Fermi National Accelerator Laboratory

FERMILAB-Pub-78/50-THY
June 1978

A Dynamical Relationship Between Baryon Structure Functions and Baryonium Trajectories

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ABSTRACT

Within the standard parton model framework, the rate of the fall-off of baryon structure functions as $x \to 1$ is shown to be related to the intercepts of four-quark $(qq\overline{qq})$ baryonium trajectories α_{M_4}. We predict that $\nu W_2^P/\nu W_2^\pi \backsim (1-x)^{\alpha_M - \alpha_{M_4}}, \alpha_M - \alpha_{M_4} \simeq 1.5$–$2.5$. It is suggested that the d/u ratio in the proton vanishes as a power of 1 - x.

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Operated by Universities Research Association Inc. under contract with the Energy Research and Development Administration

Success of the quark model in describing the observed spectrum of nonexotic mesons and baryons, including the newly discovered heavy quark resonances, has greatly strengthened our confidence in the model. A well-known prediction of the quark model approach is that of the existence of multi-quark hadrons.[1,2] Indeed, high mass resonances recently observed associated with baryon-antibaryon channels[3] appear to be long-sought baryonium ($qq\overline{qq}$) states.

In certain quark-gluon models for hadrons, e.g., the string model and the bag model, the binding force between constituent quarks is described by the same dynamics independently of the hadron they compose.[2] Therefore, we may expect that baryonium wave functions reflect certain properties of baryon wave functions. In particular, the distribution of three valence quark-partons inside a baryon is expected to be related to certain properties of baryonium states.

In this note we show that, under certain assumptions on the behavior of wee parton distribution, the rate of the fall-off of baryon structure functions $\nu W_2^B(x)$ near $x = 1$ is related to the intercept α_{M_4} of baryonium trajectories. As consequences, we obtain the following results:

i) A lower value of α_{M_4} compared with the $q\bar{q}$ meson intercept α_M implies a faster fall-off for the baryon structure function as $x \to 1$ than for the meson structure function, their ratio being $\sim (1 - x)^{\alpha_M - \alpha_{M_4}}$. Numerically, we estimate $\alpha_M - \alpha_{M_4} \simeq 1.5 - 2.5$.

ii) Different intercepts of two baryonium states $M_4(ud\overline{ud})$ and $M_4(uu\overline{uu})$ imply different power behaviors near $x = 1$ of the u quark and d quark distributions in the proton. We will give an argument that one has $\alpha_{M_4(ud\overline{ud})} > \alpha_{M_4(uu\overline{uu})}$ in the dual unitary scheme based on the linear molecule string model.[4] This inequality implies that the d/u ratio vanishes as $x \to 1$, which is consistent with the experimental indication of the ratio $\nu W_2^p / \nu W_2^n$ approaching ¼.

We first consider quarks with one flavor. The model will later be extended to the case of SU(3) symmetry. We take the standard valence-sea parton model,[5] where the valence partons are three quarks and the sea partons are sea quarks and antiquarks and gluons, and the parton wave function is given by the product of the valence and sea parts. For the sea part, partons are assumed to be distributed independently,[6] the one-parton matrix element $f_a(x)$ (a = s for the sea quark and a = g for the gluon) being a constant at $x = 0$.[5,7] We consider the sea parton matrix element squared of the form $f_a(x) = g_a{}^2$.

As for the valence part, it has been pointed out that the three quarks of a baryon are not independently distributed but are strongly correlated so that they form either a quark-diquark or a linear molecule string configuration.[4,8,9] Taking this observation seriously, we consider that the assumption of no correlation among three valence-partons made by Kuti and Weisskopf[5] is unjustified and should be modified. In fact, we have found a simple way of incorporating the correlation among three quarks into the parton model. To do this, we only assume that the valence-parton matrix element squared $f_v(x_1, x_2, x_3)$ has power behaviors for x_1, x_2 and/or $x_3 \sim \varepsilon \to 0$:

$$f_v(x_1, x_2, x_3) \sim \varepsilon^{\beta_1} \text{ for } x_3 \sim \varepsilon \quad , \tag{1}$$

$$f_v(x_1, x_2, x_3) \sim \varepsilon^{\beta_2} \text{ for } x_2 \text{ and } x_3 \sim \varepsilon \quad , \tag{2}$$

$$f_v(x_1, x_2, x_3) \sim \varepsilon^{\beta_3} \text{ for } x_1, x_2 \text{ and } x_3 \sim \varepsilon \quad . \tag{3}$$

The valence part is assumed to be otherwise a smooth function of x_i.

