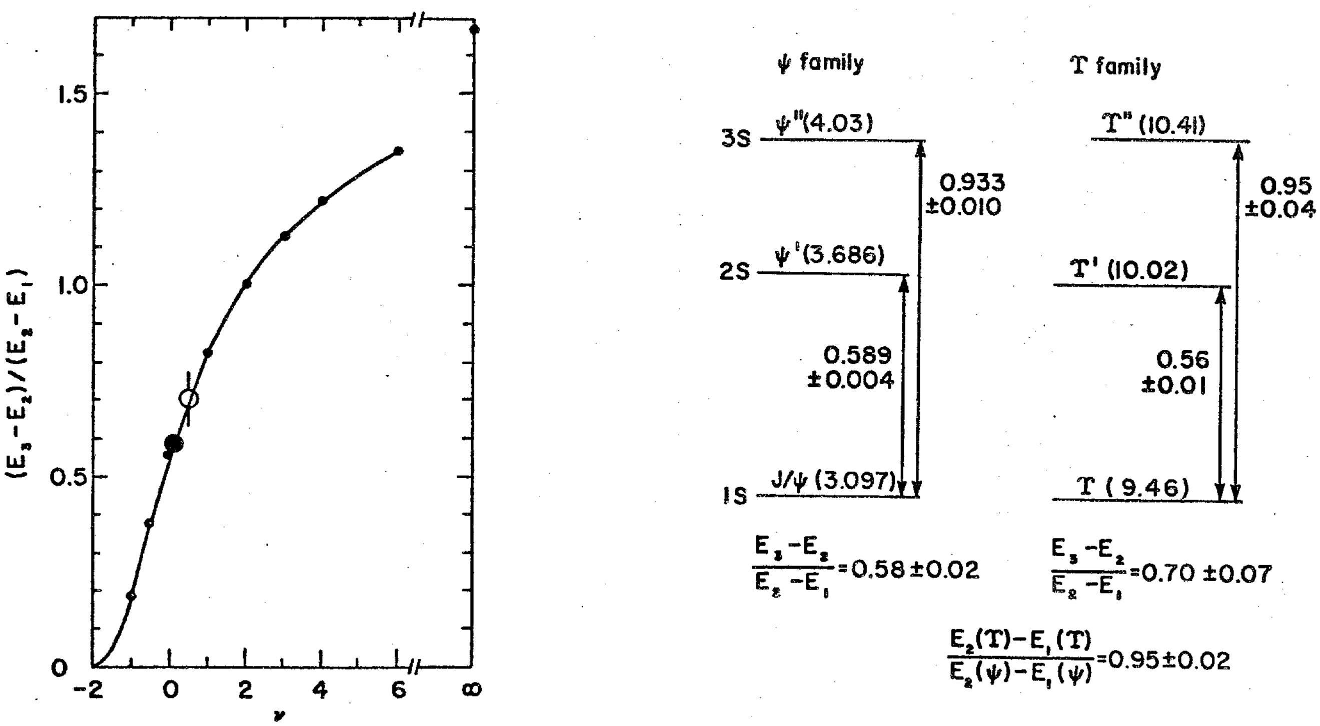


Fig. 4(a). Semiclassical (curve) and exact (small circles) ratios $(E_3 - E_2)/(E_2 - E_1)$ for s-wave levels in potentials $V(r) = \lambda r^\nu$ (from ref. 24). The data points refer to the ψ (full circle) and Υ (open circle) families. (b) Comparison of the $^3S_1$ levels of the ψ and Υ families.

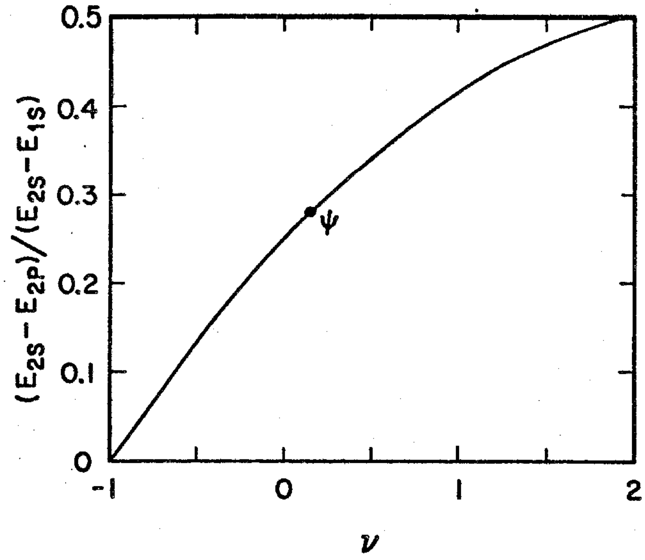


Fig. 5. The quantity $(E_{2S} - E_{2P})/(E_{2S} - E_{1S})$ for power-law potentials $V(r) = \lambda r^{\nu}$, $-1 \leq \nu \leq 2$. The datum is the value in the charmonium system (from ref. 24).

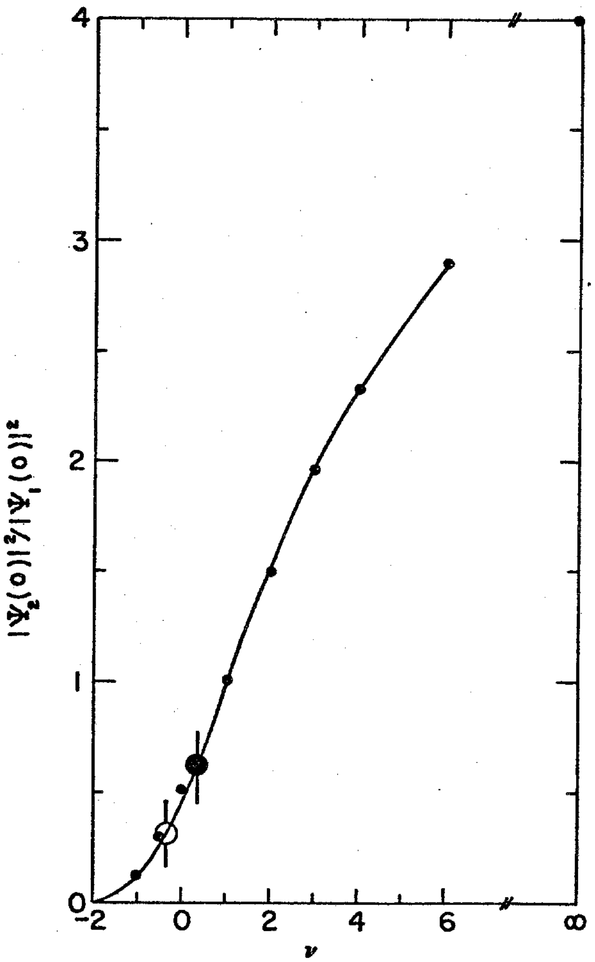


Fig. 6. Semiclassical (curve) and exact (small circles) ratios $|\Psi_2(0)|^2/|\Psi_1(0)|^2$ for s-wave levels in potentials $V(r) = \lambda r^{\nu}$ (from ref. 24). The data points refer to the ψ (full circle) and ϒ (open circle) families.

