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Topics in Perturbative QCD Beyond the Leading Order*

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ABSTRACT

The basic structure of QCD formulae for various inclusive and semi-inclusive processes is presented. Next to leading order QCD corrections to inclusive deep-inelastic scattering are discussed in some detail. The methods for calculations of QCD corrections (leading, next to leading) to semi-inclusive processes are outlined. Some results of these calculations are discussed.

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Contents

I. GENERAL OVERVIEW

Quantum Chromodynamics (QCD) is the most promising candidate for a theory of strong interactions. In these lectures we shall discuss higher order QCD predictions* for

a) Inclusive deep-inelastic scattering

$$\begin{aligned} eh &\rightarrow e + \text{anything} \quad , \\ \nu h &\rightarrow \mu^- + \text{anything} \quad , \end{aligned} \tag{1.1}$$

etc.

We shall also present the basic structure of QCD formulae for semi-inclusive processes such as:

b) $$e^+e^- \rightarrow h + \text{anything} \tag{1.2}$$

c) $$eh_1 \rightarrow e + h_2 + \text{anything} \tag{1.3}$$

d) $$h_1h_2 \rightarrow \mu^+\mu^- + \text{anything} \tag{1.4}$$

and

e) $$e^+e^- \rightarrow h_1 + h_2 + \text{anything} \tag{1.5}$$

where h_i stand for hadrons.

Finally we shall make a list of some recent higher order QCD calculations.

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*For recent reviews see refs. 1-3, 57.

In the simple parton model,[4] in which strong interactions and mass scales are neglected, the cross-sections for processes (1.1)-(1.5) are expressed in terms of parton distributions and parton fragmentation functions as follows*:

a)
$$\sigma_h^{DIS}(x) = \sum_i^f e_i^2 \left[xq_i^h(x) + x\bar{q}_i^h(x) \right] \tag{1.6}$$

for deep-inelastic scattering (1.1),

b)
$$\sigma_h^{e^+e^-}(z) = \sum_i^f e_i^2 \left[zD_{q_i}^h(z) + zD_{\bar{q}_i}^h(z) \right] \tag{1.7}$$

for semi-inclusive e^+e^- annihilation (1.2),

c)
$$\sigma_{h_1h_2}^{DIS}(x, z) = \sum_{i=1}^f e_i^2 \left[xq_i^{h_1}(x) zD_{q_i}^{h_2}(z) + x\bar{q}_i^{h_1}(x) zD_{\bar{q}_i}^{h_2}(z) \right] \tag{1.8}$$

for semi-inclusive deep-inelastic scattering (1.3),

d)
$$\frac{d\sigma}{dQ^2} = \int \frac{dx_1}{x_2} \int \frac{dx_2}{x_2} \sigma_{h_1h_2}^{\mu^+\mu^-}(x_1, x_2) \tag{1.9}$$

with ($\tau = Q^2/s$)

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*In order to simplify the presentation we suppress obvious factors such as $(4\pi\alpha^2)/(3Q^2)$ in Eqs. (1.7), (1.9) and (1.11) and color factors: "3" in Eqs. (1.7) and (1.11) and "1/3" in (1.9). Furthermore to unify notation we denote the well-known structure functions $F_2(x)$ and $F_2(x, z)$ by $\sigma_h^{DIS}(x)$ and $\sigma_{h_1h_2}^{DIS}(x, z)$ respectively. Finally, unless otherwise specified, we restrict our discussion to the transverse parts of the cross-sections for processes (1.2) and (1.5).

$$\sigma^{\mu_+\mu_-}_{h_1h_2}(x_1, x_2) = \sum_{i=1}^{f} e_i^2 \left\{ \left[x_1 q_i^{h_1}(x_1) \right] \left[x_2 \bar{q}_i^{h_2}(x_2) \right] \delta\left(1 - \frac{\tau}{x_1 x_2}\right) + \text{"}1 \leftrightarrow 2\text{"} \right\} \quad (1.10)$$

for massive μ-pair production (1.4),[5] and

$$\sigma^{e^+e^-}_{h_1h_2}(z_1, z_2) = \sum_{i=1}^{f} e_i^2 \left\{ \left[z_1 D^{h_1}_{q_i}(z_1) \right] \left[z_2 D^{h_2}_{\bar{q}_i}(z_2) \right] + \text{"}1 \leftrightarrow 2\text{"} \right\} \quad (1.11)$$

for two-hadron semi-inclusive e^+e^- annihilation (1.5.)

In the above equations $q_i^h(x)$ and $\bar{q}_i^h(x)$ are the parton distributions (quark, antiquark) which measure the probability for finding a parton of type i in a hadron h with the momentum fraction x. Similarly $D^h_{q_i}(z)$ and $D^h_{\bar{q}_i}(z)$ are the fragmentation functions which measure the probability for a quark q_i or antiquark $\bar{q}_i$ to decay into a hadron h carrying the fraction z of the quark or antiquark momentum respectively. Finally e_i stand for quark charges and f denotes number of flavors.

The following properties of Eqs. (1.6)-(1.11) deserve attention:

i) Bjorken scaling: parton distributions and parton fragmentation functions depend only on x and z respectively.

ii) Factorization between

$$x \text{ and } z \text{ in } \sigma^{DIS}_{h_1h_2}(x, z)$$

$$x_1 \text{ and } x_2 \text{ in } \sigma^{\mu^+\mu^-}_{h_1h_2}(x_1, x_2)$$

and

$$z_1 \text{ and } z_2 \text{ in } \sigma^{e^+e^-}_{h_1h_2}(z_1, z_2) \quad .$$

iii) The building blocks of all parton model formulae are universal (process independent) parton distributions and fragmentation functions. Therefore taking into account ii) we observe that if we can extract all parton distributions from inclusive deep-inelastic processes (a)) and fragmentation functions from e^+e^- annihilation (b)), then the cross-sections for the remaining processes listed above can be predicted. This implies that in the simple parton model there are relations between various processes.

iv) The gluon distribution G(x), and gluon fragmentation function $D_G^h(z)$ do not enter any of the formulae above.

v) Finally all cross-sections above can be reproduced from diagrams of Fig. 1 by using the "Feynman rules" of Fig. 2.

It is well known that in QCD quark distributions and quark fragmentation functions acquire Q^2 dependence and it is of interest and importance to ask:

–whether QCD predictions for semi-inclusive processes amount to using these Q^2 dependent functions in the parton model formulae (1.6-1.11);

–whether factorization properties listed under ii) (and correspondingly relations between various processes) are still satisfied;

–whether gluon distributions and gluon fragmentation functions explicitly enter QCD formulae, and

–whether one can find a simple extension of the rules of Fig. 2 which would allow us to construct in an easy way cross-sections for the processes a)-e) in QCD.

In these lectures we shall address these questions on two levels:

a) so-called leading order of asymptotic freedom and

b) next to leading order

with particular emphasis on the latter case.

To be more specific, if the QCD predictions for the moments of deep-inelastic structure functions are given as follows

$$\int_0^1 dx\, x^{n-2} F(x, Q^2) = A_n [\ln Q^2]^{-d_n} \left[1 + \frac{f_n}{\ln Q^2} + \ldots \right] \tag{1.12}$$

then keeping only "1" on the r.h.s. of Eq. (1.12) corresponds to the leading order whereas the second term $f_n/\ln Q^2$ stands for the next to leading order corrections. The numbers A_n, d_n and f_n will be discussed in subsequent sections.

Before going into details it is perhaps useful to get a general overview and list how the parton model properties above are modified in QCD.

The cross-sections for the processes (1.1)-(1.3) are given in QCD as follows*

$$\sigma_h^{DIS}(x, Q^2) = \sum_j \int_x^1 \frac{d\xi}{\xi} \sigma_P^j\left(\frac{x}{\xi}, Q^2\right) \left[\xi f_j^h(\xi, Q^2)\right] \tag{1.13}$$

$$\sigma_h^{e^+e^-}(z, Q^2) = \sum_j \int_z^1 \frac{d\xi}{\xi} \tilde{\sigma}_P^j\left(\frac{z}{\xi}, Q^2\right) \left[\xi D_j^h(\xi, Q^2)\right] \tag{1.14}$$

and

$$\sigma_{h_1 h_2}^{DIS}(x, z, Q^2) = \sum_{jk} \int_x^1 \frac{d\xi_1}{\xi_1} \int_z^1 \frac{d\xi_2}{\xi_2} \tilde{\sigma}_P^{jk}\left(\frac{x}{\xi_1}, \frac{z}{\xi_2}, Q^2\right) \left[\xi_1 f_j^{h_1}(\xi_1, Q^2)\right]\left[\xi_2 D_k^{h_2}(\xi_2, Q^2)\right] \quad . \tag{1.15}$$

The processes are shown schematically in Fig. 3. Similar equations exist for massive μ-pair production and two hadron semi-inclusive e^+e^- annihilation. In Eqs. (1.13)-(1.15) the sums run over quarks, antiquarks and gluons and $f_i^h(\xi, Q^2)$ denote generally parton distributions. Furthermore

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*There are so many papers on this subject that we cannot list all them here. Representative are refs. 6-8 where further references can be found.

$-\sigma_P^j$ is the photon-parton j cross-section,

$-\tilde{\sigma}_P^j$ is the cross-section for the production of the parton j in e^+e^- annihilation, and

$-\tilde{\sigma}^{jk}$ stands for the photon-parton j cross-section with the parton k in the final state.

Depending on the order considered (leading order, next to leading order) there are different rules for the parton cross-sections, parton distributions and parton fragmentation functions, which are the building blocks of the QCD formulae above.

For the first two orders the rules in question are as follows:

a) Leading Order Rules

Rule 1 (Parton cross-sections)

$$\sigma_P^j\left(\frac{x}{\xi}, Q^2\right) = \begin{cases} e_j^2\,\delta(1 - \frac{x}{\xi}) & j = q, \bar{q} \\ 0 & j = G \end{cases} \tag{1.16}$$

$$\tilde{\sigma}_P^j\left(\frac{z}{\xi}, Q^2\right) = \begin{cases} e_j^2\,\delta(1 - \frac{z}{\xi}) & j = q, \bar{q} \\ 0 & j = G \end{cases} \tag{1.17}$$

$$\tilde{\sigma}_P^{jk}\left(\frac{x}{\xi_1}, \frac{z}{\xi_2}, Q^2\right) = \begin{cases} e_j^2\,\delta_{jk}\,\delta\left(1 - \frac{x}{\xi_1}\right)\delta\left(1 - \frac{z}{\xi_2}\right) & j = q, \bar{q} \\ 0 & j = G \end{cases} \tag{1.18}$$

etc.

Rule 2 (Parton distributions and parton fragmentation functions)

The Q^2 evolution of the parton distributions is governed by certain equations. In the case of non-singlet quark distributions

$$\Delta_{ij}(x, Q^2) = q_i(x, Q^2) - q_j(x, Q^2) \tag{1.19}$$

these equations have a very simple form

$$< \Delta_{ij}(Q^2) >_n = < \Delta_{ij}(Q_o^2) >_n \left[\frac{\ln \frac{Q^2}{\Lambda^2}}{\ln \frac{Q_o^2}{\Lambda^2}} \right]^{-d_n} \tag{1.20}$$

where

$$< \Delta_{ij}(Q^2) >_n \equiv \int_0^1 dx\, x^{n-1} \Delta_{ij}(x, Q^2) \quad , \tag{1.21}$$

Q_o^2 is a reference momentum at which $\Delta_{ij}(x, Q_o^2)$ is to be taken from the data, d_n are known numbers and Λ is a scale parameter to be discussed later on. More complicated equations exist for the sums of quark and antiquark distributions (so-called singlet distributions). In the form of integro-differential equations they are often called Altarelli-Parisi equations.[9] Similar equations exist for the Q^2 evolution of the parton fragmentation functions. For nonsinglet combinations of fragmentation functions the equations are the same as the Eq. (1.20). For the singlet combinations of fragmentation functions there are slight modifications of the Altarelli-Parisi equations which are discussed in ref. 10. Using the rules 1 and 2 in Eqs. (1.13)-(1.15) we observe that the leading order corresponds to the parton model formulae (1.6)-(1.8) with $q_i^h(x)$ and $D_{q_i}^h(z)$ replaced by $q_i^h(x, Q^2)$ and $D_{q_i}^h(z, Q^2)$ which have calculable Q^2 dependence given by the equations mentioned above. Consequently all the parton model properties (except for the breakdown of scaling) are still satisfied. The following comment is however necessary.

On the basis of these results one could expect that if we extract parton distributions from inclusive deep-inelastic scattering and fragmentation functions from e^+e^- annihilation then the cross-sections for the processes (1.3)-(1.5) and in particular their Q^2 dependence can be predicted. This is not exactly true. The reason is, as we shall discuss in next sections, that in the leading order the numerical values of the scale parameter Λ extracted from the data need not be the same for different processes. Therefore a meaningful comparison of scaling violations in different processes can only be made if at least next to leading order corrections are included.[11] This point will be discussed in detail later on. The phenomenological applications of asymptotic freedom in the leading order have already been discussed in other lectures at this Summer School[12] and we shall not present them here.

b) Next to Leading Order Rules*

Rule 1' (Parton cross-sections)

$$\sigma_P^j\left(\frac{x}{\xi}, Q^2\right) = \begin{cases} \delta(1 - \frac{x}{\xi}) + \bar{g}^2(Q^2) b_q(\frac{x}{\xi}) & j = q, \bar{q} \\ \bar{g}^2(Q^2) b_G(\frac{x}{\xi}) & j = G \end{cases} \tag{1.22}$$

$$\tilde{\sigma}_P^j\left(\frac{z}{\xi}, Q^2\right) = \begin{cases} \delta(1 - \frac{z}{\xi}) + \bar{g}^2(Q^2) d_q(\frac{z}{\xi}) & j = q, \bar{q} \\ \bar{g}^2(Q^2) d_G(\frac{z}{\xi}) & j = G \end{cases} \tag{1.23}$$

$$\tilde{\sigma}_P^{jk}\left(\frac{x}{\xi_1}, \frac{z}{\xi_2}, Q^2\right) = \begin{cases} \delta\left(1 - \frac{x}{\xi_1}\right)\delta\left(1 - \frac{z}{\xi_2}\right) + \bar{g}^2(Q^2) f_{qq}\left(\frac{x}{\xi_1}, \frac{z}{\xi_2}\right) & j = k = q, \bar{q} \\ \bar{g}^2(Q^2) f_{qG}\left(\frac{x}{\xi_1}, \frac{z}{\xi_2}\right) & j = q, k = G \\ \bar{g}^2(Q^2) f_{Gq}\left(\frac{x}{\xi_1}, \frac{z}{\xi_2}\right) & j = G, k = q \end{cases} \tag{1.24}$$

where b_q, d_q, f_{qq}, f_{qG} and f_{Gq} are calculable functions to be discussed below. Similar rules exist for processes (1.4) and (1.5).