In the parton model for hadron collisions, the power behavior of the one-parton distribution near $x = 0$, eq. (1), is related to the Regge behavior $s^{\alpha_M - 1}$ of hadronic processes generated by the exchange of a wee valence parton (fig. 1a). More precisely, we have the relation[5,7] $\beta_1 = 1-\alpha_M$. Similarly, the power behavior of the two-parton distribution near x = 0, eq. (3), is related to the Regge behavior of hadronic processes generated by the exchange of two wee valence partons (fig. 1b). Assuming that the "diquark"-"antidiquark" system composes a baryonium state, we identify this Regge behavior to the exchange of a baryonium $(qq\overline{qq})$ trajectory α_{M_4}. Thus we obtain

$$\beta_2 = 1 - \alpha_{M_4} \quad . \tag{4}$$

Similarly, denoting a six-quark $(qqq\overline{qqq})$ meson by M_6, we have $\beta_3 = 1-\alpha_{M_6}$.

Inclusive parton distributions can be calculated using the phase space integral of sea partons defined by[5]

$$\Phi(x) = Z \sum_{h=0} \sum_{k=0} \frac{1}{(h!)^2 k!} \int_0^1 \prod_{i=1}^{h} \frac{dx_i}{x_i} \frac{d\bar{x}_i}{\bar{x}_i} f_s(x_i) f_s(\bar{x}_i)$$

$$\times \prod_{j=1}^{k} \frac{dz_j}{z_j} f_g(z_j) \delta\left[1 - x - \sum_{i=1}^{h}(x_i + \bar{x}_i) - \sum_{j=1}^{k} z_j\right] \quad , \tag{5}$$

where Z is a normalization coefficient. The inclusive one-parton distributions $G_a{}^B(x) \left[= \nu W^B_{2,a}(x)/x\right]$ in the baryon B are then given by

$$G^B_v(x) = x^{-1} \int_0^{1-x} \frac{dy_1}{y_1} \frac{dy_2}{y_2} f_v(x, y_1, y_2) \Phi(x + y_1 + y_2) \quad , \tag{6}$$

$$G^B_a(x) = x^{-1} f_a(x) \int_0^{1-x} \frac{dy_1}{y_1} \frac{dy_2}{y_2} \frac{dy_3}{y_3} f_v(y_1, y_2, y_3)$$

$$\times \quad \Phi(x + y_1 + y_2 + y_3), \quad a = s \text{ and } g \quad . \tag{7}$$

We now consider the inclusive parton distribution for x near 1. When the momentum fraction of one of the partons approaches unity, those of the rest of the partons have to vanish. Through this kinematical relation, the behavior of the inclusive parton distribution near $x = 1$ is related to the dynamical x dependence of the parton matrix elements squared near $x = 0$.

The phase space integral $\Phi(x)$ is known[5,7] to have a power behavior near $x = 1$, i.e., $\Phi(x) \sim (1-x)^{g^2-1}$ with $g^2 = g_s{}^2 + g_g{}^2$. Substituting this behavior of $\Phi(x)$ and the valence parton matrix elements squared (2) and (3) into eqs. (6) and (7), we derive, after a calculation which is somewhat lengthy but similar to that of ref. 5, the power behavior

