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Deep Inelastic Scattering Beyond the Leading Order in Asymptotically Free Gauge Theories

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ABSTRACT

We calculate the full order g^2 corrections to the coefficient functions which determine the Q^2 dependence of the moments of deep-inelastic structure functions. The calculation is performed in the minimal subtraction scheme of 't Hooft. The results are combined with the recent two-loop calculations of anomalous dimensions by Floratos, Ross and Sachrajda to give the full $\bar{g}^2$ corrections to the leading order of asymptotic freedom. We present results for $C_n(1, \bar{g}^2)$ relevant for electroproduction and neutrino reactions for both nonsinglet and singlet combinations of the structure functions. Phenomenological consequences of the full $\bar{g}^2$ corrections to the nonsinglet structure function are discussed. The corrections to the Gross-Llewellyn-Smith and Bjorken sum rules are estimated to be of the order of 15%.

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1. INTRODUCTION

In the last two years there has been considerable interest[1] in the comparison of asymptotically free gauge theories (ASF)[2] with the deep inelastic data. Most analyses so far have concentrated on the leading asymptotic behavior of the moments of the structure functions, in which only the $[\ln Q^2/\Lambda^2]^{d_n}$ terms are retained. The conclusion has been reached[1] that this leading behavior is consistent with the scaling violations observed in ep, μp, νN and $\bar{\nu}$N deep inelastic scattering[3] except for a large ratio σ_L/σ_T seen at SLAC.[4] However at $Q^2 \approx$ few GeV^2 the leading asymptotic behavior cannot be the whole effect and it is of interest to ask whether mass corrections and higher order corrections in the effective coupling constant $\bar{g}^2(Q^2)$ modify these results. Mass corrections when treated in the manner of Nachtmann-Georgi-Politzer[5-10] improve the agreement of the theory with the data for νW_2^{ep}, $\nu W_2^{\nu,\bar{\nu}}$ and give a small but non-vanishing σ_L/σ_T ratio.

As concerns higher order corrections in $\bar{g}^2$ only the calculation of $\sigma_L/\sigma_T \sim O(\bar{g}^2)$ is simple and the result is well known.[11-14] These corrections together with the mass corrections are unable[6,7,14] to explain the large value of σ_L/σ_T measured at SLAC at large x.

The analysis of the $\bar{g}^2$ corrections as applied to νW_2^{ep}, $\nu W_2^{\nu,\bar{\nu}}$ and $\nu W_3^{\nu,\bar{\nu}}$ is much more involved.[6,7,11] To see this consider the moments of a non-singlet structure function ($\nu W_2^{ep} - \nu W_2^{en}$, $\nu W_2^{\nu p} - \nu W_2^{\bar{\nu} p}$, etc.), which in ASF are given[2] as follows

$$M_n(Q^2) \equiv \int_0^1 dx\, x^{n-2} F_{NS}(x, Q^2) = A_n\, C_n(1, \bar{g}^2) \exp\left[-\int_{\bar{g}(Q_0^2)}^{\bar{g}(Q^2)} d\tau \frac{\gamma_n(\tau)}{\beta(\tau)} \right]. \quad (1.1)$$

Here $\bar{g}^2(Q^2)$, $\gamma_n(\bar{g}^2)$ and $\beta(\bar{g}^2)$ are the effective coupling constant, anomalous dimension of the spin n operator in the Wilson expansion of the product of two currents and the standard β function respectively. Each of these quantities and $C_n(1, \bar{g}^2)$ can be calculated in perturbation theory. The constants A_n are to be found from the data at some arbitrary value of $Q^2 = Q_0^2$.

By expanding $C_n(1, \bar{g}^2)$, $\gamma_n(\bar{g}^2)$ and $\beta(\bar{g}^2)$ in powers of $\bar{g}^2$, calculating $\bar{g}^2$ in the two-loop approximation for $\beta(\bar{g}^2)$ and inserting everything into (1.1) one finds[7,11]

$$M_n(Q^2) = A_n \left[1 + \frac{f_n^{(1)}}{\ln \frac{Q^2}{\Lambda^2}} + \frac{f_n^{(2)}}{\ln \frac{Q^2}{\Lambda^2}} \ln \ln \frac{Q^2}{\Lambda^2} \right] \left[\frac{\ln \frac{Q^2}{\Lambda^2}}{\ln \frac{Q_0^2}{\Lambda^2}} \right]^{d_n} , \tag{1.2}$$

with $f_n^{(1)}$, $f_n^{(2)}$ and d_n depending on the parameters in the expansion of C_n, γ_n and β. We shall give explicit formulae for $f_n^{(1)}$, $f_n^{(2)}$ and d_n in section 2. Here we only recall[7,11] that in order to find $f_n^{(1)}$ and $f_n^{(2)}$ the knowledge of $\gamma_n(\bar{g}^2)$ and $\beta(\bar{g}^2)$ to $\bar{g}^4$ order and of $C_n(1, \bar{g}^2)$ to $\bar{g}^2$ order is required. The $\bar{g}^2$ corrections to $C_n(1, \bar{g}^2)$ and the $\bar{g}^4$ corrections to $\gamma_n(\bar{g}^2)$ deserve particular attention. As pointed out in ref. 15 these two corrections are renormalization prescription dependent and only when both are calculated in the same renormalization scheme can a physically meaningful answer be obtained. So far only in ref. 7 have both contributions in question been included in a phenomenological analysis. To this end the results for $C_n(1, \bar{g}^2)$ of ref. 6 together with those for $\gamma_n(\bar{g}^2)$ of ref.15 have been used. Unfortunately it is now clear[16] that the renormalization scheme used in ref. 6 is not the same as that used in the two-loop calculation of $\gamma_n(\bar{g}^2)$.[15] Therefore the results of ref. 6 and 15 should not be used together.

In order to calculate $C_n(1, \bar{g}^2)$ both the $\bar{g}^2$ corrections to the virtual Compton amplitude and the $\bar{g}^2$ corrections to the matrix elements of local operators are needed.[17] In a general renormalization scheme both of these quantities are sensitive to infrared structure of the theory and are gauge dependent. We show however that in the renormalization scheme of ref. 15 the gauge dependences of the virtual Compton amplitude and of the matrix elements of local operators are the same and cancel when $C_n(1, \bar{g}^2)$ is calculated. Consequently in the renormalization scheme in question, $C_n(1, \bar{g}^2)$ is gauge independent. Of course $C_n(1, \bar{g}^2)$ remains renormalization prescription dependent. This renormalization scheme dependence of $C_n(1, \bar{g}^2)$ is then cancelled by that of two-loop contributions to $\gamma_n(\bar{g}^2)$. Since $\gamma_n(\bar{g}^2)$ in the minimal subtraction scheme[18] used in ref. 15 is gauge independent[19,20] and $\beta(g)$ to two loops is gauge and renormalization prescription independent,[19,20,21] when all corrections are combined in (1.2) the physical result is obtained.

In the present paper we shall calculate $C_n(1, \bar{g}^2)$ to $\bar{g}^2$ order in the renormalization scheme which has been used in ref. 15 to calculate $\gamma_n(\bar{g}^2)$ to $\bar{g}^4$ order. This we do not only for the non-singlet electromagnetic structure functions but also for singlet structure functions and neutrino processes.

Our paper is organized as follows. In section 2 we formulate the problem in greater detail and state a general procedure for the calculation of $C_n(1, \bar{g}^2)$. We subsequently focus on the renormalization scheme used by Floratos, Ross and Sachrajda.[15] In section 3, we calculate those quantities necessary to determine the coefficient function $C_n(1, \bar{g}^2)$ for non-singlet operators, and the significance of the scale parameter Λ is discussed.

In section 4, we calculate $C_n(1,\bar{g}^2)$ for non-singlet contributions to neutrino deep inelastic scattering and discuss $\bar{g}^2$ corrections to the various neutrino sum rules and parton model relations.

We compare our results to the recently measured[10] moments of the non-singlet structure function xF_3 in section 5. We find that the order $\bar{g}^2$ corrections do not change the conclusions of previous analyses[1,10] based on the leading order.

In section 6 we extend our calculations of $C_n(1,\bar{g}^2)$ to singlet structure functions. We use again the renormalization scheme of the authors of ref. 15. Section 7 contains a brief summary of our paper. The contributions to the virtual Compton amplitude and to the matrix elements from the individual Feynman diagrams are collected in an Appendix.