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*We drop charge factors in the following.

Rule 2' (Parton distributions and parton fragmentation functions)

The Q^2 evolution of parton distributions and parton fragmentation functions is governed by new equations which differ from the leading order equations (Rule 2) by calculable corrections of order $\bar{g}^2(Q^2)$. These new equations will be discussed in the course of these lectures.

The new features of Rules 1' and 2' not encountered in the parton model and in the leading order are as follows

i) Factorization between x and z is broken through the functions f_{ij}. Similarly the factorization between x_1 and x_2 in the process (1.4), and between z_1 and z_2 in the process (1.5) is broken by calculable corrections to the parton cross-sections.

ii) The explicit $\bar{g}^2(Q^2)$ corrections to the parton cross-sections depend on the definition of parton distributions beyond the leading order. In other words the rules 1' and 2' are not independent of each other and must be consistent with each other in order that a physical answer independent of any particular definition is obtained for the measurable quantities as $\sigma_h^{DIS}(x, Q^2), \sigma_h^{e^+e^-}(z, Q^2)$, etc.

iii) The $\bar{g}^2(Q^2)$ corrections to various parton cross-sections depend on the process considered. There exist however certain relations between some of the parton cross-sections (see Section VI).

iv) Because the parton cross-sections involving gluons are non-zero the gluon distributions and gluon fragmentation function enter the QCD formulae explicitly.

This completes the general overview. In what follows we shall show systematically how to obtain the rules 1' and 2'. In Section II we recall the ingredients of the formal approach to deep-inelastic scattering based on operator product expansion and renormalization group equations. In Sections III and IV the inclusive deep-inelastic scattering beyond the leading order is discussed in some detail. Subsequently in Section V we present the basic structure of next to leading order calculations for semi-inclusive processes. In Section VI we will list other recent higher order QCD calculations. We end our lectures with a brief summary and outlook. A detailed plan of our lectures can be found in the table of contents.

II. BASIC FORMALISM

In this section we shall recall the basic formal tools used to extract QCD predictions for deep-inelastic scattering. Generalization to other processes will be discussed in Section V.

Let us imagine that we want to find QCD predictions for deep-inelastic structure functions. We can proceed as follows:

Step 1

We consider the spin-averaged amplitude $T_{\mu\nu}$ for the forward scattering of a weak or electromagnetic current J_μ. The amplitude $T_{\mu\nu}$ can be decomposed into invariant amplitudes as follows:

$$\begin{aligned} T_{\mu\nu} &= i\int d^4x\, e^{iq\cdot x} \langle p|\, T(J_\mu(x)J_\nu(0))\,|p\rangle_{\text{spin averaged}} \\ &= e_{\mu\nu} T_L(Q^2,\nu) + d_{\mu\nu} T_2(Q^2,\nu) - i\varepsilon_{\mu\nu\alpha\beta} \frac{p_\alpha q_\beta}{\nu} T_3(Q^2,\nu) \qquad . \quad (2.1) \end{aligned}$$

Here $\nu = p\cdot q$, $Q^2 = -q^2$ and $|p\rangle$ is, for instance, a proton state. The tensors $e_{\mu\nu}$ $d_{\mu\nu}$ and $\varepsilon_{\mu\nu\alpha\beta}$ are well known.

Step 2

We expand the product of currents, which enters Eq. (2.1) as a sum of products of local operators O_i^n of definite spin n times certain coefficient functions $\tilde{C}_n^i$ called Wilson coefficient functions.[13] We write symbolically this operator product expansion (OPE) as follows:

$$\mathrm{J}(x)\mathrm{J}(0) = \sum_{i,n} \tilde{C}^i_n(x^2) O^n_i \quad . \tag{2.2}$$

The sum in Eq. (2.2) runs over spin n, twist 2 operators* such as the fermion non-singlet operator O^n_{NS} and the singlet fermion and gluon operators O^n_ψ and O^n_G respectively.

For readers less familiar with the operator product expansion we only recall that the matrix elements of local operators between proton states can be interpreted as moments of parton distributions (see Section III). Notice that any deep-inelastic structure function can be expressed through three types of parton distributions: non-singlet, singlet quark and gluon distributions. Correspondingly there are three types of operators O^n_{NS}, O^n_ψ and O^n_G. A careful discussion of operator product expansion can be found for instance in refs. 1, 13 and 14.

Step 3

Inserting Eq. (2.2) into (2.1) and using dispersion relations between deep-inelastic structure functions and the invariant amplitudes of Eq. (2.1) we obtain[15]

$$M_k(n, Q^2) \equiv \int_0^1 dx\, x^{n-2} F_k(x, Q^2) = \sum_i A^i_n(\mu^2) C^i_{k,n}\left(\frac{Q^2}{\mu^2}, g^2\right) \tag{2.3}$$

where $C^i_{k,n}$ are fourier transforms of the coefficient functions in Eq. (2.2) and A^i_n are the matrix elements of operators O^n_i between the hadronic state $|p\rangle$. Furthermore g is the renormalized quark-gluon coupling constant and μ^2 is the subtraction scale at which the theory is renormalized. The important property of Eq. (2.3) is the factorization of non-perturbative pieces $A^i_n(\mu^2)$ from perturbatively calculable coefficient functions $C^i_{k,n}(Q^2/\mu^2, g^2)$.

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*Neglecting operators of higher twist corresponds to neglecting contributions, which with increasing Q^2, decrease as inverse powers of Q^2.

Step 4

We decompose $F_k(x, Q^2)$ into a sum of singlet and non-singlet (under flavor symmetry) contributions as follows

$$F_k(x, Q^2) = F_k^{NS}(x, Q^2) + F_k^{s}(x, Q^2) \quad . \tag{2.4}$$

We have in an obvious notation:

$$M_k^{NS}(n, Q^2) = A_n^{NS}(\mu^2) C_{k,n}^{NS}\left(\frac{Q^2}{\mu^2}, g^2\right) \tag{2.5}$$

and

$$M_k^{s}(n, Q^2) = A_n^{\psi}(\mu^2) C_{k,n}^{\psi}\left(\frac{Q^2}{\mu^2}, g^2\right) + A_n^{G}(\mu^2) C_{k,n}^{G}\left(\frac{Q^2}{\mu^2}, g^2\right) \quad . \tag{2.6}$$

Step 5

We use renormalization group equations, which govern the Q^2 dependence of $C_{k,n}^{i}(Q^2/\mu^2, g^2)$. These equations are given as follows[16]:

$$\left[\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} - \gamma_{NS}^{n}(g)\right] C_{k,n}^{NS}\left(\frac{Q^2}{\mu^2}, g^2\right) = 0 \quad , \tag{2.7}$$

and

$$\left[\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g}\right] C_{k,n}^{i}\left(\frac{Q^2}{\mu^2}, g^2\right) = \sum_j \gamma_{ji}^{n}(g) C_{k,n}^{j}\left(\frac{Q^2}{\mu^2}, g^2\right) \quad i,j = \psi, G \quad . \tag{2.8}$$

Notice that the Q^2 dependence of $C^{\psi}_{k,n}$ and $C^{G}_{k,n}$ is governed by two coupled renormalization group equations due to the mixing of the operators O^n_{ψ} and O^n_G under renormalization. $\gamma^n_{NS}(g)$ is the anomalous dimension of O^n_{NS} and $\gamma^n_{ij}(g)$ are the elements of the 2×2 anomalous dimension matrix $\hat{\gamma}^n(g)$. $\beta(g)$ is the well-known renormalization group function which governs the Q^2 evolution of the effective coupling constant $\bar{g}^2(Q^2)$:

$$\frac{d\bar{g}^2}{dt} = \bar{g}\beta(g) \quad ; \quad \bar{g}(t=0) = g \equiv \bar{g}(\mu^2) \tag{2.9}$$

where $t = \ln Q^2/\mu^2$.

The solutions of Eqs. (2.7) and (2.8) can be written in terms of $\bar{g}^2(Q^2)$ as follows:

$$C^{NS}_{k,n}\left(\frac{Q^2}{\mu^2}, g^2\right) = C^{NS}_{k,n}(1, \bar{g}^2)\exp\left[-\int_{\bar{g}(\mu^2)}^{\bar{g}(Q^2)} dg' \frac{\gamma^n_{NS}(g')}{\beta(g')}\right], \tag{2.10}$$

and

$$\vec{C}_{k,n}\left(\frac{Q^2}{\mu^2}, g^2\right) = \left[T \exp\int_{\bar{g}(Q^2)}^{\bar{g}(\mu^2)} dg' \frac{\hat{\gamma}^n(g')}{\beta(g')}\right] \vec{C}_{k,n}(1, \bar{g}^2) \tag{2.11}$$

where $\vec{C}_{k,n}$ is the column vector whose components are $C^{\psi}_{k,n}$ and $C^{G}_{k,n}$. Equations (2.9)-(2.11) combined with Eqs. (2.4)-(2.6) give us <u>general</u> expressions for the Q^2 evolution of the moments of the deep-inelastic structure functions in terms of the renormalization group function $\beta(g)$, $\gamma^n_{NS}(g)$ and $\hat{\gamma}^n(g)$ and the coefficient functions $C^i_{k,n}(1, \bar{g}^2)$.

Step 6

In order to find explicit Q^2 dependence of $M_k(n, Q^2)$ we have to calculate $\beta(g)$, $\gamma^n_{NS}(g)$, $\hat{\gamma}^n(g)$ and $C^i_{k,n}(1, \bar{g}^2)$. This is done in perturbation theory in g. We shall discuss explicit examples below.

Step 7

So far our discussion was very formal. In order to have relation with the parton picture of Section I we can cast the formal expressions for the Q^2 dependent structure functions into the parton model-like formulae with effective Q^2 dependent parton distributions. We shall discuss such expressions below.

III. NEXT TO LEADING ORDER ASYMPTOTIC FREEDOM CORRECTIONS TO DEEP-INELASTIC SCATTERING (NON-SINGLET CASE)

3.1. Derivation of Basic Formulae

In order to find explicit expressions for the leading and next-to-leading contributions to $C^{NS}_{k,n}(Q^2/\mu^2, g^2)$ as given by Eq. (2.10) we expand $\gamma^n_{NS}(\bar{g})$, $\beta(\bar{g})$ and $C^{NS}_{k,n}(1, \bar{g}^2)$ in powers of $\bar{g}^2$

$$\gamma^n_{NS}(\bar{g}) = \gamma^{0,n}_{NS} \frac{\bar{g}^2}{16\pi^2} + \gamma^{(1),n}_{NS}\left(\frac{\bar{g}^2}{16\pi^2}\right)^2 + \dots \tag{3.1}$$

$$\beta(\bar{g}) = -\beta_o \frac{\bar{g}^3}{16\pi^2} - \beta_1 \frac{\bar{g}^5}{(16\pi^2)^2} + \dots \tag{3.2}$$

and (through order $\bar{g}^2$)

$$C^{NS}_{k,n}(1, \bar{g}^2) = \delta^k_{NS}\left(1 + \frac{\bar{g}^2}{16\pi^2} B^{NS}_{k,n}\right) \qquad k = 2,3 \qquad . \tag{3.3}$$

Here δ^k_{NS} are constants which depend on weak and electromagnetic charges.