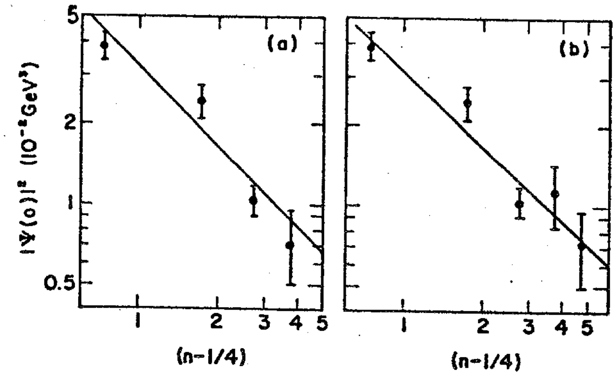


Fig. 7. Square of the wavefunction at the origin deduced from leptonic widths of the psions. Possible mixing between the $2^3S_1(3686)$ and $3^3D_1(3767)$ levels has been neglected. (a) a best fit proportional to $(n - \frac{1}{4})^p$, with $p = -1.00 \pm 0.15$, assuming the conventional 4S assignment for ψ(4414). (b) an alternative 5S assignment for ψ(4414), which corresponds to $p = -0.91 \pm 0.11$. In plotting the data against $n - \frac{1}{4}$, we have anticipated the result $p > -1$ $(\nu > 0)$ [from ref. 24].

consistent with $\nu = 0$, this time within rather large errors. At the price of adopting the WKB approximation and accepting the relevance of experimental information above the charm threshold, we may extend this analysis for the ψ family. It is straightforward to show[28,24] that for nonsingular power-law potentials,

$$|\Psi_n(0)|^2 \propto (n - \tfrac{1}{4})^{2(\nu-1)/(2+\nu)} \quad . \tag{4}$$

I show in Fig. 7 two alternative assignments of the charmonium levels, which lead to the effective powers (a) $\nu = 0 \pm 0.1$ and (b) $\nu = 0.06 \pm 0.08$.[29]

Finally, let us notice[28,24] that the dependence of level spacings on constituent mass,

$$\Delta E \propto m_Q^{-\nu/(2+\nu)} \quad , \tag{5}$$

leads to another measure of the potential. The near-equality of level spacings in the ψ and ϒ families evidenced in Fig. 4(b) leads yet again to the conclusion that $\nu \approx 0$.

We have therefore found that a great many observables are compatible with the choice of $\nu \approx 0$ for an effective power-law potential. Among many possible realizations, I shall refer only to the two simplest possibilities. These are the Coulomb-plus-linear potential

$$V(r) = -\alpha/r + ar \tag{6}$$

and the logarithmic potential

$$V(r) = C \log(r) \tag{7}$$

which are shown schematically in Fig. 8. The Coulomb-plus

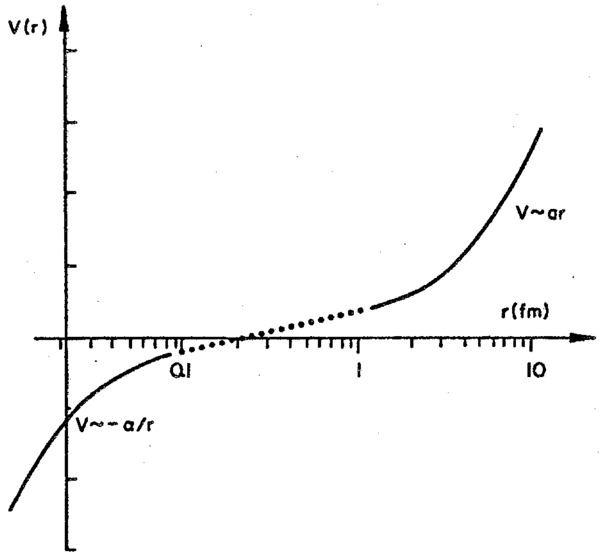


Fig. 8. Possible form of the interquark potential, showing the expected Coulomb and linear forms at short and large distances. With this choice of scale, it appears inevitable that the logarithmic potential is the appropriate interpolation in the quarkonium region.

linear form may be viewed as a caricature of theory (or of theoretical prejudice). It melds the Coulomb form implied by the idea of one gluon exchange at short distances with the linear behavior suggested by string models of the light hadron spectrum at long distances. In the same spirit, the logarithmic potential may be seen as a caricature of the current data, corresponding to the $\nu = 0$ limit of a power-law potential. Neither simple form should be expected to describe all the data in complete detail, but both should be capable of summarizing the main features both known and foreseen.

5. Theorems

An important consequence of the validity of the nonrelativistic description of quarkonium is that many powerful statements may be proved in potential theory.[30] Most of the useful theorems for quarkonium take the form of inequalities or bounds. Rather general conditions on the form of the potential lead to significant restrictions upon the properties of states within a quarkonium family. If the same potential applies to different $Q\bar{Q}$ families, important connections among observables follow.

To illustrate the application of such theorems, let us consider a single example which permits the determination of quark charges. Using the connection (3) between leptonic widths and wavefunctions at the origin, one may show[31] that for a potential satisfying $dV/dr \geq 0$, $d^2V/dr^2 \leq 0$ the leptonic widths of the psions and upsilons are related by

$$\Gamma(\Upsilon_n \to e^+e^-) \geq \frac{e_Q^2}{4/9}\,\frac{m_Q}{m_c}\,\frac{M(\psi_n)^2}{M(\Upsilon_n)^2}\,\Gamma(\psi_n \to e^+e^-) \ . \tag{8}$$

In eq. (8), e_Q is the charge of the quark which makes up the Υ family, m_Q is its mass, m_c is the charmed-quark mass, and $M(\Upsilon_n)$ is the mass of the n^3S_1 psion level. The observed masses of the ψ and Υ levels, the observed psion leptonic widths minus one standard deviation, and the plausible assumption that $m_Q/m_c \geq 2.6$ then imply the conservative lower bounds

$$\left.\begin{aligned} \Gamma(\Upsilon \to e^+e^-) &\geq 2.6\ \text{keV} \times e_Q^2 \\ \Gamma(\Upsilon' \to e^+e^-) &\geq 1.4\ \text{keV} \times e_Q^2 \end{aligned}\right\} \tag{9}$$

These bounds are shown in Fig. 9, for $|e_Q| = 1/3$ and $2/3$, together with the experimental data.[32] Also shown in Fig. 9 are the predictions based upon a variety of potentials[33] for the ψ family, assuming $|e_Q| = 1/3$. The data are inconsistent with the Υ' lower bound based upon $|e_Q| = 2/3$, and so imply that $|e_Q| = 1/3$. We therefore identify the constituent of Υ as the b-quark.

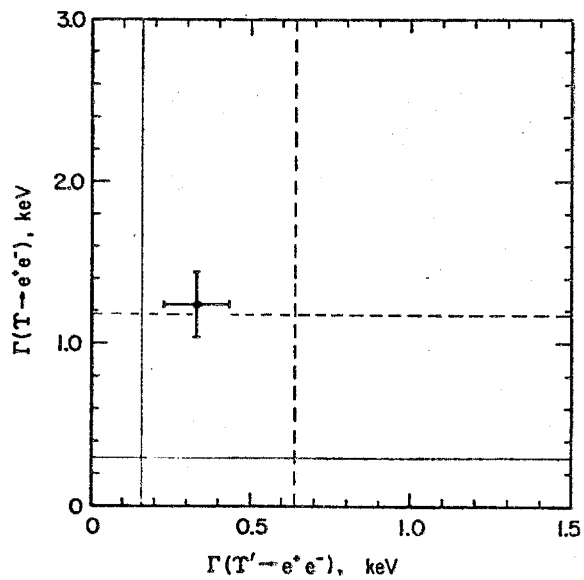


Fig. 9. Lower bounds for leptonic widths of Υ and Υ' (Ref. 31), together with data (Ref. 32). The shaded area represents the range of predictions of twenty potentials (ref. 33) reproducing the ψ and ψ' masses and leptonic widths, for $e_Q = -1/3$. Solid and dashed lines correspond to lower bounds for $e_Q = -1/3$ and 2/3, respectively.

Recent measurements at PETRA[34] of the ratio

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \qquad (10)$$

which receives a contribution

$$\Delta R = 3e_Q^2 \qquad (11)$$

from a new color-triplet quark, confirm the assignment $|e_Q| = 1/3$. This supports the application of nonrelativistic methods, and the idea that the potential is flavor-independent.

