Fermi National Accelerator Laboratory

FERMILAB-Pub-78/63-THY
August 1978

Hierarchy of Symmetry Breakings and Neutral Currents in a SU(6) Grand Unification Model

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ABSTRACT

In a SU(6) grand unification model with eight quarks and eight leptons belonging to 15-plet and singlet representations, the symmetry is spontaneously broken by the sequence, $SU(6) \to SU(3)^c \times SU(2) \times U(1) \times U(1) \to SU(3)^c \times U(1)$. For two cases of symmetry breakings the effective weak neutral current coupling constants are compared with experiment. For the $SU(3)^c \times SU(2) \times U(1) \times U(1) \to SU(3)^c \times U(1)$ symmetry breaking, the coupling constants reproduce the Weinberg-Salam model with a small correction term. Agreement with the experimental mean values are improved with the correction term. Parity-violation in atomic physics is also discussed.

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The standard SU(2) × U(1) gauge theory[1] of weak and electromagnetic interactions has been successful in predicting the existence of the charmed quark[2] in connection with GIM mechanism.[3] With the observation[4] of the predicted weak neutral current, the confidence in the extended SU(2) × U(1) model with four quarks and leptons was further raised. The recently observed Upsilon (9.4 GeV)[5] which is interpreted as a bound state of b and $\bar{b}$ quarks, and the heavy lepton τ(1.9 GeV)[6] necessitate the further extension of the model. The SU(3) × U(1) models seem to be ruled out by the analysis[7] of the weak neutral current coupling constants which favorably supports the Weinberg-Salam(W-S) model.[1] The W-S model still has one unresolved problem with its predicted parity-violation in atomic physics. The experiments in this area is still uncertain that any conclusive settlement has to wait further experimental result.

Under these circumstances, it would be meaningful to unify further with strong interaction in terms of the quantum chromodynamics[8] (QCD) with the exact color $SU(3)^c$ symmetry and an octet of colored neutral massless gluons such that the W-S model is embedded in it. There are attempts to enlarge the symmetry group G such that G contains $SU(3)^c \times SU(2) \times U(1) \times \cdots$. In the grand unification models such as SU(5)[9,10] and SU(6)[11,12] models, the symmetry group imposes mass relations among the leptons and quarks after the symmetry is broken spontaneously by Higgs mechanism. The resulting mass relations among fermions and the mixings of their states provide additional constraints on the models. SU(5) model gives mass relations[9] like $m_e=m_\mu$ and $m_\mu=m_s$, etc. after the spontaneous breaking of the symmetry down to $SU(3)^c \times U(1)$. If the electromagnetic and gluon radiative corrections are included, this may further split the fermion masses.[13] There are no other useful mass relations in SU(5) model and no explanation of Cabibbo angle in this model.

As for SU(6) model discussed in Ref. 12, we have mass relations such as $m(d_i') + m(s_i') + m(b_i') + m(h_i') = m(e^-') + m(\bar{\mu}') + m(E^-') + m(M^-')$ and $m(d_i')/m(b_i') = m(s_i')/m(h_i') = |b/a|$ and it predicts the masses of b_i' and h_i' quarks respectively $m(b_i') = 3.1 \sim 4.8$ GeV and $m(h_i') = 4.1 \sim 6.2$ GeV, where the primes indicate the diagonalized states and the color indices $i = 1,2,3$. This prediction was possible after generating Cabibbo angle by the diagonalization of mass matrix.

In this paper, we consider a hierarchy of symmetry breakings in the sequence, $SU(6) \to SU(3)^c \times SU(2) \times U(1) \times U(1) \to SU(3)^c \times U(1)$, and obtain the effective weak neutral current coupling constants for $SU(6) \to SU(3)^c \times U(1)$ and $SU(3)^c \times SU(2) \times U(1) \times U(1) \to SU(3)^c \times U(1)$ breakings. Comparing them with the experimental results in the second case, the relative mass ratios of the neutral vector bosons Z and Y to W are obtained at the level of $SU(3)^c \times U(1)$. The neutral current coupling constants in the second case of symmetry breaking are exactly the ones obtained by the W-S model in the limit of $m(Z)/m(Y) \to$ large. The correction terms to the coupling constants are related to this mass ratio.