For the exponential in Eq. (2.10) we obtain (through order $\bar{g}^2$) with $\mu^2 = Q_o^2$

$$\exp\left[-\int_{\bar{g}^2(Q_o^2)}^{\bar{g}^2(Q^2)} dg' \frac{\gamma^n_{NS}(g')}{\beta(g')}\right] = \left[1 + \frac{\left[\bar{g}^2(Q^2) - \bar{g}^2(Q_o^2)\right]}{16\pi^2} Z_n^{NS}\right]\left[\frac{\bar{g}^2(Q^2)}{\bar{g}^2(Q_o^2)}\right]^{d^n_{NS}} , \quad (3.4)$$

where

$$Z_n^{NS} = \frac{\gamma_{NS}^{(1),n}}{2\beta_o} - \frac{\gamma_{NS}^{(0),n}}{2\beta_o^2}\beta_1 \quad ; \quad d^n_{NS} = \frac{\gamma^{0,n}_{NS}}{2\beta_o} \quad . \quad (3.5)$$

Combining Eqs. (3.3), (3.4), (2.10) and (2.3) we obtain

$$M_k^{NS}(n, Q^2) = \delta^k_{NS}\bar{A}_n^{NS}(Q_o^2)\left[1 + \frac{\left[\bar{g}^2(Q^2) - \bar{g}^2(Q_o^2)\right]}{16\pi^2} R^{NS}_{k,n}\right]\left[\frac{\bar{g}^2(Q^2)}{\bar{g}^2(Q_o^2)}\right]^{d^n_{NS}} , \quad (3.6)$$

where

$$R^{NS}_{k,n} = B^{NS}_{k,n} + Z^{NS}_n \quad , \quad (3.7)$$

$$\bar{A}_n^{NS}(Q_o^2) = A_n^{NS}(Q_o^2)\left[1 + \frac{\bar{g}^2(Q_o^2)}{16\pi^2} B^{NS}_{k,n}\right] \quad (3.8)$$

and $\bar{g}^2(Q^2)$ is to be calculated by means of Eq. (2.9) with the β function given by Eq. (3.2). In phenomenological applications it is often convenient to insert into Eq. (3.6) the explicit expression for $\bar{g}^2(Q^2)$,

$$\frac{\bar{g}^2(Q^2)}{16\pi^2} = \frac{1}{\beta_o \ln\frac{Q^2}{\Lambda^2}} - \frac{\beta_1}{\beta_o^3}\frac{\ln\ln\frac{Q^2}{\Lambda^2}}{\ln^2\frac{Q^2}{\Lambda^2}} + O\left(\frac{1}{\ln^3\frac{Q^2}{\Lambda^2}}\right) \quad (3.9)$$

with the result

$$M_k^{NS}(n, Q^2) = \delta^k_{NS}\bar{A}_n^{NS}(Q_o^2)\left[1 + \frac{R_{k,n}^{NS}(Q^2)}{\beta_o \ln \frac{Q^2}{\Lambda^2}} - \frac{R_{k,n}^{NS}(Q_o^2)}{\beta_o \ln \frac{Q_o^2}{\Lambda^2}}\right]\left[\frac{\ln \frac{Q^2}{\Lambda^2}}{\ln \frac{Q_o^2}{\Lambda^2}}\right]^{-d^n_{NS}} \tag{3.10}$$

where

$$R_{k,n}^{NS}(Q^2) = R_{k,n}^{NS} - \frac{\beta_1}{2\beta_o^2}\gamma_{NS}^{0,n}\ln\ln\frac{Q^2}{\Lambda^2} \quad . \tag{3.11}$$

Equation (3.10) is the basic formula of this section. The value of Q_o^2 in Eq. (3.10) is arbitrary as required by the renormalization group equations and the predictions for $M_k^{NS}(n, Q^2)$ should be independent of it. Therefore it is sometimes convenient to get rid of Q_o^2 by writing Eq. (3.10) as

$$M_k^{NS}(n, Q^2) = \delta^k_{NS}\bar{A}_n^{NS}\left[1 + \frac{R_{k,n}^{NS}(Q^2)}{\beta_o \ln \frac{Q^2}{\Lambda^2}}\right]\left[\ln \frac{Q^2}{\Lambda^2}\right]^{-d^n_{NS}} \quad k = 2,3 \quad . \tag{3.12}$$

Here $\bar{A}_n^{NS}$ are constants (independent of Q_o^2).

3.2. Discussion of Basic Properties and Subtle Points

a) We first notice that in order to find the next to leading order corrections to the non-singlet structure functions, one has to calculate the two loop contributions to $\gamma^n_{NS}(g)$ and $\beta(g)$ and one-loop corrections to $C_{k,n}^{NS}(1, \bar{g}^2)$ i.e. the parameters $\gamma_{NS}^{(1),n}$, β_1 and $B_{k,n}^{NS}$ respectively. The parameters $\gamma_{NS}^{(0),n}$ and β_o are known already from the leading order calculations.[17]

b) The two-loop contributions to the β function, i.e. the parameter β_1, has been calculated in ref. 18 and for an $SU(3)_c$ gauge theory with f flavors is given by

$$\beta_1 = 102 - \frac{38}{3} f \quad . \tag{3.13}$$

It should be remarked that β_1 as well as $\gamma_{NS}^{0,n}$ and β_o are renormalization prescription-and gauge-independent.

c) The parameters $B_{k,n}$ and $\gamma_{NS}^{(1),n}$ are separately renormalization prescription dependent (i.e. they depend on the way one renormalizes the quantities used to calculate them (see appendix)) and in principle gauge dependent. However as shown by Floratos, Ross and Sachrajda[19] the quantity

$$B_{k,n}^{NS} + \frac{\gamma_{NS}^{(1),n}}{2\beta_o} \tag{3.14}$$

is renormalization prescription independent if (of course) $B_{k,n}^{NS}$ and $\gamma_{NS}^{(1),n}$ are calculated in the same renormalization scheme. Consequently the parameters $R_{k,n}^{NS}$ of Eq. (3.7) are renormalization prescription independent. From this we can draw two lessons:

1) care must be taken that $B_{k,n}^{NS}$ and $\gamma_{NS}^{(1),n}$ are calculated in the same renormalization scheme and

2) without doing explicit calculations one cannot a priori neglect any of the two quantities $B_{k,n}^{NS}$ and $\gamma_{NS}^{(1),n}/2\beta_o$ in any higher order formulae. The reason is that in some schemes the two-loop contribution is dominant in the sum (3.14) whereas in other schemes $B_{n,k}^{NS}$ is more important.
The method of calculation of the parameters $B_{k,n}^{NS}$ is described in the Appendix.

d) The full calculation of the sum in Eq. (3.14) has been performed in the literature only in the 't Hooft's minimal subtraction scheme (MS)*. The parameters

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*In this scheme the Feynman diagrams are evaluated, using dimensional regularization, in $d = 4 - \varepsilon$ dimensions and singularities are extracted as poles $1/\varepsilon$, $1/\varepsilon^2$ etc. The minimal subtraction then means that the amplitudes are renormalized by simply subtracting the pole parts $1/\varepsilon$, $1/\varepsilon^2$ etc.

$B^{NS}_{k,n}$ have been calculated in ref. 20 and recalculated in ref. 21. The authors of ref. 19 have calculated the two-loop anomalous dimensions $\gamma^{(1),n}_{NS}$. The latter calculation is particularly complicated. Calculations of the parameters $B^{NS}_{k,n}$ in different renormalization schemes have been done in refs. 22-25. However these results cannot be combined with the two-loop anomalous dimensions of ref. 19. In spite of this the results of refs. 22-25 will turn out to be useful in the study of QCD effects in other processes (see Section V).

e) In the minimal subtraction scheme one obtains[19,20]

$$B^{NS}_{2,n} = \bar{B}^{NS}_{2,n} + \tfrac{1}{2}\gamma^{0,n}_{NS}(\ln 4\pi - \gamma_E) \tag{3.15}$$

and

$$B^{NS}_{3,n} = B^{NS}_{2,n} - \frac{4}{3}\,\frac{4n+2}{n(n+1)} \tag{3.16}$$

where

$$\bar{B}^{NS}_{2,n} = \frac{4}{3}\left\{ 3\sum_{j=1}^{n}\frac{1}{j} - 4\sum_{j=1}^{n}\frac{1}{j^2} - \frac{2}{n(n+1)}\sum_{j=1}^{n}\frac{1}{j} + 4\sum_{s=1}^{n}\frac{1}{s}\sum_{j=1}^{s}\frac{1}{j} \right.$$
$$\left. + \frac{3}{n} + \frac{4}{(n+1)} + \frac{2}{n^2} - 9 \right\} \tag{3.17}$$

and $\gamma_E = 0.5777$ is the Euler-Mascheroni constant. The "strange" terms $(\ln 4\pi - \gamma_E)$ arise from the expansion of $\Gamma(\varepsilon/2)\cdot(4\pi)^{\varepsilon/2}$ around $\varepsilon = 0$. The numerical values of $B^{NS}_{2,n}$, $\bar{B}^{NS}_{2,n}$ and Z^{NS}_n for the renormalization scheme in question are collected for some values of n in Table 1. We observe that the terms Z^{NS}_n are small compared to the parameters $B^{NS}_{2,n}$ and $\bar{B}^{NS}_{2,n}$. In order to understand the difference between $B^{NS}_{2,n}$

and $\bar{B}^{NS}_{2,n}$ we now turn to the last important point which is related to the parameter Λ and to the arbitrariness in the definition of the effective coupling constant.

3.3. Parameter Λ and Various Definitions of $\bar{g}^2(Q^2)$

The effect of the redefinition of the scale parameter Λ is equivalent through order $\bar{g}^2(Q^2)$ to the shift of $R^{NS}_{k,n}(Q^2)$ in Eq. (3.12) by a constant amount proportional to $\gamma^{0,n}_{NS}$.[20] In fact rescaling Λ in Eq. (3.12) to Λ' by

$$\Lambda = \Lambda' \exp(-\tfrac{1}{2}\kappa) \tag{3.18}$$

where κ is a constant, and dropping terms of order $\bar{g}^4(Q^2)$ generated by this rescaling one obtains

$$M^{NS}_k(n, Q^2) = \delta^k_{NS}\bar{A}^{NS}_n \left[1 + \frac{R'^{NS}_{k,n}(Q^2)}{\beta_0 \ln \frac{Q^2}{\Lambda'^2}} \right] \left[\ln \frac{Q^2}{\Lambda'^2} \right]^{-d^n_{NS}} \tag{3.19}$$

where

$$R'^{NS}_{k,n}(Q^2) = R^{NS}_{k,n}(Q^2) - \tfrac{1}{2}\gamma^{0,n}_{NS}\,\kappa \quad . \tag{3.20}$$

The Λ' thus corresponds to the $\bar{g}^2$ corrections given by Eq. (3.20) and $\bar{g}^2(Q^2)$ having the form of Eq. (3.9) with Λ replaced by Λ'. To be more specific let us denote by Λ_{MS} the scale parameter which corresponds to $R^{NS}_{k,n}$ given by Eq. (3.7) with $B^{NS}_{k,n}$ given by Eqs. (3.15) and (3.16). This is so called minimal scheme (MS) for Λ. In the literature two other schemes have been discussed:

–$\overline{MS}$ scheme[20] for which the parameters $R^{NS}_{k,n}$ are replaced by

$$\bar{R}^{NS}_{k,n} = R^{NS}_{k,n} - \tfrac{1}{2}\gamma^{0,n}_{NS}(\ln 4\pi - \gamma_E) \tag{3.21}$$

and the corresponding Λ denoted by $\Lambda_{\overline{MS}}$, and

–MOM (momentum subtraction scheme)[26,27] for which $R^{NS}_{k,n}$ are replaced by*

$$R^{NS}_{k,n}\Big|_{MOM} = R^{NS}_{k,n} - \tfrac{1}{2}\gamma^{0,n}_{NS}(3.5) \qquad (3.22)$$

and the corresponding Λ denoted by Λ_{MOM}. Notice that the $\overline{MS}$ scheme corresponds to dropping terms in Eq. (3.15) involving factors $(\ln 4\pi - \gamma_E) \approx 1.95$.

For the three schemes considered MS, $\overline{MS}$ and MOM the effective coupling constant is given by Eq. (3.9) with Λ replaced by Λ_{MS}, $\Lambda_{\overline{MS}}$ and Λ_{MOM} respectively. It is obvious that since the functional forms of $R^{NS}_{k,n}$, $\bar{R}^{NS}_{k,n}$ and $R^{NS}_{k,n}\big|_{MOM}$ are different from each other so will be the free parameters Λ_{MS}, $\Lambda_{\overline{MS}}$ and Λ_{MOM} extracted from the data. Needless to say the three schemes considered are equivalent representations of next to leading corrections. On the other hand they correspond to different estimates of the higher order terms $O(\bar{g}^4(Q^2))$ not included in the analysis. In the next subsection we shall present numerical values of Λ_i, effective coupling constants and explicit higher order corrections for the schemes considered.

Since the explicit higher order corrections and the parameter Λ are related to each other we conclude that one cannot discuss numerical values of Λ in a theoretically meaningful way without calculating at least next to leading order corrections[11] and without specifying the definition of the effective coupling constant.[20]

Once a definition of $\bar{g}^2(Q^2)$ is made and is used in calculations of higher order corrections in various processes it is possible to make a meaningful comparison of

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*The $\bar{g}^2(Q^2)$ defined by momentum subtraction is gauge dependent but the gauge dependence is weak. The value 3.5 corresponds to the Landau gauge.

higher order corrections to various processes.[11] We shall see that these corrections are generally different for different processes. This teaches us that it is unjustified in principle to use the same value of Λ in the leading order expressions for different processes. On the other hand, once higher order corrections are included in the analysis and $\bar{g}^2(Q^2)$ is defined in a universal way, it is justified to use the same value of Λ in different processes.

Let us summarize two basic lessons of this section

i) the parameters $\gamma_{NS}^{(1),n}$ and $B_{k,n}^{NS}$ have to be calculated in the same renormalization scheme;

ii) there is a well-defined dependence of the functional form of the explicit higher order corrections on the definition of $\bar{g}^2(Q^2)$ or, equivalently, on Λ.

3.4. Phenomenology and Numerical Estimates

The following values for Λ_{MS}, $\Lambda_{\overline{MS}}$ and Λ_{MOM} have been obtained on the basis of BEBC data[28] for the moments of $F_3^{\nu,\bar{\nu}}$:[3,20]

$$\Lambda_{MS} = 0.40 \text{ GeV} \quad ; \quad \Lambda_{\overline{MS}} = 0.50 \text{ GeV} \quad ; \quad \Lambda_{MOM} = 0.85 \text{ GeV}^{*} \quad . \tag{3.23}$$

We recall that the leading order analysis leads to $\Lambda_{LO} = 0.7$ GeV.[28] The errorbars for the values of Λ are 0 (0.05 GeV). All three schemes agree with the BEBC data for $n \leq 5$. Higher moments are discussed below. The effective coupling constants in the three schemes considered for $\Lambda_{MS} = 0.40$, $\Lambda_{\overline{MS}} = 0.50$ and $\Lambda_{MOM} = 0.85$ are plotted in Fig. 4. We observe the following inequalities

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*These values are shown here as an example of a fit to a particular set of data. The CDHS data[29] lead for instance to smaller values of Λ. For a careful phenomenological study of both CDHS and BEBC data with higher order effects and mass corrections included we refer the reader to the recent paper by Abbott and Barnett.[30]

$\bar{g}^2(Q^2)|_{MOM} > \bar{g}^2(Q^2)|_{\overline{MS}} > \bar{g}^2(Q^2)|_{MS}$ which correspond to $R^{NS}_{2,n} > \bar{R}^{NS}_{2,n} > R^{NS}_{2,n}|_{MOM}$. Furthermore in all cases considered the effective coupling constant is smaller than that given by the leading order expression (the first term on the r.h.s. of Eq. (3.9)) with $\Lambda_{LO} = 0.7$ GeV.