Inequalities of the form (8) may be used to bound the integrated cross sections for the production of new quarkonium states in the reaction

$$e^+e^- \to V^0 \to \text{hadrons} \quad . \qquad (12)$$

The integrated cross section for V^0 production and hadronic decay can be written as

$$\int_{V^0\text{peak}} \sigma(M)dM = \frac{6\pi^2\Gamma(V^0 \to e^+e^-)}{M(V)^2} \geq$$

$$\frac{e_Q^2}{4/9} \frac{m_Q}{m_c} \frac{M(\psi)^2}{M(V)^4} 6\pi^2\Gamma(V^0 \to e^+e^-)$$

$$\approx \frac{e_Q^2}{4/9} \left(\frac{M(\psi)}{M(V)}\right)^3 \int_{\psi\ \text{peak}} \sigma(M)dM \quad , \qquad (13)$$

assuming $\Gamma_{had}/\Gamma_{total} \approx 1$, where (neglecting binding energy) I have approximated $m_Q/m_c \approx M(V)/M(\psi)$. The lower bound derived from the ψ is shown in Fig. 10. The integrated cross section for Υ lies above this lower bound, as required. The Υ cross section thus implies a more restrictive lower bound on the integrated cross section for the ground state of a more massive quarkonium family, as shown in Fig. 10. Similarly, we may infer from the integrated cross section for Υ production an upper bound on the cross section to produce the ground state of a hypothetical family between the ψ and Υ. Many authors[35] have noticed (see also Question and Answer) that the leptonic widths of $\rho, \omega, \phi, \psi, \Upsilon$ are described by

$$\frac{\Gamma(V^0 \to e^+e^-)}{e_Q^2} = (11.9 \pm 0.8)\ \text{keV} \quad . \qquad (14)$$

The cross section which follows from this Ansatz falls as $1/m_Q^2$; it lies between the broken lines in Fig. 10. In a power-law potential (2), the dependence of leptonic width upon the quark mass is given by[28,24]

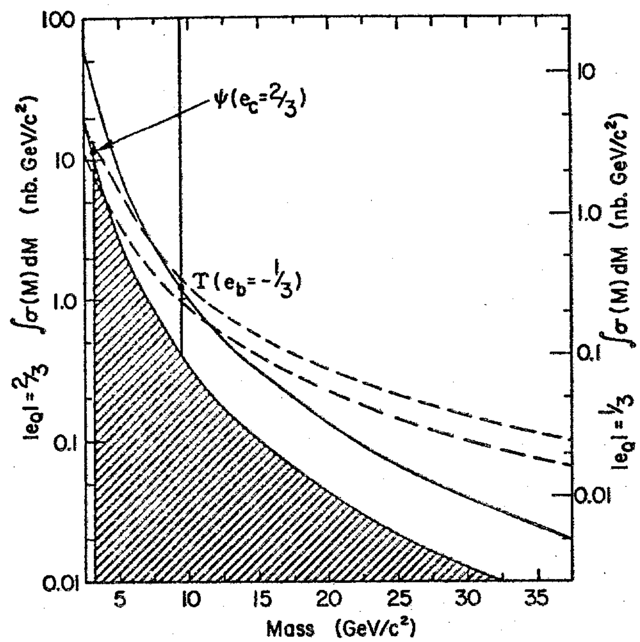


Fig. 10. Lower bounds for the integrated cross section for the production of new (heavier) quarkonium ground states in e^+e^- annihilations. Bounds are based on the observed cross sections for ψ ( ) and Υ ( ) production, and eq. (13). The expectation based upon the regularity $\Gamma(e^+e^-)/e_Q^2$ = constant, eq. (14), is shown by the dashed lines.

$$\Gamma(V^0 \to \ell^+\ell^-) \propto m_Q^{-(1+2\nu)/(2+\nu)} \quad , \nu \geq -1 \qquad (15)$$

(again neglecting binding energies), which implies that

$$\int_{V^0\text{peak}} \sigma(M)dM \propto m_Q^{-(5+4\nu)/(2+\nu)} \quad . \qquad (16)$$

Thus for $\nu = 0$, which roughly describes observables in the present regime, one expects

$$\int_{V^0\text{peak}} \sigma(M)dM \sim m_Q^{-5/2} \quad , \qquad (17)$$

and for $\nu = -1$, the expected behavior for very massive quarks,[36]

$$\int_{V^0\text{peak}} \sigma(M)dM \sim m_Q^{-1} \quad . \qquad (18)$$

6. Problems for Potential Models

While the nonrelativistic description of quarkonium spectroscopy is generally successful, as I have discussed in §4, potential models do suffer from some ambiguities and quantitative difficulties. These will be addressed briefly in this and the following section.

Two circumstances cloud tests of theoretical predictions: the interquark potential is not known from fundamental theory in the region of space probed by the data, and the quark mass is essentially a free parameter. These facts make for a certain degree of theoretical flexibility which is not altogether welcome if we are to strive for ever more incisive tests of the picture. Furthermore, the ψ/J family is only marginally nonrelativistic in all explicit potential models, so there is an important ambiguity about the significance of relativistic corrections.[37]

All explicit potentials for the charmonium family encounter a quantitative challenge. They lead to predicted E1 transition rates for the decays $\psi' \to \gamma + {}^3P_J$ which are larger than observed, or to predicted leptonic decay widths of ψ and ψ' which are larger than observed, or both. This failure is not easily fiddled away, as we may see by considering again the consequences of an effective power-law potential $V(r) = \lambda r^{\nu}$. Let us suppose the exponent ν is fixed (its value is irrelevant for this discussion) and consider the coupling strength λ and the quark mass m as adjustable parameters.

If we require that level spacings be unaffected by parameter changes, the usual scaling laws[24] demand that

$$|\lambda| \propto m^{\nu/2} \quad , \tag{19}$$

from which it follows that

$$\beta^2 \propto m^{-1} \quad , \tag{20}$$

where β is the speed of a bound quark. Since it would be comforting to make β^2 acceptably small,[38] it is instructive to express in terms of β^2 the effect of parameter changes on other observables. Elementary scaling arguments show that

$$\Gamma(\mathrm{E1}) \propto \beta^2 \quad , \tag{21}$$

whereas

$$\Gamma(\ell^+\ell^-) \propto (\beta^2)^{-3/2} \quad . \tag{22}$$

Thus it is not possible to diminish both the E1 rates and the leptonic widths by machinations of this kind. Although this discussion is in terms of power-law potentials, the same behavior is displayed by more general forms, including the Coulomb-plus-linear potential of Eichten, et al.[23] The price of reducing β^2 and Γ(E1) is a significant increase in $\Gamma(\ell^+\ell^-)$.

If it is not possible to describe all observables precisely using a simple form for the potential, a subjective element—the choice of how to weight various pieces of data—enters comparisons of theory and experiment. It is worth considering a few examples which illustrate the available options. Four cases are shown in Table 1.

Table 1. What observables are to be fitted?

Potential	Cornell I weak Coulomb Ref. 9	Cornell II strong Coulomb Ref. 23	log Ref. 23	log Ref. 23
m_c(GeV/c^2)	1.65	1.84	1.1	1.84
β_ψ^2	0.23	0.2	0.32	0.2
Level spacings				
ψ(2S - 1S)	✓	✓	✓	✓
Υ(2S - 1S)	0	✓	✓	✓
ψ(2S - 2P)	0	✓	✓	✓
$\Gamma(\overline{ee})$				
ψ, ψ'	✓	x3	✓	x2
Υ, Υ'		x3	✓	x2
Γ(E1)	x(2-3)	x2	x(3-4)	x2

The original Cornell potential[9] [$V(r) = -0.30/r + (0.23\ \mathrm{GeV}^2)r$, with $m_c = 1.65\ \mathrm{GeV}/c^2$] gave an adequate description of the ψ and ψ' spacing and leptonic widths, but failed badly on the $\Upsilon - \Upsilon'$ splitting and was significantly in error on the ψ(2S - 2P) splittings. It led to E1 transition rates which were too large by a factor of two to three.

