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Toward a More Profound Theory of Electromagnetic Interactions

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ABSTRACT

In this paper, the new gauge formulation of the electromagnetic interaction theory, containing the "fundamental length" ℓ as a universal scale like $\hbar$ and c, is worked out. If an interaction is switching off, the resulting free theory, written in terms of momenta, happens to be formulated in the 4-dimensional de Sitter p-space, with the curvature radius $\hbar/\ell c$. On one hand, it means that the configurational space of one particle can be treated as a quantized manifold, the size of granularities being $\sim\ell$. On the other hand, due to 3-dimensionality of the mass shell $p^2 = m^2$, such a scheme is equivalent to the conventional free theory, based on the concept of the Minkowskian 4-momentum.

In the new approach the electromagnetic potential becomes a 5-vector associated with de Sitter group O(4,1). The extra fifth component, called the τ-photon, similar to scalar and longitudinal photons, does not correspond to an independent dynamical degree of freedom. Respectively, the new local gauge group is larger than the ordinary one and depends intrinsically on the fundamental length ℓ.

The gauge invariant equations of motion, replacing the Dirac-Maxwell equations, are set up in the framework of an appropriate Lagrangian formalism. The new formulation is minimal with respect to the 5-potential but is not so in terms of the usual 4-potential. As a result, the underlying physics looks much richer than the ordinary electromagnetic phenomena. The new scheme predicts the

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existence of the electric dipole moments for charged particles, leading to a direct violation of P- and CP-symmetries, and the new universal correction to the (g - 2)-anomaly. Further, some new group of internal symmetry, $SU_\tau(2)$, arises that can be used to describe the μe-symmetry of the electromagnetic interactions. It turns out that $SU_\tau(2)$-symmetry is violated by the 4-fermion type interaction, induced by τ-photons, with associated coupling constant $\sim\alpha\ell^2$. This novel interaction might give rise to the μe-mass difference and processes like $\mu \to 3e$, $\mu \to e\gamma$, etc.

In the limit $\ell \to 0$, the new field equations turn into the Dirac-Maxwell equations for the electron, muon, and electromagnetic fields. So, one may consider our approach as a generalization in a profound way of the standard theory of electromagnetic interactions at small distances $\lesssim \ell$ (high energies $\gtrsim 1/\ell$).

The upper bound for the fundamental length ℓ is discussed taking into account the various experimental data.

I. INTRODUCTION

In this paper we shall discuss a generalization of the theory of electromagnetic interactions which is based on a concept of fundamental length. This new hypothetical constant we denote as ℓ. Together with $\hbar$ and c it is expected to regulate all microscopic phenomena. The quantity

$$M = \hbar/\ell c \tag{1.1}$$

is called the fundamental mass.

The idea of the existence of a new universal length, and therefore mass, that would fix a scale in 4-dimensional space-time, and therefore 4-dimensional momentum-energy space, was discussed in the literature of the last four decades in different contexts.[1-14]

Most of the people who tried to introduce a fundamental length into field theory, pursued a quite clear and practical goal: to cure the theory from the ultraviolet divergences. But it turned out that a theory can survive with this chronic disease, and work as a quantitative scheme, if it possesses genetically a renormalizability property. Nowadays, the principle of renormalizability has imperceptibly become one of the corner stones of the quantum field theory. As a result, interest in a fundamental length has almost died out (see, however, [15-18]).

The greatest triumph of the renormalization approach to the formulation of the quantum field theory is certainly quantum electrodynamics (QED). The predictions of QED agree with a number of highly precise experiments. An upper bound for the magnitude of the fundamental length, established in experiments on the test of QED at high energies, now is given by

$$\ell \lesssim 10^{-15}\ \mathrm{cm} \qquad . \tag{1.2}$$

The harmony and elegance of QED makes an impression which cannot be darkened even by the obviously algorithmic character of the renormalization procedure. It should be clear that the fundamental length hypothesis is first of all a challenge to contemporary QED. In other words, this hypothesis can survive only if it will lead naturally to modifying QED in a <u>profound</u> way. This would, of course, preserve or enhance its aesthetic appeal.

The crucial advantage of QED is that the form of the interaction in this theory is dictated by gauge symmetry arguments. It is called the minimal interaction principle and is symbolized by the following substitution law:

$$p_\mu \to p_\mu - e_o A_\mu(x) \qquad . \tag{1.3}$$

This substitution leads one to the inhomogeneous Dirac-Maxwell equations for "bare" fields

$$(i\partial\!\!\!/ - e_0 A\!\!\!/(x) - m_0)\psi(x) = 0 \tag{1.4a}$$

$$\frac{\partial F^{\mu\nu}(x)}{\partial x^{\nu}} = e_0 \overline{\psi}(x)\gamma^{\mu}\psi(x) \tag{1.4b}$$

where the field strengths are defined as

$$F^{\mu\nu}(x) = \frac{\partial A^{\mu}(x)}{\partial x_{\nu}} - \frac{\partial A^{\nu}(x)}{\partial x_{\mu}} \quad . \tag{1.5}$$

Let us mention, to be complete, that local gauge transformations of the fields $\psi(x)$, $\overline{\psi}(x)$ and $A(x)$ leaving Eqs. (1.4a)-(1.4b) invariant are given by:

$$\psi(x) \to e^{ie_0\lambda(x)}\psi(x)$$

$$\overline{\psi}(x) \to e^{-ie_0\lambda(x)}\overline{\psi}(x) \tag{1.6a}$$

$$A_{\mu}(x) \to A_{\mu}(x) - \frac{\partial\lambda(x)}{\partial x^{\mu}} \quad ; \quad \lambda(x) = \lambda^{\dagger}(x) \quad . \tag{1.6b}$$

The rule (1.3) does not contain any scale like ℓ or M and for this reason is universally applied to <u>all</u> space-time intervals and to <u>all</u> values of 4-momenta.* Therefore, if one adopts the fundamental length hypothesis it means that in the domains

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*In p-representation the hermiticity condition of λ-function, evidently, becomes

$$\lambda^{\dagger}(p) = \lambda(-p) \quad . \tag{1.6c}$$

Excluding the constraint (1.6c), the function $\lambda(p)$ is completely <u>arbitrary</u> in the Minkowskian p-space.

$$|x| \lesssim \ell \tag{1.7a}$$

$$|p| \gtrsim M \tag{1.7b}$$

the substitution law (1.3) and its consequences are probably invalid or incomplete.

Let us consider just one consequence of (1.3). Choosing $e_o A_\mu = \text{const} = k_\mu$ we obtain, obviously, zero field strengths: $F_{\mu\nu} = 0$. But the corresponding substitution (1.3) is not yet an identity transformation, namely

$$p_\mu \rightarrow p_\mu - k_\mu \quad . \tag{1.8}$$

This is a pseudoeuclidean parallel shift transformation of the 4-dimensional p-space, testifying that a geometry of this space is a Minkowskian one.*

One may conclude now that our fundamental length hypothesis challenges the Minkowskian structure of the momentum 4-space in the region (1.7b). But if the momentum 4-space is not everywhere pseudoeuclidean, then what is a reasonable

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*Let us point out in this connection, that the condition $e_o A_\mu = \text{const.}$, due to (1.6b), admits only those functions $\lambda(x)$ that are linear in x:

$$\lambda(x) = \lambda_o + (qx) \quad .$$

It gives for $\psi(x)$ and $\overline{\psi}(x)$:

$$\psi(x) \rightarrow e^{ie_o\lambda_o} e^{iqx} \psi(x)$$

$$\overline{\psi}(x) \rightarrow e^{-ie_o\lambda_o} e^{-iqx} \overline{\psi}(x) \quad .$$

Up to an unimportant phase factor this is the transformation law of the wave functions under translations (1.8) with $k_\mu = q_\mu$.

alternative? According to a general geometrical classification, the (pseudo)-euclidean spaces are those with zero curvature. Their closest neighbors are spaces with nonzero constant curvature. In the present 4-dimensional case, these curved spaces are so-called "de Sitter spaces."

Let us try to impose on 4-momentum space the de Sitter geometry realized on the one-sheeted 5-hyperboloid

$$p_o^2 - p_1^2 - p_2^2 - p_3^2 - M^2 p_4^2 = -M^2 \quad . \tag{1.9}$$

The curvature radius M we identify with the fundamental mass (1.1), assuming that this quantity is large enough (cf. (1.2)). Note that Eq. (1.9) places no constraint on timelike 4-momenta, and it is therefore not in conflict with the construction of Fock space and Poincaré invariance of the S-matrix.*

Since the mass shell hyperboloids

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*Besides (1.9), there exist only one more de Sitter space satisfying the correspondence principle at $M \to \infty$, namely

$$p_o^2 - p_1^2 - p_2^2 - p_3^2 + M^2 p_4^2 = M^2 \quad .$$

But in this geometry we are faced with the universal upper bound for time-like momenta

$$p_o^2 - \vec{p}^2 \leq M^2$$

which is inconsistent with the implementation of a unitary representation of the Poincaré group on Fock space.

$$p^2 = m_1{}^2$$

$$p^2 = m_2{}^2$$

$$\cdots$$

can be equally well embedded into de Sitter p-space (1.9) or in flat Minkowskian p-space, free physical particles cannot distinguish these two geometries. Actually, only virtual (interacting) particles can probe the geometrical structure of 4-momentum space.

In the "flat limit," i.e., in the region of small virtual momenta

$$|p| \ll M \qquad , \tag{1.10}$$

one can neglect the curvature of de Sitter p-space, and therefore the new formalism reduces to the ordinary theory. In this domain the parallel shift (1.8) is, up to terms of order $1/M^2$, a symmetry transformation of de Sitter momentum space (1.9).

For virtual momenta belonging to the region (1.7b), the curvature of de Sitter p-space becomes a crucial factor. It means that the old (Minkowskian) and new (de Sitterian) formalisms should lead to quite different descriptions of particle interactions at small space-time intervals.

A general approach to the construction of quantum field theory on the basis of de Sitter p-space has been put forward and investigated in the papers [19-29].* Now we are interested in the QED case and therefore we would like to point out that in the region (1.7b) the p-space (1.9) does not possess even an approximate symmetry under the shift (1.8). This indicates indirectly that the standard local gauge theory techniques based on the relations (1.6a), (1.6b) and (1.3), should be given up in a new (de Sitterian) version of QED.

It is clear, of course, independently of arguments connected with p-space geometry, that, in a theory that is based on a concept of a fundamental length, the notion of a local gauge group should be revised or generalized in some nontrivial manner. However, geometrical or group theoretical arguments allow it to be done in an essentially unique way.

Indeed, one can easily realize that, in de Sitter p-space (1.9), λ-functions parametrizing the gauge transformation in question, may be written as follows:

$$\lambda(p_o, \vec{p}, p_4) = \delta\left(p_o^{\,2} - \vec{p}^{\,2} - M^2 p_4^{\,2} + M^2\right) \tilde{\lambda}(p_o, \vec{p}, p_4) \tag{1.11}$$

with the hermiticity condition inherited from (1.6c):

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*The use of such a momentum space in a field theory was pioneered in Refs. 4 and 8. The list of other papers on this subject can be found in Ref. 20. In Ref. 10, the field theory with the momentum space of variable curvature was discussed. We should point out that in all previous attempts to employ a non-euclidian p-space, Poincaré invariance of the theory was not maintained. The concept of nonlocal electromagnetic field based on ideas which were close to a non-euclidean momentum space hypothesis, holding Poincaré invariance, was developed many years ago in Ref. 3.

$$\lambda^{\dagger}(p, p_4) = \lambda(-p, p_4) \quad . \tag{1.12}$$

The next step is connected with the following observation: if

$$\lambda(p, p_4) = \frac{1}{2\pi} \int e^{-ipx - ip_4\tau} \lambda(x, \tau) d^4x d\tau \quad , \tag{1.13}$$

then

$$\left(\Box - M^2 \frac{\partial^2}{\partial\tau^2} - M^2\right)\lambda(x, \tau) = 0$$

$$\lambda(x, \tau)^{\dagger} = \lambda(x, -\tau) \quad . \tag{1.14}$$

So, in the new scheme, the gauge functions λ may be treated as local functions of five variables (x^{μ}, τ), with the obligatory constriants (1.14). The extra space-like variable τ can be interpreted, due to its commutativity with the Poincaré group generators, as some internal parameter of the theory.