It is instructive to calculate the term

$$1 + \frac{R^{NS}_{k,n}(Q^2)}{\beta_o \ln \frac{Q^2}{\Lambda_i^2}} \qquad i = MS, \overline{MS}, MOM \tag{3.24}$$

in Eq. (3.12) which is equal unity in the leading order. The result is shown in Table 2.

We conclude that in the expansion in the inverse powers of logarithms the next-to-leading order corrections to $M_2^{NS}(n, Q^2)$ calculated in the momentum subtraction scheme with a = 3.5 are larger than those in the $\overline{MS}$ scheme but smaller than in the MS scheme.

On the other hand in the expansion in $\bar{g}^2(Q^2)$ (see Eq. (3.6)) the momentum subtraction scheme leads to a better convergence of the perturbative series than $\overline{MS}$ and MS schemes.[26, 27,58]

From the discussion in subsec. 3.3 it is clear that if the coefficient of $1/(\beta_o \ln Q^2/\Lambda^2)$ in Eq. (3.12) were independent of Q^2 and had exactly the same n dependence as $\gamma^{0,n}_{NS}$, then all $\bar{g}^2$ corrections could be absorbed in the parameter Λ, and the higher order formula would look like the leading order expression. Conversely, we could say that the leading order formula assumes that the next-to-leading order corrections have the same n dependence as the $\gamma^{0,n}_{NS}$. Therefore it is of interest to see whether the next-to-leading order corrections, which we have calculated in this Section, exhibit a non-trivial n-dependence different from $\gamma^{0,n}_{NS}$.

This is most conveniently done by putting Eq. (3.12) into a form of a leading order expression[11,20]

$$M_k^{NS}(n, Q^2) = \delta^k_{NS} \bar{A}_n^{NS} \left[\ln \frac{Q^2}{\Lambda_n^2(Q^2)} \right]^{-d^n_{NS}} \tag{3.25}$$

with

$$\Lambda_n(Q^2) = \Lambda \exp\left[\frac{R^{NS}_{k,n}(Q^2)}{\gamma^{0,n}_{NS}} \right] \approx \Lambda \exp\left[\frac{B^{NS}_{k,n}}{\gamma^{0,n}_{NS}} \right] \equiv \Lambda_n \quad . \tag{3.26}$$

The second relation in Eq. (3.26) is a good approximation if the quantities Z_n^{NS} in Eq. (3.7) are calculated in 't Hooft's minimal subtraction scheme. Needless to say the n dependence of $\Lambda_n(Q^2)$ or Λ_n is independent of the definition of $\bar{g}^2(Q^2)$. In Fig. 5 the formula (3.26) is compared[31] with Λ_n extracted from the data of BEBC and CDHS for F_3^ν and Fermilab and SLAC ep, en and μp and μn data for F_2^{p-n}. We observe a remarkable agreement of Eq. (3.26) with the data for F_2^{p-n}. The BEBC and in particular CDHS data do not show very clear n dependence of Λ_n, although at lower values of n they are consistent with Eq. (3.26). We may conclude in particular, on the basis of F_2^{p-n}, that there are indications in the data for the n dependence of Λ_n as predicted by QCD. It is of interest to see whether the new μ-experiments at CERN and Fermilab will confirm these results.

3.5. Parton Distributions Beyond Leading Order

So far our discussion of higher order corrections was very formal. We shall now express Eq. (3.6) in terms of parton distributions and parton cross-sections to obtain a more intuitive formula (1.13).* Let us first recall that the parametrization of the QCD predictions (3.6), (3.10) or (3.12) in terms of an <u>effective</u> $\bar{g}^2(Q^2)$

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*In this section we discuss only non-singlet parts of Eq. (1.13).

and explicit higher order corrections depends on the definition of $\bar{g}^2(Q^2)$. Similarly the parametrization of QCD predictions in terms of "effective" parton distributions and parton cross-sections depends on the definition of parton distributions.

In order to illustrate this point, we consider the moments of a non-singlet structure function which in the leading order is expressed through the moments of a non-singlet quark distribution $\Delta(x, Q^2)$ as follows

$$M_k^{NS}(n, Q^2) = \delta_{NS}^{(k)} A_n^{NS}(Q_o^2) \left[\frac{\ln \frac{Q^2}{\Lambda^2}}{\ln \frac{Q_o^2}{\Lambda^2}} \right]^{-d_{NS}^n} \equiv \delta_{NS}^{(k)} < \Delta(Q^2) >_n \tag{3.27}$$

with $\delta_{NS}^{(k)}$ being a charge factor; e.g. $\delta_{NS}^{(k)} = 1/6$ for F_2^{ep}. Notice that

$$< \Delta(Q^2) >_n = A_n^{NS}(Q_o^2) \left[\frac{\ln \frac{Q^2}{\Lambda^2}}{\ln \frac{Q_o^2}{\Lambda^2}} \right]^{-d_{NS}^n} \equiv A_n^{NS}(Q^2) \tag{3.28}$$

i.e. the moments of a non-singlet parton distribution are equal to matrix elements of a non-singlet operator normalized at Q^2.

Beyond the leading order there are various ways of defining parton distributions. We shall discuss here two examples:

a) Generalization of Eq. (3.28)[32]

$$< \Delta(Q^2) >_n^{(a)} = A_n^{NS}(Q_o^2) \exp \left[- \int_{\bar{g}^2(Q_o^2)}^{\bar{g}^2(Q^2)} dg' \frac{\gamma_{NS}^n(g')}{\beta(g')} \right] = A_n^{NS}(Q^2) \tag{3.29}$$

where through order $\bar{g}^2$, the exp ( ) is given by Eq. (3.4). Furthermore $A_n^{NS}(Q_o^2) = < \Delta(Q_o^2)>_n^{(a)}$. In terms of $<\Delta(Q^2)>_n^{(a)}$ we have, on the basis of Eq. (2.5) and (2.10),

$$M_k^{NS}(n, Q^2) = < \Delta (Q^2)>_n^{(a)} C_{k,n}^{NS}(1, \bar{g}^2) \tag{3.30}$$

which when inverted gives the form of Eq. (1.13)

$$F_k^{NS}(x, Q^2) = \int_x^1 \frac{d\xi}{\xi} \left[\xi \, \Delta^{(a)}(\xi, Q^2) \right] \sigma_{P,k}^{NS}(\frac{x}{\xi}, Q^2) \quad . \tag{3.31}$$

Here

$$\int_0^1 dx \, x^{n-2} \sigma_{P,k}^{NS}(x, Q^2) = C_{k,n}^{NS}(1, \bar{g}^2) \tag{3.32}$$

with $C_{k,n}^{NS}(1, \bar{g}^2)$ given by Eqs. (3.3), (3.15) and (3.16).

b) In the second example[21,24] one absorbs all higher order corrections to $M_2^{NS}(n, Q^2)$ into the parton distributions i.e.

$$M_2^{NS}(n, Q^2) = < \Delta (Q^2) >_n^{(b)} \delta_{NS}^{(2)} \quad . \tag{3.33}$$

$< \Delta(Q^2) >_n^{(b)}$ is obtained by comparing Eq. (3.33) with Eq. (3.10) for $k = 2$.

Notice that the Q^2 dependence of $\Delta^{(a)}(x, Q^2)$ and of $\Delta^{(b)}(x, Q^2)$ are different from each other and so are the corresponding parton cross-sections. In the second example the parton cross-section is trivial for $k = 2$. (b_q in Eq. (1.22) is zero.) For $k \neq 2$ there are some non-zero $\bar{g}^2$ corrections to parton cross-sections even in the second example due to the fact that $B_{k,n}^{NS}$ are different for different k. The parton distributions and parton cross-sections in the example b) are separately renormalization prescription and gauge independent. This is not the case in the example a) but the renormalization prescription and gauge dependences of $\Delta^{(a)}(\xi, Q^2)$ and of $\sigma^{NS}_{P,k}$ cancel in the final expression for $F^{NS}_k(x, Q^2)$ (Eq. 3.31). Since one can define parton distributions in many ways anyhow, one should not worry about this renormalization prescription dependence of parton distributions in Example a).

We next notice that whereas the input distributions at $Q^2 = Q_o^2$ in the example b) will be (for k = 2) the same as in the leading order (i.e. the data does not change), the input distributions in the example a) will differ considerably at low Q^2 and large x from those used in the leading order phenomenology. The reason is that even in the $\overline{MS}$ scheme $C^{NS}_{k,n}(1, \bar{g}^2)$ differs considerably from 1 for low Q^2 and large n. On the other hand the example a) turns out to be useful for inversion of moments if $Z^{NS}_{n,k}$ are calculated in the 't Hooft's scheme. In this scheme $Z^{NS}_{n,k}$ are small (see Table 1) and the equation for the Q^2 dependence of the parton distributions are essentially the same as the leading order equation (3.27) with $\bar{g}^2(Q^2)$ now given by Eq. (3.9). Therefore the standard techniques used to invert moments in the leading order can also be used successfully here. In particular one can find analytic expressions for $\Delta^{(a)}(x, Q^2)$ which to a good approximation represent the exact QCD predictions. The function $C^{NS}_{k,n}(1, \bar{g}^2)$, which contains all non-trivial n-dependence of higher order corrections, can be inverted exactly (analytically). As a result of this procedure one obtains approximate analytic expressions for $\Delta^{(a)}(x, Q^2)$ and exact expressions for $\sigma^{NS}_k(x, Q^2)$. Inserting these two functions into Eq. (3.31) leads to $F^{NS}_k(x, Q^2)$. Details of this inversion method can be found in ref. 33.

IV. SINGLET SECTOR BEYOND THE LEADING ORDER

The study of next to leading order QCD corrections to the singlet structure functions is much more complicated than for the non-singlet structure functions due to the mixing between fermion singlet and gluon operators (see Eq. 2.8). The derivation of the formal and parton model like expressions can be found in refs. 3, 21, 34 and 35. Here we shall only make a few remarks.

i) The analysis of singlet contributions requires the calculation of the two-loop anomalous dimension matrix and of the one loop corrections to the fermion singlet and gluon Wilson coefficient functions $C^{\psi}_{k,n}(1, \bar{g}^2)$ and $C^{G}_{k,n}(1, \bar{g}^2)$. As in the non-singlet case, one has to take care that all these quantities are calculated in the same renormalization scheme. The full answer has been obtained in the literature again only in 't Hooft's scheme.[20,21*] The calculation of the two-loop anomalous dimension matrix performed in ref. 21 is particularly complicated and involves 0 (100) two-loop diagrams.

ii) It turns out that the formal expressions for the Q^2 evolution of the moments of singlet structure functions are very simple[34] e.g.

$$M_2^s(n, Q^2) = \delta_{\psi}^{(2)}\bar{A}_n^- \left[1 + \frac{R_{2,n}^-(Q^2)}{\beta_o \ln \frac{Q^2}{\Lambda^2}} \right] \left[\ln \frac{Q^2}{\Lambda^2} \right]^{-d_-^n}$$

$$+ \delta_{\psi}^{(2)}\bar{A}_n^+ \left[1 + \frac{R_{2,n}^+(Q^2)}{\beta_o \ln \frac{Q^2}{\Lambda^2}} \right] \left[\ln \frac{Q^2}{\Lambda^2} \right]^{-d_+^n} \qquad (4.1)$$

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*For calculations of $C^{G}_{k,n}(1, \bar{g}^2)$ in other renormalization schemes we refer the reader to refs. 22-25, and 36.

where $\bar{A}_n^{\mp}$ and Λ are the only free parameters. The next to leading order corrections turn out to be of the same order as in the non-singlet sector although their n dependence in particular at low values of n is different due to the mixing. Numerical estimates of these corrections are given in ref. 34.

iii) The equations for the Q^2 evolution of the singlet parton distributions turn out to be much more complicated than Eq. (4.1). As in the non-singlet sector they depend on the definition of parton distributions. Explicit expressions for the singlet analogs of examples a) and b) of the previous section are presented in refs. 3 and 21 respectively.

iv) Equation (3.31) is now generalized to

$$F_2^s(x, Q^2) = \int_x^1 \frac{d\xi}{\xi} \Big[\xi \Sigma(\xi, Q^2) \sigma^{\psi}_{P,2}(\frac{x}{\xi}, Q^2) + \xi\, G(\xi, Q^2) \sigma^{G}_{P,2}(\frac{x}{\xi}, Q^2) \Big] \qquad (4.2)$$

where $\Sigma(\xi, Q^2)$ and $G(\xi, Q^2)$ are singlet fermion and gluon distributions respectively. The important point is that only the sum of the two terms in Eq. (4.2) does not depend on the definition of parton distributions. The separation of F_2^s into quark and gluon contributions depends on the other hand on the definition of parton distributions. Some phenomenological applications of the singlet formulae can be found in the paper by Anderson et al.[31] and in ref. 35.