2. BASIC FORMALISM

2.1. Preliminaries

In what follows we shall discuss the spin averaged amplitude $T_{\mu\nu}{}^{a\bar{a}}$ for the forward scattering of a current $J_\mu{}^a$. In our case $J_\mu{}^a$ will stand either for the electromagnetic current (ep scattering) or a weak current ($\nu,\bar{\nu}$ scattering). The amplitude $T_{\mu\nu}{}^{a\bar{a}}$ can be decomposed into invariant amplitudes as follows

$$T_{\mu\nu}{}^{a\bar{a}}(q^2,\nu) = i\int d^4x\, e^{iq\cdot x} \langle p|T(J_\mu{}^a(x)J_\nu{}^{\bar{a}}(0))|p\rangle_{\text{spin averaged}}$$

$$= e_{\mu\nu}T_L{}^{a\bar{a}}(q^2,\nu) + d_{\mu\nu}T_2{}^{a\bar{a}}(q^2,\nu) - i\,\varepsilon_{\mu\nu\alpha\beta}\frac{p_\alpha q_\beta}{\nu}\,T_3{}^{a\bar{a}}(q^2,\nu) \tag{2.1}$$

with $\nu = p\cdot q$. Here $J_\mu{}^{\bar{a}} = (J_\mu{}^a)^+$ and $|p\rangle$ is for instance a proton state. The tensors $e_{\mu\nu}$ and $d_{\mu\nu}$ are defined as follows

$$e_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \tag{2.2}$$

$$d_{\mu\nu} = -\frac{p_\mu p_\nu}{\nu^2} q^2 + \frac{p_\mu q_\nu + p_\nu q_\mu}{\nu} - g_{\mu\nu} \quad . \tag{2.3}$$

Using the operator product expansion for currents[2] we can write equation (2.1) as

$$\begin{aligned} T_{\mu\nu}^{a\bar{a}}(q^2, \nu) = \sum_n \frac{1}{x^n} \Bigg\{ e_{\mu\nu} \sum_i C_{L,n}^{i,a\bar{a}}\left(\frac{Q^2}{\mu^2}, g^2\right) \\ + d_{\mu\nu} \sum_i C_{2,n}^{i,a\bar{a}}\left(\frac{Q^2}{\mu^2}, g^2\right) - i\epsilon_{\mu\nu\alpha\beta} \frac{p_\alpha q_\beta}{\nu} \sum_i C_{3,n}^{i,a\bar{a}}\left(\frac{Q^2}{\mu^2}, g^2\right) \Bigg\} A_n^i\left(\frac{p^2}{\mu^2}, g^2\right) \end{aligned} \tag{2.4}$$

where $Q^2 = -q^2$ and p^2 is the target four-momentum squared. Furthermore, x is the Bjorken variable $(Q^2/2\nu)$, g^2 is the renormalized coupling constant, μ^2 is the subtraction scale at which the theory is renormalized and A_i^n are constants specified in equation (2.8). The sum on the r.h.s. of equation (2.4) runs over spin n, twist 2 operators such as[22]

$$O_\beta^{\mu_1 \cdots \mu_n} = \frac{i^{n-1}}{n!} \left[\bar{\psi} \lambda_\beta \gamma^{\mu_1} \mathscr{D}^{\mu_2} \ldots \mathscr{D}^{\mu_n} \psi + \text{permutations} \right] \tag{2.5}$$

$$O_\psi^{\mu_1 \cdots \mu_n} = \frac{i^{n-1}}{n!} \left[\bar{\psi} \gamma^{\mu_1} \mathscr{D}^{\mu_2} \ldots \mathscr{D}^{\mu_n} \psi + \text{permutations} \right] \tag{2.6}$$

and

$$O_G^{\mu_1 \cdots \mu_n} = \frac{i^{n-2}}{2n!} \left[F^{\alpha\mu_1} \mathscr{D}^{\mu_2} \ldots \mathscr{D}^{\mu_{n-1}} F^{\alpha\mu_n} + \text{permutations} \right] \quad . \tag{2.7}$$

$O_\beta^{\mu_1 \cdots \mu_n}$ are the fermion non-singlet (under physical symmetries) operators whereas $O_\psi^{\mu_1 \cdots \mu_n}$ and $O_G^{\mu_1 \cdots \mu_n}$ are singlet fermion and gluon operators respectively. C_i^n are the corresponding coefficient functions of the Wilson expansion. The constants A_n^i are related to the spin averaged proton matrix elements of $O_i^{\mu_1 \cdots \mu_n}$ as follows

$$<p|O_i^{\mu_1 \cdots \mu_n}|p> = A_n^i p_{\mu_1} \cdots p_{\mu_n} + \text{trace terms} \quad . \tag{2.8}$$

In our field theoretical calculations we shall deal with the matrix elements of the operators in question between quark and gluon states rather than between proton states and therefore it is convenient to generalize (2.8) to

$$<p; j|O_i^{\mu_1 \cdots \mu_n}|p;j> = A_{nj}^i p_{\mu_1} \cdots p_{\mu_n} + \text{trace terms} \tag{2.9}$$

with $i,j = \beta, \psi, G$.

The coefficients $C_{2,n}^{i,a\bar{a}}$, $C_{L,n}^{i,a\bar{a}}$ and $C_{3,n}^{i,a\bar{a}}$ as defined in equation (2.4) are related to the moments of the standard structure functions

$$F_1^{a\bar{a}}(x, Q^2) \equiv W_1^{a\bar{a}}(x, Q^2),\ F_2^{a\bar{a}}(x, Q^2) \equiv \nu W_2^{a\bar{a}}(x, Q^2),\ F_3^{a\bar{a}}(x, Q^2) \equiv \nu W_3^{a\bar{a}}(x, Q^2) \tag{2.10}$$

as follows [23]

$$\int_0^1 dx\, x^{n-1} F_1^{a\bar{a}}(x, Q^2) = \sum_i A_n^i \left\{ C_{2,n}^{i,a\bar{a}}\left(\frac{Q^2}{\mu^2}, g^2\right) - C_{L,n}^{i,a\bar{a}}\left(\frac{Q^2}{\mu^2}, g^2\right) \right\} \tag{2.11}$$

$$\int_0^1 dx\, x^{n-2} F_2^{a\bar{a}}(x, Q^2) = 2 \sum_i A_n^i C_{2,n}^{i,a\bar{a}}\left(\frac{Q^2}{\mu^2}, g^2\right) \tag{2.12}$$

and

$$\int_0^1 dx\, x^{n-1} F_3(x, Q^2) = 2 \sum_i A_n{}^i C_{3,n}{}^{i,a\bar{a}} \left(\frac{Q^2}{\mu^2}, g^2\right) \quad . \tag{2.13}$$

In spite of the fact that there is a set of non-singlet operators corresponding to various λ_β, the Q^2 dependence of their coefficient functions in the Wilson expansion is in common since they neither mix under renormalization with each other nor with the singlet operators. Therefore it will be convenient to factor out the dependence on the SU(n) nature of the operator as well as the current involved and write

$$C_{r,n}{}^{a\bar{a},\beta} \left(\frac{Q^2}{\mu^2}, g^2\right) = K^{a\bar{a},\beta} C_{r,n}{}^{NS} \left(\frac{Q^2}{\mu^2}, g^2\right) \qquad r = 1,2,3 \tag{2.14}$$

with $C_{r,n}{}^{NS}(Q^2/\mu^2, g^2)$ being common for all non-singlet operators. In what follows we shall directly work with $C_{r,n}{}^{NS}(Q^2/\mu^2, g^2)$. In addition we shall choose the overall normalization of $C_{r,n}{}^{NS}$ in such a way that in the g^0 order

$$C_{r,n}{}^{(0)NS} = 1 \quad . \tag{2.15}$$

The relative normalization of the Born term and the g^2 correction is then fixed by the minimal subtraction scheme used.

Correspondingly for the coefficient function of the singlet fermion operator we shall write

$$C_{r,n}{}^{a\bar{a},\psi} \left(\frac{Q^2}{\mu^2}, g^2\right) = K^{a\bar{a},\psi} C_{r,n}{}^{\psi} \left(\frac{Q^2}{\mu^2}, g^2\right) \qquad r = 1,2,3 \tag{2.16}$$

where the overall normalization of $C_{r,n}{}^{\psi}$ is chosen in such a way that in the g^0 order

$$C_{r,n}^{(0)\psi} = 1 \quad . \tag{2.17}$$

Similarly we shall choose (see 2.5-2.7), the overall normalization for the matrix elements of the local operators so that in g^0 order

$$A_{nj}^{(0)i} = \delta_{ij} \quad . \tag{2.18}$$

As in the case of coefficient functions the relative normalization of the Born term and the g^2 corrections to the matrix element of local operators is fixed by the minimal subtraction scheme.

We shall now present in simple terms the general structure of our calculations. To simplify matters we shall neglect Lorentz indices whenever possible and only write formulae for structure functions which do not vanish in the leading order (e.g. νW_2). For completeness however we shall comment from time to time how the formulae change when applied to the longitudinal structure function which vanishes in the leading order. We shall keep all Lorentz indices in section 3.

2.2. Wilson Coefficient Function to Order $\bar{g}^2$

For the non-singlet combinations of the structure functions (e.g. $F_2^{ep}-F_2^{en}$, $F_2^{\bar{\nu} p}-F_2^{\nu p}$ etc.) equations (2.11-2.13) simplify to the following general expression

$$M_n^{NS}(Q^2) = \int_0^1 dx\, x^{n-2} F_{NS}(x, Q^2) = C_n^{NS}\left(\frac{Q^2}{\mu^2}, g^2\right) A_n^{NS} \quad . \tag{2.19}$$

The Q^2 dependence of $C_n^{NS}(Q^2/\mu^2, g^2)$ is governed by the renormalization group equation[2]

$$\left(\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} - \gamma_n(g)\right) C_n^{NS}\left(\frac{Q^2}{\mu^2}, g^2\right) = 0 \tag{2.20}$$

which has the following solution

$$C_n^{NS}\left(\frac{Q^2}{\mu^2}, g^2\right) = C_n^{NS}(1, \bar{g}^2) \exp\left[- \int_{\bar{g}(\mu^2)}^{\bar{g}(Q^2)} d\tau \frac{\gamma^n(\tau)}{\beta(\tau)} \right] \quad . \tag{2.21}$$

Here $\bar{g}^2(Q^2)$ is the effective coupling constant which satisfies the equation

$$\frac{d\bar{g}^2}{dt} = \bar{g}\beta(\bar{g}) \qquad ; \qquad \bar{g}(t = 0) = g \tag{2.22}$$

where $t = \ln Q^2/\mu^2$. $\gamma_n(\bar{g})$ is the anomalous dimension of the operator $O_{NS}^{\mu_1 \cdots \mu_n}$.