The parameters of the Coulomb and linear terms have been readjusted by many authors to account for the observed $\Upsilon - \Upsilon'$ splitting.[23] To be specific, let us consider the 1979 Cornell potential [$V(r) = -0.52/r + (0.18\ \mathrm{GeV}^2)r$, with $m_c = 1.84\ \mathrm{GeV}/c^2$]. The large quark mass implies a relatively small value of β^2 for the ψ ground state and somewhat reduced values for the E1 rates. However the leptonic widths are too large by a factor of three. It is possible that this problem can be mitigated by radiative corrections to the Van Royen-Weisskopf formula (3). A transcription from QED to QCD yields[39]

$$\Gamma(V \to e^+e^-) = \frac{16\pi\alpha^2 e_Q^{\ 2}}{M_V^{\ 2}} \, |\Psi(0)|^2 \times \left(1 - \frac{16\alpha_s}{3\pi}\right) , \tag{23}$$

where α_s is the strong (QCD) coupling strength. If we interpret as $4\alpha_s/3$ the coefficient of $(-1/r)$ in the potential,[40] this first-order radiative correction reduces the predicted leptonic widths by a factor of three. This brings them into agreement with experiment, but if the lowest-order radiative corrections are so important, can higher-order effects safely be neglected? The entire realm of radiative corrections to quarkonium decays (into gluons as well as real or virtual photons) is an important problem area which deserves a systematic and definitive treatment. Capable persons may consider themselves exhorted to the task!

The logarithmic potential [$V(r) = (.73\ \mathrm{GeV})\ln r$] provides additional insight into the dilemma of observables. Its consequences for the spectrum do not depend upon the quark mass, and it performs satisfactorily in this regard. With a charmed quark mass $m_c = 1.1\ \mathrm{GeV}/c^2$, the logarithmic potential predicts leptonic widths in acceptable agreement with the data, but leads to E1 rates that are too large by a factor of three or four. The value of β_ψ^2 is also large by the standards of the Coulomb-plus-linear examples.[41] We may instead choose $m_c = 1.84\ \mathrm{GeV}/c^2$, to obtain a smaller value of β_ψ^2. This choice leads to smaller E1 rates (only twice the observed rates) and to leptonic decay rates which are twice the experimental values. Radiative corrections may perhaps be invoked to excuse the latter. It is worth noting that all the analyses which deduce small values of the quark mass[42] are sensitive to radiative corrections.

7. Fine Structure and Hyperfine Structure

For the past several years, fine structure and hyperfine structure splittings have seemed a stumbling block for the elementary nonrelativistic description. Many imaginative (but largely unconvincing) interpretations were given of the large hyperfine $^3S_1 - {^1S_0}$ intervals suggested by early experiments. These have received considerable attention at previous conferences[18] and will not be reviewed here, because the experimental evidence for large hyperfine intervals has evaporated. Instead, I will briefly discuss the problem of fine structure in the ^3P_J charmonium levels

$$\left.\begin{array}{ll} {^3P_2} & \chi(3552) \\ {^3P_1} & \chi(3508) \\ {^3P_0} & \chi(3415) \end{array}\right\} \qquad (24)$$

which are split from the "center of gravity" at 3522 MeV/c^2.

The fine-structure splitting results in general from the combined effects of spin-orbit and tensor forces. In the usual nonrelativistic reduction of the Dirac equation, these may be related to the static central potential in a manner dependent upon the Lorentz properties of the exchanged quantum, as shown in Table 2. It is by no means certain that this framework is appropriate for the strong interactions as described by QCD,[43] because color-electric and color-magnetic components of the interaction are thought to play very different roles in confinement. In any case, we do not know the Lorentz structure of the interaction in the interesting region of space.

Table 2. Static and quasi-static interactions

Type of interaction	Lorentz property		
	4-vector $\gamma_\mu \otimes \gamma_\mu$	4-scalar $1 \otimes 1$	4-pseudoscalar $\gamma_5 \otimes \gamma_5$
Static potential	$V(r)$	$S(r)$	$P(r)$
Spin-orbit	$\frac{3}{2m^2}\frac{1}{r}\frac{dV}{dr}\vec{L}\cdot\vec{S}$	$-\frac{1}{2m^2}\frac{1}{r}\frac{dS}{dr}\vec{L}\cdot\vec{S}$	0
Tensor force[a]	$\frac{S_{12}}{12m^2}\left[\frac{1}{r}\frac{dV}{dr} - \frac{d^2V}{dr^2}\right]$	0	$-\frac{S_{12}}{12m^2}\left[\frac{1}{r}\frac{dP}{dr} - \frac{d^2P}{dr^2}\right]$
Fermi hyperfine	$\frac{\vec{\sigma}_1\cdot\vec{\sigma}_2}{6m^2}\nabla^2 V$	0	$\frac{\vec{\sigma}_1\cdot\vec{\sigma}_2}{12m^2}\nabla^2 P$

a) $S_{12} \equiv 3\vec{\sigma}_1\cdot\hat{r}\,\vec{\sigma}_2\cdot\hat{r} - \vec{\sigma}_1\cdot\vec{\sigma}_2$

Let us therefore consider a few simple cases, and the patterns to be expected from them. For a pure spin-orbit interaction, one expects

$$R_{FS} \equiv \frac{M(^3P_2) - M(^3P_1)}{M(^3P_1) - M(^3P_0)} = 2 \quad , \tag{25}$$

while for a pure tensor force,

$$R_{FS} = -2/5 \quad . \tag{26}$$

A static potential $V(r) = \lambda r^{\nu}$ arising from the exchange of vector quanta would imply[44]

$$R_{FS}(\nu) = \frac{2(13+\nu)}{5(5-\nu)} \quad . \tag{27}$$

The data yield

$$R_{FS}\Big|_{exp} = 0.47 \quad , \tag{28}$$

a value inconsistent with a pure $\vec{L}\cdot\vec{S}$ or pure tensor origin. Interpreted in terms of the $\gamma_\mu \otimes \gamma_\mu$ effective power-law potential, the data would require a power $\nu = -3.25$, which is both nonsensical and different from the choice $\nu \approx 0$ that served so well for other observables in §4.

I conclude that fine structure is not simple! There are many speculations of what might be, but no firm predictions of what must be. Within the quarkonium picture, we may parametrize the masses of the charmonium P-states as

$$M(^3P_J) = (3522 + 34\langle\vec{L}\cdot\vec{S}\rangle + 10\langle S_{12}\rangle)\ \mathrm{MeV}/c^2 \quad . \tag{29}$$

At present, fine structure represents not a failure of the model, but the absence of a definite prediction. Much insight may be expected from a comparison of ^3P_J fine structure in the Υ and ψ families.

8. Where Are the Spin-Singlet States?

Although much is known about the spin-triplet charmonium spectrum (cf. Fig. 2), none of the spin-singlet states

$$\begin{array}{lll} 1^1S_0 & (\eta_c) & J^{PC} = 0^{-+} \\ 2^1S_0 & (\eta_c') & 0^{-+} \\ 2^1P_1 & & 1^{+-} \end{array}$$

has yet been established. There are several reasons to care about the spin-singlet spectrum. First, it is of interest to complete the charmonium spectroscopy for its own sake. We may note that no isoscalar 1^{+-} state is known in any hadronic system. Second, a comparison of the hadronic decay widths of η_c and ψ would illuminate the QCD description of strong decays and test the ortho/parapositronium analogy. Third, the hyperfine separation is sensitive to the Lorentz structure of the interquark interaction.