V. SEMI-INCLUSIVE PROCESSES

5.1. Massive μ-pair Production

We begin our discussion of semi-inclusive processes with the massive μ-pair production in hadron-hadron collisions. In the leading order of asymptotic freedom one sums the QCD diagrams contributing to this process to all orders in g^2 and keeps only the leading logarithms. In the course of the calculation one encounters mass singularities which are factored out and absorbed in the wave functions of the incoming hadrons.* As a result[6,38] the standard Drell-Yan formula (1.10) is reproduced with the scale independent quark distributions replaced by the Q^2 dependent quark distributions with Q^2 dependence governed by the leading order formula (1.20) and its generalization to the singlet sector.

In order to discuss the structure of the corresponding calculations in the next to leading order it is useful to take moments in τ

$$\sigma_n(Q^2) = \int d\tau\, \tau^n \frac{d\sigma}{dQ^2} \quad . \tag{5.1}$$

Formally $\sigma_n(Q^2)$ can be written as follows[8]

$$\sigma_n(Q^2) = \sum_{i,j} A^{(1)}_{n,i}(\mu^2) A^{(2)}_{n,j}(\mu^2) C^{ij}_n\left(\frac{Q^2}{\mu^2}, g^2\right) \tag{5.2}$$

where the sums run over $\bar{q}$, q and G, and the indices (1) and (2) label the incoming hadrons.

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*Since factorization of mass singularities have been discussed in other lectures at this Summer School in detail,[12] we do not demonstrate it here. Proofs of factorization of mass singularities to all orders can be found in refs. 7, 8 and 37.

The expansion in Eqs. (5.2) is analogous to the operator product expansion of Eq. (2.3). The A_n's which correspond to matrix elements of local operators in Eq. (2.3) are called cut vertices.[8] They are incalculable in perturbation theory. In more intuitive language they can be related to the moments of the parton distributions at $Q^2 = \mu^2$. The $C_n^{ij}(Q^2/\mu^2, g^2)$ are the coefficient functions in the cut vertex expansion in Eq. (5.2). They satisfy[8] renormalization group equations similar to the ones satisfied by $C_n^i(Q^2/\mu^2, g^2)$. $C_n^{ij}(Q^2/\mu^2, g^2)$ are calculable in perturbation theory. Expansion (5.2) expresses the factorization of the non-perturbative pieces (cut vertices) from the perturbatively calculable pieces (coefficient functions) just as the operator product expansion. In what follows we shall discuss the calculation of $C_n^{ij}(Q^2/\mu^2, g^2)$ in the framework of two different approaches.

Approach I[39]

We begin by expressing Eq. (5.2) in terms of parton distributions. It turns out that the cut vertices for the incoming hadrons are the same as the hadronic matrix elements of local operators which enter the discussion of deep-inelastic scattering.

Consequently the anomalous dimensions which enter the calculation of $C_n^{ij}(Q^2/\mu^2, g^2)$ are the ones which we encountered already in deep-inelastic scattering. Therefore if we put $\mu^2 = Q^2$ in Eq. (5.2) we can write

$$\sigma_n(Q^2) = \sum_{i,j} A_{n,i}^{(1)}(Q^2) A_{n,j}^{(2)}(Q^2) C_n^{ij}(1, \bar{g}^2) \tag{5.3}$$

$$\equiv \sum_{i,j} \langle f_i(Q^2)\rangle_n^{(1)} \langle f_j(Q^2)\rangle_n^{(2)} C_n^{ij}(1, \bar{g}^2) \tag{5.4}$$

where $\langle f_i(Q^2)\rangle_n$ are the moments of parton distributions (quarks and gluons) defined as in example a) of Section 3 (Eqs. (3.29, 3.30)). $C_n^{ij}(1, \bar{g}^2)$ can be interpreted in analogy with Eq. (3.32) as the moments of cross-sections for parton j - parton i scattering or annihilation with a $\mu^+\mu^-$ pair in the final state. We know already the Q^2 dependence of $\langle f_i(Q^2)\rangle_n$ with next to leading order corrections included (example a) of Sections III and IV). What remains to be done is to calculate the coefficients $C_n^{ij}(1,\bar{g}^2)$. The procedure for calculation of $C_n^{ij}(1,\bar{g}^2)$ is a straightforward generalization of the procedure used in the calculation of $C_n^{i}(1,\bar{g}^2)$ for deep-inelastic scattering which we outlined in the Appendix. Let us illustrate this procedure with $i = q$ and $j = \bar{q}$, i.e. $C_n^{q\bar{q}}(1,\bar{g}^2)$ which through order $\bar{g}^2$ has the expansion

$$C_n^{q\bar{q}}(1,\bar{g}^2) = 1 + \frac{\bar{g}^2}{16\pi^2} B_n^{q\bar{q}} \quad . \tag{5.5}$$

In order to find $B_n^{q\bar{q}}$ one considers $q\bar{q}$ annihilation in which case we have

$$\sigma_n^{q\bar{q}}(Q^2) = A^{(1)}_{n,q}(\mu^2) A^{(2)}_{n,\bar{q}}(\mu^2) C_n^{q\bar{q}}\left(\frac{Q^2}{\mu^2}, g^2\right) \quad . \tag{5.6}$$

Taking incoming quarks off-shell ($p_1^2 < 0$, $p_2^2 < 0$) one obtains in analogy with (A.2)

$$\sigma_n^{q\bar{q}}(Q^2) = 1 + \frac{g^2}{16\pi^2}\left[-\frac{1}{2}\gamma_{qq}^{0,n}\ln\frac{Q^2}{-p_1^2} - \frac{1}{2}\gamma_{qq}^{0,n}\ln\frac{Q^2}{-p_2^2} + \bar{\sigma}_n^{q\bar{q}}\right] \tag{5.7}$$

where $\bar{\sigma}_n^{q\bar{q}}$ are independent of Q^2 and p_i^2, and $\gamma_{qq}^{0,n} = \gamma_{NS}^{0,n}$. Next using Eq. (A.4)* for $A^{(1)}_{n,q}$ and $A^{(2)}_{n,\bar{q}}$ we obtain from (5.6) and (5.7), after putting $Q^2 = \mu^2$

$$B_n^{q\bar{q}} = \bar{\sigma}_n^{q\bar{q}} - 2A^{q}_{nq} \quad . \tag{5.8}$$

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* g^2 corrections to $A^{(1)}_{n,q}$ and $A^{(2)}_{n,\bar{q}}$ are equal to each other in this order and equal to g^2 corrections to the matrix elements of non-singlet operator taken between quark states (see Eq. (A.4)).

$B_n^{q\bar{q}}$ depends on the renormalization scheme through A_{nq}^q. This renormalization prescription dependence is however cancelled in the final expression (5.4) by that of $<f_i(Q^2)>_n$ if the two loop anomalous dimensions which enter Eq. (3.29) are calculated in the same scheme as A_{nq}^q. To complete the calculation of next to leading order corrections to massive μ-pair production one has to calculate

$$C_n^{qG}(1,\bar{g}^2) = \frac{\bar{g}^2}{16\pi^2} B_n^{qG} \quad . \tag{5.9}$$

As the reader may easily check in this case in an obvious notation

$$B_n^{qG} = B_n^{\bar{q}G} = \bar{\sigma}_n^{qG} - A_{nG}^q \tag{5.10}$$

with A_{nG}^q known already from the analysis of deep-inelastic scattering (Eq. A.6). In summary in this approach (dropping summation over flavors) we have

$$\begin{aligned} \sigma_n(Q^2) &= <q(Q^2)>_n <\bar{q}(Q^2)>_n \left[1 + \frac{\bar{g}^2}{16\pi^2} B_n^{q\bar{q}} \right] \\ &+ <G(Q^2)>_n \left[<q(Q^2)>_n + <\bar{q}(Q^2)>_n \right] \left[\frac{\bar{g}^2}{16\pi^2} B_n^{qG} \right] + O(\bar{g}^4) \end{aligned} \tag{5.11}$$

with $B_n^{q\bar{q}}$ and B_n^{qG} given by Eqs. (5.8) and (5.10) respectively, and the parton distributions defined as in example a) of Section III.

<u>Approach II</u>[24,25]

If the parton distributions are defined as in example b) of Sections III and IV (i.e. the next to leading order corrections to deep-inelastic scattering are (for F_2) absorbed totally in the definition of <u>quark</u> distributions) then Eq. (5.11) is replaced by

$$\sigma_n(Q^2) = \langle\tilde{q}(Q^2)\rangle_n \langle\tilde{\bar{q}}(Q^2)\rangle_n \left[1 + \frac{\bar{g}^2}{16\pi^2}\tilde{B}_n^{q\bar{q}} \right]$$

$$+ \langle\tilde{G}(Q^2)\rangle_n \left[\langle\tilde{q}(Q^2)\rangle_n + \langle\tilde{\bar{q}}(Q^2)\rangle_n \right] \left[\frac{\bar{g}^2}{16\pi^2} \tilde{B}_n^{qG} \right] \tag{5.12}$$

where the parton distributions are modified relative to those in (5.11) and $\tilde{B}_n^{q\bar{q}}$ and $\tilde{B}_n^{qG}$ are

$$\tilde{B}_n^{q\bar{q}} = B_n^{q\bar{q}} - 2B_n^q = \bar{\sigma}_n^{q\bar{q}} - 2\bar{\sigma}_n^q \tag{5.13}$$

and

$$\tilde{B}_n^{qG} = B_n^{qG} - B_n^G = \bar{\sigma}_n^{qG} - \bar{\sigma}_n^G \quad . \tag{5.14}$$

B_n^q, B_n^G, $\bar{\sigma}_n^q$ and $\bar{\sigma}_n^G$ are defined in the Appendix. We observe that in this approach the A_n's do not need to be calculated to obtain $\tilde{B}_n^{q\bar{q}}$ and $\tilde{B}_n^{qG}$. Furthermore contrary to the previous approach, the parton distributions and parton cross-sections ($\tilde{B}_n^{q\bar{q}}$, $\tilde{B}_n^{qG}$) are separately renormalization and regularization scheme* independent. It should be however stressed that in both (5.11) and (5.12) only $\sigma_n(Q^2)$ is independent of the definition of parton distributions. The separation of $\sigma_n(Q^2)$ into qq and qG term depends as we have seen on the definition of parton distributions used. In particular the qG term and the next to leading order corrections to the Q^2 evolution of quark distributions in the $q\bar{q}$ term depend on each other.

As yet nobody has presented the full numerical calculation of next to leading order corrections to $\sigma_n(Q^2)$.** Numerical estimates of the separate corrections to

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*If the same regularization schemes are used for $\bar{\sigma}_n^{q\bar{q}}$ and $\bar{\sigma}_n^q$, and $\bar{\sigma}_n^{qG}$ and $\bar{\sigma}_n^G$. This regularization scheme independence has been recently questioned by Humpert and Van Neerven.[41]

**In all papers on higher order corrections to Drell-Yan process the leading order formulae for the Q^2 evolution of parton distributions have been used.

parton cross-sections are however known. In particular the parameters $\tilde{B}_n^{q\bar{q}}$ and $\tilde{B}_n^{q\bar{q}}$ obtained by various groups[24,25,39,40,41] are so large that the authors conclude that at present values of Q^2 the perturbative calculations cannot be trusted. Further details can be found in refs. 24, 25, 39, 40.

5.2. Processes Involving Fragmentation Functions[8,42,43,44]

Here we shall comment briefly on the processes (1.2), (1.3) and (1.5). We begin with the semi-inclusve e^+e^- annihilation. The expression for the moments of $\sigma_h^{e^+e^-}(z, Q^2)$ which we denote by $\sigma_h^{e^+e^-}(n, Q^2)$ can be written formally as[8]

$$\sigma_h^{e^+e^-}(n, Q^2) = \sum_i V_n^i(\mu^2)\tilde{C}_n^i\left(\frac{Q^2}{\mu^2}, g^2\right) \tag{5.15}$$

$$\equiv \sum_i V_n^i(Q^2)\tilde{C}_n^i(1, \bar{g}^2) \quad i = q, \bar{q}, G \quad . \tag{5.16}$$

Equation (5.15) is the analogue of Eqs. (2.3) and (5.2) with $V_n^i(\mu^2)$ being called time-like cut vertices.[8] $\tilde{C}_n^i(Q^2/\mu^2, g^2)$ are the corresponding coefficient functions. In analogy with Eq. (5.4), $V_n^i(Q^2)$ can be interpreted as the moments of Q^2 dependent fragmentation functions and $\tilde{C}_n^i(1, \bar{g}^2)$ as the moments of the cross-section for the production of parton i in e^+e^- annihilation. Inverting the moments one obtains Eq. (1.14). The structure of next to leading order QCD corrections to the process in question is similar to that in deep-inelastic scattering. The questions of renormalization prescription dependence of separate quantities entering (5.16) and the freedom in defining fragmentation functions beyond the leading order also arise here. The full study of next to leading order corrections to the process above has not yet been completed. Missing is the calculation of two-loop anomalous dimensions of time-like cut vertices. This implies that we do not know at present the full next to leading order corrections to the Q^2 evolution of fragmentation functions, which enter the QCD formulae for processes (1.2), (1.3) and (1.5). What we know however are the relevant parton cross-sections which contribute to the processes in question. They have been most extensively studied by the authors of ref. 43. The calculations of next to leading order corrections to parton cross-sections relevant for the processes (1.2) and (1.3) can also be found in refs. 42 and 44. The method used in these calculations is the extension of the approach II discussed previously to processes involving fragmentation functions. One defines fragmentation functions by absorbing all next to leading order QCD corrections to $e^+e^- \to h$ + anything into quark fragmentation functions. The cross-sections for the processes (1.3) and (1.5) can then be expressed through so defined fragmentation functions and parton distributions which we discussed previously. The form of the resulting expression for $eh_1 \to eh_2$ + anything is shown in Eq. (1.15). A similar equation exists for $e^+e^- \to h_1 + h_2$ + anything with $f_j^{h_1}$ replaced by $D_j^{h_1}$ and $\tilde{\sigma}_P^{jk}$ by parton cross-sections for production of partons j and k in e^+e^- annihilation. The latter parton cross-section we shall denote by $\hat{\sigma}_P^{jk}$. As in the case of massive μ-pair production $\tilde{\sigma}_P^{jk}$ and $\hat{\sigma}_P^{jk}$ calculated in the approach in question are regularization and renormalization scheme independent.