We consider the following two left-handed (right-handed) antisymmetric 15-plets and two left-handed (right-handed) singlets of fermions.

$$L_1 = \frac{1}{\sqrt{2}} \left[\begin{array}{ccc|ccc} 0 & \bar{u}_3 & -\bar{u}_2 & -u_1 & -d_1 & -b_1 \\ -\bar{u}_3 & 0 & \bar{u}_1 & -u_2 & -d_2 & -b_2 \\ \bar{u}_2 & -\bar{u}_1 & 0 & -u_3 & -d_3 & -b_3 \\ \hline u_1 & u_2 & u_3 & 0 & e^+ & -E^+ \\ d_1 & d_2 & d_3 & -e^+ & 0 & \bar{E}^0 \\ b_1 & b_2 & b_3 & E^+ & -\bar{E}^0 & 0 \end{array} \right]_L \qquad R_1 = \frac{1}{\sqrt{2}} \left[\begin{array}{ccc|ccc} 0 & \bar{t}_3 & -\bar{t}_2 & -t_1 & -b_1 & -d_1 \\ -\bar{t}_3 & 0 & \bar{t}_1 & -t_2 & -b_2 & -d_2 \\ \bar{t}_2 & -\bar{t}_1 & 0 & -t_3 & -b_3 & -d_3 \\ \hline t_1 & t_2 & t_3 & 0 & E^+ & -e^+ \\ b_1 & b_2 & b_3 & -E^+ & 0 & \bar{\nu}_e \\ d_1 & d_2 & d_3 & e^+ & -\bar{\nu}_e & 0 \end{array} \right]_R$$

$$L_2 = \frac{1}{\sqrt{2}} \left[\begin{array}{ccc|ccc} 0 & \bar{c}_3 & -\bar{c}_2 & -c_1 & -s_1 & -h_1 \\ -\bar{c}_3 & 0 & \bar{c}_1 & -c_2 & -s_2 & -h_2 \\ \bar{c}_2 & -\bar{c}_1 & 0 & -c_3 & -s_3 & -h_3 \\ \hline c_1 & c_2 & c_3 & 0 & \mu^+ & -M^+ \\ s_1 & s_2 & s_3 & -\mu^+ & 0 & M^0 \\ h_1 & h_2 & h_3 & M^+ & \bar{M}^0 & 0 \end{array} \right]_L \qquad R_2 = \frac{1}{\sqrt{2}} \left[\begin{array}{ccc|ccc} 0 & \bar{g}_3 & -\bar{g}_2 & -g_1 & -h_1 & -s_1 \\ -\bar{g}_3 & 0 & \bar{g}_1 & -g_2 & -h_2 & -s_2 \\ \bar{g}_2 & -\bar{g}_1 & 0 & -g_3 & -h_3 & -s_3 \\ \hline g_1 & g_2 & g_3 & 0 & M^+ & -\mu^+ \\ h_1 & h_2 & h_3 & -M^+ & 0 & \bar{\nu}_\mu \\ s_1 & s_2 & s_3 & \mu^+ & -\bar{\nu}_\mu & 0 \end{array} \right]_R$$

$$L_3 = (\bar{\nu}_e)_L \qquad\qquad R_3 = (\bar{E}^0)_R$$

$$L_4 = (\bar{\nu}_\mu)_L \qquad\qquad R_4 = (\bar{M}^0)_R \quad .$$

The SU(6) invariant Lagrangian is

$$\mathcal{L} = \tfrac{1}{2} i\, g_6 \sum_i \mathrm{Tr}(\bar{L}_i \gamma_\mu \lambda^\alpha L_i + \bar{R}_i \gamma_\mu \lambda^\alpha R) V_\alpha^{\ \mu} \quad , \tag{2}$$

where $\alpha = 1, \cdots, 35$.