All λ-functions which parametrize the conventional gauge transformations (1.6a)-(1.6b), can be found among the functions

$$\lambda(x, 0) = \left\{ \frac{1}{(2\pi)^4} \int e^{ipx + ip_4\tau} \delta(p_o{}^2 - \vec{p}^2 - M^2 p_4{}^2 + M^2) \tilde{\lambda}(p, p_4) d^5p \right\}_{\tau = 0} \equiv$$

$$\equiv \frac{1}{(2\pi)^4} \int e^{ipx} \lambda(p) d^4p \tag{1.15}$$

where

$$\lambda(p) = \frac{\tilde{\lambda}\left(p, \sqrt{1+\frac{p^2}{M^2}}\right) + \tilde{\lambda}\left(p, -\sqrt{1+\frac{p^2}{M^2}}\right)}{2M^2\sqrt{1+\frac{p^2}{M^2}}} \quad .$$

The inverse statement is not true: knowing $\lambda(p)$ one cannot reconstruct entirely the function $\tilde{\lambda}(p, p_4)$ and, respectively, $\lambda(x, \tau)$. Hence, the class of λ-functions designed to describe new gauge group transformations is larger than the conventional one.

Since the localization of the new gauge group happens to be connected with the dependence of its parameters on the five coordinates (x^μ, τ), the relevant gauge vector field, i.e., the electromagnetic potential, has to be a 5-vector as well.

The concept of electromagnetic 5-potential, local gauge transformations in terms of $\lambda(x, \tau)$ with the constraints (1.4) and appropriate gauge invariant theory of free electromagnetic field, were worked out in Ref. 30. The goal of the present paper is to transfer to the new geometrical arena the principle of the minimal electromagnetic interaction and to derive, along these lines, the appropriate generalization of the Dirac-Maxwell equations (1.4a)-(1.4b). Since the ordinary 4-dimensional space-time is embedded in our 5-dimensional manifold and new gauge group parameters remain local functions of $(x^o, \vec{x})$, one may expect that the conceived theory can be projected on the 4-dimensional space-time and turned into some sort of a local field theory containing the fundamental length ℓ as a parameter.

Later on we shall employ, as a rule, units in which

$$\hbar = c = \ell = M = 1 \quad . \tag{1.16}$$

The following 5-dimensional notations will also be applied:

1. Metric 5-tensor

$$(g^{LM}) = (g_{LM}) = \begin{pmatrix} 1 & & & & \\ & -1 & & 0 & \\ & & -1 & & \\ & 0 & & -1 & \\ & & & & -1 \end{pmatrix}; \; L,M = 0,1,2,3,4 \quad . \tag{1.17}$$

2. 5-dimensional momentum

$$(p^L) = (p^\lambda, p^4) = (E, \vec{p}, p^4) = (g^{LM} p_M) \quad . \tag{1.18}$$

3. The de Sitter surface (1.9)

$$g^{LM} p_L p_M = g_{LM} p^L p^M = -1 \quad . \tag{1.19}$$

4. 5-dimensional radius-vector

$$(x^L) = (x^\lambda, x^4) = (t, \vec{x}, \tau) = (g^{LM} x_M) \; . \tag{1.20}$$

Relevant differential operators:

$$\partial_L \equiv \frac{\partial}{\partial x^L} \; ; \; L = 0,1,2,3,4$$

$$g^{LM} \partial_L \partial_M = \Box - \frac{\partial^2}{\partial \tau^2} \equiv \pentagon \quad . \tag{1.21}$$

5. The 5-vector corresponding to the origin of a coordinate system in de Sitter p-space (1.19) ("vacuum momentum"*)

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*This very useful "spurious" 5-vector was introduced in the field theory with the de Sitter p-space by I.E. Tamm.[20]

$$(V^L) = (0, 0, 0, 0, 1) \quad . \tag{1.22}$$

6. 5-dimensional electromagnetic potential

$$(A^L(x, \tau)) = (A^\lambda(x, \tau), A^4(x, \tau)) \quad . \tag{1.23}$$

The more fundamental object, having a clear geometrical meaning in the fiber bundle theory* context, turns out to be the following 5-vector

$$B^L(x, \tau) = e^{i(Vx)} A^L(x, \tau) = e^{-i\tau} A^L(x, \tau)$$

$$= g^{LM} B_M(x, \tau) \quad . \tag{1.24}$$

This quantity also will be referred to as the 5-potential. The extra component $B^4(x, \tau)$ (or $A^4(x, \tau)$) is called the τ-photon.

7. 5-dimensional field strengths[30]

$$\mathscr{F}_{LM}(x, \tau) = \frac{\partial B_L(x, \tau)}{\partial x^M} - \frac{\partial B_M(x, \tau)}{\partial x^L} =$$

$$= g_{LR} g_{MS} \mathscr{F}^{RS}(x, \tau) \qquad L,M,R,S = 0,1,2,3,4 \quad . \tag{1.25}$$

The strategy that will be followed further on is based on transferring the new features of the free electromagnetic theory developed in Ref. 30 to the theory of massive particles. This is prompted by past experience.

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* Another name for this formalism: layered space theory.

Indeed, let us give a retrospective look to the relatively recent history of the fundamental theory of matter. The Maxwell equations for a free electromagnetic field are relativistically invariant and gauge invariant by their nature. Both of these invariance properties were transferred to the theory of particles with mass. The first one led to the special theory of relativity. The second gave rise to the minimal interaction principle which we discussed above.

The quantum theory of electromagnetic radiation and the concept of the photon itself were the first signs of the contemporary quantum theory. Let us recall also what heuristic meaning the wave properties of light quanta had for de Brogle's hypothesis. And, of course, the expression "optical-mechanical analogy" speaks eloquently for itself.

What can this historical excursion teach us? It shows that a careful study of the pure electromagnetic field seems to be very instructive preparation before venturing to take the next step in the development of a massive particle theory.

Therefore, in Section II we summarize the most important concepts related to photons in our approach, based on the De Sitter p-space. The rest of the paper is arranged as follows. Section III is devoted to the formulation of the free Dirac theory in new terms. In Section IV, we discuss the interaction between electromagnetic and charged fields governed by the new generalized gauge principle. Section V contains the physical analysis of the picture obtained. In Section VI we make concluding remarks.

II. THE FREE ELECTROMAGNETIC FIELD

As we already mentioned, the equations of motion for all five components of electromagnetic potential in a free case have been set up in Ref. 30:

$$2\left(A^{\mu}(x,\tau)+i\frac{\partial A^{\mu}}{\partial\tau}+i\frac{\partial A^{4}(x,\tau)}{\partial x_{\mu}}\right) = 0 \tag{2.1a}$$

$$2\left(A^{4}(x,\tau)-i\frac{\partial A^{4}(x,\tau)}{\partial\tau}-i\frac{\partial A^{\nu}(x,\tau)}{\partial x^{\nu}}\right) = 0 \tag{2.1b}$$

$$(\Box - 1)A^{\mu}(x,\tau) = 0 \quad ; \quad \mu,\nu = 0,1,2,3 \quad . \tag{2.1c}$$

It is easy to see that Eqs. (2.1) can also be written as follows:

$$i\mathscr{F}_{\mu 4}(x,\tau) = 0 \tag{2.2a}$$

$$2B_{4}(x,\tau)-i\frac{\partial}{\partial\tau}B_{4}(x,\tau)+i\frac{\partial B_{\nu}(x,\tau)}{\partial x_{\nu}} = 0 \tag{2.2b}$$

$$\frac{\partial\mathscr{F}^{\mu\nu}(x,\tau)}{\partial x^{\nu}} = 0 \quad . \tag{2.2c}$$

From Eqs. (2.1a)-(2.1b) or Eqs. (2.2a)-(2.2b), one obtains automatically

$$(\Box - 1)A^{4}(x,\tau) = (\Box - 1)(e^{i\tau}B^{4}(x,\tau)) = 0 \quad . \tag{2.3}$$

Further, one can verify that Eqs. (2.2) are invariant under the common gauge transformation

$$B^{M}(x,\tau) \to B^{M}(x,\tau)-\frac{\partial}{\partial x_{M}}(e^{-i\tau}\lambda(x,\tau)) \quad ; \quad M = 0,1,2,3,4 \tag{2.4}$$

with the following constraint on functions $\lambda(x, \tau)$:

$$(\square - 1)\lambda(x,\tau) = 0 \tag{2.5a}$$

(c.f. (1.14)). The second condition from (1.14)

$$\lambda(x, \tau)^{\dagger} = \lambda(x, -\tau) \tag{2.5b}$$

together with (2.4) leads to the relation:

$$B^{M^{\dagger}}(x,\tau) = (B^{\mu}(x, -\tau), -B^{4}(x, -\tau)) \tag{2.6}$$

that can be given two-fold interpretation, namely:

i) generalized neutrality condition of the electromagnetic field;

ii) transformation law of the 5-potential $B^M(x, \tau)$ under <u>τ-inversion</u>.

$$\tau \to -\tau \quad . \tag{2.7}$$

Further, it is easy to check using (2.6) that all new equations of motion (2.2)-(2.3) are invariant with respect to (2.7). Observing that

$$\frac{\partial}{\partial x_M}(e^{-i\tau}\lambda(x,\tau)) = \frac{1}{ie_o}\left(\frac{\partial}{\partial x_M}\Omega(x,\tau)\right)\Omega^{-1}(x,\tau) \tag{2.8}$$

where

$$\Omega(x,\tau) = e^{ie_o e^{-i\tau}\lambda(x,\tau)} \quad , \tag{2.9}$$

one can consider (2.9) as a "matrix" form of new gauge group transformation. Due to (2.5b), the plane

$$\tau = 0 \tag{2.10}$$

is remarkable in that the quantities

$$\Omega(x, 0) = e^{ie_0\lambda(x, 0)} \tag{2.11}$$

are unitary and describe the conventional U(1) gauge transformations (c.f. (1.15)). Further,

$$B^\mu(x, 0) \rightarrow B^\mu(x, 0) - \frac{\partial\lambda(x, 0)}{\partial x_\mu} \tag{2.12}$$

and therefore one can treat $B^\mu(x, 0)$ as an ordinary electromagnetic 4-potential:

$$B^\mu(x, 0) \equiv A^\mu(x) \quad . \tag{2.13}$$

Correspondingly, the 4-tensor $\mathscr{F}^{\mu\nu}(x, \tau)$ on the plane (2.10) should be identified with the Maxwell field strengths (1.5):

$$\mathscr{F}^{\mu\nu}(x, 0) = F^{\mu\nu}(x) \quad . \tag{2.14}$$

But as a matter of fact, owing to the identity

$$\frac{\partial\mathscr{F}^{\mu\nu}}{\partial x_4} + \frac{\partial\mathscr{F}^{\nu 4}}{\partial x_\mu} + \frac{\partial\mathscr{F}^{4\mu}}{\partial x_\nu} = 0 \tag{2.15}$$

and Eq. (2.2a), the quantity $\mathcal{F}^{\mu\nu}(x, \tau)$ does not depend on τ at all:

$$\frac{\partial \mathcal{F}^{\mu\nu}(x,\tau)}{\partial \tau} = 0 \qquad . \tag{2.16a}$$

Hence,

$$\mathcal{F}^{\mu\nu}(x,\tau) = \mathcal{F}^{\mu\nu}(x, 0) \tag{2.16b}$$

and Eq. (2.2c) is just the free Maxwell equation for the field strengths (2.14):

$$\frac{\partial F^{\mu\nu}(x)}{\partial x^{\nu}} = 0 \qquad . \tag{2.17}$$

Later on we shall call (2.10) the physical plane. The gauge transformation (2.12), attached to (2.10), together with the τ-independent Maxwell equation (2.17) will be referred to as the 4-sector, and the general gauge transformation (2.4), together with Eqs. (2.2a)-(2.2b), as the 5-sector, respectively. One may say that 4-sector Eq. (2.17) plays the role of a "boundary condition" at $\tau = 0$ that completes the 5-sector Eqs. (2.2a)-(2.2b).