In order to find explicit expressions for $f_n^{(1)}$ and $f_n^{(2)}$ in equation (1.2) we follow ref. 7 and expand first $\gamma_n(\bar{g})$, $\beta(\bar{g})$ and $C_n^{NS}(1, \bar{g}^2)$ in powers of $\bar{g}$

$$\gamma^n(\bar{g}) = \gamma_0^n \frac{\bar{g}^2}{16\pi^2} + \gamma_1^n \left(\frac{\bar{g}^2}{16\pi^2}\right)^2 + \cdots \tag{2.23}$$

$$\beta(\bar{g}) = -\beta_0 \frac{\bar{g}^3}{16\pi^2} - \beta_1 \frac{\bar{g}^5}{(16\pi^2)^2} + \cdots \tag{2.24}$$

$$C_n^{NS}(1, \bar{g}^2) = 1 + B_n^{NS} \frac{\bar{g}^2}{16\pi^2} + \cdots \quad . \tag{2.25}$$

For the longitudinal structure function the first term on the r.h.s. of eq. (2.25) is zero.

We next expand $\bar{g}(Q^2)$, the solution of equation (2.22) with $\beta(g)$ given by (2.24), in powers of $\bar{g}_0^2(Q^2)$, the effective coupling constant calculated in the one loop approximation with the result

$$\bar{g}^2(Q^2) = \bar{g}_0^2(Q^2) - \frac{\beta_1}{\beta_0} \frac{\bar{g}_0^4(Q^2)}{16\pi^2} \ln\ln \frac{Q^2}{\Lambda^2} + O(\bar{g}_0^6) \tag{2.26}$$

where

$$\bar{g}_0^{\,2}(Q^2) = \frac{48\pi^2}{(33-2f)\ln\frac{Q^2}{\Lambda^2}} \qquad (2.27)$$

with f being number of flavors.

In equation (2.26) and following ref. 7, the constant Λ has been arbitrarily chosen so that there are no further terms of order $\bar{g}_0^{\,4}$. A little algebra shows that μ^2, Λ^2 and g^2 are related to each other by

$$\Lambda^2 = \mu^2 \exp\left[-\frac{16\pi^2}{\beta_0 g^2} + \frac{\beta_1}{\beta_0^2}\ln\left(\beta_0 g^2\right)\right] \quad . \qquad (2.28)$$

Clearly this choice of Λ is not unique and we shall discuss in detail in section 3.4 definitions for Λ which lead to additional terms of $O(\bar{g}_0^{\,4})$ in equation (2.26).

Inserting (2.23)-(2.26) into (2.21) and expanding in powers of $\bar{g}_0^{\,2}$ we obtain[7]

$$C_n^{NS}\left(\frac{Q^2}{\mu^2}, g^2\right) = C_n\left[1 + \frac{1}{\beta_0 \ln\frac{Q^2}{\Lambda^2}}\left(B_n^{NS} + P_n + L_n\right)\right]\left[\ln\frac{Q^2}{\Lambda^2}\right]^{-\gamma_0^n/2\beta_0} \qquad (2.29)$$

where

$$P_n = \frac{\gamma_1^n}{2\beta_0} - \frac{\beta_1\gamma_0^n}{2\beta_0^2} \quad , \qquad (2.30)$$

$$L_n = -\frac{\beta_1\gamma_0^n}{2\beta_0^2}\ln\ln\frac{Q^2}{\Lambda^2} \qquad (2.31)$$

and C_n is an overall Q^2 independent constant. In the case of the longitudinal structure function P_n, L_n and the leading term 1 are absent and B_n^{NS} is replaced by B_{nL}^{NS}.

The parameters $\gamma_0{}^n$, β_0 and β_1 are gauge and renormalization prescription independent and are given for an SU(3) gauge theory with f flavors as follows[2,21]

$$\gamma_0{}^n = \frac{8}{3}\left[1 - \frac{2}{n(n+1)} + 4 \sum_{j=2}^{n} \frac{1}{j}\right] \tag{2.32}$$

$$\beta_0 = 11 - \frac{2}{3}f \tag{2.33}$$

$$\beta_1 = 102 - \frac{38}{3}f \quad . \tag{2.34}$$

The quantities B_n and $\gamma_1{}^n$ are renormalization prescription dependent (and in principle also gauge dependent). However as pointed out in ref. 15 the renormalization prescription dependence of B_n is related to that of $\gamma_1{}^n$ and when these quantities are inserted in equation (2.29) a physical, renormalization prescription independent answer is obtained. In other words, the calculation of B_n and $\gamma_1{}^n$ may be performed in any renormalization scheme <u>but</u> care must be taken that these two quantities are calculated in the same scheme.

The $\gamma_1{}^n$ for non-singlet operators has been calculated by Floratos, Ross and Sachrajda,[15] who have used 't Hooft's[18] minimal subtraction scheme to renormalize the amplitudes. In this scheme the Feynman diagrams are evaluated, using dimensional regularization, in $d = 4 - \varepsilon$ dimensions and singularities are extracted as poles $1/\varepsilon$, $1/\varepsilon^2$, etc. The minimal subtraction then means that the amplitudes are renormalized by simply subtracting the pole parts $1/\varepsilon$, $1/\varepsilon^2$, etc.

The coefficients B_n^{NS} existing in the literature[6,11] have been calculated in schemes which differ from the one above and cannot be combined with the results of ref. 15. Therefore in our paper we shall calculate them in the minimal subtraction scheme. We now outline a method for calculating B_n^{NS}.

2.3. Procedure for the Calculation of B_n^{NS}

We first remark that in order to find B_n as defined in equation (2.25) it is sufficient to calculate $C_n(Q^2/\mu^2, g^2)$ in perturbation theory to order g^2 and put $Q^2 = \mu^2$. This is obvious from equations (2.21 and 2.22).

Writing next the non-singlet version of (2.4) symbolically as follows

$$T^{NS}(Q^2, \nu) = \sum_n \left(\frac{1}{x}\right)^n C_n^{NS}\left(\frac{q^2}{\mu^2}, g^2\right) A_n^{NS}\left(\frac{p^2}{\mu^2}, g^2\right) \tag{2.35}$$

we see that in order to find $C_n^{NS}(Q^2/\mu^2, g^2)$ we generally have to calculate both $T^{NS}(Q^2, \nu)$ and $A_n^{NS}(p^2/\mu^2, g^2)$. Since the coefficient functions do not depend on the target we can choose as a target any state for which calculations can be easily performed. In what follows we shall choose either quarks or gluons as targets.[24] In order to be consistent with the calculations of ref. 15, we shall deal with massless, off-shell quarks or gluons with space-like momenta $p^2 < 0$.

Having all information at hand we can now state the procedure for extracting $C_n^{NS}(Q^2/\mu^2, g^2)$ and specifically the constants B_n^{NS}.

The procedure is as follows:

i) Calculate $T^{NS}(q^2, \nu)$ (forward amplitude for scattering off quarks) in perturbation theory to order g^2 and expand in powers $(1/x)^n$ for $x > 1$. The coefficients of this expansion will have the form

$$T_n^{NS} = 1 + \frac{g^2}{16\pi^2}\left[-\frac{1}{2}\gamma_0^n \ln\frac{Q^2}{-p^2} - \frac{1}{2}\gamma_F \ln\frac{-p^2}{\mu^2} + T_n^{(2)NS}\right] \tag{2.36}$$

with γ_0^n given by (2.32) and γ_F being the anomalous dimension of the quark field. The overall normalization has been chosen in accordance with equations (2.15 -2.18).

ii) Calculate $A_n^{NS}(p^2/\mu^2, g^2)$ by considering the matrix elements of non-singlet operator between quark states with the result

$$A_n^{NS} = 1 + \frac{g^2}{16\pi^2}\left[\frac{1}{2}\gamma_0^n \ln\frac{-p^2}{\mu^2} - \frac{1}{2}\gamma_F \ln\frac{-p^2}{\mu^2} + A_n^{(2)NS}\right] . \tag{2.37}$$

iii) Insert (2.37) and (2.36) into (2.35) to obtain

$$C_n^{NS}\left(\frac{Q^2}{\mu^2}, g^2\right) = 1 + \frac{g^2}{16\pi^2}\left[-\frac{1}{2}\gamma_0^n \ln\frac{Q^2}{\mu^2} + T_n^{(2)NS} - A_n^{(2)NS}\right] . \tag{2.38}$$

iv) Finally evaluate (2.38) for $Q^2 = \mu^2$ and compare with (2.25) to find

$$B_n^{NS} = T_n^{(2)NS} - A_n^{(2)NS} . \tag{2.39}$$

This equation applies to the non-singlet components of the structure functions F_1, F_2 and F_3. In the case of longitudinal structure function, which vanishes in g^0 order the corresponding expression is

$$B_{L,n}^{NS} = T_{L,n}^{(2)NS} . \tag{2.40}$$

The above procedure applies for any renormalization scheme. However as we already stated above different renormalization schemes lead generally to different values of B_n. If B_n and γ_1^n are calculated in the same scheme, a renormalization prescription independent result for the moments of the structure functions is obtained.