The charge operator Q is

$$Q = I_3 + \frac{Y}{2} + Y = \tfrac{1}{2}(\lambda_3 + \frac{1}{\sqrt{3}} \lambda_8 + \frac{4}{\sqrt{3}} \lambda_0) \quad , \tag{3}$$

where

$$\lambda_3 = \left[\begin{array}{ccc|ccc} 0 & & & & & \\ & 0 & & & 0 & \\ & & 0 & & & \\ \hline & & & 1 & & \\ & 0 & & & -1 & \\ & & & & & 0 \end{array}\right] \quad \lambda = \frac{1}{\sqrt{3}}\left[\begin{array}{ccc|ccc} 0 & & & & & \\ & 0 & & & 0 & \\ & & 0 & & & \\ \hline & & & 1 & & \\ & 0 & & & 1 & \\ & & & & & -2 \end{array}\right] \quad \lambda_0 = \frac{1}{\sqrt{3}}\left[\begin{array}{ccc|ccc} -1 & & & & & \\ & -1 & & & 0 & \\ & & -1 & & & \\ \hline & & & 1 & & \\ & 0 & & & 1 & \\ & & & & & 1 \end{array}\right]$$

This is due to the charge structure of fermions which is shown in Table 1. The 6×6 matrix representation of the 35 vector bosons $V_j^{\ i}$ where $i,j = 1, \cdots, 6$, has the following diagonal elements,

$$V_1^1 = V_2^2 = V_3^3 = -\frac{1}{\sqrt{3}} V_0$$

$$V_4^4 = V_3 + \frac{1}{\sqrt{3}} V_8 + \frac{1}{\sqrt{3}} V_0$$

(continued)

$$V_5^5 = -V_3 + \frac{1}{\sqrt{3}} V_8 + \frac{1}{\sqrt{3}} V_0$$

$$V_6^6 = -\frac{2}{\sqrt{3}} V_8 + \frac{1}{\sqrt{3}} V_0 \quad . \tag{4}$$

With the 35-plet Higgs particles whose vacuum expectation values (VEV) are

$$\langle \phi_j^i \rangle = a_{35} \left[\begin{array}{ccc|ccc} 1 & & & & & \\ & 1 & & & 0 & \\ & & 1 & & & \\ \hline & & & -1 & & \\ & 0 & & & -1 & \\ & & & & & -1 \end{array}\right]$$

the gluons are kept massless while the lepto-quark (or super-heavy) vector bosons which have color as well as flavor acquire masses as given in Ref. 12. This breaks SU(6) down to $SU(3)^c \times SU(3) \times U(1)$ separating QCD and weak interaction. The second 35-plet Higgs particles with the VEV of

$$\langle \Phi_j^i \rangle = \left[\begin{array}{ccc|ccc} 0 & & & & & \\ & 0 & & & 0 & \\ & & 0 & & & \\ \hline & & & c & 0 & 0 \\ & 0 & & 0 & d & a \\ & & & 0 & b & d \end{array}\right], \text{ where } c + 2d = 0$$

boosts some of the weak vector bosons massive while keeping the photon massless. A 15-plet Higgs bosons with the VeV of

$$\langle \Psi_j^i \rangle = a_{15} \left[\begin{array}{ccc|ccc} 0 & & & & & \\ & 0 & & & 0 & \\ & & c & & & \\ \hline & & & 0 & 0 & 0 \\ & 0 & & 0 & 0 & -1 \\ & & & 0 & 1 & 0 \end{array}\right]$$

and a singlet Higgs boson $\langle\psi_1\rangle = a_1$ make M^0 massive while keeping $\bar{\nu}_e$ and $\bar{\nu}_\mu$ massless. This further breaks the symmetry down to $SU(3)^c \times SU(2) \times U(1) \times U(1)$ and in turn to $SU(3)^c \times U(1)$. The resulting masses are given in Ref. 12, except that here we have (u_i, c_i, t_i, g_i) mixing all among themselves, rather than the separate (u_i, c_i) and (t_i, g_i) mixings. In particular, the states of the weak neutral vector bosons are

$$A = -\frac{1}{\sqrt{2}}\left[\tfrac{1}{2}(\sqrt{3}\,V_3 + V_8) + V_0\right]$$
$$Z = \frac{1}{\sqrt{2}}\left[\tfrac{1}{2}(\sqrt{3}\,V_3 + V_8) - V_0\right]$$
$$Y = \tfrac{1}{2}(V_3 - \sqrt{3}\,V_8) \quad . \tag{5}$$