Note that, due to (2.16)-(2.17), we actually can weaken the de Sitterian constraint (2.1c) and require its maintenance on the physical plane $\tau = 0$ only:

$$[(\bigcirc - 1)A_{\mu}(x, \tau)]_{\tau = 0} = 0 \qquad . \tag{2.18}$$

As was shown in Ref. 30, the τ-photon component $B_4(x,\tau)$, like the scalar and longitudinal components, does not correspond to an independent dynamical degree of freedom and can be excluded by an appropriate gauge transformation (2.4). The new gauge group turns out to be large enough to carry out such a procedure.

To make this statement clearer let us decompose $B_\mu(x, \tau)$ in terms of its transverse and longitudinal 4-parts

$$B_\mu(x, \tau) = B_\mu^{\perp}(x, \tau) + B_\mu^{\parallel}(x, \tau)$$

$$\frac{\partial B_\mu^{\perp}(x, \tau)}{\partial x_\mu} = 0 \quad , \quad \frac{\partial B_\mu^{\parallel}(x, \tau)}{\partial x_\mu} = \frac{\partial B_\mu(x, \tau)}{\partial x_\mu} \quad . \tag{2.19}$$

Then the 5-sector equations (2.2a)-(2.2b) become:

$$\frac{\partial B_\mu^{\perp}(x, \tau)}{\partial \tau} = 0 \tag{2.20}$$

and

$$\frac{\partial B_\mu^{\parallel}(x, \tau)}{\partial \tau} - \frac{\partial B_4(x, \tau)}{\partial x^\mu} = 0$$

$$2B_4(x, \tau) - i\frac{\partial B_4(x, \tau)}{\partial \tau} + i\frac{\partial B_\nu^{\parallel}(x, \tau)}{\partial x_\nu} = 0 \quad . \tag{2.21}$$

We can see now that the spin one components $B_\mu^{\perp}(x, \tau)$ actually do not depend on τ (c.f. Eq. (2.16a)). All τ-dependence is concentrated in Eqs. (2.21) describing simultaneously spin zero photons and τ-photons. These equations look like a gauge constraint imposed on the 5-potential $B_M(x, \tau)$. It is readily verified using the gauge transformations (2.4), that the components $B_\mu^{\parallel}$ and B_4 can be eliminated, so each of Eqs. (2.21) converts into the identity 0 = 0.

The following question seems to be relevant now: why is a consideration of the 5-sector necessary? The 4-sector evidently provides us with the conventional description of the free electromagnetic field. Therefore, the 5-sector appears to overdescribe this field and looks superfluous.

The existence of two or more mathematical formulations of the same free field theory is familiar. A deviation of one approach from another may be revealed when one begins to describe interactions. It is usually connected with a difference in symmetry properties embodied in the free equations.

Obviously, if we ignore the 5-sector completely, then we will deal with the standard 4-dimensional gauge transformation (2.12) and finally be led to the Maxwell-Dirac theory of electromagnetic interactions based on Eqs. (1.4). On the other hand, as one already knows, 5-sector Eqs. (2.2a)-(2.2b) possess the new symmetry properties in which our fundamental length hypothesis is essentially reflected, namely:

i) They are invariant under the more general gauge transformations (2.4);

ii) They are invariant under τ-inversion (2.7).

Therefore, keeping in mind our general strategy (see the end of Section I), we adopt the formalism of a 5-component* free electromagnetic field, described by the whole set of Eqs. (2.2a), (2.2b) and (2.17), as a pattern for a construction of a general theory of electromagnetic interactions. So the new local gauge transformations (2.4) for the 5-potential $B_M(x, \tau)$ and those with the "matrix" (2.9) for charged particle fields and the τ-inversion transformation are destined to play a fundamental role in the new approach. Further, one may expect that the theory in question should be split, as in the just considered free case, in 5-sector and 4-sector. The 5-sector equations of motion have to be set up in all 5-space and be

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*By analyzing interacting fields, we shall see that the τ-photon degree of freedom, like other pseudophoton components, will revive to play an important mediating role in an interaction.

invariant under new gauge transformations and the τ-inversion. The 4-sector equations, ascribed to the physical plane $\tau = 0$ where the conventional gauge group is operating (see (2.11) and (2.12)), have to be consistent with the 5-sector equations and serve as a specific "boundary condition" for them at $\tau = 0$. It is also crucial to realize that the 4-sector equations should play the role of a generalization of the Dirac-Maxwell equations (1.4a)-(1.4b).

Coming back to the theory of free electromagnetic field, we may observe that the 5-sector Eqs. (2.2a)-(2.2b) are the Euler-Lagrange equations for the 5-dimensional action integral

$$\int \mathscr{L}_{\text{MAXWELL}}(x,\tau)\, dx d\tau \tag{2.22}$$

where the lagrangian 5-density $\mathscr{L}_{\text{MAXWELL}}(x,\tau)$ is given by*

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*Note that due to (2.6)

$$\frac{\delta}{\delta B_\mu{}^{\dagger}(x,\tau)} = \frac{\delta}{\delta B_\mu(x,-\tau)} \quad , \quad \frac{\delta}{\delta (B^4(x,\tau))^{\dagger}} = -\frac{\delta}{\delta B^4(x,-\tau)}$$

The increments of the field derivatives are defined in a usual way

$$\delta\left(\frac{\partial B^\mu(x,\tau)}{\partial x^\nu}\right) = \frac{\partial}{\partial x^\nu}(\delta B^\mu(x,\tau)), \ldots \tag{2.23a}$$

$$\delta\left(\frac{\partial B^\mu(x,\tau)}{\partial \tau}\right) = \frac{\partial}{\partial \tau}\left(\delta B^\mu(x,\tau)\right), \ \delta\left(\frac{\partial B^4(x,\tau)}{\partial \tau}\right) = \frac{\partial}{\partial \tau}\left(\delta B^4(x,\tau)\right), \ldots \tag{2.23b}$$

$$\mathscr{L}_{MAXWELL}(x,\tau) = \mathscr{L}^{\dagger}_{MAXWELL}(x,\tau) = -2(B^4(x,\tau))^{\dagger} B^4(x,\tau) +$$

$$+\frac{i}{2}\left[B^{\mu\dagger}(x,\tau)\frac{\partial B_\mu(x,\tau)}{\partial\tau} - \frac{\partial B_\mu^{\dagger}(x,\tau)}{\partial\tau} B^\mu(x,\tau)\right] +$$

$$+\frac{i}{2}\left[(B^4(x,\tau))^{\dagger}\frac{\partial B^4}{\partial\tau} - B^4(x,\tau)\left(\frac{\partial B^4(x,\tau)}{\partial\tau}\right)^{\dagger}\right] +$$

$$+ i\left[(B^\mu(x,\tau))^{\dagger}\frac{\partial B^4(x,\tau)}{\partial x^\mu} - B^\mu(x,\tau)\left(\frac{\partial B^4(x,\tau)}{\partial x^\mu}\right)^{\dagger}\right] \quad . \qquad (2.24)$$

Since

$$\frac{\partial \mathscr{L}_{MAXWELL}(x,\tau)}{\partial \dfrac{\partial B_\mu(x,\tau)}{\partial x^0}} = 0 \qquad (2.26)$$

we are dealing with a so-called singular lagrangian.[31-36] This circumstance will be essential for the quantization procedure.[37]

Let us consider now the 4-sector, i.e. the Maxwell equation (2.17). The relevant action integral is well known:

$$-\int \tfrac{1}{4} F_{\mu\nu}(x)F^{\mu\nu}(x)dx \equiv \int L_{MAXWELL}(x)dx \quad . \qquad (2.27)$$

In contrast to (2.22), the increments that we should give here to the functional arguments $A_\mu(x) = B_\mu(x,0)$ do not depend on τ:

$$\frac{\partial}{\partial\tau}\left(\delta A_\mu(x)\right) = 0 \quad . \qquad (2.28)$$

As a result, due to (2.23b),

$$\delta\left(\frac{\partial B_\mu(x,0)}{\partial\tau}\right) = 0 \qquad . \qquad (2.29)$$

Projecting the 5-sector Eqs. (2.2a)-(2.2b) onto the physical plane $\tau = 0$, one finds

$$i\mathscr{F}_{\mu 4}(x,0) = i\left(\frac{\partial B_\mu(x,0)}{\partial\tau} - \frac{\partial B_4(x,0)}{\partial x^\mu}\right) = 0 \qquad (2.30a)$$

$$2B_4(x,0) - i\frac{\partial B_4(x,0)}{\partial\tau} + i\frac{\partial A_\nu(x)}{\partial x_\nu} = 0 \qquad . \qquad (2.30b)$$

The relation (2.30b) just fixes a gauge of electromagnetic 4-potential $A_\mu(x)$* and evidently does not conflict with Eq. (2.1). Owing to (2.29)

---

*Actually $B_4(x, 0)$ and $\frac{\partial B_4(x,0)}{\partial\tau}$ in (2.30b) are completely arbitrary functions of x.

Putting

$$B_4(x,0) = \frac{i}{(2\pi)^{3/2}} \int_{1+p^2\geq 0} c_1(p)e^{ipx}\, d^4p$$

$$B_4(x,0) - i\frac{\partial}{\partial\tau}B_4(x,0) = \frac{i}{(2\pi)^{3/2}} \int_{1+p^2\geq 0} c_2(p)e^{ipx} d^4p \qquad (2.31)$$

with $c_1^\dagger(p) = c_1(-p)$ and $c_2^\dagger(p) = c_2(-p)$, one can express in terms (2.31) any solution of Eqs. (2.3):

$$B_4(x,\tau) = \frac{i\,e^{-i\tau}}{(2\pi)^{3/2}}\left\{\int_{1+p^2\geq 0} e^{ipx} c_1(p)\cos\tau\sqrt{1+p^2}\, d^4p + \right.$$
$$\left. + i\int_{1+p^2\geq 0} e^{ipx}c_2(p)\frac{\sin\tau\sqrt{1+p^2}}{\sqrt{1+p^2}}\, d^4p\right\} \qquad . \qquad (2.32)$$

We shall see in Sec. 4 that the τ-photon field $B_4(x, \tau)$ continues to satisfy Eq. (2.3) in the presence of interactions as well.

$$\frac{\delta \mathscr{F}_{\mu 4}(x, 0)}{\delta A_{\nu}(x)} = 0 \tag{2.33}$$

and therefore (2.30a) is consistent with Eq. (2.17) as well.