Let us enumerate some properties of $\tilde{\sigma}_P^{jk}$ and $\hat{\sigma}_P^{jk}$:

i) The $\bar{g}^2$ corrections to these cross-sections are only large at kinematical boundaries.

ii) There is a breakdown of factorization in x and z in $\tilde{\sigma}_P^{jk}$ which is of order 10-20% for small and moderate x and z but larger when both x and z are large.

iii) The authors of ref. 43 have found the following relations between $\tilde{\sigma}_P^{jk}$ and $\hat{\sigma}_P^{jk}$

$$\tilde{\sigma}_P^{jk}\left(\frac{1}{x}, z, Q^2\right) = -\frac{1}{x}\hat{\sigma}_P^{jk}(x, z, Q^2) \qquad j,k = q,\bar{q},G \quad . \tag{5.17}$$

Equation (5.17) is analogous to the parton model and leading order relations connecting deep-inelastic structure functions and structure functions for $e^+e^- \to h$ + anything, which have been proposed by Gribov and Lipatov[45] and Drell, Levy and Yan.[46] It should be remarked that the relations like (5.17) are shown to hold only for properly defined parton cross-sections and are not expected to hold beyond the leading order for parton distributions and parton fragmentation functions themselves.

For further details related to the processes (1.2), (1.3) and (1.5) we refer the interested reader to refs. 42.-44.

VI. OTHER HIGHER ORDER CALCULATIONS

There are a few recent higher order calculations which we did not discuss in the previous section. We shall comment here on them very briefly.

a) Violations of Parton Model Sum Rules

Gross-Llewellyn-Smith relation and Bjorken Sum Rule are violated beyond the leading order as follows[20,24]

$$\int_0^1 dx \left[F_3^{\bar{\nu} p} + F_3^{\nu p} \right] = 6 \left[1 - \frac{12}{(33 - 2f) \ln \frac{Q^2}{\Lambda^2}} \right] \tag{6.1}$$

$$\int_0^1 dx \left[F_1^{\bar{\nu} p} - F_1^{\nu p} \right] = 1 - \frac{8}{(33 - 2f) \ln \frac{Q^2}{\Lambda^2}} \quad . \tag{6.2}$$

This leads to 20% and 10% corrections respectively at values of $Q^2 \sim O(10\ \mathrm{GeV}^2)$.

There are also corrections to Callan-Gross relation[47] which turn out to be much smaller than the violation of this relation seen in the data.

b) Higher Order Corrections to Photon-Photon Scattering

The process $\gamma + \gamma \to$ hadrons can be measured in $e^+e^- \to e^+e^- +$ hadrons.[48] When one photon has large Q^2 and the other is close to its mass-shell the photon-photon process can be viewed as deep-inelastic scattering on a photon target. It turns out that due to the point-like character of the photon the dominant contribution to photon-photon scattering at large Q^2 can be exactly calculated in QCD with the result

$$\int_0^1 dx\, x^{n-2} F_2^\gamma(x, Q^2) = \alpha^2 \left[a_n \ln \frac{Q^2}{\Lambda^2} + \tilde{a}_n \ln\ln \frac{Q^2}{\Lambda^2} + b_n + O\left(\frac{1}{\ln \frac{Q^2}{\Lambda^2}} \right) \right] \quad . \qquad (6.3)$$

Here F_2^γ is the photon structure function. a_n have been calculated in ref. 49. The $\tilde{a}_n$ and b_n have been obtained in ref. 50. The exact values of b_n depend on the definition of $\bar{g}^2(Q^2)$ or equivalently Λ. Taking the values of Λ extracted from deep-inelastic scattering in the same scheme for $\bar{g}^2(Q^2)$ in which b_n's are calculated,[50] one can make definite predictions about the moments of Eq. (6.3). The corrections turn out to be slightly bigger than corresponding corrections in deep-inelastic scattering and suppress F_2^γ at large values of x. In the $\overline{MS}$ scheme b_n's are negative but in the MOM scheme they are positive. However when the corresponding values of Λ($\Lambda_{\overline{MS}} = 0.5$, $\Lambda_{MOM} = 0.85$ GeV) are inserted in Eq. (6.3) the same predictions[3] are obtained for $F_2^\gamma(x, Q^2)$. Bigger corrections are found in the MS scheme.

c) α_s^2 Corrections to $e^+e \to$ annihilation

Recently α_s^2 corrections to $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ have been calculated with the result:[51]

$$R = 3\,\Sigma Q_i^2 \left[1 + \frac{\alpha_s}{\pi} + A\left(\frac{\alpha_s}{\pi}\right)^2 \right] \qquad (6.4)$$

where (for four flavors) A = 5.6, 1.5 and -1.7 for MS, $\overline{\mathrm{MS}}$ and MOM schemes respectively. Taking the corresponding values for Λ from deep-inelastic scattering we observe that α_s^2 corrections to R are small.

d) Large QCD corrections to $Q\bar{Q} \to 2$ gluon decay have been reported by the authors of ref. 52.

e) Higher order corrections to the polarized electroproduction structure functions have been calculated in ref. 53.

f) qq contributions (order $\bar{g}^4(Q^2)$) to massive muon production have been calculated in ref. 54. They turn out to be small.

g) Finally the next to leading order corrections to large $p_\perp$ processes are being performed.[55]

VII. SUMMARY

In these review lectures we have discussed higher order QCD predictions for inclusive deep-inelastic scattering. We have also presented the basic structure of QCD formulae and the methods of corresponding calculations for semi-inclusive processes. We have seen that the structure of QCD formulae with higher order corrections taken into account is fairly complicated and involves many features not encountered in the leading order. These new features include:

i) gauge and renormalization-prescription dependences of separate elements of the physical expressions;

ii) well-defined dependence of the functional form of the explicit higher order corrections on the definition of $\bar{g}^2(Q^2)$ or, equivalently, on Λ;

iii) freedom in the definition of parton distributions and parton fragmentation functions beyond the leading order approximation.

These features have to be kept in mind when carrying out calculations to make sure that various parts of the higher order calculations are compatible with each other. Only then can a physical result be obtained which is independent of gauge, renormalization scheme, particular definition of $\bar{g}^2(Q^2)$, and particular definition of the parton distributions.

We have seen that the higher order corrections are quite large and, moreover, that there are some indications for their presence in the deep-inelastic scattering data. This is most clearly seen in the n-dependence of the parameter Λ_n extracted from the data on the basis of the leading order formulae. This n-dependence agrees well with that obtained from higher order calculations.

In some processes such as massive μ-pair production the next to leading order corrections turn out to be too large at present values of Q^2 that the perturbative calculations could be trusted. This is also the case of η_c decay. We think these processes deserve further study.

The calculation of two-loop anomalous dimensions to the Q^2 evolution of fragmentation functions is very desirable. Also more phenomenology of next to leading order corrections to various processes should be done.

In our lectures we have not discussed the mass effects and higher twist operators effects, which at values of Q^2 of 0 (5 GeV^2) are of some importance. They have been recently discussed in refs. 30 and 56 which the interested reader may consult. At low values of Q^2 also nonperturbative effects should be taken into account.

In spite of the fact that there is still much to be done, both theoretically and phenomenologically, we believe that a lot of progress has been done in the past few years in understanding QCD effects in the inclusive and semi-inclusive processes and we are looking forward to the summer schools next year when surely more progress will be reported.

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APPENDIX Procedure for the Calculation of $B^{NS}_{k,n}$

We first notice that in order to find $B^{NS}_{k,n}$ as defined in Eq. (3.3) it is sufficient to calculate $C^{NS}_{k,n}(Q^2/\mu^2, g^2)$ in perturbation theory to order g^2 and put $Q^2 = \mu^2$. This is obvious from Eqs. (2.9) and (2.10). In order to calculate $C^{NS}_{k,n}(Q^2/\mu^2, g^2)$ in perturbation theory we calculate first the virtual Compton amplitude for quark photon scattering. Taking the incoming quarks slightly off-shell ($p^2 < 0$) in order to regulate mass-singularities we obtain for $T_k(Q^2,\nu)$

$$T_k(Q^2,\nu) \equiv \sum_n \frac{1}{x^n} \sigma^q_{k,n}\left(\frac{Q^2}{p^2}, g^2\right) \tag{A.1}$$

with*

$$\sigma^q_{k,n}\left(\frac{Q^2}{p^2}, g^2\right) \equiv \sigma^q_n\left(\frac{Q^2}{p^2}, g^2\right) = 1 + \frac{g^2}{16\pi^2}\left[-\frac{1}{2}\gamma^{0,n}_{NS} \ln\frac{Q^2}{-p^2} + \bar{\sigma}^q_n\right] \tag{A.2}$$

where $\bar{\sigma}^q_n$ are constant terms and $x = Q^2/(2p\cdot q)$. The diagrams contributing to σ^q_n in order g^2 are shown in Fig. 6. Now in accordance with the operator product expansion

$$\sigma^q_n\left(\frac{Q^2}{p^2}, g^2\right) = C^{NS}_n\left(\frac{Q^2}{\mu^2}, g^2\right) A^{NS}_n\left(\frac{p^2}{\mu^2}, g^2\right) \tag{A.3}$$

where $C^{NS}_n(Q^2/\mu^2, g^2)$ is the same coefficient function as in Eq. (2.5) but $A^{NS}_n(p^2/\mu^2, g^2)$ are the matrix elements of non-singlet operator sandwiched between quark states instead of proton states as in Eq. (2.5). The coefficient functions are the same as before because they <u>do not</u> depend on the state between

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*We drop the index k and the charge factor δ^k_{NS} to simplify notation.

which the operator product expansion is sandwiched. $A_n^{NS}(p^2/\mu^2, g^2)$ can be evaluated in perturbation theory (see diagrams in Fig. 7) with the result*

$$A_n^{NS}\left(\frac{p^2}{\mu^2}, g^2\right) = 1 + \frac{g^2}{16\pi^2}\left[\frac{1}{2}\gamma_{NS}^{0,n}\ln\frac{-p^2}{\mu^2} + A_{nq}^q\right] \tag{A.4}$$

where A_{nq}^q are independent of p^2. Combining Eqs. (A.2)-(A.4) and using (3.3) we obtain

$$B_n^{NS} = B_n^q = \bar{\sigma}_n^q - A_{nq}^q \quad . \tag{A.5}$$

The point is that A_{nq}^q depends on the renormalization scheme used to render finite the result of the calculation of diagrams in Fig. 7. In so-called $p^2 = -\mu^2$ subtraction schemes A_{nq}^q is put to zero. In the 't Hooft's scheme it is non-zero. The evaluation of the g^2 corrections to the gluon coefficient function proceeds in a similar way with the result

$$C_n^G(1, \bar{g}^2) = \frac{\bar{g}^2}{16\pi^2} B_n^G \quad ; \quad B_n^G = \bar{\sigma}_n^G - A_{nG}^q \tag{A.6}$$

where $\bar{\sigma}_n^G$ is obtained by calculating photon-gluon scattering to order g^2 and A_{nG}^q is the constant piece in the matrix element of quark operator between gluon states. Eq. (A.6) is explicitly derived in refs. 3, 20. Details of the calculations using this method can be found for instance in refs. 3 and 20.

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*Generally A_{nj}^i are the constant pieces in order g^2 of the matrix element of operator O_i sandwiched between j state.

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n	$B_{2,n}$	$\bar{B}_{2,n}$	Z_n^{NS}
2	7.39	0.44	1.65
4	19.70	6.07	2.05
6	28.77	11.18	2.16
8	35.96	15.53	2.25

Table 1. Numerical values of the parameters $B_{2,n}$, $\bar{B}_{2,n}$ and Z_n^{NS} (see Eqs. (3.15), (3.17) and (3.5) respectively) for various values of n and $f = 4$. The values of Z_n^{NS} are obtained on the basis of results of ref. 19 and correspond to minimal subtraction scheme.

n	Scheme	Q^2 [GeV^2] 5	10	50	200
	$\overline{\text{MS}}$	0.97	0.96	0.95	0.95
2	MOM	0.68	0.73	0.80	0.83
	MS	1.20	1.15	1.09	1.07
	$\overline{\text{MS}}$	1.10	1.05	0.99	0.97
4	MOM	0.63	0.65	0.71	0.75
	MS	1.53	1.42	1.26	1.20
	$\overline{\text{MS}}$	1.24	1.15	1.05	1.01
6	MOM	0.69	0.68	0.71	0.74
	MS	1.79	1.62	1.40	1.30
	$\overline{\text{MS}}$	1.37	1.25	1.11	1.06
8	MOM	0.79	0.74	0.73	0.75
	MS	2.00	1.79	1.52	1.39

Table 2. The values of the quantity $1 + \dfrac{R^{NS}_{2,n}(Q^2)}{\beta_o \ln \frac{Q^2}{\Lambda^2}}$ as a function of n and Q^2 in various schemes: $\overline{\text{MS}}$ ($\overline{\Lambda}$ = 0.5 GeV), MOM (Λ = 0.85 GeV) and MS (Λ = 0.4 GeV).

FIGURE CAPTIONS

Fig. 1: Parton model diagrams for processes (1.1)-(1.5).

Fig. 2: Parton model rules.

Fig. 3: Illustration of the r.h.s. of Eqs. (1.13), (1.14) and (1.15). The sums run over quarks and gluons. The circles stand for parton distributions or parton fragmentation functions. The squares denote the parton cross-sections.