We consider now the neutral current coupling constants in the case of symmetry breaking, $SU(6) \to SU(3)^c \times U(1)$. For this symmetry breaking, the effective weak neutral coupling constants are given in view of Eqs. (2) and (5),

$$L_{\text{neutral}} = \frac{G}{\sqrt{2}}\,\bar{\nu}_e\gamma_\mu(1+\gamma_5)\nu_e\Big[C_{u_L}\bar{u}\gamma_\mu(1+\gamma_5)u + C_{d_L}\,\bar{d}\,\gamma_\mu(1+\gamma_5)d + C_{u_R}\,\bar{u}\,\gamma_\mu(1-\gamma_5)u + C_{d_R}\,\bar{d}\,\gamma_\mu(1-\gamma_5)d + \cdots\Big] \quad , \tag{6}$$

where

$$\frac{G}{\sqrt{2}} = \frac{g^2}{8m^2(w)}$$
$$C_{u_L} = C_{c_L} = \frac{1}{3}\,A$$

(cont.)

$$C_{d_L} = C_{s_L} = -\frac{1}{3}A$$

$$C_{u_R} = C_{c_R} = -\frac{1}{3}A$$

$$C_{d_R} = C_{s_R} = -\frac{1}{6}A \quad , \qquad (7)$$

where $A \equiv \left(\frac{m^2(w)}{2m^2(Z)}\right)$, and there is no simple relation between m(w) and m(Z) in this case[12]. Note that Y doesn't contribute here. By comparing with experimental values given in Table 2 we see that the coupling constants for u_R and d_R are not as good as the ones for u_L and d_L for the value of $A \approx 1$. The symmetry breaking of $SU(6) \rightarrow SU(3)^c \times U(1)$ gives poor values for right-handed couplings. The neutral current couplings of electron in this case is such that $\bar{e}e$ is pure vector, and $\bar{u}u$ and $\bar{d}d$ are pure axial vectors as far as their couplings through Z are concerned. However through Y , $\bar{e}e$ and $\bar{d}d$ are pure axial vectors with vanishing $\bar{u}u$. Overall, parity-violation in atomic physics is suppressed in this case of symmetry breaking.

At the level of $SU(3)^c \times SU(2) \times U(1) \times U(1)$ symmetry, the 15-plet fermion states are broken down to SU(2) doublets and singlets, as follows:

$$\begin{pmatrix} u_i \\ d_i \end{pmatrix}_L \quad \begin{pmatrix} t_i \\ b_i \end{pmatrix}_R \quad \begin{pmatrix} c_i \\ s_i \end{pmatrix}_L \quad \begin{pmatrix} g_i \\ h_i \end{pmatrix}_R$$

$$b_{i_L} \quad d_{i_R} \quad h_{i_L} \quad s_{i_R}$$

$$\bar{u}_{i_L} \quad \bar{t}_{i_R} \quad \bar{c}_{i_L} \quad \bar{g}_{i_R}$$

$$\begin{pmatrix} E^+ \\ \bar{E}^0 \end{pmatrix}_L \quad \begin{pmatrix} e^+ \\ \bar{\nu}_e \end{pmatrix}_R \quad \begin{pmatrix} M^+ \\ \bar{M}^0 \end{pmatrix}_L \quad \begin{pmatrix} \mu^+ \\ \bar{\nu}_\mu \end{pmatrix}_R$$

$$e^+_L \quad E^+_R \quad \mu^+_L \quad M^+_R$$

$$\bar{\nu}_{e_L} \quad \bar{E}^0_R \quad \bar{\nu}_{\mu_L} \quad M^0_R \qquad (8)$$

Their quantum numbers are shown in Table 1. The SU(2) ×U(1) × U(1) invariant weak interaction is given by

$$L_W = (\tfrac{1}{2}i) \left[g\bar{\psi}\gamma_\mu \vec{\tau}\psi \; \vec{A}^\mu + g'y\bar{\psi}\gamma_\mu\psi B^\mu + g''\psi\gamma_\mu 2Y\psi C^\mu \right], \qquad (9)$$

where ψ is the SU(2) doublets or singlets given in (8). It is noted here that (d_i, s_i, b_i, h_i) as well as leptons are already mixed at this stage as shown in Ref. 12. Thus heavy quarks such as b_i decays to light quarks via these mixed states. Due to the undetermined mixing angles, there is not much we can say at this moment. If we regard E^+ as τ^+ heavy lepton, this model may have predominantly right-handed coupling even after mixing, which may be a problem. One way to modify is to add another 15-plet which contains τ^+ and ν_τ in the right-handed state.