In the scheme of de Rujula, et al.,[6] the matrix elements of operators are normalized so that $A_n^{(2)} = 0$. Therefore in that scheme we have simply $B_n^{NS} = T_n^{(2)NS}$. Unfortunately we cannot use this scheme for our calculations since the only existing results for γ_1^n have been obtained[15] using the minimal subtraction scheme. As we shall see in the next section in the latter scheme $A_n^{(2)NS} \neq 0$ and must be calculated in order to extract the coefficient function.

3. $C_n(Q^2/\mu^2, g^2)$ TO ORDER g^2 FOR ELECTROMAGNETIC CURRENTS (NON-SINGLET CONTRIBUTIONS)

For electromagnetic currents, the calculation of the non-singlet contributions using equation (2.4) simplifies to

$$T_{\mu\nu}^{NS}(q^2, \nu, p^2) = \sum_n \left(\frac{1}{x}\right)^n \left\{ e_{\mu\nu} C_{L,n}^{NS}\left(\frac{Q^2}{\mu^2}, g^2\right) + \right.$$
$$\left. + d_{\mu\nu} C_{2,n}^{NS}\left(\frac{Q^2}{\mu^2}, g^2\right) \right\} A_n^{NS}\left(\frac{p^2}{\mu^2}, g^2\right) \quad . \tag{3.1}$$

$T_{\mu\nu}^{NS}$ is the forward spin averaged amplitude for scattering of photons off off-shell massless quarks with space-like momentum $p^2 < 0$ and A_n^{NS} are the matrix elements of the non-singlet operators of equation (2.5) between the quark states in question. $C_{L,n}^{NS}$ and $C_{2,n}^{NS}$ are the coefficient functions which we want to calculate in perturbation theory to order g^2. To this end according to the procedure of section 2.3 we begin by calculating $T_{\mu\nu}^{NS}$ to order g^2.

3.1. Calculation of the Virtual Compton Amplitude to Order g^2

We first recall that the calculation of $T_{\mu\nu}^{NS}$ in g^0 order involves diagrams of Fig. 1 and when the result is expanded in powers of $1/x$ we find in the normalization of (2.15-2.18)

$$T_{L,n}^{(0)NS} = 0 \quad , \quad T_{2,n}^{(0)NS} = 1 \quad (n \text{ even}) \quad . \tag{3.2}$$

The calculation of $T_{\mu\nu}^{NS}$ in g^2 order involves diagrams of Fig. 2. We use an arbitrary covariant gauge α where the gluon propagator is given by

$$-\frac{i}{k^2}\left[g_{\mu\nu}-(1-\alpha)\frac{k_\mu k_\nu}{k^2}\right] \quad . \tag{3.3}$$

Using the minimal subtraction scheme and dropping terms of order p^2 we obtain the following result for the coefficients $T_n^{(2)NS}$ defined in equation (2.36)

$$\begin{aligned} T_{2,n}^{(2)NS} = C_2(R)\Big\{ & 3\sum_{j=1}^{n}\frac{1}{j} - 8\sum_{j=1}^{n}\frac{1}{j^2} + \frac{6}{n+1} + \frac{4}{n^2} - \frac{4}{(n+1)^2} \\ & - 1 - \frac{1}{n} + (\ln 4\pi - \gamma_E) + (1-\alpha)\left(\frac{1}{n} + \sum_{j=1}^{n}\frac{1}{j} - 1 - \ln 4\pi + \gamma_E\right)\Big\} \quad n \text{ even} \end{aligned} \tag{3.4}$$

and

$$T_{L,n}^{(2)NS} = C_2(R)\frac{4}{n+1} \quad , \quad C_2(R) = \frac{4}{3} \quad . \tag{3.5}$$

The contributions from individual diagrams are collected in the Appendix. The gauge dependence and the presence of the term $(\ln 4\pi - \gamma_E)$ where γ_E is the Euler-Mascheroni constant are due to the normalization procedure and will be discussed in section 3.4.

3.2. Matrix Elements of Non-singlet Operators to Order g^2

The calculation involves the diagrams of Fig. 3. Using again minimal subtraction scheme, we determine the constants $A_n^{(2)NS}$ defined in equation (2.37) with the result

$$\begin{aligned} A_n^{(2)NS} = C_2(R)\Big[& 8 - \frac{4}{n} + \frac{2}{n+1} + \frac{2}{n^2} - \frac{4}{(n+1)^2} \\ & - 4\sum_{j=1}^{n}\frac{1}{j^2} + \frac{2}{n(n+1)}\sum_{j=1}^{n}\frac{1}{j} - 4\sum_{s=1}^{n}\frac{1}{s}\sum_{j=1}^{s}\frac{1}{j} + \\ & +(\ln 4\pi - \gamma_E)\left(\frac{2}{n(n+1)} - 4\sum_{j=2}^{n}\frac{1}{j}\right) + (1-\alpha)\left(\frac{1}{n} + \sum_{j=1}^{n}\frac{1}{j} - 1 - \ln 4\pi + \gamma_E\right)\Big] \quad n\text{-even} \quad . \end{aligned} \tag{3.6}$$

Details of the calculation are given in the Appendix.

3.3. Final Result for $C_{L,n}^{NS}(1,\bar{g}^2)$ and $C_{2,n}^{NS}(1,\bar{g}^2)$

Inserting (3.4), (3.5) and (3.6) into equations (2.39) and (2.40) and using the definition (2.25) we obtain the following results for $C_{L,n}^{NS}(1,\bar{g}^2)$ and $C_{2,n}^{NS}(1,\bar{g}^2)$ to order $\bar{g}^2$

$$C_{2,n}^{NS}(1,\bar{g}^2) = 1 + \frac{\bar{g}^2}{16\pi^2} B_{2,n}^{NS} \tag{3.7}$$

and

$$C_{L,n}^{NS}(1,\bar{g}^2) = \frac{\bar{g}^2}{16\pi^2} C_2(R) \frac{4}{n+1} \tag{3.8}$$

where

$$\begin{aligned} B_{2,n}^{NS} = C_2(R) \Big\{ & 3 \sum_{j=1}^{n} \frac{1}{j} - 4 \sum_{j=1}^{n} \frac{1}{j^2} - \frac{2}{n(n+1)} \sum_{j=1}^{n} \frac{1}{j} \\ & + 4 \sum_{s=1}^{n} \frac{1}{s} \sum_{j=1}^{s} \frac{1}{j} + \frac{3}{n} + \frac{4}{n+1} + \frac{2}{n^2} - 9 \\ & + \tilde{\gamma}_0^n (\ln 4\pi - \gamma_E) \Big\}, \quad n \text{ even} \end{aligned} \tag{3.9}$$

and

$$\tilde{\gamma}_0^n = 1 - \frac{2}{n(n+1)} + 4 \sum_{j=2}^{n} \frac{1}{j} = \frac{3}{8} \gamma_0^n \quad . \tag{3.10}$$

The result (3.8) is well known.[11-14] On the other hand the result for $B_{2,n}^{NS}$ in the particular renormalization scheme considered is new.

Notice that the gauge dependence of $A_n^{(2)NS}$ cancelled that of $T_n^{(2)NS}$ leaving $B_{2,n}^{NS}$ gauge independent in the minimal subtraction scheme. Numerical values of $B_{2,n}^{NS}$ together with those for γ_1^n and γ_0^n are collected in the Table.

There are only two calculations of $B_{2,n}^{NS}$ in the present literature. One by Calvo[11] and another by de Rujula, Georgi and Politzer.[6] None of these results for $B_{2,n}^{NS}$ can be directly compared with equation (3.9) because the renormalization schemes used in refs. 6 and 11 are different from ours. On the other hand we can directly compare our result for $T_{2,n}^{(2)NS}$ with that obtained by de Rujula et al.[6] Setting $\alpha = 0$ in equation (3.4) we obtain their result except for the numerator of the term $1/(n + 1)$ which in our case is 6 and theirs 10.

3.4. Discussion of the Results for Non-singlet Structure Functions

In this section, we discuss some technical aspects of our calculations including infrared sensitivity, gauge dependence, and normalization dependence. We defer discussion of phenomenological aspects to section 5.

In the previous sections, we have calculated the coefficient functions, C_n, using 't Hooft's minimal subtraction scheme. This normalization procedure is insensitive to the infrared structure of the theory for large Q^2; corrections are of order p^2/Q^2 or $m_q{}^2/Q^2$ for the coefficient functions. The amplitudes, T_n and A_n, which were used for our calculation of C_n are sensitive to the infrared behavior since they involve matrix elements in particular states, free quarks in our case. Although our states are only logarithmically off shell ($p^2 \to 0$, $\ln p^2$ finite), numerator factors of order p^2 may still contribute as the Feynman integrals can yield terms of order $1/p^2$. Therefore one must be careful when dropping p^2 terms. Such terms are responsible for the gauge dependence of the amplitudes, T_n(3.4) and A_n(3.6).