III. THE FREE DIRAC FIELD

Let us consider first a scalar field of massive particles just to acquire some terminology and notations. In the de Sitter p-space (1.19), the mass shell equation

$$p^2 - m^2 = 0 \tag{3.1}$$

can be factorized in the following way

$$p^2 - m^2 = (p^4 - \cosh\mu)(p^4 + \cosh\mu) \tag{3.2}$$

where

$$\cosh\mu \equiv \sqrt{1 + m^2} \quad . \tag{3.3}$$

Due to the relation

$$\cosh\mu = | g_{LM} p^L v^M | \ ; \quad L,M = 0,1,2,3,4 \tag{3.4}$$

μ is just the "noneuclidean" 4-distance between the point p and the origin of the coordinate system (1.22), i.e. the exact geometrical analog of m. In the flat limit, correspondingly,

$$\mu \simeq m \quad . \tag{3.5}$$

The factorization formula (3.2) and evident symmetry of the de Sitter p-space (1.19) under the inversion

$$p^4 \to -p^4 \tag{3.6}$$

lead to a possibility of an existence in the new scheme of two "Klein-Gordon type" equations:[21,24,25]

$$2(p^4 - \cosh \mu)\phi_1(p, p^4) = 0 \tag{3.7a}$$

$$2(-p^4 - \cosh \mu)\phi_2(p, p^4) = 0 \quad . \tag{3.7b}$$

After the 5-dimensional Fourier transformation one obtains from (3.7a)-(3.7b):

$$2\left(-i\frac{\partial}{\partial\tau} - \cosh\mu\right)\phi_1(x,\tau) = 0 \tag{3.8a}$$

$$2\left(i\frac{\partial}{\partial\tau} - \cosh\mu\right)\phi_2(x,\tau) = 0 \quad . \tag{3.8b}$$

So

$$\phi_1(x, \tau) = e^{i\tau\cosh\mu}\,\phi_1(x, 0)$$

$$\phi_2(x, \tau) = e^{-i\tau\cosh\mu}\,\phi_2(x, 0) \quad . \tag{3.9}$$

We call ϕ_1 and ϕ_2 the normal and abnormal fields, respectively. This "flavor" clearly has no analog in the ordinary theory with $\ell = 0$ and is of a great importance for the present approach.*

To specify $\phi_1(x, 0)$ and $\phi_2(x, 0)$ we need to take into account the de Sitterian constraint (c.f. (2.1c))

$$(\bigcirc - 1)\phi_a(x,\tau) = 0 \quad , \quad a = 1,2 \quad . \tag{3.10}$$

Since

$$(\bigcirc - 1)\phi_a(x,\tau) = e^{i\varepsilon_a \tau \cosh\mu} (\square + m^2)\, \phi_a(x, 0) \quad ,$$

$$a = 1, 2 \quad ; \quad \varepsilon_1 = 1 \quad , \quad \varepsilon_2 = -1$$

Eq. (3.10) is equivalent to the "weakened" de Sitterian constraint (c.f. (2.18))

$$\left[(\bigcirc - 1)\,\phi_a(x, \tau)\right]_{\tau = 0} = 0 \quad , \quad a = 1,2 \tag{3.11}$$

or

$$(\square + m^2)\phi_a(x, 0) = 0 \quad , \quad a = 1,2 \quad . \tag{3.12}$$

Thus, in our terms, the conventional Klein-Gordon equations (3.12) for $\phi_1(x, 0)$ and $\phi_2(x, 0)$ play the role of the 4-sector of the free scalar theory.

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*It follows from Eqs. (2.1a)-(2.1b) that in the Lorentz gauge, $\frac{\partial A_\nu(x,\tau)}{\partial x_\nu} = 0$, the 4-potential $A_\mu(x,\tau)$ and τ-photon component $A_4(x,\tau)$ satisfy, correspondingly, the normal Eq. (3.8a) and abnormal Eq. (3.8b), with $\mu = 0$ in both cases.

Correspondingly, Eqs. (3.8a)-(3.8b) are the 5-sector in this case, with $\phi_1(x, \tau)$ and $\phi_2(x, \tau)$ replacing each other under the τ-inversion (2.7) by definition:*

$$\begin{pmatrix} \phi_1(x, \tau) \\ \phi_2(x, \tau) \end{pmatrix} \to \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \phi_1(x, -\tau) \\ \phi_2(x, -\tau) \end{pmatrix} \quad . \tag{3.13}$$

Proceeding to a spinor field case, one might think that the corresponding 5-sector equations can be found as a result of the "extraction of the square root" from each bracket in the right-hand side of (3.2). To carry out such an operation, note, first, that the following identities

$$2(p^4 - \cosh \mu) = g_{KL}(p - V)^K(p - V)^L - 4 \sinh^2 \mu/2$$

$$-2(p^4 + \cosh \mu) = g_{KL}(p + V)^K(p + V)^L - 4 \sinh^2 \mu/2 \quad , \tag{3.14}$$

are held in the de Sitter p-space (1.19). Second, let us introduce the five matrices

$$\Gamma^L = (\Gamma^o, \Gamma^1, \Gamma^2, \Gamma^3, \Gamma^4)$$

with properties

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*It turns out if ϕ is a neutral scalar field then within the framework of electromagnetic interaction theory

$$\phi_1(x) = \phi_2(x)$$

and the transformation law under τ-inversion is given by $\phi^\dagger(x, \tau) = \phi(x, -\tau)$ (c.f. (2.6)).

$$\Gamma^K\Gamma^L + \Gamma^L\Gamma^K = 2g^{KL}$$

$$(\Gamma^K)^\dagger = \Gamma^o\Gamma^K\Gamma^o \quad ; \quad K,L = 0,1,2,3,4 \quad . \tag{3.15}$$

The minimal order of such Γ-matrices is equal to four. This corresponds to the fundamental (spinor) representation of the de Sitter group SO(4, 1). As is known, this representation is simultaneously an irreducible spinor representation of the improper Lorentz group O(3, 1). Therefore, we choose

$$\Gamma^L = (\gamma^o, \gamma^1, \gamma^2, \gamma^3, -i\gamma^5) \tag{3.16}$$

where $\gamma^\lambda = (\gamma^o, \vec{\gamma})$ and γ^5 are ordinary γ-matrices:

$$\gamma^o = \begin{pmatrix} E & 0 \\ 0 & -E \end{pmatrix} \; , \; \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix} \; , \; \gamma^5 = \begin{pmatrix} 0 & E \\ E & 0 \end{pmatrix} \quad . \tag{3.17}$$

Using the identity

$$g_{KL}A^KA^L = (A_K\Gamma^K)(A_L\Gamma^L) \tag{3.18}$$

that is valid for an arbitrary 5-vector A^L, one can obtain from (3.13) the pair of "Dirac type" equations

$$[\not{p} - (p^4 - 1)\Gamma^4 - 2\sinh\mu/2\,]\,\psi_1(p, p^4) = 0 \tag{3.19a}$$

$$[\not{p} + (p^4 + 1)\Gamma^4 - 2\sinh\mu/2\,]\,\psi_2(p, p^4) = 0 \quad , \tag{3.19b}$$

normal and abnormal, respectively.[29-30] In (x, τ)-representation Eqs. (3.19a)-(3.19b) become

$$\left[i \frac{\partial}{\partial x} + i \frac{\partial}{\partial \tau} \Gamma^4 + \Gamma^4 - 2 \sinh \mu/2\right] \psi_1(x, \tau) = 0 \tag{3.20a}$$

$$\left[i \frac{\partial}{\partial x} - i \frac{\partial}{\partial \tau} \Gamma^4 + \Gamma^4 - 2 \sinh \mu/2\right] \psi_2(x, \tau) = 0 \quad . \tag{3.20b}$$

As for the de Sitterian constraint that should be imposed on $\psi_1(x, \tau)$ and $\psi_2(x, \tau)$, we shall use at once its "weakened" form that is tied to the physical plane $\tau = 0$:

$$[(\pentagon - 1)\psi_a(x, \tau)]_{\tau = 0} = 0 \quad ; \quad a = 1,2 \tag{3.21}$$

(c.f. (2.18) and(3.11)).* Combining Eqs. (3.19a)-(3.19b) and (3.21) leads one to the following equation of motion on the plane $\tau = 0$;

$$\left(i \not\partial - i\gamma^5(\cosh \mu - 1) - 2 \sinh \mu/2\right) \psi_a(x, 0) = 0; \quad a = 1,2 \tag{3.23}$$

Evidently, we should adopt Eq. (3.23) as the 4-sector equation for the Dirac free field in our approach. With the notation

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*It is easy to check that solutions of Eqs. (3.20a)-(3.20b), subjected to (3.21), satisfy automatically the exact de Sitterian constraint

$$(\pentagon - 1)\psi_a(x, \tau) = 0 \quad ; \quad a = 1,2 \quad . \tag{3.22}$$

As we have seen, a similar situation takes place in the theories of electromagnetic and scalar fields.

$$\tanh \mu/2 = \sin\theta \qquad , \tag{3.24}$$

Eq. (3.23) may be rewritten as follows:

$$\left(i\not{\partial} - m\, e^{-i\theta\gamma^5} \right) \psi_a(x,0) = 0 \quad ; \quad a = 1,2 \qquad .$$

This equation is clearly <u>equivalent</u> to the ordinary Dirac equation for the wave function

$$\psi_a(x) = e^{-i\frac{\theta}{2}\gamma^5}\, \psi_a(x,0) \quad , \quad a = 1,2 \qquad . \tag{3.25}$$

Thus, our 4-sector is nothing more than the conventional theory of two Dirac free fields $\psi_a(x)$ $(a = 1,2)$ with equal masses, where a is a label of the flavor distinguishing the normal and abnormal fields. The corresponding lagrangian density can be written in terms of $\psi_a(x)$ and $\overline{\psi}_a(x)$ simply as

$$L_{DIRAC}(x) = \sum_{a=1,2} \overline{\psi}_a(x)(i\not{\partial} - m)\psi_a(x) \qquad . \tag{3.26}$$

Actually we are dealing here with a new kind of internal symmetry described by SU(2)-group, that will be further denoted as $SU_\tau(2)$. It should be rather clear that the $SU_\tau(2)$-symmetry is one of the direct consequences of our fundamental length hypothesis.

Coming back to Eqs. (3.20a)-(3.20b), one can guess that they represent the 5-sector of the free Dirac theory in our formalism. The key point here is a question of an invariance under τ-inversion (2.7). Each of these equations taken separately is not invariant under (2.7) (c.f. Eqs. (3.8a)-(3.8b)). As a matter of fact 4-dimensional de Sitterian spinors like $\psi_1(x,\tau)$ and $\psi_2(x,\tau)$ do not possess linear transformation properties with respect to the inversion of the τ-axis. It is qualitatively the same situation which occurs in the 2-component Lorentz spinor formalism when a space inversion operation is introduced. In that case, one needs to construct bispinors from 2-component wave functions. Here, we have to go to 8-component spinors which will transform linearly with respect to the improper de Sitter group O(4, 1).

Putting

$$\begin{pmatrix} \psi_1(x,\tau) \\ \psi_2(x,\tau) \end{pmatrix} \equiv \Psi(x,\tau) \quad , \tag{3.27}$$

one can combine Eqs. (3.20a)-(3.20b) as one equation for the 8-component object (3.27):

$$\left(i\partial_M G^M + \sigma_o \times \Gamma^4 - 2\sinh\mu/2 \right) \Psi(x,\tau) = 0 \tag{3.28}$$

where

$$\sigma_o = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$G^\mu = \sigma_o \times \gamma^\mu = \begin{pmatrix} \gamma^\mu & 0 \\ 0 & \gamma^\mu \end{pmatrix}$$

$$G^4 = \sigma_3 \times \Gamma^4 = \begin{pmatrix} \Gamma^4 & 0 \\ 0 & -\Gamma^4 \end{pmatrix} = -i \begin{pmatrix} \gamma^5 & 0 \\ 0 & -\gamma^5 \end{pmatrix}$$

$$G^M G^N + G^N G^M = 2 g^{MN}$$

$$(G^M)^\dagger = G^o G^M G^o \quad . \tag{3.29}$$

Eq. (3.28) will remain invariant under the τ-inversion (2.7) if one defines the following linear transformation law for the wave function $\Psi(x,\tau)$ (c.f. (2.13)):

$$\Psi'(x,\tau) = T\Psi(x,-\tau) \tag{3.30}$$

where T is the following 8×8matrix

$$T = \sigma_1 \times E = \begin{pmatrix} 0 & E \\ E & 0 \end{pmatrix} \quad . \tag{3.31}$$

It is easily seen that

$$\text{i)}\ T^2 = 1 \quad , \quad T^\dagger = T;$$

$$\text{ii)}\ T(\sigma_o \times \Gamma^M)T = \sigma_o \times \Gamma^M; \ \ M = 0,1,2,3,4;$$

$$\text{iii)}\ TG^4T = -G^4 \quad . \tag{3.32}$$

Later on it will be convenient for us to use a special notation

$$\Phi(x,\tau) = T\Psi(x,-\tau) \tag{3.33}$$

and to write, in addition to Eq. (3.28),

$$(i\partial_M G^M + \sigma_o \times \Gamma^4 - 2\sinh\mu/2)\Phi(x,\ \tau) = 0 \quad . \tag{3.34}$$