We now introduce Higgs mechanism by a SU(2) doublet and a singlet,

$$\begin{array}{lcccc} & I_3 & y & Y & Q \\ \langle\phi_1\rangle = \begin{pmatrix} 0 \\ \lambda_1 \end{pmatrix} & \begin{array}{c} \frac{1}{2} \\ -\frac{1}{2} \end{array} & \begin{array}{c} \frac{1}{3} \\ \frac{1}{3} \end{array} & \begin{array}{c} \frac{1}{3} \\ \frac{1}{3} \end{array} & \begin{array}{c} +1 \\ 0 \end{array} \\ \langle\phi_2\rangle = (\lambda_2) & 0 & \frac{2}{3} & -\frac{1}{3} & 0 \end{array} \quad . \tag{10}$$

The neutral vector bosons are mixed as

$$\begin{aligned} A_3 &= (Z\cos\phi - A\sin\phi) \\ B &= \left[(Z\sin\phi + A\cos\phi)\cos\alpha - Y\sin\alpha\right] \\ C &= \left[(Z\sin\phi + A\cos\phi)\sin\alpha + Y\cos\alpha\right]. \end{aligned} \tag{11}$$

The charge structure of fermions imposes that

$$\begin{aligned} \frac{1}{e^2} &= \frac{1}{g^2} + \frac{1}{g'^2} + \frac{1}{g''^2} \\ e &= -g\sin\phi = g'\cos\phi\cos\alpha = g''\cos\phi\sin\alpha, \end{aligned} \tag{12}$$

and the diagonalization of mass matrix for the neutral vector bosons Z and Y demands

$$\cos^2\alpha = \frac{1}{3} \quad . \tag{13}$$

The masses of the vector bosons are acquired by

$$\begin{aligned} L_\phi = &\left|(\partial_\mu - i\frac{g}{2}\vec{\tau}\cdot\vec{A}_\mu - i\frac{g'}{2} y B_\mu - i\frac{g''}{2} 2Y C_\mu)\phi_1\right|^2 \\ &+ \left|(\partial_\mu - i\frac{g'}{2} y B_\mu - i\frac{g''}{2} 2Y C_\mu)\phi_I\right|^2 \quad , \end{aligned} \tag{14}$$

and

$$m(W) = \frac{1}{\sqrt{2}}\,\lambda_1 g$$

$$m(Z) = \frac{m(W)}{\cos\phi}$$

$$m(Y) = \sqrt{\frac{2}{3}}\,\lambda_2 a \quad , \qquad (15)$$

where

$$a \equiv (g'^2 + g''^2)^{\frac{1}{2}}$$

$$\cos\phi = \frac{ag}{b}$$

$$b \equiv (a^2 g^2 + g'^2 g''^2)^{\frac{1}{2}} \quad . \qquad (16)$$

The effective weak neutral coupling constants of quarks through Z and Y bosons given in Eq. (6) are obtained by eqs. (9) and (11).

$$C_{u_L} = C_{c_L} = (\tfrac{1}{2} = \tfrac{2}{3}\sin^2\phi) + \frac{1}{18}\left(\frac{\lambda_1}{\lambda_2}\right)^2$$

$$C_{d_L} = C_{s_L} = -\,(\tfrac{1}{2} - \tfrac{1}{3}\sin^2\phi) + \frac{1}{18}\left(\frac{\lambda_1}{\lambda_2}\right)^2$$

$$C_{u_R} = C_{c_R} = (-\,\tfrac{2}{3}\sin^2\phi) - \frac{1}{9}\left(\frac{\lambda_1}{\lambda_2}\right)^2$$

$$C_{d_R} = C_{s_R} = (\tfrac{1}{3}\sin^2\phi) - \frac{1}{9}\left(\frac{\lambda_1}{\lambda_2}\right)^2 \quad . \qquad (17)$$

These results reproduce the ones for the W-S model in the limit of $\left(\frac{\lambda_1}{\lambda_2}\right)^2 \approx$ small.