$$G_a^B(x) \sim (1-x)^{N_a^B} \quad , \tag{8}$$

where for the valence parton

$$N_v^B = g^2 - 1 + (1 - \alpha_{M_4}) \quad , \tag{9}$$

and for the sea parton $N_s^B = g^2 - 1 + (1 - \alpha_{M_6})$

The power behavior of the form (8) and (9) can be derived in QED and two-dimensional QCD. In QED, considering a system of an electron (the only valence parton of the system) and soft photons whose distribution behaves like $g^2 dx/x$, where g^2 depends weakly on the transverse momentum of the electron, one obtains $N_v = g^2 - 1$.[10] In the leading order of $1/N$ in two-dimensional QCD with SU(N) gauge symmetry, there are no sea partons. One finds $N_v = -\alpha_M$,[11] which corresponds to eq. (9) with $g^2 = 0$ and α_{M_4} replaced by α_M.

We now discuss the physical consequences of the result obtained above. Let us first compare the structure function of a baryon to that of a meson. The calculation of the meson structure function is similar to that for the baryon. The result is $G_v^M(x) \sim (1-x)^{N_v^M}$ with $N_v^M = g^2 - 1 + (1 - \alpha_M)$. Under the usual assumption[7,9] of the same sea parton matrix element at $x = 0$ for all hadrons,[12] we obtain the ratio of the two structure functions at x near 1,

$$G_V^B(x)/G_V^M(x) \sim (1-x)^{\alpha_M - \alpha_{M_4}} \quad . \tag{10}$$

The baryonium trajectory is expected to lie much lower than the ordinary meson trajectory. From eq. (10) it follows that the baryon structure function should fall off much faster as $x \to 1$ than the meson structure function.

The spectrum of multi-quark hadrons has recently been studied by Chan and Hogaasen in a quark-gluon model and by Jaffe in the bag model.[2] Using linear trajectories and their estimates of the spectrum, we find $\alpha_{M_4} \simeq -1 - -1.5$. Igi and Yazaki have derived from unitarity an inequality involving a baryonium trajectory and a baryon trajectory α_B, i.e., $\alpha_{M_4} > 2(\alpha_B - ½) - \alpha_M$.[13] Using $\alpha_B = \alpha_N \simeq -0.3$ gives $\alpha_{M_4} \gtrsim -2$. Considering these two analyses, we estimate the power difference to be $N_V^B - N_V^M = \alpha_M - \alpha_{M_4} \simeq 1.5 - 2.5$.

Given the observed behavior of the proton structure function near $x = 1$, i.e., $\nu W_2^P(x) \sim (1-x)^{N^P}$ with $N^P \simeq 3$,[14] we predict for the pion that $\nu W_2^\pi(x) \sim (1-x)^{N^\pi}$ with N^π 0.5 - 1.5. Farrar and Jackson have predicted that $\nu W_2^\pi \sim (1-x)^2$ in a quark-gluon model, while Field and Feynman have made the conjecture that $\nu W_2 \sim$ const.[15] Our prediction for the power of $1 - x$ lies in between these two, and differs from them in the respect that the power is not necessarily an integer.

Brodsky and Farrar have argued on dimensional considerations that structure functions have integer-power behaviors, e.g., $N_V^B = 3$ and $N_V^M = 1$.[16] In our scheme their result corresponds to assuming a specific dependence of the intercept of n-quark meson M_n on the number of quarks n, i.e., $\alpha_{M_n} = g^2 + 1 - n$. Although such a relation seems plausible qualitatively, there is no a priori reason that intercepts of multi-quark mesons have to be integer-spaced.

Let us introduce the flavor SU(3) symmetry into our model and consider the square of the valence parton matrix element $f_{ijk}^B(x_1, x_2, x_3)$, where B denotes a member of the octet baryon, and i, j and k denote u, d and s (strange) quarks. Its small x behavior can be parametrized as

$$f^B_{ijk}(x_1, x_2, x_3) \sim \varepsilon^{\beta^B_k} \quad \text{for } x_3 \sim \varepsilon \to 0 \quad , \tag{11}$$

$$f^B_{ijk}(x_1, x_2, x_3) \sim \varepsilon^{\beta^B_{jk}} \quad \text{for } x_2 \text{ and } x_3 \sim \varepsilon \to 0 \quad , \tag{12}$$

where the power β is related to the meson intercept of appropriate quantum numbers. We have $\beta^B_u = \beta^B_d = 1 - \alpha_f$ and $\beta^B_s = 1 - \alpha_{f'}$, and $B^B_{uu} = 1 \quad \alpha_{M_4(uu\overline{uu})}$, $\beta^B_{ud} = 1 - \alpha_{M_4(ud\overline{ud})}$,[17] etc. instead of eq. (4).