For the moment, one related test of the QCD picture may be carried out.[45] The hadronic decays of the ^3P_J levels proceed via

$$\left.\begin{array}{l} ^3P_{0,2} \to gg \\ ^3P_1 \to gq\bar{q} \end{array}\right\} \quad . \tag{30}$$

For vector gluons, the hadronic decay widths are in the ratio

$$\Gamma_h(2^{++}) : \Gamma_h(1^{++}) : \Gamma_h(0^{++}) :: 4 : \mathcal{O}(1) : 15 \quad . \tag{31}$$

The ratio 4/15 results from Clebsch-Gordanry; the $\mathcal{O}(1)$ is sensitive to calculational details. Under the assumption that the transitions $\chi_J \to \gamma\psi$ arise from a common matrix element so that decay rates scale with photon energy as k^3, the Crystal Ball data[13] imply that

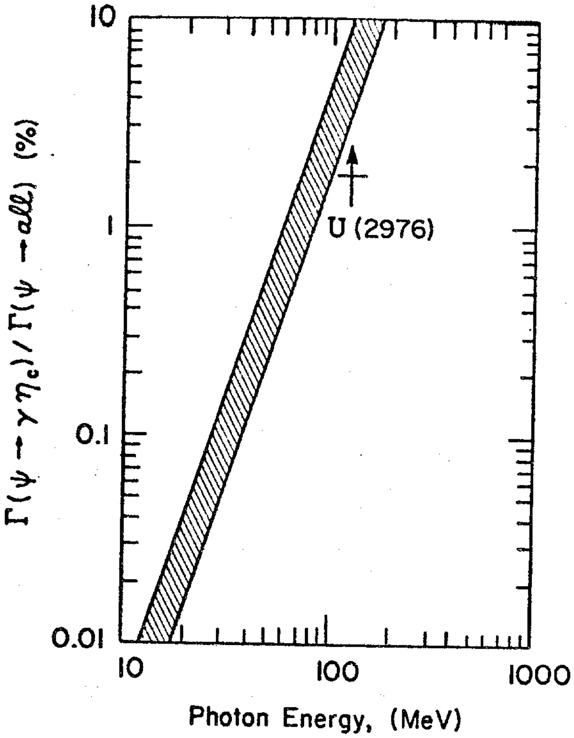
$$\Gamma_h(2^{++}) : \Gamma_h(1^{++}) : \Gamma_h(0^{++}) :: (2.9 \pm 2.0) : 1 : (26^{+\infty}_{-13}) \ , \tag{32}$$

where the quoted errors are impressionistic (to say the least). Thus the data give

$$\Gamma_h(2^{++})/\Gamma_h(0^{++}) = 0.11^{+0.05}_{-0.11} \quad , \tag{33}$$

to be compared with the QCD prediction of 0.27. The "agreement" between theory and experiment leaves something to be desired, but the effect of radiative corrections has not been explored.

Uncertainty about the form and Lorentz structure of the quarkonium potential leads to ambiguity in the predicted hyperfine splitting, but it is generally expected that $M(\psi) - M(\eta_c) \approx (30\text{-}150)\ \mathrm{MeV}/c^2$. Given the $\psi - \eta_c$ splitting, it is expected that $M(\psi') - M(\eta_c') = [\,|\Psi_{2S}(0)|^2/|\Psi_{1S}(0)|^2] \times [M(\psi) - M(\eta_c)] \approx \frac{3}{5}[M(\psi) - M(\eta_c)]$. In the absence of non-Coulombic contributions to hyperfine splitting, the $^1P_1$ level would be degenerate with ^3P_J center of gravity. How can the η_c be observed? The traditional search mode has been to look for sharp lines in inclusive γ-spectra, specifically for the "allowed" $(\psi \to \gamma\eta_c, \psi' \to \gamma\eta_c')$ and "hindered" $(\psi' \to \gamma\eta_c)$ M1 transitions. The allowed M1 rates are given[46] by



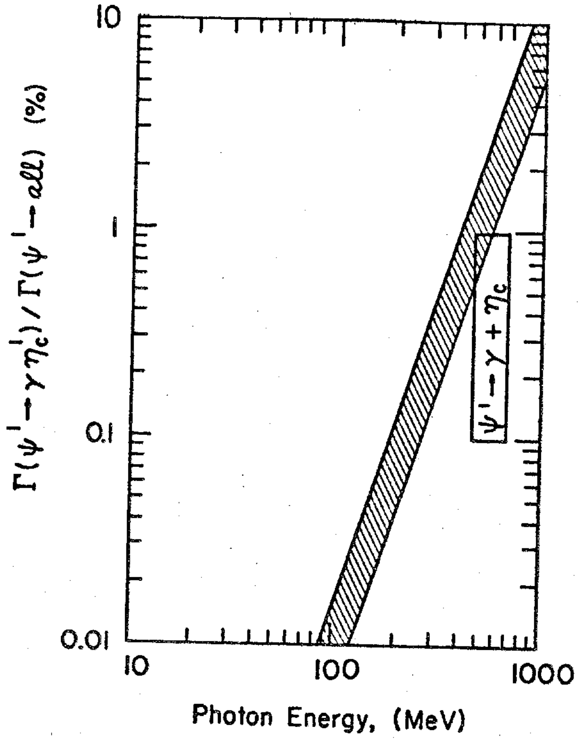


Fig. 11(a). Expected branching ratio for the allowed M1 transition $\psi \to \gamma + \eta_c$; the shaded band represents an estimate of the theoretical uncertainty. The Crystal Ball experiment has a sensitivity at about the 1% level. (b) Same for $\psi' \to \gamma + \eta_c'$. Also shown is the expected rate for the hindered M1 transition $\psi' \to \gamma + \eta_c$.

$$\frac{\Gamma(n^3S_1 \to \gamma + n^1S_0)}{k^3} = 4\alpha e_Q^{\,2}/3m_Q^{\,2} \quad . \tag{34}$$

The predicted rates (with an uncertainty imposed by the arbitrariness of the quark mass) are shown in Fig. 11. If the U(2976) is indeed the η_c, one would expect the transition $\psi \to \gamma + U$ to occur with a branching fraction of a few percent. This would seem to be comfortably within the capabilities of the Crystal Ball, unless the η_c is unexpectedly broad. For the hindered rate for $\psi' \to \gamma\eta_c$, only an order-of-magnitude estimate is possible. Typical values for the branching ratio $\Gamma(\psi' \to \gamma\eta_c)/\Gamma(\psi' \to \text{all})$ lie between 10^{-3} and 10^{-2}, if η_c is identified as U(2976).

The η_c should also be observed as a peak in multimeson mass distributions, either in the debris of ψ and ψ' or by direct η_c production in hadron collisions. A typical estimate for the production cross section[47] in pp collisions is 1 μb at 400 GeV/c. A large number of positive-G-parity final states can be expected to result from η_c decay. One exercise in fortunetelling[48] is summarized in Fig. 12. Unless η_c events can be tagged with high efficiency, many of these modes may be contaminated by the second-order electromagnetic decays $\psi \to \gamma \to$ hadrons. Whether this poses a serious problem will depend upon the $\psi - \eta_c$ separation and experimental resolution. Decays which are forbidden by charge conjugation for the ψ are free from such confusion. These include $\eta_c \to \eta\pi^0\pi^0$, $K_SK_S\pi^0$, $\phi\phi$, etc.

Whether or not η_c is first established elsewhere, it may be profitable to study its production by the two-photon mechanism

$$e^+e^- \to e^+e^-\eta_c \quad , \tag{35}$$

in which the partial decay width $\Gamma(\eta_c \to \gamma\gamma)$ can be measured. This belief is encouraged by the recent observations of the sequence

$$e^+e^- \quad e^+e^-\eta' \atop \qquad\qquad \hookrightarrow \rho^0\gamma \tag{36}$$

carried out using the Mark II detector at SPEAR.[49,12] As shown in Fig. 13, standard estimates[50] of $\Gamma(\gamma\gamma)$ lead[51] to ample production cross sections at energies accessible to PETRA, PEP and CESR. One may imagine double tagging two photon events with excellent resolution in missing mass, or by single tagging and reconstructing characteristic decay modes. The "backgrounds" from other charmonium states, also indicated in Fig. 13, may provide an additional largesse.

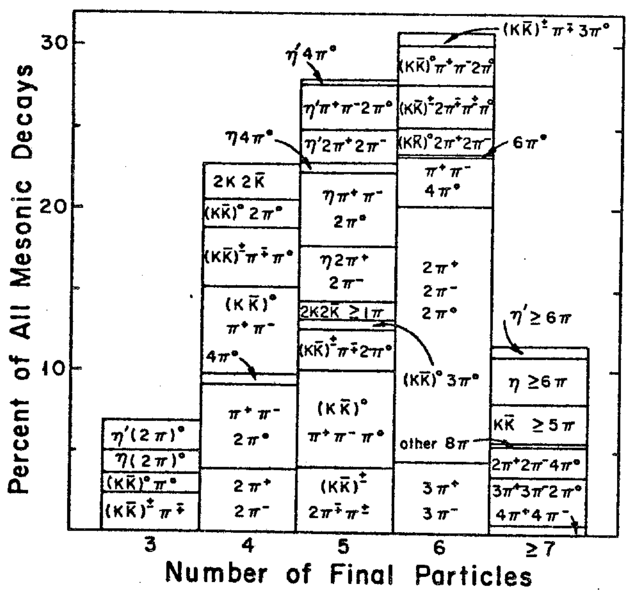


Fig. 12. Predictions of the constant-matrix-element (phase space) model of ref. 48 for mesonic decays of a hypothetical $\eta_c(3095)$.