The question of gauge dependence involves a number of aspects. We have noted that the spin averaged matrix elements of the correlation function, T_n, and the local operators, A_n, are found to be gauge dependent. On the other hand, the coefficient functions, C_n(3.9), are found to be gauge invariant. This result is expected as the C_n appear as coefficients in the Wilson expansion involving gauge invariant operators. The coefficient functions are not automatically gauge invariant as the local operators may be given a gauge dependent normalization in some renormalization schemes.

The gauge dependence of the theory also affects the renormalization group equations which are used to convert the perturbation theory results into the true asymptotic behavior of the theory. Except in the Landau gauge, the usual renormalization group equations involve a derivative with respect to the renormalized gauge parameter.[19] The solution of the renormalization group equations usually requires full knowledge of the gauge dependence of the renormalized group parameters. However, the two loop anomalous dimensions have only been computed in the Feynman gauge.[15] The renormalization group equations for amplitudes in Feynman gauge must be modified by the inclusion of inhomogenous terms. The Feynman gauge amplitudes for T_n and A_n will satisfy renormalization group equations of the forms

$$[\mu \partial_\mu + \beta(g)\partial_g - 2\gamma_F]T_n = \Delta T_n$$

$$[\mu \partial_\mu + \beta(g)\partial_g + \gamma_n - 2\gamma_F]A_n = \Delta A_n \qquad . \tag{3.11}$$

The inhomogenous terms result from the variation of the bare gauge parameter as we change the normalization scale while remaining in Feynman gauge. In the minimal subtraction scheme, these terms may be computed for the relations in (3.11). However the analogous relations for the coefficient functions C_n will not involve inhomogenous terms. Hence the naive renormalization group equations used in (2.20) for the coefficient functions are correct for Feynman gauge calculations in the minimal subtraction scheme.

We now turn to problems associated with the solution of renormalization group equations which relate to the significance of the parameter Λ and the presence of terms like $\ln 4\pi$ and γ_E in our expressions for the coefficient functions. The coefficient functions for nonsinglet operators satisfy homogenous renormalization group equations in the form

$$[\mu \partial_\mu + \beta(g)\partial_g - \gamma_n(g)]\, C_n\left(\frac{Q^2}{\mu^2}, g^2\right) = 0 \tag{3.12}$$

where μ is the normalization scale and g the renormalized coupling constant. These equations may be solved in a standard manner by introducing a renormalization group invariant coupling constant, $\bar{g}(Q^2/\mu^2, g)$. The result (2.21-2.22) may be expressed as

$$C_n\left(\frac{Q^2}{\mu^2}, g\right) = C_n(1, \bar{g}^2)\exp\left[\int_{\bar{g}}^{g} d\tau\, \gamma^n(\tau)/\beta(\tau)\right] \tag{3.13}$$

$$-\frac{1}{2}\ln\frac{Q^2}{\mu^2} = \int_{\bar{g}}^{g} d\tau (1/\beta(\tau)) \quad . \tag{3.14}$$

As noted in section 2, $C_n(1, \bar{g}^2)$ is simply evaluated in perturbation theory by computing $C_n(Q^2/\mu^2, g^2)$ at the normalization point $Q^2 = \mu^2$ as the exponential factor in (3.13) is one if $\bar{g}$ is defined as in (3.14).

The leading order results are obtained by evaluating $C_n(1, \bar{g}^2)$ in zeroth order and truncating $\gamma^n(g)$ and $\beta(g)$ in one loop order. Using the expansions in Eqs. (2.23) and (2.24), eqs. (3.14) and (3.13) may be integrated to obtain

$$\frac{16\pi^2}{\beta_0}\frac{1}{\bar{g}^2} = \frac{16\pi^2}{\beta_0}\frac{1}{g^2} + \ln\frac{Q^2}{\mu^2} \equiv \ln\frac{Q^2}{\Lambda^2} \tag{3.15}$$

and

$$C_n\left(\frac{Q^2}{\mu^2}, \bar{g}^2\right) = e^{-(\gamma_0^n/\beta_0)\ln g/\bar{g}} = \left[\frac{\ln Q^2/\Lambda^2}{\ln \mu^2/\Lambda^2}\right]^{-\gamma_0^n/2\beta_0} \quad . \tag{3.16}$$

For asymptotically free theories ($\beta_0 > 0$), this result represents the true asymptotic behavior of the coefficient functions as $Q^2 \to \infty$. If the full theory were precisely given by the truncated theory, then the parameter, Λ, would have significance as the scale of strong interactions. However, in the full theory there are corrections of order $(1/\ln Q^2)$. These corrections cannot be uniquely specified independent of the scale, Λ, as changes in the value of Λ may also be represented as order $(1/\ln Q^2)$ for large Q^2. This fact becomes more apparent when considering the first systematic corrections to the behavior of the coefficient function.

This next order calculation requires the knowledge of the one and two loop contributions to the anomalous dimension, γ^n, and the β-function as well as the entire one loop contributions to the coefficient functions. As discussed in section 2, the calculation of these quantities must be done in a consistent manner. The two loop β-function[2,21] and the two loop anomalous dimension for nonsinglet operators[15] have been computed using 't Hooft's minimal subtraction scheme.[18] We have presented our results for the full one loop coefficient function using this same scheme in section 3.3.

These results may be combined to obtain a solution to the renormalization group consistent to second order. One such solution is discussed in section 2, equations (2.26 - 2.29), with the result

$$C_n\left(\frac{Q^2}{\mu^2}, g^2\right) = C_n\left[1 + \frac{1}{\beta_0 \ln \frac{Q^2}{\Lambda^2}}\left\{B_n^{NS} + \frac{1}{2}\left(\frac{\gamma_1^{\,n}}{\beta_0} - \frac{\beta_1\gamma_0^{\,n}}{\beta_0^{\,2}}\right) - \frac{\gamma_0^n\beta_1}{2\beta_0^{\,2}} \ln\ln\frac{Q^2}{\Lambda^2}\right\}\right]\left(\ln\frac{Q^2}{\Lambda^2}\right)^{-\gamma_0^n/2\beta_0} \quad . \tag{3.17}$$

The equation (3.17) is exact through order $O(1/\ln Q^2/\Lambda^2)$ in the expansion for large Q^2 with the scale Λ. Asymptotic expansions of this type are also possible for some other choice of scale, say Λ_0. To be specific, take $\Lambda_0 = 1$ GeV and re-expand (3.17) in powers of $1/\ln Q^2$ to find

$$C_n\left(\frac{Q^2}{\mu^2}, g\right) = C_n\left[1 + \frac{1}{\beta_0 \ln Q^2}\left\{B_n^{NS} + \frac{1}{2}\gamma_0^n \ln\Lambda^2 + \frac{1}{2}\left(\frac{\gamma_1^{\,n}}{\beta_0} - \frac{\beta_1\gamma_0^n}{\beta_0^{\,2}}\right) - \frac{\gamma_0^n\beta_1}{2\beta_0^{\,2}} \ln\ln Q^2\right\}\right](\ln Q^2)^{-\gamma_0^n/2\beta_0} \quad , \tag{3.18}$$

through order $O(1/\ln Q^2)$. An important observation is that the effect of the change of scale is equivalent through order $1/\ln Q^2$ to the shift of the constant B_n^{NS} by an amount proportional to the one-loop anomalous dimension $\gamma_0^{\,n}$. This may be seen in (3.18) explicitly. Thus any term proportional to γ_0^n in B_n^{NS} can be absorbed by the redefinition of the scale Λ. In particular the term $\ln 4\pi - \gamma_E$ in (3.9) may be absorbed by choosing the new scale $\Lambda \exp \frac{1}{2}(\ln 4\pi - \gamma_E)$.

From the point of view of the renormalization group equation, the ambiguity of $\bar{g}^2$ corrections in terms proportional to γ_0^n as well as in the redefinition of Λ are related to the freedom of defining the effective coupling constant in solving the renormalization group equations (3.12). In fact a new solution for $\bar{g}$ may be obtained by adding a constant b to the left-hand side of (3.14), yielding

$$-\frac{1}{2}\ln\frac{Q^2}{\mu^2} + b = \int_{\bar{g}}^{g} d\tau \frac{1}{\beta(\tau)} \quad . \tag{3.19}$$

With this normalization $\bar{g}(Q^2)$ is equal to the renormalized coupling constant g at $Q^2 = \mu^2 e^{-2b}$ $(= \mu'^2)$ and not at $Q^2 = \mu^2$ as defined in (3.14). Since Λ is proportional to μ (see (2.28) and (3.15)), we observe that any redefinition of the scale Λ corresponds to redefinition of the effective coupling constant. If the new renormalization scale μ' is used, the solution of the renormalization group equations takes the following form,

$$C_n\left(\frac{Q^2}{\mu^2}, g^2\right) = C_n\left(\frac{\mu'^2}{\mu^2}, \bar{g}^2\right) \exp\int_{\bar{g}}^{g} d\tau \frac{\gamma_n(\tau)}{\beta(\tau)} \quad , \tag{3.20}$$

with $\bar{g}(\mu'^2) = g$. Although the expression of $C_n(Q^2/\mu^2,\ g^2)$ in terms of g and μ in (3.20) is different from that in (2.21), the physical result is identical.