With the inputs of

$$\sin^2\phi = 0.22$$

$$\left(\frac{\lambda_1}{\lambda_2}\right)^2 = 0.435 \qquad (18)$$

the neutral current coupling constants are compared with experiment in Table 2. Our results are as good as W-S model or closer to the mean values. The correction term in Eq. (17) due to Y contribution is small. By (15) and (18), we obtain

$$m(Z) = 1.13\ m(W)$$
$$m(Y) = 1.98\ m(W) \quad . \tag{19}$$

The neutral current coupling of electron through Z and Y is given by

$$L_e = (\tfrac{1}{4}\ i)\ \frac{g}{\cos\ \phi}\left\{\left[-(1-2\sin^2\phi)\bar{e}\gamma_\mu(1+\gamma_5)e + (2\sin^2\phi)\bar{e}\gamma_\mu(1-\gamma_5)e + \cdots\right]Z + \left[-\frac{2}{3}\frac{a^2}{b}\ \bar{e}\ \gamma_\mu(1+\gamma_5)e + \cdots\right]Y\right\} \quad . \tag{20}$$

If we neglect the small Y contribution, the parity violation (PV) in atomic physics is the same as W-S model.

Thus, in the case of the symmetry breaking $SU(6) \to SU(3)^c \times U(1)$, the effective weak coupling constants are not in good agreement with experiment. But for the symmetry breaking $SU(3)^c \times SU(2) \times U(1) \times U(1) \to SU(3)^c \times U(1)$, the agreement is closer to the experimental mean values than W-S model.

ACKNOWLEDGEMENT

The author wishes to thank H.D.I. Abarbanel for the arrangement of his visit and also thank the members of Theoretical Physics Department at Fermi National Accelerator Laboratory for their hospitality. He also thanks A.J. Buras, M.K. Gaillard, and C.H. Albright for reading the manuscript and making their helpful comments.

TABLE 1

The charge structures of fermions.

	I_3	y	Y	Q	$SU(3)^{color} \times SU(3)^{flavor}$ Content
$u_{i_L}, c_{i_L}, t_{i_R}, g_{i_R}$	$\frac{1}{2}$	$\frac{1}{3}$	0	$\frac{2}{3}$	(3,3)
$d_{i_L}, s_{i_L}, b_{i_R}, h_{i_R}$	$-\frac{1}{2}$	$\frac{1}{3}$	0	$-\frac{1}{3}$	
$b_{i_L}, h_{i_L}, d_{i_R}, s_{i_R}$	0	$-\frac{2}{3}$	0	$-\frac{1}{3}$	
$\bar{u}_{i_L}, \bar{c}_{i_L}, \bar{t}_{i_R}, \bar{g}_{i_R}$	0	0	$-\frac{2}{3}$	$-\frac{2}{3}$	$(\bar{3},1)$
$E_L^+, M_L^+, e_R^+, \mu_R^+$	$\frac{1}{2}$	$-\frac{1}{3}$	$\frac{2}{3}$	1	$(1,\bar{3})$
$\bar{E}_L^0, \bar{M}_L^0, \bar{\nu}_{e_R}, \bar{\nu}_{\mu_R}$	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{2}{3}$	0	
$e_L^+, \mu_L^+, E_R^+, M_R^+$	0	$\frac{2}{3}$	$\frac{2}{3}$	1	
$\bar{\nu}_{e_L}, \bar{\nu}_{\mu_L}, \bar{E}_L^0, \bar{M}_L^0$	0	0	0	0	(1,1)

TABLE 2

Comparison of weak neutral current coupling constants in the case of the symmetry breaking $SU(3)^c \times SU(2) \times U(1) \times U(1) \to SU(3)^c \times U(1)$.

	The present model (Eq. (16))	W-S Model	Experiment[7]
$C_{u_L} = C_{c_L}$	0.377	0.353	0.33 ± 0.07
$C_{d_L} = C_{s_L}$	-0.403	-0.427	-0.40 ± 0.07
$C_{u_R} = C_{c_R}$	-0.195	-0.147	-0.18 ± 0.06
$C_{d_R} = C_{s_R}$	0.025	0.073	0 ± 0.11

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