9. Prospects for Υ Spectroscopy

Expected properties of the upsilon family have been extensively surveyed,[52] but a few specific comments are in order here. The anticipated spectrum[53] is summarized in

Table 3. Prospects for Υ Spectroscopy

	Expected spectrum	Experiment	
1S	9.46	9.46 ± 0.01	Refs. 19, 20
2P	9.92		
2S	10.02	10.02 ± 0.02	Refs. 19, 21
3D	10.20		
3P	10.27		
4F	10.36		
3S	10.39	10.41 ± 0.05	Ref. 22
4D	10.45		
⋮			
4S	10.61		Flavor Threshold
⋮			
5S	10.85		

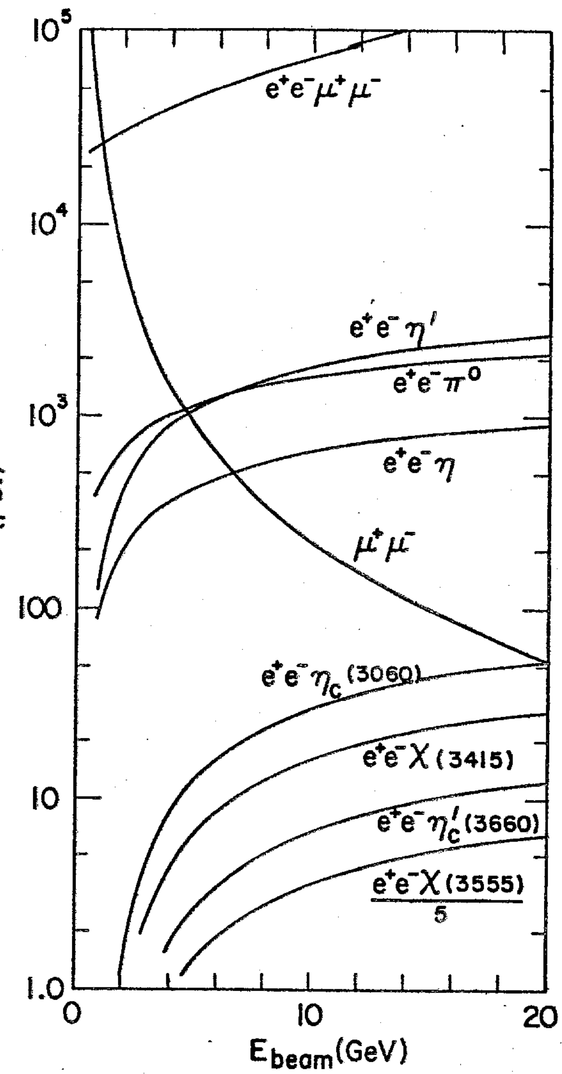


Fig. 13. Predicted cross sections for various two-photon reactions leading to the final states indicated. The cross section for $e^+e^- \to \mu^+\mu^-$ ("one unit of R") is indicated for reference. The cross section for production of $\chi(3555)$ has been divided by 5 to avoid crowding the curve for $\chi(3415)$.

Table 3. The positions of Υ and Υ' are by now well known, and first measurements of their leptonic widths have been made at DORIS.[20,21] The Υ''(10.4)[22] has not yet been observed in e^+e^- annihilations. The ease with which this is accomplished will depend upon the leptonic width of Υ'', for which some educated guesses are possible. A number of such guesses are shown in Fig. 14. Assuming that the enhancement identified as Υ'' in the reaction $pN \to \mu^+\mu^- +$ anything is a single resonance, one may infer from its prominence a lower bound, $\Gamma(\Upsilon'' \to \ell^+\ell^-) \geq 0.14$ keV. In the semiclassical approximation the quantity $\ln|\Psi_3(0)/\Psi_2(0)|/\ln|\Psi_2(0)/\Psi_1(0)|$ varies within narrow bounds for simple potentials.[28,24] Knowing the leptonic widths of Υ and Υ', one may consequently estimate[54] that $\Gamma(\Upsilon'' \to \ell^+\ell^-) = (0.16 \pm 0.10)$ keV. The other predictions shown in Fig. 14 are based upon an inverse scattering

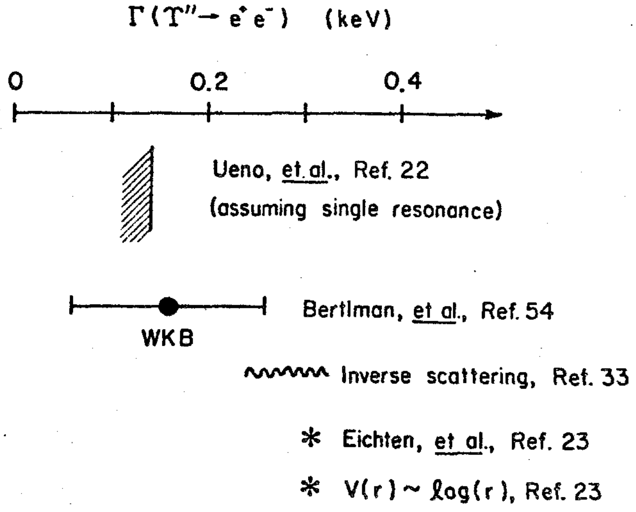


Fig. 14. Various expectations for the leptonic width of $\Upsilon''(10.41)$.

approach explained in §12, and upon two specific potentials:[23,24] the 1979 Cornell model and the logarithmic potential. It should be noted that the Υ' leptonic width predicted by these two potentials is one standard deviation larger than the experimental value.

It is expected that E1 decay rates will be an order of magnitude smaller than the corresponding rates in charmonium. (A factor of ¼ arises from the ratio of quark charges; the rest of the suppression is due to a reduction in the scale of the matrix element and small changes in Q-values.) However, the total widths of excited Υ levels may also be considerably smaller than their charmonium counterparts. Thus we anticipate roughly comparable E1 branching ratios for the ψ and Υ systems. In this regard, the study of hadronic cascades such as $\Upsilon' \to \Upsilon\pi\pi$ is of considerable interest.

If, as seems plausible,[55] the interval between flavor threshold and twice the heavy-quark mass is independent of the quark mass, a WKB argument[56] shows that the number of $^3S_1$ levels below flavor threshold is

$$n \approx 2\sqrt{m_Q/m_c} \quad . \tag{37}$$

The derivation does not depend on details of the potential, so long as the potential is flavor-independent. For the upsilon family this result leads us to expect 3 or 4 narrow $^3S_1$ levels, which is in accord with the predictions of specific models. A corollary of this reasoning is that flavor threshold corresponds to a fixed impact parameter, independent of quark mass. Heavier families may therefore allow us to see deeper within the potential well, but narrow levels will never be sensitive to larger distances than in the charmonium system.

In the upsilon family, the flavor threshold will occur near 10.55 GeV. The 4S level will therefore lie just below or just above the threshold. According to its position it will either be narrow or will be a factory for b-flavored particles, analogous to $\psi(3767)$.

At the last lepton/photon symposium, Gottfried[18] quite properly pleaded for names of new-flavored particles which would be acceptable in polite company. It is obvious that hadrons with manifest beauty should be called <u>godivas</u>, and those with manifest truth should be called <u>verities</u>.[57]

10. The Next Quarkonium Family

Using our knowledge of the ψ and Υ families, we may anticipate the properties of the next quarkonium system, and identify important questions to be addressed by the new spectroscopy. To be specific, I will discuss the characteristics of a hypothetical ζ family with a mass of about 30 GeV/c^2.