The baryonium trajectories $\alpha_{M_4(jk\overline{jk})}$ with different quantum numbers are likely to have different intercepts. Therefore, we expect that the u, d and s quark distributions in the octet baryon may have different power behaviors. For the d/u ratio in the proton, in particular, we have

$$G^P_d(x)/G^P_u(x) \sim (1-x)^{\alpha_{M_4(ud\overline{ud})} - \alpha_{M_4(uu\overline{uu})}} \quad . \tag{13}$$

We have recently shown[4] in the linear molecule string picture for baryons that baryon trajectories have the exchange degeneracy breaking patterns $\alpha_{1_\gamma} > \alpha_{8_\alpha} > \alpha_{8_\gamma} > \alpha_{10_\alpha}$[18] (I) and $\alpha_{10_\delta} > \alpha_{8_\beta} > \alpha_{8_\delta} > \alpha_{1_\beta}$ (II). Such mass splitting of hadrons arises from the interchange interaction between the middle quark k and one of the other two quarks i and j sitting at the ends of the linear string (fig. 2), and can also be described in terms of mass splitting of the "diquark" system (jk), e.g., m(ud) < m(uu) [m(uu) < m(ud)] inside the octet for the splitting I [II].

The same "diquark" mass splitting should also give rise to mass splitting of baryonium states. Therefore, we infer that $\alpha_{M_4(ud\overline{ud})} > \alpha_{M_4(uu\overline{uu})}$ $[\alpha_{M_4(uu\overline{uu})} = \alpha_{M_4(ud\overline{ud})}]$ for the case I [II]. For the octet baryon of $J^P = 1/2^+, 5/2^+, \ldots$, to which the nucleon belongs, we have the mass splitting I. Therefore, for the baryonium trajectories which contribute to the structure functions of these octet baryons, we have $\alpha_{M_4(ud\overline{ud})} > \alpha_{M_4(uu\overline{uu})}$.

The measured value of $\nu W_2^n/\nu W_2^p$ for large x falls much below 2/3, the value predicted from the symmetric quark model. An interesting possibility is that it approaches ¼ as $x \to 1$, which means the d/u ratio vanishes there. Implications of this possibility on the baryon wave function have been discussed by several authors.[9,19] We have suggested above that the rate of fall-off of the d/u ratio as $x \to 1$ is related to the splitting of two baryonium trajectories.

If the dynamics which describe the force between hadronic constituents is to be the same in all hadrons, certain relationships should exist between wave functions of different hadrons. In this note, we have derived within the standard parton model approach simple relations between the rate of fall-off of meson and baryon structure functions near x = 1 and the intercepts of baryonium ($qq\overline{qq}$) Regge trajectories. We predict that the pion structure function falls off as $\sim(1 - x)^{N^\pi}$, $N^\pi \simeq 0.5 - 1.5$ and can be determined from the baryonium trajectory and that the d to u quark ratio in a proton vanishes as $x \to 1$. It still remains a question to be investigated further whether the relation betweeen baryon structure functions and baryonium intercepts can be derived in certain field theoretical models.

The authors would like to thank Hannu Miettinen, Chris Quigg and Hank Thacker for reading the manuscript and for remarks. Conversations with Dennis Duke, Hannu Miettinen and Hank Thacker strengthened our motivation to study the problem of parton distributions in baryons. We have also benefitted from discussions with Henry Abarbanel and Tohru Eguchi. One of the authors (T.I.) wishes to thank Chris Quigg for the kind hospitality at Fermilab.