It is clear that the addition of the constant, b, in definition of $\bar{g}^2$ (3.19) just reproduces the effect of shifting Λ to absorb terms proportional to γ_0^n in B_n. The complete second order contributions to the coefficient function given in Eq. (3.18) are, of course, independent of this redefinition as would be full all order calculations. However if we truncate the calculation in second order in $\bar{g}^2$, different choices for b correspond to different estimates of the higher order terms in $\bar{g}^2$ for the coefficient functions. The choice for b must give results consistent with the asymptotic perturbation expansion and the normalization scheme used.

This freedom in choosing b can be exercised separately for each n when solving the renormalization group equation. One possible procedure, therefore, involves the introduction of a b_n in Eq. (3.19) such that in Eq. (3.20)

$$C_n\left(\frac{\mu_n'^2}{\mu^2}, \bar{g}_n^2\right) = 1 \qquad ,$$

with $\mu_n' = \mu e^{-b_n}$ and $\bar{g}_n^{\,2}$ depending on n through

$$\Lambda_n = \Lambda \exp\left(B_n^{NS}/\gamma_0^n\right) \qquad ,$$

so the n dependence of Λ_n is calculable. This procedure has the nice property that the relative n dependence between B_n^{NS} and γ_0^n, i.e. the difference in n dependence between order $\bar{g}^2$ and leading order, is isolated in Λ_n and in the two-loop anomalous dimensions (the effect of the latter is small for $n < 12$). This procedure is similar to one proposed by Bacé.[25] The phenomenological implications of our calculations are discussed in section V.

4. g^2 CORRECTIONS TO ν AND $\bar{\nu}$ SCATTERING (NON-SINGLET CONTRIBUTIONS)

4.1. Calculation of g^2 Corrections

The evaluation of g^2 corrections to ν and $\bar{\nu}$ deep inelastic scattering proceeds as in sections 2 and 3 except that now we must also deal with axial-vector currents. It is convenient to consider certain combinations of the $\nu, \bar{\nu}$ structure functions which have simple properties under crossing. These are

$$F_2^{\bar{\nu} p} - F_2^{\nu p} \tag{4.1}$$

$$F_2^{\bar{\nu} p} + F_2^{\nu p} \tag{4.2}$$

$$F_3^{\bar{\nu} p} - F_3^{\nu p} \tag{4.3}$$

and

$$F_3^{\bar{\nu} p} + F_3^{\nu p} \quad . \tag{4.4}$$

The remaining structure functions for scattering off neutron or nuclear targets can be directly obtained from (4.1)-(4.4) using charge symmetry. For instance

$$F_2^{\bar{\nu} p} - F_2^{\nu p} = F_2^{\nu n} - F_2^{\nu p} = F_2^{\nu n} - F_2^{\bar{\nu} n} = F_2^{\bar{\nu} p} - F_2^{\bar{\nu} n} \quad . \tag{4.5}$$

In order to calculate g^2 corrections to F_2 one considers again the diagrams of Fig. 2 except that now diagrams with both vector currents replaced by axial-vector currents also contribute. But since we have put masses to zero the axial-vector-axial-vector contributions are equal to vector-vector contributions. Obviously calculation of the g^2 corrections to the combinations (4.1) and (4.2) corresponds to subtracting and adding crossed diagrams respectively.

The structure function F_3 corresponds to the vector-axial-vector interference and therefore the diagrams contributing to it are obtained from Fig. 2 by replacing one of the vector currents by an axial vector current. Again the calculation of the g^2 corrections to the combinations (4.3) and (4.4) corresponds to subtracting and adding crossed diagrams respectively.

By inspecting the diagrams directly or by considering the decomposition (2.1) and taking into account known properties of various structure functions under the transformations $\mu \to \nu$, $x \leftrightarrow -x$ one can easily find whether even or odd spin operators contribute to each of the combinations (4.1-4.4). It turns out[2,26] that to $F_2^{\bar{\nu}-\nu}$ and $F_3^{\bar{\nu}+\nu}$ only odd spin and to $F_2^{\bar{\nu}+\nu}$ and $F_3^{\bar{\nu}-\nu}$ only even spin operators contribute.

Finally we have to determine which combinations are independent of gluon operators and therefore satisfy simple renormalization group equations as given in eq. (2.20). The combinations (4.1) and (4.3) transform obviously as non-singlets under flavor symmetry and therefore satisfy equations like (2.20). $F_2^{\bar{\nu}p} + F_2^{\nu p}$ is a singlet combination which can be seen, for instance, by writing it in terms of quark distributions. Therefore because of mixing between gluon and fermion singlet operators this combination will satisfy more complicated renormalization group equations, which we shall discuss in section 6. On the other hand, $F_3^{\bar{\nu}+\nu}$ still satisfies equation (2.20) in spite of having contributions from singlet fermion operators. This is because the gluon operators of odd spin[27] transform differently under charge conjugation than the corresponding singlet fermion operators and therefore there is no mixing.[26]

In this section we shall restrict the calculation to the combinations (4.1), (4.3) and (4.4) and come back to the combination (4.2) in the next section.

For $F_2^{\bar{\nu}p} - F_2^{\nu p}$ the calculation is exactly as in section 3 and we obtain for the corresponding coefficient functions defined in equation (2.4) the following final result

$$C_{2,n}^{\bar{\nu}-\nu}(1, \bar{g}^2) = C_{2,n}^{NS}(1, \bar{g}^2) \quad , \quad n \text{ odd} \tag{4.7}$$

$$C_{L,n}^{\bar{\nu}-\nu}(1, \bar{g}^2) = C_{L,n}^{NS}(1, \bar{g}^2) \quad , \quad n \text{ odd} \tag{4.8}$$

where $C_{2,n}^{NS}$ and $C_{L,n}^{NS}$ are given by equations (3.7-3.10).

On the other hand, we find

$$C_{3,n}^{\bar{\nu}\pm\nu}(1, \bar{g}^2) = C_{2,n}^{NS}(1, \bar{g}^2) - \frac{\bar{g}^2}{16\pi^2} C_2(R) \frac{4n+2}{n(n+1)} \quad , \quad \begin{matrix} n \text{ odd} \\ \\ n \text{ even} \end{matrix} \quad . \tag{4.9}$$

4.2. Corrections to Sum Rules and Parton Model Relations

It is well known[2] that in the leading order of asymptotic freedom parton model relations and sum rules are satisfied. The $\bar{g}^2$ corrections calculated in this section can generally introduce violations of the sum rules and relations in question.

Evaluating the formulae (4.7-4.9) for n = 1 and recalling that $\gamma_{n=1} = 0$ due to current conservation so that except for B_n^{NS} calculated here, all contributions in Eq. (2.29) vanish, we obtain

$$\int_0^1 \frac{dx}{x} \left[F_2^{\bar{\nu}p} - F_2^{\nu p} \right] = 2 \tag{4.10}$$

$$\int_0^1 dx \left[F_3^{\bar{\nu}p} + F_3^{\nu p} \right] = -6 \left[1 - \frac{12}{(33 - 2f) \ln \frac{Q^2}{\Lambda^2}} \right] \tag{4.11}$$

and

$$\int_0^1 dx \left[F_1^{\bar{\nu}p} - F_1^{\nu p}\right] = \left[1 - \frac{8}{(33-2f)\ln \frac{Q^2}{\Lambda^2}}\right] . \tag{4.12}$$

These results disagree with those obtained by Calvo.[11] Notice that the Adler sum rule[28] (4.10) is exactly satisfied, whereas both the Gross-Llewellyn-Smith[29] sum rule (4.11) and the Bjorken sum rule[30] (4.12) are violated. In Fig. 4 we have plotted predictions of (4.11) and (4.12) versus Q^2/Λ^2. We observe that the deviations from the two sum rules in question are predicted to be non-negligible and accurate measurements should detect them.

Equations (3.8) and (4.8) imply violation of the Callen-Gross relation $2xF_1 = F_2$. Previous investigation[6,7,14] has shown, however, that the violations of the relations in question seen in the data at large x are larger than predicted by the theory.

5. PHENOMENOLOGY OF THE ORDER $\bar{g}^2$ CORRECTIONS

We discuss in this section the phenomenological application of our calculation. The theory predicts directly the Q^2 dependence of the moments of structure functions; we will use in our analysis the combined Gargamelle-BEBC data[10] for the non-singlet structure function $xF_3(x, Q^2)$ to obtain the experimental values of the moments, thereby avoiding the somewhat involved problem of inverting the moments. Bosetti et al.[10] have already analyzed the moments of xF_3 using only the leading order effects of asymptotic freedom. It is our purpose here to investigate the effect of including the order $\bar{g}^2$ corrections in the analysis.

The moments of xF_3 most appropriate for comparison with our calculation are the Nachtmann moments[5]

$$M_n(Q^2) = \int_0^1 dx \frac{\xi^{n+1}}{x^3} xF_3(x, Q^2) \left[1 + \frac{n+1}{n+2} 2 \left(\frac{x}{\xi} - 1\right)\right] \tag{5.1}$$

with

$$\xi = 2x/(1 + \sqrt{1 + 4M^2x^2/Q^2})$$

and M the nucleon mass. The moments are obtained by straightforward numerical evaluation of the integral using the data of Ref. 10.