If the interquark potential is flavor-independent, we expect that 6 or 7 narrow $^3S_1$ levels will lie below the new-flavor threshold. Under the same assumption, the leptonic width of the ground state is subject to a rigorous bound,

$$\Gamma(\zeta \to e^+e^-) \geq 1.6 \text{ keV} \quad , \tag{38}$$

if the quark charge is $e_Q = 2/3$. In specific potential models, it is expected that

$$\Gamma(\zeta \to e^+e^-) \approx 5 \text{ keV} \quad , \tag{39}$$

As a function of the principal quantum number, the leptonic widths of the zetas will decrease more rapidly than $1/n$:

$$\Gamma(\zeta_n \to e^+e^-) < \frac{\text{constant}}{n} \quad . \tag{40}$$

Because heavier quarks probe the potential at increasingly short distances, it is of interest to ask whether observables become more Coulomb-like. Two Coulomb characteristics to be looked for are the 2S-2P degeneracy and the Landé interval rule for ^3P_J fine structure splitting, $R_{FS} = 0.8$ [cf. eq. (27)].

Since the yield of hadrons produced in e^+e^- annihilations indicates no threshold for the production of a new flavor with $e_Q = 2/3$ below about $E_{cm} = 31.6$ GeV, it is of interest to know how large should be the interval between the ground state and flavor threshold. In the charmonium system, $2M_D - M_\psi = 630$ MeV/c^2; for the upsilon family we expect godiva threshold to lie about 1100 MeV/c^2 above $\Upsilon(9.46)$. If the interquark potential continues to be characterized by $\nu \approx 0$, the interval between $\zeta(30)$ and flavor threshold would be about 1550 MeV/c^2. A "worst case" is perhaps represented by an effective potential with $\nu = -1$, for which an interval slightly in excess of 2 GeV/c^2 is thinkable.

11. Extra Levels

Because hadron physics is ultimately much richer than the $Q\bar{Q}$ sector alone, it is important to be alert for extra states which cannot be described in terms of a nonrelativistic potential picture. I will do little more than enumerate some possibilities. Multiquark ($Q\bar{q}\bar{Q}q$) states[58] or states with constituent gluons ($Q\bar{Q}g$)[59] may well occur in the midst of the narrow quarkonium levels. The identification of quarkless states would be an event of considerable significance,[60] but it is difficult to specify an unambiguous experimental signature.[61]

It is conceivable that the discovery of neutral Higgs bosons may be intimately connected with quarkonium spectroscopy, either by the decay[62]

$$^3S_1(Q\bar{Q}) \rightarrow H + \gamma \tag{41}$$

or the complementary process[63]

$$H \rightarrow {}^3S_1(Q\bar{Q}) + \gamma \tag{42}$$

The possibility of mixing between a Higgs scalar and a $^3P_0(Q\bar{Q})$ level has also been raised.[64]

Giles, Ng and Tye[65] have emphasized the possibility of vibrational modes in the quarkonium spectrum. These correspond to collective excitations of the color gluon flux, which are implied by relativistic invariance within the string picture of confinement. The vector meson vibrational excitations typically will have leptonic widths which are only about 10% of the leptonic widths of the normal radial excitations. Giles and Tye have suggested that extra vector states of this type should occur in the charmonium spectrum within 50 MeV/c^2 of 4.00 and 4.41 GeV/c^2. They anticipate the first extra Υ level near 10.45 GeV/c^2. Extensive experimentation will be required to distinguish interlopers of this kind from $^3D_1$ ($Q\bar{Q}$) levels, among others.

12. Inverse Bound-State Problem in Quantum Mechanics

The direct problem of quantum mechanics entails the computation of bound state properties and scattering amplitudes from a given potential. Less familiar is the inverse scattering (or inverse-bound-state) problem in which the potential is deduced from the scattering data.[66] In one space dimension, the binding energies and phase shifts uniquely define a symmetric potential $V(x) = V(-x)$, for which $V(\infty)$ approaches a constant (finite) value.[67] For the case of a trivial phase shift ("reflectionless potential"), $V(x)$ is given as an algebraic function of the binding energies.[68] Recently, applications have been made to confining potentials:[69] a potential $V_N(x)$ which reproduces the first N levels of the true potential $V(x)$ is constructed, and may be shown[69,70] to approach the true potential as $N \rightarrow \infty$.

Rather than discuss the details of this technique, I show two representative examples of the results in Fig. 15. There the 8-level reconstructions of the harmonic oscillator (a) and linear (b) potential are compared with the true potentials. I regard the agreement as extremely impressive.

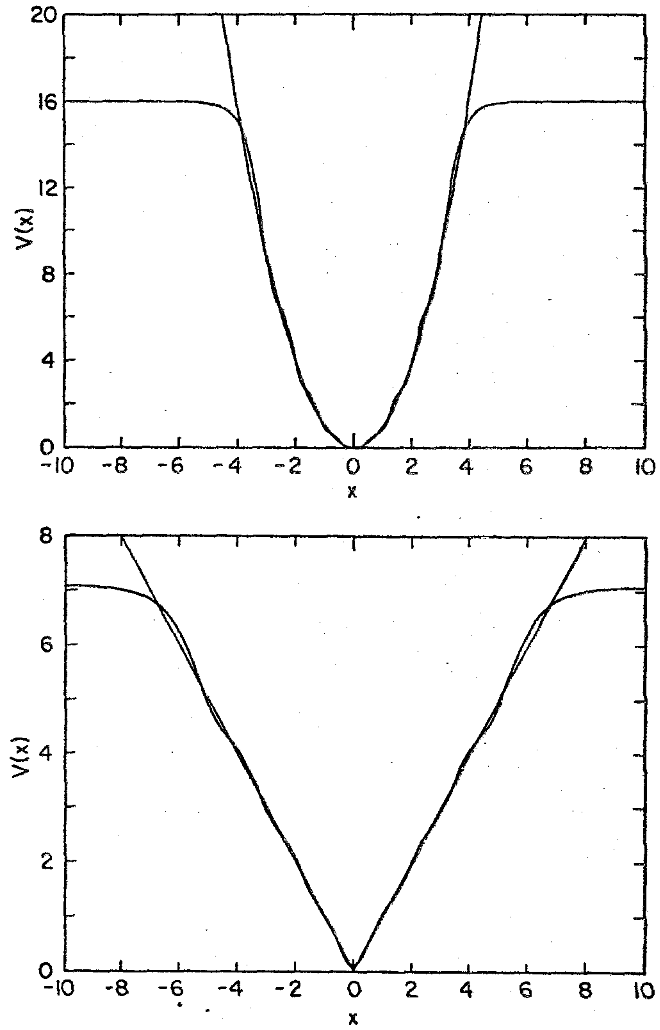


Fig. 15(a). Eight level reconstruction of the one-dimensional harmonic oscillator potential, $V(x) = x^2$. (b) eight-level reconstruction of the linear potential, $V(x) = |x|$ in one dimension. From Schonfeld, et al., ref. 70.