It is clear now that either of the two equations, Eq. (3.28) or Eq. (3.33), can be referred to as the 5-sector equation of the free Dirac theory in our terms. Introducing the conjugated 8-component spinors

$$\overline{\Psi}(x, \tau) = \Psi^{\dagger}(x, \tau) G^{o}$$

$$\overline{\Phi}(x, \tau) = \Phi^{\dagger}(x, \tau) G^{o} \tag{3.35}$$

one can construct the appropriate lagrangian 5-density:

$$\mathscr{L}_{\mathrm{DIRAC}}(x, \tau) = \mathscr{L}^{\dagger}_{\mathrm{DIRAC}}(x, \tau) =$$

$$= \frac{i}{4}\Big[\overline{\Phi}(x, \tau) G^{M}(\partial_{M}\Psi(x, \tau)) - (\partial_{M}\overline{\Psi}(x, \tau)) G^{M}\Phi(x, \tau) +$$

$$+ \overline{\Psi}(x, \tau) G^{M}(\partial_{M}\Phi(x, \tau)) - (\partial_{M}\overline{\Phi}(x, \tau)) G^{M}\Psi(x, \tau)\Big] +$$

$$+ \tfrac{1}{2}\overline{\Phi}(x, \tau)(\sigma_{o} \times \Gamma^{4} - 2 \sinh \mu/2)\Psi(x, \tau) +$$

$$+ \tfrac{1}{2}\overline{\Psi}(x, \tau)(\sigma_{o} \times \Gamma^{4} - 2 \sinh \mu/2)\Phi(x, \tau) \quad . \tag{3.36}$$

Note, that, due to Eq. (3.28) and Eq. (3.34), we have a conserving 5-current:

$$J^{M}(x, \tau) = \overline{\Psi}(x, \tau) G^{M}\Phi(x, \tau) \quad ; \quad M = 0,1,2,3,4 \tag{3.37a}$$

$$\frac{\partial J^{M}(x, \tau)}{\partial x^{M}} = 0 \quad . \tag{3.37b}$$

Further (c.f. (2.6)),

$$(J^M(x, \tau))^\dagger = (J^\mu(x, -\tau), -J^4(x, -\tau)) \quad \mu = 0,1,2,3 \quad . \tag{3.37c}$$

Thus, we have reformulated the free Dirac theory along our lines. The new description is based on the set of equations for the 8-component wave function $\psi(x, \tau)$ in which <u>all</u> components of the 5-gradient $\partial_M = (\partial_\mu, \partial_4)$ are presented on an equal footing:

$$\begin{cases} (i\partial_M G^M + \sigma_0 \times \Gamma^4 - 2 \sinh \mu/2)\Psi(x, \tau) = 0 \\ [(\bigcirc - 1)\Psi(x, \tau)]_{\tau=0} = 0 \end{cases} \quad . \tag{3.38}$$

The physical plane $\tau = 0$ in Eqs. (3.38) is singled out. It indicates that the largest group of continuous symmetry transformations admitted by Eqs. (3.38) is the standard Poincaré group in the 4-dimensional configurational space:

$$x^{\mu'} = L^\mu{}_\nu x^\nu + a^\mu \quad . \tag{3.39}$$

As one knows, on the plane $\tau = 0$, due to Eqs. (3.38), the 4-sector Eq. (3.23) for the wave function

$$\Psi(x, 0) = \begin{pmatrix} \psi_1(x, 0) \\ \psi_2(x, 0) \end{pmatrix} \tag{3.40}$$

is held. Further, this equation is equivalent to the conventional Dirac equation for each component of the $SU_T(2)$-doublet

$$\begin{pmatrix} \psi_1(x) \\ \psi_2(x) \end{pmatrix} = \begin{pmatrix} e^{-i\frac{\theta}{2}\gamma^5} & 0 \\ 0 & e^{-i\frac{\theta}{2}\gamma^5} \end{pmatrix} \begin{pmatrix} \psi_1(x, 0) \\ \psi_2(x, 0) \end{pmatrix}$$

$$\sin\theta = \tanh \mu/2 \tag{3.41}$$

(see (3.25)). It is of primary importance that the equivalency transformation (3.41) turned out to be a chiral one. Let us note also that the $SU_\tau(2)$-symmetry of the free 4-sector happens to be the direct consequence of the invariance of the free 5-sector under the τ-inversion. This is why we use the subscript τ in denoting the group considered.

IV. NEW GAUGE SYMMETRY GROUP AND INTERACTING FIELDS

According to our general conception (see Secs. I and II) within the 5-sector the Dirac field has to share the new symmetry properties with the electromagnetic field. One such a symmetry connected with the τ-inversion has been transferred to Dirac particles in the previous section. As a result we have the $SU_\tau(2)$-symmetry of the free Dirac 4-sector.

Now we turn to the new gauge symmetry properties related to the transformation (2.4). As was pointed out already (2.4) is associated with a gauge group element (2.9). Therefore, let us require that the general theory of electromagnetic interactions be invariant under the following simultaneous local gauge transformations:

$$\Psi(x,\tau) \to e^{ie_o e^{-i\tau}\lambda(x,\tau)}\,\Psi(x,\tau)$$

$$\Phi(x,\tau) \to e^{ie_o e^{i\tau}\lambda^{\dagger}(x,\tau)}\,\Phi(x,\tau)$$

$$\overline{\Psi}(x,\tau) \to e^{-ie_o e^{i\tau}\lambda^{\dagger}(x,\tau)}\,\overline{\Psi}(x,\tau)$$

$$\overline{\Phi}(x,\tau) \to e^{-ie_o e^{-i\tau}\lambda(x,\tau)}\,\overline{\Phi}(x,\tau)$$

$$B_M(x,\tau) \to B_M(x,\tau) - \frac{\partial}{\partial x^M}\left(e^{-i\tau}\lambda(x,\tau)\right) \quad ; \quad M = 0,1,2,3,4 \tag{4.1}$$

where, as before (see (2.5a)-(2.5b)),

$$(\bigcirc - 1)\lambda(x,\tau) = 0$$

$$\lambda^{\dagger}(x,\tau) = \lambda(x,-\tau) \quad . \tag{4.2}$$

Further, to make our approach as unambiguous as the conventional theory of electromagnetic interactions, we should generalize the minimal interaction principle. It is clear that the new formulation of this principle, adequate to the local gauge transformations (4.1), can be associated with the following substitution law [c.f. (1.3)]

$$i\frac{\partial}{\partial x^M} \to i\frac{\partial}{\partial x^M} - e_o B_M(x,\tau) \quad . \tag{4.3}$$

Applying (4.3) to Eqs. (3.38) leads one to the following gauge invariant set of equations:

$$\begin{cases} [(i\partial_M - e_o B_M(x,\tau))G^M + \sigma_o \times \Gamma^4 - 2\sinh\mu/2]\,\Psi(x,\tau) = 0 & (4.4a) \\ \left\{[g^{LM}(\partial_L + ie_o B_L(x,\tau))(\partial_M + ie_o B_M(x,\tau)) - 1]\,\Psi(x,\tau)\right\}_{\tau=0} = 0 & (4.4b) \end{cases}$$

Due to (2.6), (3.32) and (3.33), we have also

$$\begin{cases} [(i\partial_M - e_o B_M^{\dagger}(x,\tau))G^M + \sigma_o \times \Gamma^4 - 2\sinh\mu/2]\,\Phi(x,\tau) = 0 & (4.5a) \\ \left\{[g^{LM}(\partial_L + ie_o B_L^{\dagger}(x,\tau))(\partial_M + ie_o B_M^{\dagger}(x,\tau)) - 1]\,\Phi(x,\tau)\right\}_{\tau=0} = 0\ . & (4.5b) \end{cases}$$

Eqs. (4.4a) and (4.5a) can be derived from the 5-dimensional action integral with the lagrangian 5-density

$$\mathscr{L}_{DIRAC}(x,\tau) - \frac{e_o}{2}\left[J^M(x,\tau)B_M^{\dagger}(x,\tau) + J_M^{\dagger}(x,\tau)B^M(x,\tau)\right] \qquad (4.6)$$

where $\mathscr{L}_{DIRAC}(x,\tau)$ is given by (3.36) and $J^M(x,\tau)$ is the 5-current (3.37a).

Adding to (4.6) the 5-dimensional lagrangian density (2.24) that corresponds to a free electromagnetic field, one obtains

$$\mathscr{L}_{TOTAL}(x,\tau) = \mathscr{L}_{MAXWELL}(x,\tau) + \mathscr{L}_{DIRAC}(x,\tau) - \frac{e_o}{2}\left[J^M(x,\tau)B_M^{\dagger}(x,\tau) + J_M^{\dagger}(x,\tau)B^M(x,\tau)\right] \quad . \qquad (4.7)$$

This lagrangian density is obviously invariant under the new gauge transformations (4.1) and the τ-inversion. The variation of the action integral

$$\int \mathscr{L}_{TOTAL}(x, \tau)dxd\tau \tag{4.8}$$

with respect to $B_M^{\dagger}(x, \tau)$ and $(\partial B_M^{\dagger}(x, \tau))/(\partial x^N)$ leads to the inhomogeneous first-order equations for the 5-potential $B_M(x, \tau)$:

$$\left\{\begin{aligned} & i\mathscr{F}_{\mu 4}(x, \tau) = \frac{e_o}{4} J_{\mu}(x, \tau) && (4.9a) \\ & 2B_4(x, \tau) - i \frac{\partial}{\partial \tau} B_4(x, \tau) + i \frac{\partial B_{\mu}(x, \tau)}{\partial x_{\nu}} = \frac{e_o}{2} J_4(x, \tau) \quad . && (4.9b) \end{aligned}\right.$$

Eqs. (4.4a), (4.5a), (4.9a) and (4.9b), being invariant to the new gauge transformations and to the τ-inversion, play the role of the 5-sector equations in our description of interacting electromagnetic and charged spinor fields. It is easily seen from Eqs. (4.4a) and (4.5a) that the 5-current (3.37a) satisfies, as in a free case, the continuity equation (3.37b) and generalized hermiticity condition (3.37c). Due to (2.6) the condition (3.37c) follows automatically from Eqs. (4.9a)-(4.9b). In order that Eq. (3.37b) be consistent with Eqs. (4.9a)-(4.9b) we have to require a validity of Eq. (2.3). Hence, the τ-photon field permanently satisfies the exact de Sitterian constraint regardless of a presence or an absence of an interaction (see also the footnote on page 22).

Let us proceed now to a derivation of the 4-sector equations attached to the physical plane $\tau = 0$. First we project Eq. (4.4a) onto this plane using Eq. (4.4b). It gives:

$$\Big[(i\partial_{\mu} - e_o A_{\mu}(x)G^{\mu}) - (\cosh\mu - 1)\sigma_o \times \Gamma^4 + \frac{e_o}{4}(\sigma_o \times \Gamma^4)(\Sigma^{LM}\mathscr{F}_{LM}(x, 0)) - \\ - 2\sinh\mu/2\Big]\Psi(x, 0) = 0 \quad , \tag{4.10}$$

where

$$\Sigma^{LM} = \frac{i}{2}(G^L G^M - G^M G^L) \quad , \tag{4.11}$$

$A_\mu(x)$ is the electromagnetic 4-potential (2.13), and $\mathscr{F}_{LM}(x, 0)$ are the 5-dimensional field strengths (1.25) at $\tau = 0$.