In order to investigate the effect of the order $\bar{g}^2$ corrections, we have chosen four different schemes with which to compare theory and experiment. The first scheme, denoted LO, uses simply the leading order prediction of asymptotic freedom, which for the non-singlet moments is

$$M_n(Q^2) = A_n \left(\ln \frac{Q^2}{\Lambda_{LO}^2}\right)^{-\gamma_0^n/2\beta_0} , \tag{5.2}$$

where the A_n are related to matrix elements of the relevant operators between target states and are taken as free parameters, $\gamma_0{}^n$ and β_0 are defined in Eqs. (2.32) and (2.33) and Λ_{LO} is the scale parameter defined by the leading order expression (2.27) for $\bar{g}^2(Q^2)$.

The second and third schemes include the order $\bar{g}^2$ corrections which multiply (5.2) by an additional term, yielding

$$M_n(Q^2) = A_n \left[1 + \frac{1}{\beta_0 \ln \frac{Q^2}{\Lambda^2{}_{MS}}} (B_n + P_n + L_n) \right] \left(\ln \frac{Q^2}{\Lambda^2{}_{MS}} \right)^{-\gamma_0^n/2\beta_0} , \quad (5.3)$$

where

$$P_n = \frac{\gamma_1{}^n}{2\beta_0} - \frac{\beta_1 \gamma_0{}^n}{2\beta_0{}^2} ,$$

$$L_n = - \frac{\beta_1 \gamma_0{}^n}{2\beta_0{}^2} \ln \ln \frac{Q^2}{\Lambda^2{}_{MS}} ,$$

and B_n is obtained for xF_3 from Eqs. (3.7), (3.9), and (4.9). This expression uses a scale parameter Λ_{MS} (MS for minimal scheme) corresponding to the definition (2.26) of $\bar{g}^2(Q^2)$ introduced in Ref. 7. As discussed in section 3.4, however, B_n is actually determined only up to a term proportional to $\gamma_0{}^n$, corresponding to an arbitrariness in the normalization involved in introducing $\bar{g}$ when solving the renormalization group equation. For the purposes of illustration, therefore, we will also use (5.3) with B_n replaced by $\bar{B}_n = B_n - \frac{1}{2}\gamma_0{}^n(\ln 4\pi - \gamma_E)$ and Λ_{MS} replaced by $\Lambda_{\overline{MS}}$, thereby defining the scheme $\overline{MS}$. It is easy to see that the schemes MS and $\overline{MS}$ are equivalent (through order $\bar{g}^2$) provided

$$\Lambda_{\overline{MS}} = \Lambda_{MS}\, e^{\frac{1}{2}(\ln 4\pi - \gamma_E)} \quad . \tag{5.4}$$

Finally, we introduce a fourth scheme which, as discussed in section 3.4, treats the n-dependence of $\gamma_1{}^n$ exactly but absorbs the n-dependence of B_n into Λ. Thus using Eqs. (2.23) and (2.24), we evaluate the integral in Eq. (1.1) to obtain

$$M_n(Q^2) = A_n\left(1 + \frac{\beta_1}{\beta_0}\frac{\bar{g}^2}{16\pi^2}\right)^{\frac{\gamma_1{}^n}{2\beta_1} - \frac{\gamma_0{}^n}{2\beta_0}} \left(\frac{\bar{g}^2}{16\pi^2}\right)^{\frac{\gamma_0{}^n}{2\beta_0}} \quad , \tag{5.5}$$

where $\bar{g}^2$ is the exact solution of Eq. (2.22) appropriate to our choice of Λ_n,

$$\frac{16\pi^2}{\beta_0\bar{g}^2} - \frac{\beta_1}{\beta_0{}^2}\ln\frac{16\pi^2 + \frac{\beta_1}{\beta_0}\bar{g}^2}{\beta_0\bar{g}^2} = \ln\frac{Q^2}{\Lambda_n{}^2} \quad , \tag{5.6}$$

$$\Lambda_n = \Lambda e^{\bar{B}_n/\gamma_0^n} \quad . \tag{5.7}$$

This fourth scheme, which we denote as the Λ_n scheme, is clearly equivalent (through order $\bar{g}^2$) to the schemes MS and $\overline{MS}$. This scheme has the particularly nice property that if Eq. (5.5) is used to determine "experimental" Λ_n's separately for each n, then the resulting Λ_n's should follow the pattern of n-dependence predicted by Eq. (5.7). As a corollary to this scheme, we remark that the second factor in Eq. (5.5) is always very near unity (see Table) in the region of interest, hence Eq. (5.5) has essentially the same form as the leading order Eq. (5.2), and therefore this scheme is quite similar in spirit to one proposed by Bacé.[25]

We have used each of the four schemes LO, MS, $\overline{MS}$, and Λ_n discussed above to determine the unknown scale parameter in each scheme (the constants A_n are also fitted but their values are uninteresting and are not given). We find

$$\Lambda_{LO} = 0.73 \text{ GeV}$$

$$\Lambda_{MS} = 0.40 \text{ GeV}$$

$$\Lambda_{\overline{MS}} = 0.52 \text{ GeV}$$

and

$$\Lambda = 0.40 \text{ GeV} \quad .$$

These values are obtained using β_0, β_1, $\gamma_0{}^n$, $\gamma_1{}^n$ appropriate to 4 flavors (using 3 flavors shifts each Λ to a slightly larger value). The fitting program estimates the errors on each Λ at about ten percent. In each case we used moments for $n \leq 8$ and $Q^2 > 1 \text{ GeV}^2$. The results of the fits do not change significantly if we restrict our considerations to $Q^2 > 2 \text{ GeV}^2$. In all cases the quality of the fit was very good (χ^2/D.F. $\lesssim 1$).

We now proceed to discuss several important points related to the phenomenology: (1) Our fits for the four schemes are shown in Fig. 5. Somewhat surprisingly (and disappointingly), we find that the LO, MS, and $\overline{MS}$ schemes are virtually indistinguishable for $Q^2 > 1.5 \text{ GeV}^2$; thus they are represented by the same (solid) line in the figure. Of course, the fits differ for $Q^2 < 1 \text{ GeV}^2$ but then $\bar{g}^2$ is large and the perturbation theory is meaningless. The fit using the Λ_n scheme does not fit quite as well as the others, but the quality of the fit is still fairly good. The similarity of the LO, MS, and $\overline{MS}$ fits simply indicates that it is possible for the A_n's and Λ in each case to conspire to mask the combined n- and Q^2-dependence of the order $\bar{g}^2$ corrections. This is in spite of the fact that the corrections in question are not necessarily small for low values of Q^2 and $n > 3$. For instance, for $Q^2 \approx 10 \text{ GeV}^2$ and $n = 5$ the second factor in equation (5.3) is roughly 1.2 and 1.5 for $\overline{MS}$ and MS respectively as compared to 1 in the leading order. The situation is much worse for higher moments since for fixed Q^2 the corrections in question grow due to $\gamma_n^{(1)}$ like $(\ln n)^3$ [15,20] and perturbation theory breaks down.

(2) The most important result of the analysis of Bosetti et al.[10] is the quantitative verification of QCD based on the leading order prediction

$$\frac{d \ln M_n}{d \ln M_m} = \frac{\gamma_0^n}{\gamma_0^m} \quad . \tag{5.8}$$

It is demonstrated in Ref. 10 that the experimental values of the l.h.s. of (5.8) agree remarkably well with the QCD predictions for the ratios γ_0^n/γ_0^m. We find that the agreement between theory and experiment is not disturbed even when the order $\bar{g}^2$ corrections are large, i.e. as discussed in point (1) above, the schemes LO and MS, $\overline{MS}$ are indistinguishable (except that the A_n's and Λ are very different in each case).

(3) Finally, we have fitted the moments for each n separately using the Λ_n scheme, and the "experimental" results for Λ_n are shown in Fig. 6, along with the prediction of Eq. (5.7) using the value $\Lambda = 0.4$ GeV determined previously in the Λ_n scheme by fitting together all moments with $n \leq 8$. The "data" points do not seem to follow the theoretical prediction, but we cannot claim that the disagreement is significant since, after all, the Λ_n scheme is equivalent (through the order $\bar{g}^2$ we have computed) with the MS, $\overline{MS}$ schemes, and the latter fit the data quite well. We believe, however, that the Λ_n scheme is particularly well suited to comparison with experiment because the n-dependence predicted by the theory is strictly enforced in the fit (thus prohibiting the kind of conspiracy between the A_n's and Λ which occurs in the MS, $\overline{MS}$ schemes).

(4) We have also shown (as a dashed line) in Fig. 6 the prediction of Eq. (5.7) for $\Lambda = 0.5$ GeV. This value of Λ gives good agreement with the Λ_n "data" for $n \leq 5$ and isolates the disagreement at large n. We are motivated here by the conjecture of De Rujula, et al.,[6] which claims that the effect of higher twist operators in the operator product expansion is most strongly felt at large n, or more specifically whenever

$$n\frac{M_0^2}{Q^2} \sim O(1)$$

with $M_0^2 \sim O(\Lambda^2)$ an appropriate (but unknown) scale. Mainly as a curiosity, we have included a term of the form

$$1 + n\frac{M_0^2}{Q^2}$$

as an additional factor in (5.5) and redetermined the "experimental" Λ_n's. With $M_0^2 \sim -0.16$ GeV^2, the resulting Λ_n "data" do in fact follow the trend predicted by Eq. (5.7), i.e. Λ_n increases with n. Due to large theoretical uncertainties in the higher twist corrections, it is not appropriate to pursue this farther.