Fig. 4: The effective coupling constant $\bar{\alpha}(Q^2)$ as extracted from the BEBC data for the leading order (L.O.), MS scheme, $\overline{\text{MS}}$ scheme and momentum subtraction scheme MOM.

Fig. 5: Experimental Λ_n values obtained by Duke and Roberts using the data of BEBC (open box), CDHS (open diamond) and the entire SLAC data. The solid line is the QCD prediction of Eq. (3.26).

Fig. 6: Diagrams entering the calculation of $\sigma^q_{k,n}(Q^2/p^2, g^2)$ of Eq. (A.2).

Fig. 7: Diagrams entering the calculation of $A_n^{NS}(p^2/\mu^2, g^2)$ of Eq. (A.4).

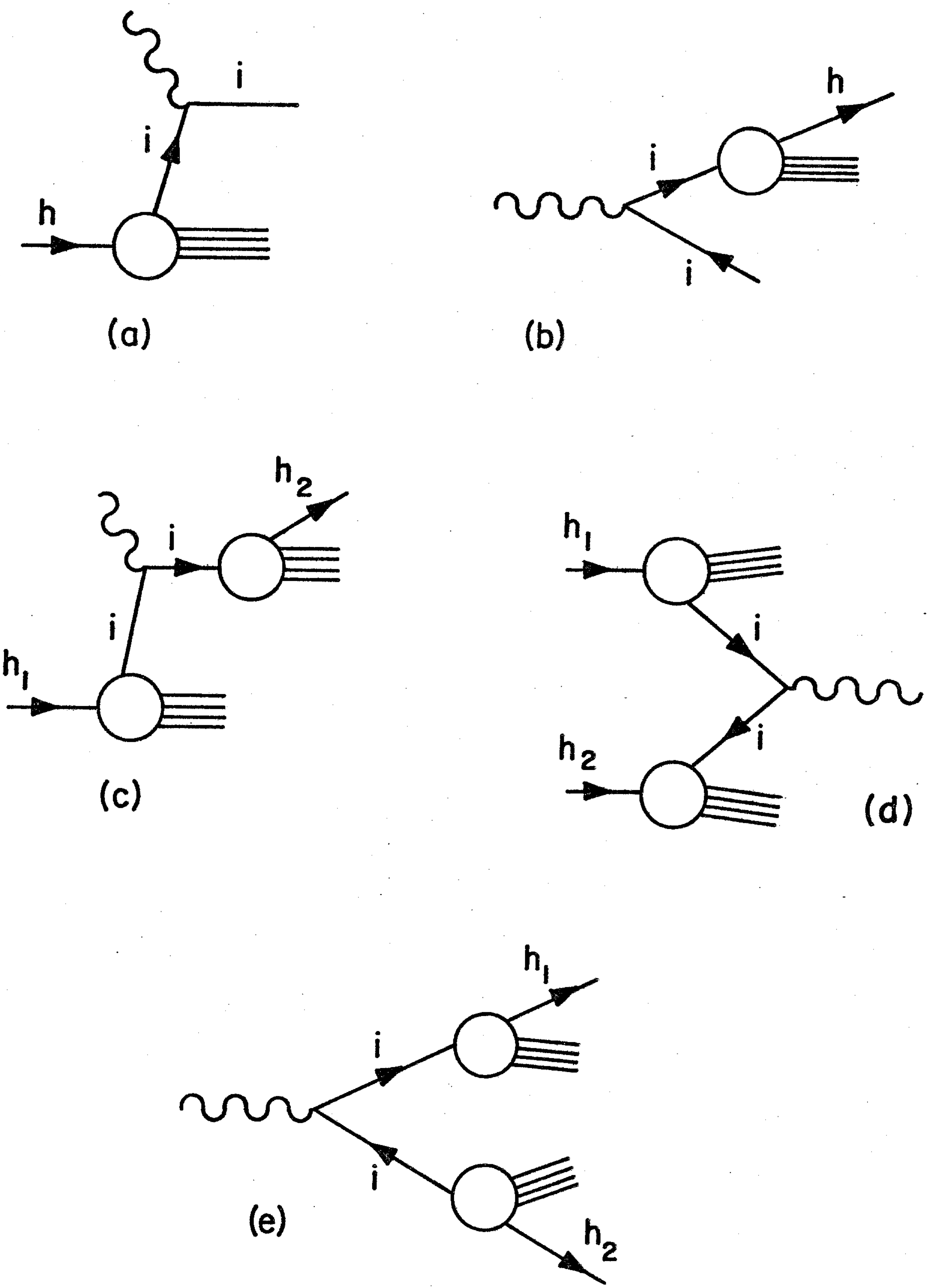


Fig. 1

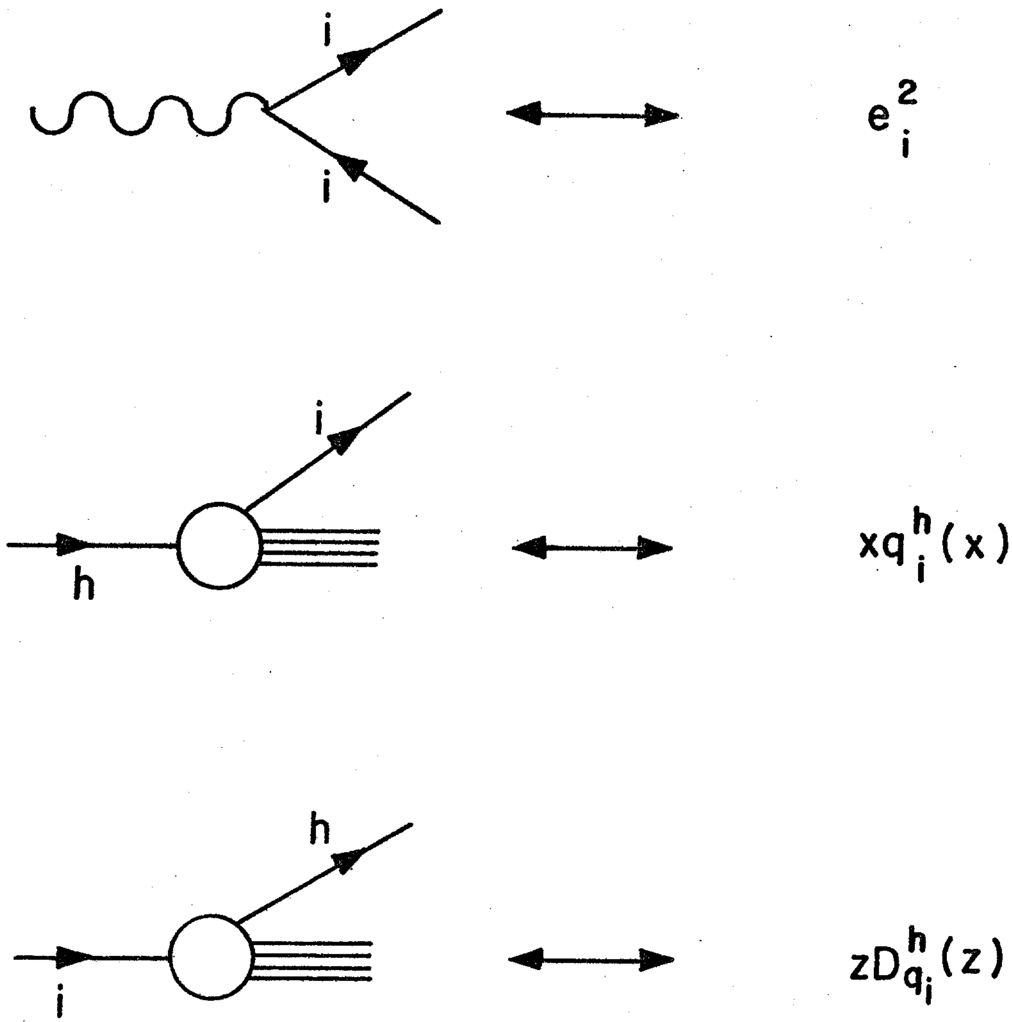
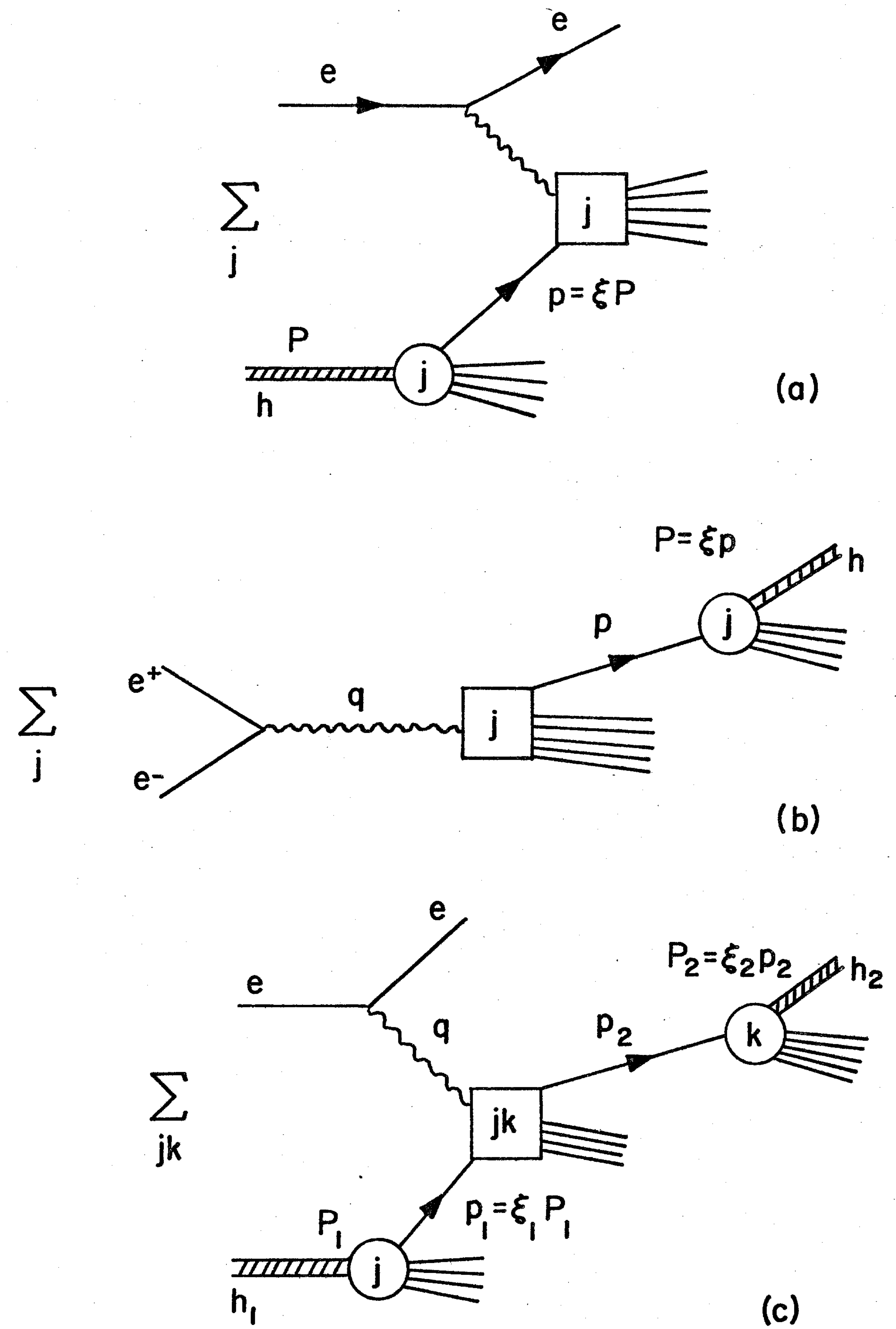
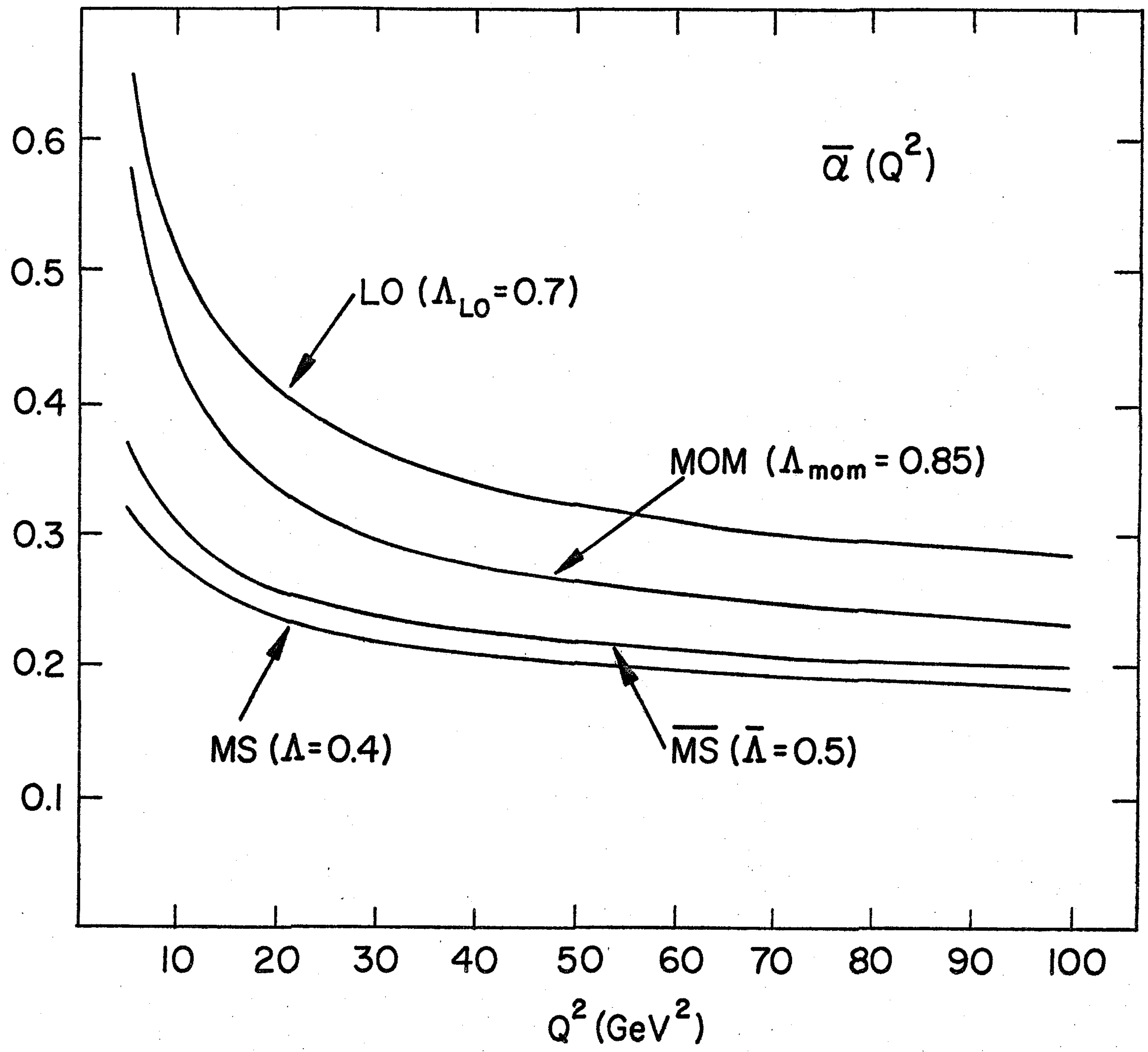