When the interaction is switching off ($e_o \to 0$) Eq. (4.10) turns into free Eq. (3.23) with the mass operator containing the pseudoscalar γ^5-term. If we want the conventional Dirac equation to emerge in this limit, then, in advance, we should subject Eq. (4.10) to the chiral equivalence transformation (3.41). This leads to the following set of equations, instead of Eq. (4.10):

$$(i\partial\!\!\!/ - e_o A\!\!\!/ - m_o)\psi_1(x) = \frac{ie_o \cos\theta_o}{4} \gamma^5 \sigma^{\mu\nu} F_{\mu\nu}(x)\psi_1(x) -$$

$$- \frac{e_o \sin\theta_o}{4} \sigma^{\mu\nu} F_{\mu\nu}(x)\psi_1(x) - \frac{ie_o}{2} \gamma^\mu \mathscr{F}_{\mu 4}(x, 0)\psi_1(x) \quad , \tag{4.12a}$$

$$(i\partial\!\!\!/ - e_o A\!\!\!/ - m_o)\psi_2(x) = \frac{ie_o \cos\theta_o}{4} \gamma^5 \sigma^{\mu\nu} F_{\mu\nu}(x)\psi_2(x) -$$

$$- \frac{e_o \sin\theta_o}{4} \sigma^{\mu\nu} F_{\mu\nu}(x)\psi_2(x) + \frac{ie_o}{2} \gamma^\mu \mathscr{F}_{\mu 4}(x, 0)\psi_2(x) \quad , \tag{4.12b}$$

where $\sigma^{\mu\nu} = i/2\,(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$, $F_{\mu\nu}(x)$ are the Maxwell field strengths (1.5) and

$$\mathscr{F}_{\mu 4}(x, 0) = \frac{\partial B_\mu(x, 0)}{\partial \tau} - \frac{\partial B_4(x, 0)}{\partial x^\mu} \tag{4.13}$$

are the components of 5-dimensional field strengths at $\tau = 0$ to which τ-photons directly contribute. The particle mass is denoted as m_o to emphasize that we are still dealing with "bare" fields. Correspondingly (see (3.3) and (3.24)),

$$\sin\theta_o = \tanh \mu_o/2 = \frac{\sqrt{1 + m_o^2} - 1}{m_o} \quad ;$$

$$\cos\theta_o = \frac{1}{\cosh\frac{\mu_o}{2}} = \sqrt{\frac{2}{1+\sqrt{1+m_o^2}}} \quad . \tag{4.13}$$

Eqs. (4.12a)-(4.12b) look like a nontrivial generalization of the Dirac equation (1.4a). Let us point out here some evident properties of equations obtained, postponing a more detailed analysis until the next Section:

1. Eqs. (4.12a)-(4.12b) are consistent with 5-sector Eq. (4.4a). They can be treated simply as a boundary condition at $\tau = 0$, completed Eq. (4.4a).

2. Eqs. (4.12a)-(4.12b) are invariant under the ordinary gauge transformations [c.f. (1.6a)-(1.6b)]

$$\psi_a(x) \to e^{ie_o\lambda(x,0)}\psi_a(x) \quad ;$$

$$\overline{\psi}_a(x) \to e^{-ie_o\lambda(x,0)}\overline{\psi}_a(x) \quad , \quad a = 1,2 \quad ;$$

$$A_\mu(x) \to A_\mu(x) - \frac{\partial\lambda(x,0)}{\partial x^\mu} \quad , \quad \lambda(x,0) = \lambda^\dagger(x,0) \quad , \tag{4.14}$$

which are a projection of the new gauge group (4.1) onto the physical plane $\tau = 0$. We do not need to consider here the gauge transformation of the τ-photon component $B_4(x, 0)$ because it is involved in the gauge invariant structure (4.13).

3. The left-hand sides of the considered equations contain the conventional minimal interaction term, whereas the right-hand sides consist entirely of non-minimal, namely:

i) electric dipole moment (EDM) interaction ($\sim \gamma^5\sigma^{\mu\nu}F_{\mu\nu}(x)$);

ii) magnetic dipole moment (MDM) interaction also called the Pauli interaction term ($\sim \sigma^{\mu\nu}F_{\mu\nu}(x)$);

iii) τ-photon interaction ($\sim i\gamma^\mu\mathscr{F}_{\mu 4}(x, 0)$). Such a name is chosen because this interaction survives even if there are no "ordinary" photons ($A_\mu(x) = 0$) and, therefore, it could be induced only by the τ-photons. Note, that the τ-photon interaction terms enter with different signs in Eq. (4.12a) and Eq. (4.12b). This is the only source for a violation of the $SU_\tau(2)$-symmetry in the considered equations.

The EDM term inevitably leads to the parity (P) violation and CP-violation. Let us notice that in free Eq. (3.23) one faces the fictitious P- and CP-violations disappearing after the chiral equivalence transformation (3.41). But applying (3.41) to Eq. (4.10), that contains an interaction, cannot eliminate the γ^5 entirely.

It is crucial to understand that the conventional minimal interaction as all new non-minimal interactions in Eqs. (4.12a) and (4.12b) are originated by the minimal interaction in terms of the 5-potential $B_M(x, \tau)$ that was introduced by the substitution (4.3). One may also think that the P- and CP-violations revealed in present theory are a "price" for its invariance with respect to the larger gauge group (4.1).*

*However, C- and CPT-symmetries are held in our scheme and can be introduced naturally on a 5-dimensional level. For instance, the charge conjugation C is defined as follows

$$\Psi(x,\tau) \to G_2\Psi^*(x,-\tau) \quad ,$$
$$B_M(x,\tau) \to -B_M(x,\tau)$$

(* denotes the complex conjugation). It is readily verified that Eqs. (4.4a)-(4.4b) remain unaltered after this transformation. Note, that all five components of the electromagnetic 5-potential change their signs under the charge conjugation. On the physical plane $\tau = 0$ we obtain, obviously, the ordinary C-conjugation rule

$$\psi_a(x) \to \gamma_2\psi_a^*(x) \quad , \quad a = 1,2 \quad ;$$
$$A_\mu(x) \to -A_\mu(x)$$

and, in addition,

$$\mathscr{F}_{\mu 4}(x,0) \to -\mathscr{F}_{\mu 4}(x,0) \quad .$$

The invariance of Eqs. (4.12a)-(4.12b) under these transformations is evident.

iv. If $F_{\mu\nu}(x)$ and $\mathcal{F}_{\mu 4}(x,0)$ in Eqs. (4.12a)-(4.12b) are fixed, then one can consider them as the motion equations of charged Dirac particles in an <u>external electromagnetic field</u>. The appropriate lagrangian density is of the form:

$$L_{DIRAC}(x) + L_{INT}(x) \tag{4.15}$$

where $L_{DIRAC}(x)$ is given by (3.26) and

$$\begin{aligned} L_{INT}(x) = - \sum_{a=1,2} \Big[& e_o \overline{\psi}_a(x) \gamma^{\mu} \psi_a(x) A_{\mu}(x) + \\ + \frac{ie_o \cos\theta_o}{4} \overline{\psi}_a(x) \gamma^5 \sigma^{\mu\nu} \psi_a(x) F_{\mu\nu}(x) - & \frac{e_o}{4} \sin\theta_o \overline{\psi}_a(x) \sigma^{\mu\nu} \psi_a(x) F_{\mu\nu}(x) \Big] + \\ & + \frac{ie_o}{2} \left[\overline{\psi}_1(x) \gamma^{\mu} \psi_1(x) - \overline{\psi}_2(x) \gamma^{\mu} \psi_2(x) \right] \mathcal{F}_{\mu 4}(x, 0) \end{aligned} \tag{4.16}$$

Let us proceed now to the general case when the electromagnetic field is included in our dynamical system. As dynamical variables describing this field on the physical plane $\tau = 0$ we choose the quantities

$$(A_{\mu}(x), \quad \mathcal{F}_{\mu 4}(x,0)) \tag{4.17}$$

which are remaining independent variables if even an interaction is switched on (cf. (2.33)).

Next, adding the term $L_{MAXWELL} = -\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x)$ to (4.15), we may write the following action integral for our system:

$$= \int d^4x \, [L_{MAXWELL} + L_{DIRAC} + L_{INT}(x)] = \tag{4.18}$$

$$= \int d^4x \, L_{TOTAL}(x).$$

Further, one should realize that the dynamical variables $\psi_a(x)$, $\bar{\psi}_a(x)$ and $\mathcal{F}_{\mu 4}(x,0)$ are not completely independent. They are subjected to the constraint that appears as a result of projecting 5-sector equation (4.9a) onto the plane $\tau = 0$ *

$$i\mathcal{F}_{\mu 4}(x,0) = \frac{e_0}{4} J_\mu(x,0) \tag{4.19}$$

where, because of (3.37a), (3.33), (3.40) and (3.41)

$$J_\mu(x,0) = \bar{\psi}_1(x)\gamma_\mu\psi_2(x) + \bar{\psi}_2(x)\gamma_\mu\psi_1(x) \tag{4.20}$$

Now the motion equations for our fields emerge from a stationary condition of the functional

$$\int d^4x \{L_{\text{TOTAL}}(x) + \xi^\mu(x)[i\mathcal{F}_{\mu 4}(x,0) - \frac{e_0}{4} J_\mu(x,0)]\} \tag{4.21}$$

where $\xi^\mu(x)$ are Lagrange multipliers. Thus, one obtains

$$(i\not\partial - e_0\not A(x) - m_0)\psi_1(x) = \frac{ie_0\cos\theta_0}{4}\gamma^5\sigma^{\mu\nu}F_{\mu\nu}(x)\psi_1(x) -$$

$$- \frac{e_0\sin\theta_0}{4}\sigma^{\mu\nu}F_{\mu\nu}(x)\psi_1(x) - \frac{ie_0}{2}\gamma^\mu\mathcal{F}_{\mu 4}(x,0)\psi_1(x) + \frac{e_0}{4}\xi^\mu\frac{\partial J_\mu(x,0)}{\partial\bar{\psi}_1(x)}; \tag{4.22a}$$

---

*Cf. Eq. (2.30a). Instead of Eq. (2.30b) we have now

$$2B_4(x,0) - i\frac{\partial}{\partial\tau}B_4(x,0) + i\frac{\partial A_\nu(x)}{\partial x_\nu(x)} = \frac{e_0}{4}J_4(x,0)$$

Similar to the free case, this relation due to our choice of independent variables, is not a constraint but just describes an <u>arbitrary</u> <u>gauge</u> of the electromagnetic 4-potential $A_\mu(x)$.

$$(i\partial\!\!\!/ - e_o A\!\!\!/(x) - m_o)\psi_2(x) = \frac{ie_o\cos\theta_o}{4}\gamma^5\sigma^{\mu\nu}F_{\mu\nu}(x)\psi_2(x) -$$

$$-\frac{e_o\sin\theta_o}{4}\sigma^{\mu\nu}F_{\mu\nu}(x)\psi_2(x) + \frac{ie_o}{2}\gamma^\mu\mathcal{F}_{\mu 4}(x,0)\psi_2(x) + \frac{e_o}{4}\xi^\mu\frac{\partial J_\mu(x,0)}{\partial\bar{\psi}_2(x)}; \tag{4.22b}$$

$$\frac{\partial F^{\mu\nu}(x)}{\partial x^\nu} = e_o\sum_{a=1,2}[\bar{\psi}_a(x)\gamma^\mu\psi_a(x) - \frac{i\cos\theta_o}{2}\frac{\partial}{\partial x^\nu}(\bar{\psi}_a(x)\gamma^5\sigma^{\mu\nu}\psi_a(x)) +$$

$$+\frac{\sin\theta_o}{2}\frac{\partial}{\partial x^\nu}(\bar{\psi}_a(x)\sigma^{\mu\nu}\psi_a(x)]; \tag{4.22c}$$

$$\frac{e_o}{2}[\bar{\psi}_1(x)\gamma^\mu\psi_1(x) - \bar{\psi}_2(x)\gamma^\mu\psi_2(x)] + \xi^\mu(x) = 0. \tag{4.23}$$