The s-wave inverse problem in three dimensions can be attacked in analogous fashion. In this case, the (central) potential is implied by the bound-state energies and the squares of s-wave wavefunctions at the origin. For applications to the quarkonium problem, $|\psi(0)|^2$ has been inferred from the Van Royen-Weisskopf formula (3). In this manner, information on ψ and ψ' may be used to reconstruct a charmonium potential which should prove useful in the region of space affecting the data.[71] Such a potential is shown in Fig. 16. By solving the Schrödinger equation with a larger quark mass, one may derive expectations for the Υ spectrum implied by this potential. The predictions thus obtained[33] are in good agreement with experiment.[72]

Like the existence of analytic expressions for potentials that reproduce the principal features of the ψ and Υ families, this provides evidence that the interquark potential is flavor independent. The issue of flavor-independence can be addressed somewhat more objectively, by using information on Υ and Υ' to reconstruct a potential without reference to the psions. A representative potential is compared in Fig. 17 with the charmonium potential of Fig. 16. In the region of space where both have been given experimental information, i.e. for $0.5\ \mathrm{GeV}^{-1} \lesssim x \lesssim 4\ \mathrm{GeV}^{-1}$, the two potentials are in excellent agreement. The successful comparison yields constructive evidence for flavor-independence of the interquark force assuming the constituent of Υ is a color-triplet quark with charge $|e_b| = 1/3$. The charge assignment $|e_Q| = 2/3$ does not lead to a flavor-independent potential. The interpretation[73] of the fifth quark as a color sextet with charge $-1/3$ requires that near the origin $V_{sextet}(x) = 5/2\, V_{triplet}(x)$, whereas the inverse technique yields a putative sextet potential which is weaker than the charmonium potential near the origin. Consequently, the most plausible alternatives to the conventional b-quark assignment yield unacceptable results.[72]

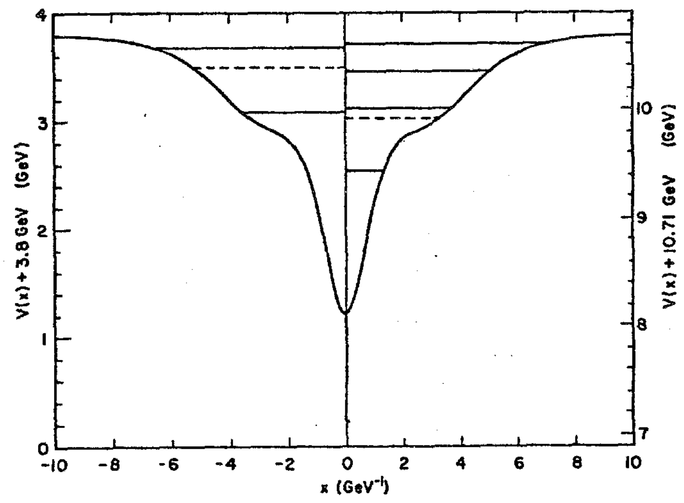


Fig. 16. Interquark potential constructed from the masses and leptonic widths of $\psi(3.095)$ and $\psi'(3.684)$ [from ref. 72]. The levels of charmonium are indicated on the lefthand side of the graph, while those of the Υ family are on the right-hand side. The solid lines denote $^3S_1$ levels; dashed lines indicate the 2^3P_J levels.

13. Conclusions

Quarkonium families and nonrelativistic quantum mechanics provide an important means for learning about the strong interactions and the color properties of quarks. The upsilon experience shows that quantum mechanical techniques allow us reliably to infer quark charges from bound-state properties. The topic most in need of systematic theoretical attention is that of radiative and relativistic corrections to the elementary bound-state picture. Flavor-independence of the interquark potential is indicated by fits to the ψ and Υ families and by an inverse scattering exercise. Heavier quarkonium families may be exploited even more fully than the ψ and Υ families: more narrow levels will exist, the nonrelativistic approximation is more reliable, and heavier quarks probe the potential at shorter distances. Upsilon spectroscopy of

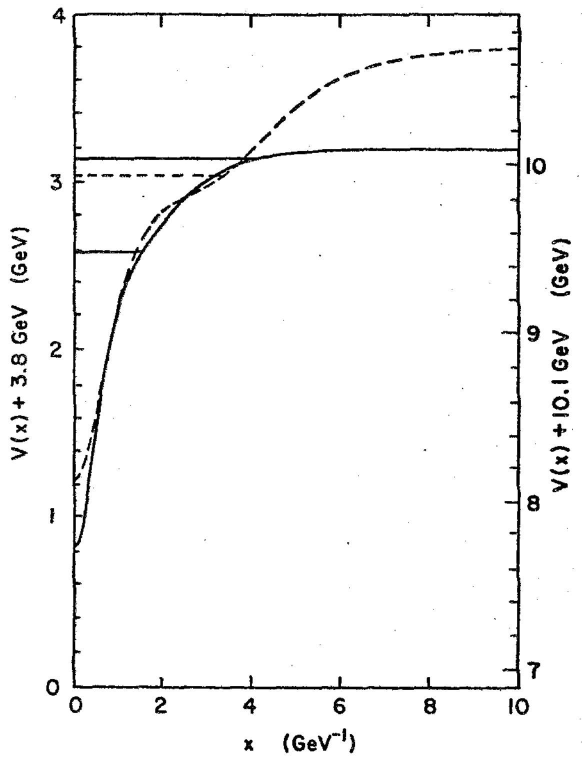


Fig. 17. Interquark potential (solid line) constructed from Υ and Υ' (from ref. 72). The charmonium potential of Fig. 16 is indicated by the dashed line for comparison. The 1S and 2S Υ levels are horizontal solid lines; the 2P $\langle\chi_b\rangle$ level is the horizontal dashed line. Relative scales of charmonium and upsilon levels are shifted as in Fig. 16.

considerable richness and significance awaits the new detectors at CESR.

Acknowledgments

Jonathan Rosner has collaborated in all of my work on the subject of this report; it is a particular pleasure to recognize his contribution to my understanding of quarkonium quantum mechanics. Special thanks are also owed to H.B. Thacker and J. Schonfeld, who have taught me much about the inverse scattering problem. Many others have been generous with their insights, including John Bell, Kurt Gottfried, J.D. Jackson, André Martin, and L.B. Okun. In preparing this talk I have also benefitted from the advice of Harry Lipkin, Vera Lüth, J. Namysłowski, Jack Smith, and Henry Tye. With great delight and appreciation, I commend my Fermilab colleagues who organized and executed this splendid symposium. Finally, it is fitting to pay tribute to the father of quarkonium quantum mechanics [Fig. 18].

Fig. 18. Erwin Schrödinger (courtesy of the AIP Center for the History of Physics Meggars Gallery of Nobel Laureates).

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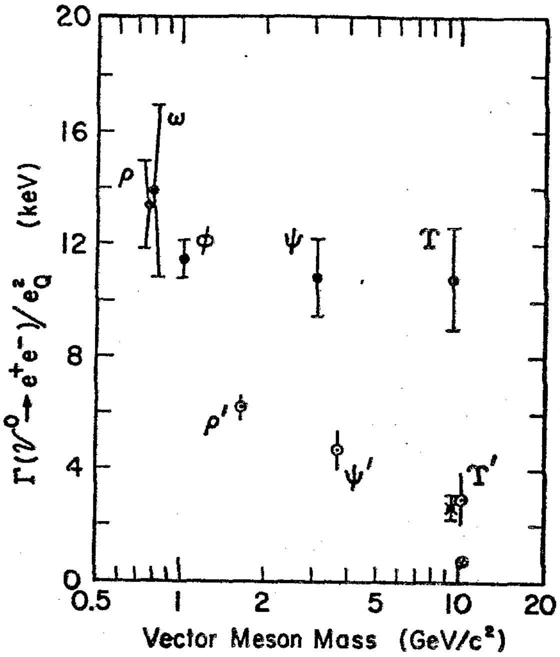

Fig. 19: Leptonic widths $\Gamma(\mathcal{V}^0 \to e^+e^-)$ normalized by squares of quark charges e_Q^2, as functions of vector meson mass. The solid points correspond to the ground states. Open circles correspond to 2S levels. For the Υ and Υ' a quark charge $|e_Q| = 1/3$ has been assumed. The crossed points refer to the alternative assignment $|e_Q| = 2/3$.

Discussion

Q. (Margolis, McGill) There's a well-known systematic for leptonic widths, namely that they just depend on the quark charge squared. This suggests a generality beyond potential models and also raises the question of whether the radiative corrections that you mentioned are really that important. Do you have any comment?

A. The regularity that Margolis points out is the leptonic widths divided by the quark charge squared are apparently universal for ρ, ω, φ, ψ, and Υ, as shown in Fig. 19. It is amusing that they appear much less universal for ρ', ψ', Υ'. Potential models have nothing to say about the light mesons, but (as we have seen in §4-6) can easily accommodate the observed behavior of ψ and Υ, either with or without large radiative corrections. In Coulomb-plus-linear potentials the mass-independence of leptonic widths is decidedly a transitory phenomenon; for very heavy quarkonia, leptonic widths become proportional to the quark mass. [See, e.g. Krasemann and Ono, ref. 23.]