Fig. 2



e
e
Σ j
j
p = ξ P
P
j
h
(a)
P = ξ p
h
p
j
Σ j
e+
q
j
e−
(b)
e
P2 = ξ2 p2
e
h2
q
p2
k
Σ jk
jk
P1
p1 = ξ1 P1
j
h1
(c)

Fig. 3



$\bar{\alpha}(Q^2)$
LO (Λ_{LO}=0.7)
MOM (Λ_{mom} = 0.85)
MS (Λ=0.4)
$\overline{MS}$ ($\bar{\Lambda}$=0.5)
0.6
0.5
0.4
0.3
0.2
0.1
10
20
30
40
50
60
70
80
90
100
Q^2 (GeV^2)

Fig. 4

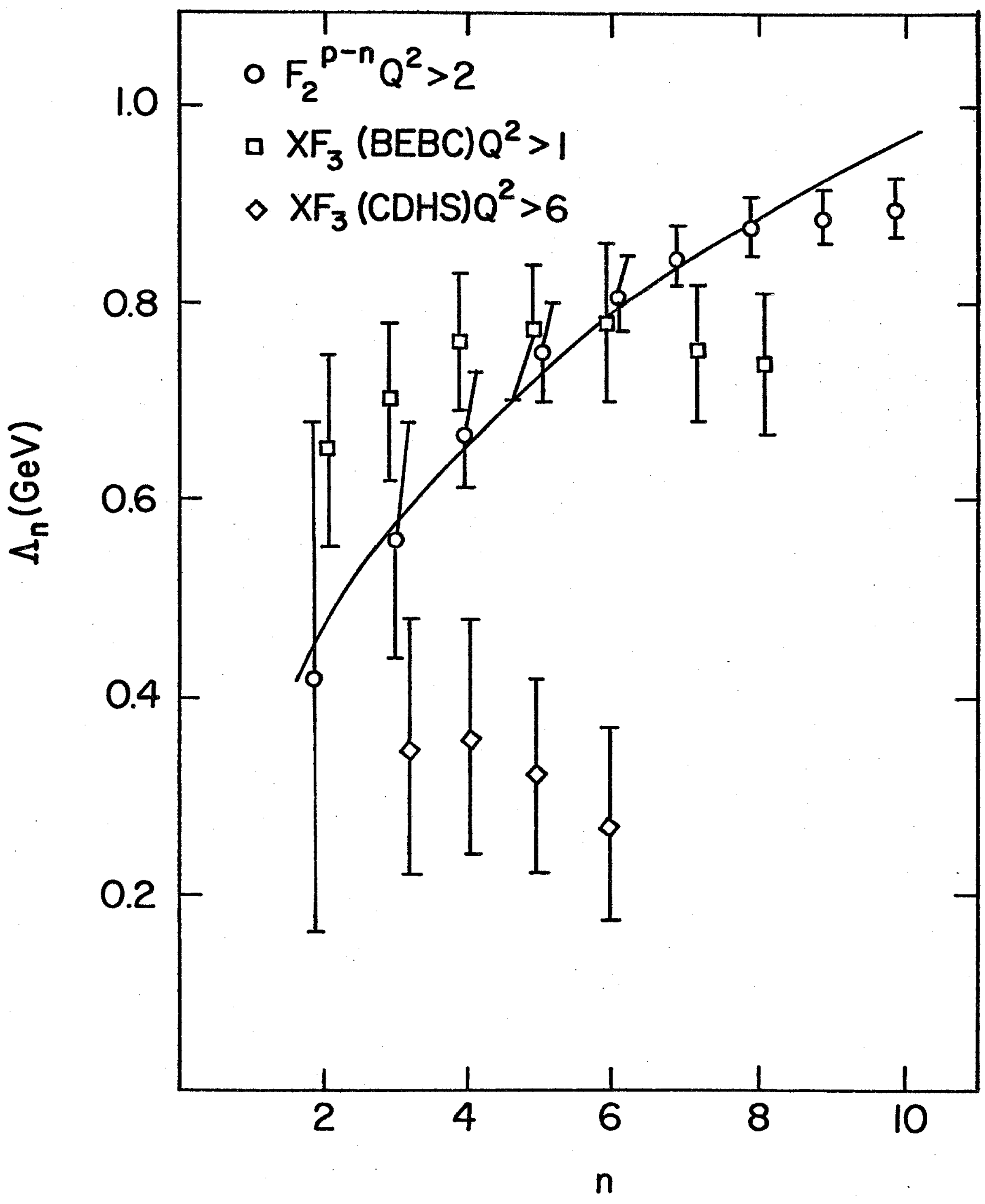

F_2^{p-n} $Q^2>2$
XF_3 (BEBC) $Q^2>1$
XF_3 (CDHS) $Q^2>6$
Λ_n (GeV)
1.0
0.8
0.6
0.4
0.2
2
4
6
8
10
n

Fig. 5

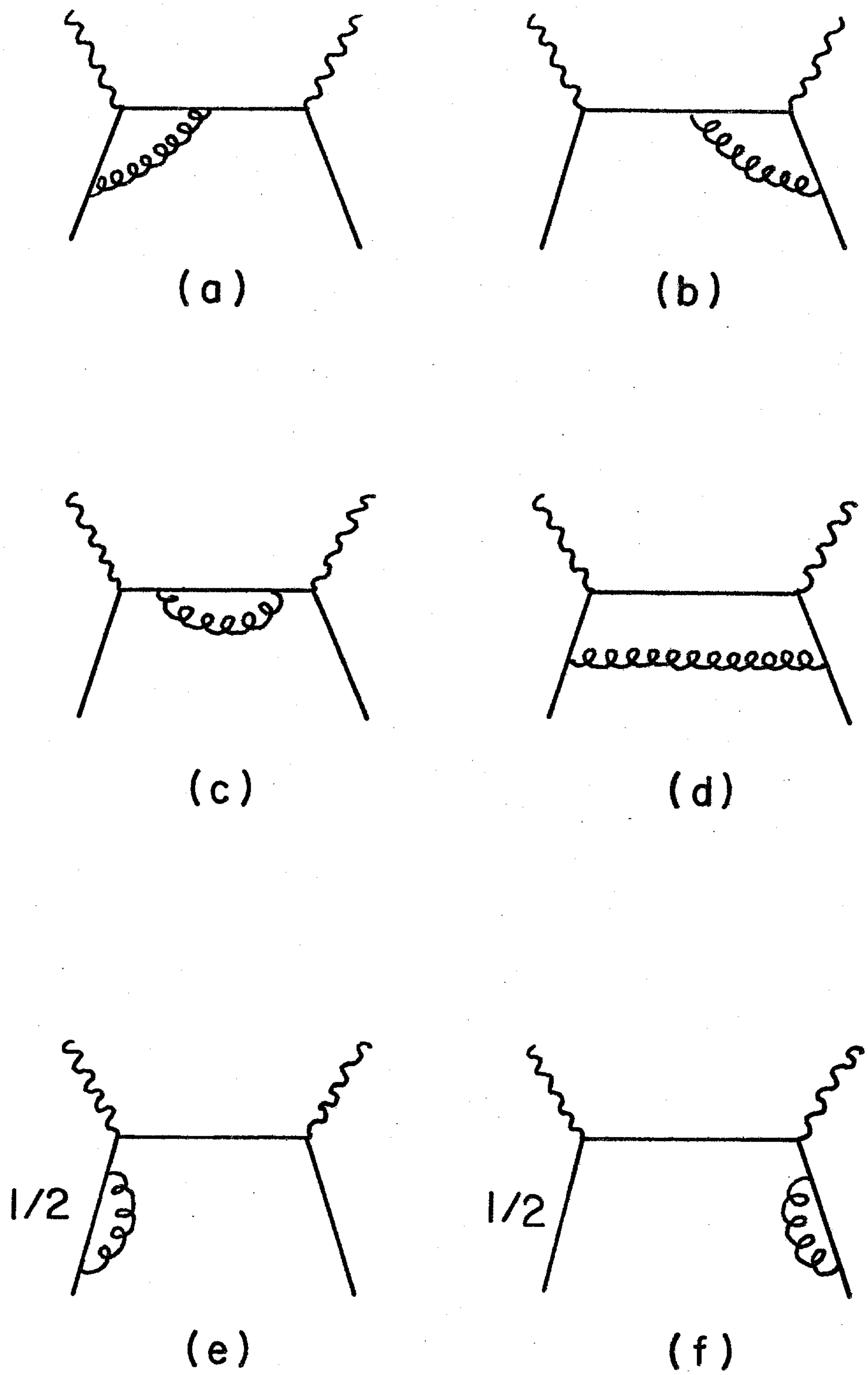


Fig. 6

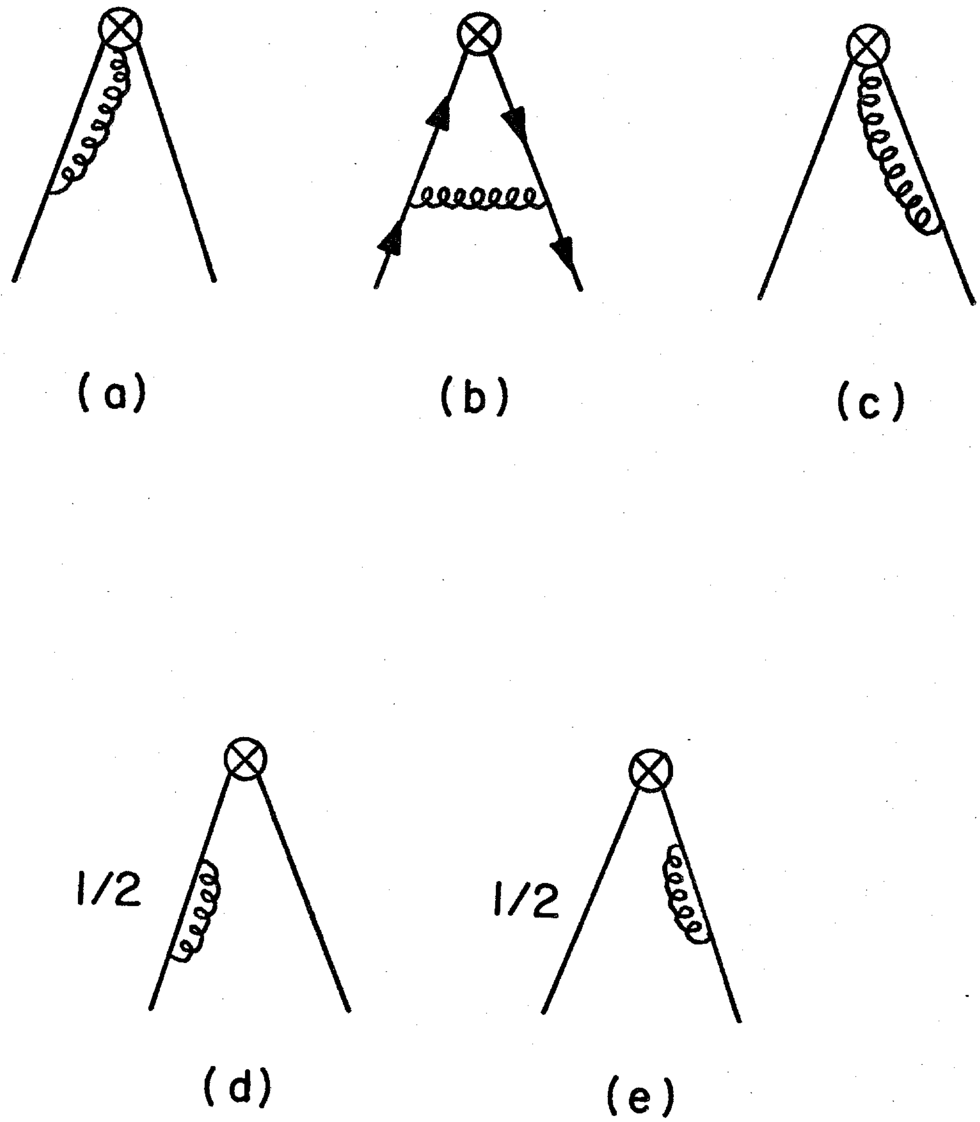


Fig. 7