These equations, completed by the constraint (4.19), can be used to determine the functions $\psi_a(x)$ and $A_\mu(x)$. Thus, altogether these are the 4-sector equations in question that should play the same role in our approach as the Dirac-Maxwell equations (1.4a)-(1.4b) do in the conventional theory of electromagnetic interactions. It is instructive to write these generalized Dirac-Maxwell equations once more, in terms of $\psi_a(x)$, $\bar{\psi}_a(x)$, $A_\mu(x)$ and $F_{\mu\nu}(x)$ only, introducing explicitly the fundamental length ℓ :

$$(i\partial\!\!\!/ - e_o A\!\!\!/ - m_o)\psi_1(x) = \frac{i e_o \ell \cos\theta_o}{4}\gamma^5 \sigma^{\mu\nu} F_{\mu\nu}(x)\psi_1(x) -$$

$$- \frac{e_o \ell \sin\theta_o}{4}\sigma^{\mu\nu} F_{\mu\nu}(x)\psi_1(x) - \frac{e_o^2 \ell^2}{8}[(\bar{\psi}_1(x)\gamma_\mu\psi_2(x) + \bar{\psi}_2(x)\gamma_\mu\psi_1(x))\gamma^\mu\psi_1(x) +$$

$$+ (\bar{\psi}_1(x)\gamma_\mu\psi_1(x) - \bar{\psi}_2(x)\gamma_\mu\psi_2(x))\gamma^\mu\psi_2(x)]; \tag{4.24a}$$

$$(i\partial\!\!\!/ - e_o A\!\!\!/ - m_o)\psi_2(x) = \frac{i e_o \ell \cos\theta_o}{4}\gamma^5 \sigma^{\mu\nu} F_{\mu\nu}(x)\psi_2(x) -$$

$$- \frac{e_o \ell \sin\theta}{4}\sigma^{\mu\nu} F_{\mu\nu}(x)\psi_2(x) + \frac{e_o^2 \ell^2}{8}[(\bar{\psi}_1(x)\gamma_\mu\psi_2(x) + \bar{\psi}_2(x)\gamma_\mu\psi_1(x))\gamma^\mu\psi_2(x) -$$

$$-(\bar{\psi}_1(x)\gamma_\mu\psi_1(x) - \bar{\psi}_2(x)\gamma_\mu\psi_2(x))\gamma^\mu\psi_1(x)] \tag{4.24b}$$

$$\frac{\partial F^{\mu\nu}(x)}{\partial x^\nu} = e_o[j^\mu(x) + \ell\cos\theta_o j^\mu_{EDM}(x) + \ell\sin\theta_o j^\mu_{MDM}(x)] \tag{4.24c}$$

where

$$\sin\theta_o = \tanh\mu_o/2 = \frac{\sqrt{1+m_o^2\ell^2}-1}{m_o\ell}$$

$$\cos\theta_o = \frac{1}{\cosh\mu/2} = \sqrt{\frac{2}{1+\sqrt{1+m_o^2\ell^2}}} \tag{4.25}$$

and

$$j^\mu(x) = \sum_{a=1,2}\bar{\psi}_a(x)\gamma^\mu\psi_a(x) \tag{4.26}$$

$$j^\mu_{EDM}(x) = -\frac{i}{2}\sum_{a=1,2}\frac{\partial}{\partial x^\nu}(\bar{\psi}_a(x)\gamma^5\sigma^{\mu\nu}\psi_a(x)) \tag{4.27}$$

$$j^\mu_{MDM}(x) = \frac{1}{2}\sum_{a=1,2}\frac{\partial}{\partial x^\nu}(\bar{\psi}_a(x)\sigma^{\mu\nu}\psi_a(x)) \tag{4.28}$$

From Eqs. (4.24a)-(4.24b) one can derive the continuity equation for the total electromagnetic current (4.26) of normal and abnormal particles:

$$\frac{\partial j^\mu(x)}{\partial x^\mu} = 0 \tag{4.29}$$

Note that the currents (4.27)-(4.28) satisfy Eq. (4.29) independently of any equation of motion.

Evidently, as $\ell \to 0$ our equations (4.24a), (4.24b) and (4.24c) turn into Dirac-Maxwell equations (1.4a)-(1.4b) up to an additional degeneracy connected with $SU_\tau(2)$-symmetry. Let us emphasize that the non-linear spinor terms, originated by the τ-photon interaction, violate this symmetry in Eqs. (4.24a)-(4.24b). At that, the associated coupling consta nts are relatively small $(\sim e^2\ell^2)$.

It is easily seen that the lagrangian density corresponding to Eqs. (4.24a), (4.24b) and (4.24c) will coincide with $L_{TOT}(x)$ from (4.18) if the variable $\mathcal{F}_{\mu 4}(x,0)$ in this expression is substituted using (4.19). So

$$L_{TOT}(x) = -\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) + \bar{\Psi}(x)(i\partial\!\!\!/ - m)\Psi(x) -$$

$$- e_0 \bar{\Psi}(x)\gamma^\mu \Psi(x) A_\mu(x) - \frac{ie_0\ell}{4}\cos\theta_0 \bar{\Psi}(x)\gamma^5\sigma^{\mu\nu}\Psi(x)F_{\mu\nu}(x) +$$

$$+ \frac{e_0\ell}{4}\sin\theta_0 \bar{\Psi}(x)\sigma^{\mu\nu}\Psi(x)F_{\mu\nu}(x) + \frac{e^2\ell^2}{8}(\bar{\Psi}(x)\sigma_3\gamma^\mu\Psi)(\bar{\Psi}(x)\sigma_1\gamma_\mu\Psi(x)) ,$$

(4.30)

where

$$\Psi(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \end{pmatrix} \quad ; \quad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad , \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad .$$

V. DISCUSSION AND INTERPRETATION

Let us start with $SU_\tau(2)$-symmetry. It is clear that the theory developed here, after an appropriate quantization procedure, has to be applied first to a description of the electromagnetic interactions of leptons, since this is the arena of the extremely precise tests of QED. As is well known, the electron and muon have been found, up to now, to have identical electromagnetic properties, described universally by the Dirac-Maxwell equations (1.4a)-(1.4b)*), the only difference being the value of the mass. The origin of this universality is an old puzzle.

But now we seem to be in a position to say something about it. In our scheme, the new flavor, connected with the invariance requirement under τ-inversion (see Sec. III), is deeply ingrained. In the 5-sector, it is expressed by the obligatory use of 8-component spinors. On the 4-sector level, we have inevitably equations of motion for the pair of spinor fields $\psi_1(x)$ and $\psi_2(x)$.

Therefore, let us interpret these two fields as follows:

$$\psi_1(x) = e$$

$$\psi_2(x) = \mu \qquad . \tag{5.1}$$

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*The renormalization technicalities of electron QED coincide with those of muon QED.

Now we have a right to say that the new approach is basically μe-universal.* The theory will be damaged if, for instance, the muon field $\psi_2(x)$ is removed.

In the 4-sector framework μe-universality is extended to $SU_\tau(2)$-symmetry when one neglects the τ-photon interaction terms. But even in the presence of these terms, the interactions of the electron and muon with the ordinary photons (the conventional minimal coupling, EDM and MDM interactions) remain identical.

If all terms containing the fundamental length in Eqs. (4.34a), (4.34b) and (4.34c) are omitted then two copies of the Dirac-Maxwell equations (1.4a)-(1.4b) emerge, one to provide the conventional QED description of the electron and the other for the same purpose concerning the muon. In this approximation the masses of the electron and of the muon are equal.

It is tempting to speculate on the τ-photon interaction as a possible origin of the muon-electron mass difference.** Further, this interaction should give rise to the decay***

*The recently discovered τ-lepton needs a partner to complete another $SU_\tau(2)$-doublet. The neutrinos ν_e and ν_μ already can be treated as an $SU_\tau(2)$-doublet. Let us point out that the new flavor described by $SU_\tau(2)$ is not, of course, a privilege of leptons only. It is a new universal degree of freedom which is intrinsically connected with the fundamental length and which is relevant to all particles including hadrons, hypothetical intermediate W-boson and so on. As for hadrons, this flavor has to be introduced on the quark level. So one is now asked to believe in the existence of electron-type and muon-type quarks, probably with the large difference in their masses. Perhaps to explain the existence of the Υ-family, and possible similar discoveries in the future, one should think along these lines. Further, we can also expect particles that will be components of $SU_\tau(2)$-triplets and so on.

**The author is grateful to Prof. R. Feynman for a valuable discussion of this point.

***Note, that in this reaction, parity is conserving.

$$\mu \to 3e \tag{5.2}$$

and, effectively, to such processes as

$$\mu \to e\gamma \tag{5.3a}$$

$$\mu \to e\gamma\gamma \tag{5.3b}$$

$$\mu + Z \to e + Z, \text{ etc.} \tag{5.3c}$$

The most accurate experiment on a search for the decay (5.2) gives the following upper bound for its branching ratio[38]:

$$\frac{\Gamma(\mu \to 3e)}{\Gamma(\mu \to e\nu\bar{\nu})} < 1.9 \times 10^{-9} \quad . \tag{5.4}$$

If we shall forget for a while that our scheme is not yet secondary quantized, then we can estimate, with the help of (5.4), the allowed magnitude of the coupling constant in the τ-photon interaction, i.e. the upper bound* of the fundamental length:

$$\ell \lesssim 3 \times 10^{-18} \text{ cm} \quad . \tag{5.5}$$

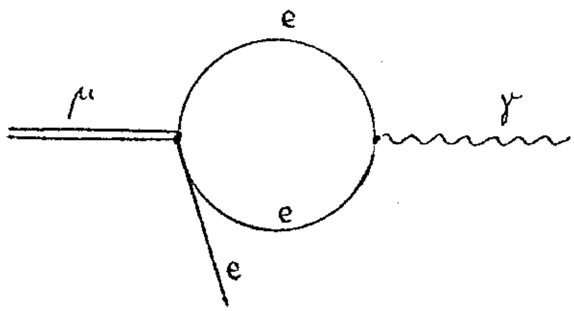
This is consistent with the recent data on the decay (5.3a)[39]

$$\frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to \text{all})} \lesssim 2.0 \times 10^{-10} \quad . \tag{5.6}$$

---

*Let us pay attention that its magnitude is three orders less than in (1.2).

In our scheme the considered reaction is described by the graph



It is interesting to notice that the τ-photon interaction is characterized by the finite range although the τ-photon field is massless.[30] One should realize that, due to this interaction, the picture of the electromagnetic phenomena will change drastically at super-high energies $\gtrsim M/e_o$.

As explained earlier[30] our approach to a formulation of the electromagnetic interaction theory with the de Sitter momentum space in terms of <u>5-dimensional quantities</u> ($\partial/(\partial x^M)$, $B_M(x,\tau)$, $\mathscr{F}_{MN}(x,\tau)$, $\Psi(x,\tau)$, etc.) is an <u>alternative</u> to the manifestly <u>4-dimensional formalism</u> based on 4-dimensional Fourier analysis on the hyperboloid (1.19). The relevant configurational 4-space turns out to be continuous along the time direction and quantized along any space axis.* [4,9,20-21] For instance, the analog of the Yukawa potential, produced by the propagator (c.f. Eq. (3.7a))

$$D^c(p) = \frac{1}{2p^4 - 2\cosh\mu + io} \quad , \tag{5.7}$$

is of the form

$$V_{YUKAWA}(n) = \text{const.}\ \frac{e^{-\mu(n+1)}}{n+1} \quad ; \quad n = 0,1,2,3,\ldots \tag{5.8}$$

---

*In the euclidean formulation, the curved 4-momentum space becomes compact and, hence, the configurational 4-space is totally quantized.

So the space which separates particles possesses some granular structure that can be treated as one of the novel properties of the vacuum in this version of a theory. Meanwhile, from the 5-dimensional approach we know that the charged fields $\psi_1(x)$ and $\psi_2(x)$, due to the τ-photon mediation, are interacting even if $A_\mu(x) \equiv 0$. Supposing that both approaches are equivalent to each other* we might figure that the τ-photon interaction is caused by the scattering of our particles on granularities of the space structure. In other words, the τ-photon may be treated as a quasiparticle which, in the framework of the 5-dimensional approach, is a specific substitute for the granularity of space.