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[18] α, β, γ and δ denote states of $J^P = 1/2^+, 5/2^+, \ldots$, $J^P = 1/2^-, 5/2^-, \ldots$, $J^P = 3/2, 7/2^-, \ldots$ and $J^P = 3/2^+, 7/2^+, \ldots$

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FIGURE CAPTIONS

Fig. 1: Graphs of hadron collisions $AB \rightarrow AB$ due to exchange of a wee valence parton (a) and two wee valence partons (b).

Fig. 2: Pictorial representation of a baryon in the linear molecule string model with orbital angular momentum L between the two quarks i and j (a). The interchange interaction between the middle quark k and the quark j (b).

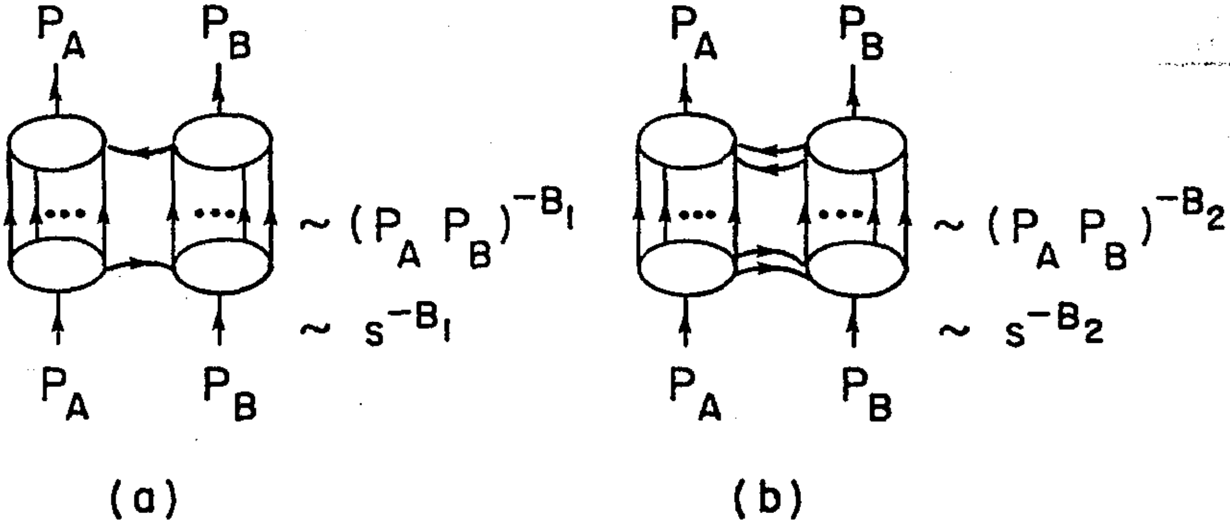
P_A
P_B
$\sim (P_A\,P_B)^{-B_1}$
$\sim s^{-B_1}$
P_A
P_B
(a)
P_A
P_B
$\sim (P_A\,P_B)^{-B_2}$
$\sim s^{-B_2}$
P_A
P_B
(b)

Fig. 1

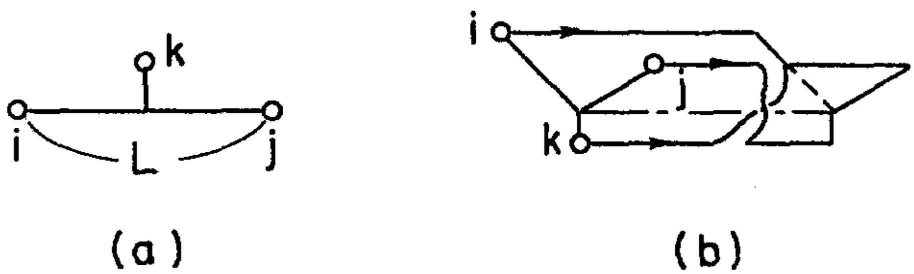
k
i
L
j
(a)
i
j
k
(b)

Fig. 2