Let us consider now the MDM and EDM interactions emerged in our scheme. Introducing the standard notation $(e_o\kappa)/(4m_o)\sigma^{\mu\nu}F_{\mu\nu}(x)\psi_a(x)$ for Pauli terms in Eqs. (4.34a)-(4.34b) we obtain, using (4.35), the following expression for the intrinsic anomalous magnetic moment of the charged Dirac particle

$$\kappa = \cosh \mu_o - 1 = \sqrt{1 + m_o^2 \ell^2} - 1 = \\ = \sqrt{1 + \frac{m_o^2}{M^2}} - 1 \qquad . \tag{5.9}$$

The covariant EDM-term is less familiar. Therefore, we continue our analysis non-relativistically. Assuming that our Dirac fields interact with an external electromagnetic field $A^\mu(x)$ such that

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*The mapping between these two formalisms is carried out by the Fourier transforms similar to

$$\psi_1(x) = \frac{1}{(2\pi)^{3/2}} \int_{p^4>0} \delta(p^2 - p_4^2 + 1)\psi(p, p^4)e^{i(px)}d^5p \qquad ,$$

$$\psi_2(x) = \frac{1}{(2\pi)^{3/2}} \int_{p^4<0} \delta(p^2 - p_4^2 + 1)\psi(p, p^4)e^{i(px)}d^5p \ ; \ (px) = p^0x^0 - \vec{p}\,\vec{x} \qquad .$$

$$A^{o}(x) = \phi(r) = \text{arbitrary}$$

$$\vec{A}(x) = \tfrac{1}{2}[\vec{H} \times \vec{r}] \quad ; \quad \vec{H} = \text{const.} \quad , \tag{5.10}$$

and expanding Eqs. (4.34a)-(4.34b) in powers of $1/c$, one finds the following generalized Pauli equation:

$$i\hbar \frac{\partial}{\partial \tau} \phi_a(\vec{r}, t) = \left[\frac{\vec{p}^2}{2m_o} - e_o\phi(r) - e_o \frac{(\vec{L} + g\vec{S})\cdot\vec{H}}{2m_o c} + \frac{e_o \cos\theta_o}{Mc} (\vec{S}\cdot\vec{E}) \right] \phi_a(\vec{r})$$

$$(a = 1,2) \tag{5.11}$$

where

$$\vec{p} = -i\hbar \frac{\partial}{\partial \vec{r}} \quad ; \quad \vec{L} = [\vec{r} \times \vec{p}] \quad ; \quad \vec{S} = \frac{\hbar\vec{\sigma}}{2}$$

$$\vec{E} = -\frac{\partial}{\partial \vec{r}} \phi(r) \tag{5.12}$$

and

$$g = 2(1 + \kappa) = 2\sqrt{1 + \frac{m_o^2}{M^2}}$$

$$\cos\theta_o = \sqrt{\frac{2M}{M + \sqrt{M^2 + m_o^2}}} \quad . \tag{5.13}$$

Further, writing the EDM-interaction in the canonical form

$$U_{EDM} = -(\vec{d} \cdot \vec{E}) \quad , \tag{5.14}$$

one obtains the following expression for the <u>intrinsic</u> electric dipole moment of the charged Dirac particle:

$$\vec{d} = -e_o \frac{\cos\theta_o}{Mc} \vec{S} = -e_o \ell \frac{\cos\theta_o}{2} \vec{\sigma} \quad . \tag{5.15}$$

Let us point out that EDM $\vec{d}$ is antiparallel to the particle magnetic moment

$$\vec{\mu} = \frac{e_o g}{2m_o c} \vec{S} \quad . \tag{5.16}$$

Eq. (5.11) holds in the general case of arbitrary ratio between the particle mass m_o and the fundamental mass M. For leptons, anyhow,

$$\frac{m_o}{M} << 1 \quad . \tag{5.17}$$

From here and from (5.13) and (5.16) it follows that

$$\left(\frac{g-2}{2}\right)_{\text{lepton}} \simeq \frac{m_o^2}{2M^2} \tag{5.18a}$$

$$|\vec{d}|_{\text{lepton}} \simeq \frac{e_o \ell}{2} \quad . \tag{5.18b}$$

Comparing (5.18a) with the current theoretical and experimental uncertainties in this quantity[40]

$$\Delta\left(\frac{g-2}{2}\right)_{\text{electron}} = (2 \div 6) \times 10^{-10}$$

$$\Delta\left(\frac{g-2}{2}\right)_{\text{muon}} = (10 \div 12) \times 10^{-9} \tag{5.19}$$

we obtain one more upper bound for the fundamental length*:

$$\ell \lesssim 2.6 \times 10^{-17}\ \mathrm{cm} \tag{5.20}$$

that is not very far from (5.5).

Using (5.5), (5.20) and (5.18b) we can conclude that the upper bound for lepton EDM, in order of magnitude, is at least

$$|\vec{d}| \lesssim [(10^{-17} \div 10^{-18})\mathrm{cm}]\, e_o \quad . \tag{5.21}$$

This is consistent with the old experimental data on a direct measurement of the electron and muon EDM,[41] with observed shifts of atomic levels,[41] with parity violation effects in atoms,[42-43] with the recent search for the parity violation in the polarized electron scattering.[44]

On the other hand, a number of experiments was performed on indirect estimation of the electron EDM through the measurements of EDM that it induces in atoms. The results obtained are the following:

$$|\vec{d}_{\mathrm{electron}}| \lesssim \begin{cases} \lesssim (3\times 10^{-24}\ \mathrm{cm})e_o^{(45)} & (5.22a) \\ = [(0.7 \pm 2.2)\times 10^{-24}\mathrm{cm}]\, e_o^{(46)} & (5.22b) \\ = [(1.9 \pm 3.4)\times 10^{-24}\ \mathrm{cm}]\, e_o^{(47)} & (5.22c) \\ = [(8.1 \pm 11.6)\times 10^{-23}\ \mathrm{cm}]\, e_o^{(48)} \quad . & (5.22d) \end{cases}$$

The results (5.22a)-(5.22c)** lie in the same range of values as the latest measured upper bound for the neutron EDM[49]

*To our knowledge, it is the first time a highly precise experiment, designed for a test of a validity of QED, is used to estimate the fundamental length.

**The method employed in Ref. 48 is independent of that used in Refs. 45-47.

$$|\vec{d}_{\text{neutron}}| \lesssim 3 \times 10^{-24}\ \text{cm} \quad . \tag{5.23}$$

From (5.18b) and (5.22a)-(5.22c) one obtains a considerably more restrictive bound on ℓ than in (5.5):

$$\ell \lesssim 10^{-24} - 10^{-23}\ \text{cm} \quad . \tag{5.24}$$

Assuming that the electromagnetic interactions of quarks are described by our equations (4.34a)-(4.34c), a similar estimate for ℓ can be established from (5.23) in the framework of the relativistic quark model.[50]

As is known,[51] the existence of non-zero EDM for elementary particles would be direct evidence of the violation of the CP-symmetry. Thus, according to our approach, the mechanism of the CP-violation might be purely electromagnetic if the theory of the electromagnetic interactions is based on the fundamental length hypothesis.* It does not need any comment that the experimental discovery of the particle EDM would be of great importance for the present theory. Concerning leptons which have not undergone strong interactions, one should realize that, due to (5.18b), the measurement of their EDM is the straight measurement of the fundamental length.

Since in this approach even "bare" Dirac particles possess EDM and anomalous MDM, it means that they are <u>extended</u> objects having some structure.** As follows from (5.15), (5.16) and (5.13), if the ratio m_0/M runs from 0 to ∞, $|\vec{d}|$ and $|\vec{\mu}|$ vary in the following limits

---

*As was pointed out[30] non-abelian gauge theories can be reformulated in our terms as well. It would be interesting to see, for instance, new versions of the QCD and the Salam-Weinberg model, containing the fundamental length. What situation will finally take place in these theories concerning CP-symmetry?

**It somehow testifies to the existence of the ultraviolet cut-off in our scheme (see the next Section).

$$\frac{e_o \hbar}{2Mc} \geq |\vec{d}| \geq 0$$

$$\frac{e_o \hbar}{2m_o c} \geq |\vec{\mu}| \geq \frac{e_o \hbar}{2Mc} \quad . \tag{5.25}$$

So the quantity

$$\frac{e_o \hbar}{2Mc} = \frac{e_o \ell}{2} \tag{5.26}$$

plays the role of a minimal magneton attainable only when

$$m_o \gg M \quad . \tag{5.27}$$

Hence, superheavy Dirac particles, if such objects somewhere exist*, should serve not only as sources of the static Coulomb field but also as sources of the static magnetic field, produced by the MDM

$$\frac{e_o \hbar}{2Mc} \vec{\sigma} \quad . \tag{5.28}$$

*C.f. the "maximon" considered by Markov.[52]

VI. CONCLUDING REMARKS

In this paper we derived generalized classical equations of motion for the interacting electromagnetic and Dirac fields, containing the fundamental length as a new universal scale. The secondary quantization procedure, the appropriate diagram techniques, the renormalization, etc. will be worked out separately.[37] However, in a preliminary manner, we may say that the problem of ultraviolet divergences seems to lose here its acuteness. Although the non-minimal interactions in Eqs. (4.34a)-(4.34c), conventionally speaking, are non-renormalizable, the underlying symmetry with respect to larger gauge group leads to a situation which is similar to what one faces in so-called superrenormalizable theories. As a result, the renormalization procedure in this case should deal with finite quantities only.

Such things could be expected from the comparison of our 5-dimensional approach with the 4-dimensional one[19-29] that we outlined briefly in Sec. V. Indeed, due to (5.8), the "scalar" Coulomb potential in the 4-dimensional framework reads

$$V_{COULOMB} = \frac{\text{const}}{(n+1)\ell} \qquad . \tag{6.1}$$

This expression is free of singularities at small distances. It is already some indication of an absence of divergences in the relevant field theory. Furthermore, in the euclidean formulation of the 4-dimensional approach one needs to use the compact p-space (see footnote on p. 50):

$$p_1{}^2 + p_2{}^2 + p_3{}^2 + p_4{}^2 + p_5{}^2 = 1$$

$$p_5 = ip_o \qquad . \tag{6.2}$$

In other words, the theory intrinsically contains a universal cut-off. Formulae like (5.8) and (6.1) are just one of the manifestations of this cut-off.

Turning back to the 5-dimensional formalism we would like to emphasize that in the presence of an interaction the de Sitterian constraint of the ($\bigcirc$-1)-type remains true only for the gauge functions $\lambda(x, \tau)$ and the τ-photon field $A_4(x, \tau) = e^{i\tau} B_4(x, \tau)$. The Dirac field $\Psi(x, \tau)$, for instance, satisfies instead the constraint (4.4b). As a matter of fact, the relation (4.4b) plays the key role in our approach because it combines three important things altogether:

i) de Sitterian geometry of p-space when an interaction is turned off ($e_o \to 0$);

ii) generalized minimal substitution law based on the new gauge group;

iii) preferable role of the physical plane $\tau = 0$ that originates the correspondence principle.

Putting

$$\Pi_M = i\frac{\partial}{\partial x^M} - e_o B_M(x, \tau) \tag{6.3}$$

we can write (4.4b) as

$$\left[\left(\Pi_o^2 - \vec{\Pi}^2 - \Pi_4^2 + 1\right) \Psi(x, \tau)\right]_{\tau=0} = 0 \quad . \tag{6.4}$$

So the main geometrical point of our fundamental length hypothesis may be adjusted as follows: the "generalized momentum" Π is declared the de Sitterian vector in a sense of relation (6.4). If an interaction is switched off then Π coincides with the ordinary momentum which belongs to the de Sitter space (1.19) as is due to (3.22). Hence, finally, our 5-scheme is a local gauge theory of electromagnetic interactions based on the assumption that free fields are described in terms of the de Sitter p-space and the "generalized momentum" Π operates as de Sitterian vector on the physical plane $\tau = 0$. As was pointed out in Sec. I, in the free case the 3-dimensionality of the mass shell screens the difference in a geometrical structure between 4-dimensional Minkowsky p-space and 4-dimensional de Sitter p-space. The only de Sitterian attribute that survives and makes one theory different from another is the flavor labelling the normal (electron-type) and abnormal (muon-type) fields.

Our last remark concerns a global structure of the de Sitter p-space. As is known,[53] two spaces of constant curvature may possess identical metric properties but differ by topological ones. For instance, if on the surface (1.19) the diametrically opposite points $(p^o, \vec{p}, p^4)$ and $(-p^o, -\vec{p}, -p^4)$ will be identified, we shall obtain the so-called "pseudoelliptic" de Sitter space that possesses a more exquisite topology than the initial one, called "pseudospherical" de Sitter space.

Pseudoelliptic p-space was employed in first attempts to use de Sitter geometry in the field theory.[8,9,11] It would be interesting to see what changes will occur in our formalism if we adopt this new topology. Certainly, the concept of normal and abnormal fields and definitions of discrete symmetry transformations (C, P, T and CPT) should be modified in such a scheme.

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