We conclude this section with the following words of caution and recommendation. If the n-dependence of Λ_n as predicted by Eq. (5.7) is not eventually found in higher statistics experimental data, then one must conclude (short of abandoning QCD) that certain possibly strong effects not included in the analysis are present in the data. Possible sources for these effects would be higher twist operators, higher order gluon corrections to twist-two operators, or perhaps even nonperturbative (instanton) effects. It will be very helpful to isolate these effects phenomenologically, since the theoretical calculations are apparently rather difficult. Finally, it should be apparent by now that due to the fact that B_n is determined only up to a term proportional to $\gamma_0^{\ n}$, any attempts to extract the scale parameter Λ of the theory from the experimental data must be viewed with a certain amount of reservation.

6. $C_n(Q^2/\mu^2,\ g^2)$ TO ORDER g^2 FOR SINGLET OPERATORS

6.1. Preliminaries

In sections 3 and 4 we have calculated the coefficient function to order $\bar{g}^2$ for non-singlet operators relevant for ep and ν scattering. Here we extend our analysis to singlet operators. We begin with electromagnetic currents.

The Q^2 dependence of the Wilson coefficient functions C_n^{ψ} and C_n^{G} corresponding to singlet fermion O_n^{ψ} and O_n^{G} operators is governed by the following renormalization group equations[2]

$$\left[\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g}\right] C_n^{\,i}\left(\frac{Q^2}{\mu^2}, g^2\right) = \sum_j \gamma_{ij}^{\,n}(g^2) C_n^{\,j}\left(\frac{Q^2}{\mu^2}, g^2\right) \tag{6.1}$$

where i,j = ψ, G and $\gamma_{ij}^{\,n}$ is the anomalous dimension matrix.

The solution to (6.1) is given as follows

$$C_n^{\,i}\left(\frac{Q^2}{\mu^2}, g^2\right) = \sum_j \left\{T_g \exp\left[+\int_{\bar{g}}^{g} \frac{\gamma^{\,n}(\bar{g}')}{\beta(\bar{g}')}\, d\bar{g}'\right]\right\}_{ij} C_n^{\,j}(1, \bar{g}^2) \tag{6.2}$$

with $C_n^{\,j}(1,\bar{g}^2)$ having the expansion in powers of $\bar{g}^2$

$$C_n^{\,i}(1,\bar{g}^2) = C_n^{(0)i} + B_n^{\,i}\frac{\bar{g}^2}{16\pi^2} + O(\bar{g}^4) \quad . \tag{6.3}$$

It is instructive to expand $C_n^{\,i}(Q^2/\mu^2,\ g^2)$ as given by (6.2) in powers of g. Recalling that in the g^0 order in the normalization of equation (2.17)

$$C_n^{(0)\psi} = 1, \qquad C_n^{(0)G} = 0 \tag{6.4}$$

we obtain

$$C_n^{\psi}\left(\frac{Q^2}{\mu^2}, g^2\right) = 1 + \frac{g^2}{16\pi^2}\left\{-\frac{1}{2}\gamma_{\psi\psi}^{n} \ln\frac{Q^2}{\mu^2} + B_n^{\psi}\right\} \tag{6.5}$$

and

$$C_n^{G}\left(\frac{Q^2}{\mu^2}, g^2\right) = \frac{g^2}{16\pi^2}\left\{-\frac{1}{2}\gamma_{\psi G}^{n} \ln\frac{Q^2}{\mu^2} + B_n^{G}\right\} . \tag{6.6}$$

Here $\gamma_{\psi\psi}{}^n$ is equal to $\gamma_0{}^n$ of equation (2.32). $\gamma_{\psi G}{}^n$ is the non-diagonal element of the anomalous dimension matrix given by[2]

$$\gamma_{\psi G}{}^{n} = -4f\frac{n^2+n+2}{n(n+1)(n+2)} , \tag{6.7}$$

f being the number of flavors. In order to calculate $B_n{}^i$ in perturbation theory we consider the two forward Compton amplitudes

$$T^{\psi}(Q^2, \nu) = i\int d^4x\, e^{iq\cdot x} \langle\psi ; p|\, T(j(x)j(0))\,|\psi;p\rangle \tag{6.8}$$

and

$$T^{G}(Q^2, \nu) = i\int d^4x\, e^{iq\cdot x} \langle G; p|\, T(j(x)j(0))\,|G; p\rangle \tag{6.9}$$

where $p^2 < 0$.

Using the operator product expansion for the appropriate currents we obtain the following generalizations of equation (2.35)

$$T^{\psi}(Q^2, \nu) = \sum_n \left(\frac{1}{x}\right)^n \left\{C_n^{\psi}\left(\frac{Q^2}{\mu^2}, g^2\right) A_{n\psi}^{\psi}\left(\frac{p^2}{\mu^2}, g^2\right) + \right.$$

$$\left. + C_n^{G}\left(\frac{Q^2}{\mu^2}, g^2\right) A_{n\psi}^{G}\left(\frac{p^2}{\mu^2}, g^2\right)\right\} . \tag{6.10}$$

$$T^G(Q^2,\nu) = \sum_n \left(\frac{1}{x}\right)^n \left\{ C_n^\psi\left(\frac{Q^2}{\mu^2}, g^2\right) A_{nG}^\psi\left(\frac{p^2}{\mu^2}, g^2\right) + C_n^G\left(\frac{Q^2}{\mu^2}, g^2\right) A_{nG}^G\left(\frac{p^2}{\mu^2}, g^2\right)\right\} \quad . \tag{6.11}$$

The reduced matrix elements $A_{nj}^{\ i}$ defined in equation (2.9) can be calculated in perturbation theory and the result can be written as

$$A_{nj}^{\ i}\left(\frac{p^2}{\mu^2}, g^2\right) = A_{nj}^{(0)i} + \frac{g^2}{16\pi^2}\left[\frac{1}{2}\gamma_{ij}^n \ \ln\frac{-p^2}{\mu^2} + A_{nj}^{(2)i}\right] \tag{6.12}$$

where $A_{nj}^{(0)i}$ in the normalization of equation (2.18) take the following values

$$A_{n\psi}^{(0)\psi} = A_G^{(0)G} = 1 \qquad A_{n\psi}^{(0)G} = A_{nG}^{(0)\psi} = 0 \quad . \tag{6.13}$$

Inserting (6.5), (6.6) and (6.12) into (6.10) and (6.11) and using (6.4) and (6.13) we obtain in an obvious notation

$$T_n^\psi = 1 + \frac{g^2}{16\pi^2}\left[-\frac{1}{2}\gamma_{\psi\psi}^n \ \ln\frac{Q^2}{-p^2} + B_n^\psi + A_{n\psi}^{(2)\psi}\right] \tag{6.14}$$

and

$$T_n^G = \frac{g^2}{16\pi^2}\left[-\frac{1}{2}\gamma_{\psi G}^n \ \ln\frac{Q^2}{-p^2} + B_n^G + A_{nG}^{(2)\psi}\right] \quad . \tag{6.15}$$

Consequently

$$B_n^\psi = T_n^{(2)\psi} - A_{n\psi}^{(2)\psi} \tag{6.16}$$

$$B_n^G = T_n^{(2)G} - A_{nG}^{(2)\psi} \tag{6.17}$$

where $T_n^{(2)\psi}$ and $T_n^{(2)G}$ are the constant parts of order g^2 in the Compton amplitudes (6.8) and (6.9). For longitudinal structure functions which vanish in zeroth order only the first terms on the r.h.s. of equations (6.16) and (6.17) are present.

Equation (6.16) is equivalent to (2.39) and using (6.3) we obtain to order $\bar{g}^2$

$$C_{2,n}^{\psi}(1,\bar{g}^2) = C_{2,n}^{NS}(1,\bar{g}^2) \tag{6.18}$$

and

$$C_{L,n}^{\psi}(1,\bar{g}^2) = C_{L,n}^{NS}(1,\bar{g}^2) \tag{6.19}$$

with the non-singlet structure functions calculated in section 3.3.

On the other hand equation (6.17) tells us that in order to calculate B_n^G we have to find the forward Compton amplitude for a photon scattering off a gluon and subtract from it the matrix element of the fermion singlet operator (2.6) between gluon states.

6.2. Calculation of $T_n^{(2)G}$

The calculation of $T_n^{(2)G}$ involves diagrams of Fig. 7. We use again the dimensional regularization scheme and keep the gluons off-shell with space-like momentum $p^2 < 0$. Both gluons and quarks are kept massless. The diagrams separately are divergent but when they are combined the divergences cancel and no renormalization is needed. The result for $T_{L,n}^{(2)G}$ and $T_{2,n}^{(2)G}$ is

$$T_{L,n}^{(2)G} = \frac{g^2}{16\pi^2} T(R) \frac{16}{(n+1)(n+2)} \qquad n \text{ even} \tag{6.20}$$

and

$$T_{2,n}^{(2)G} = \frac{g^2}{16\pi^2} T(R)\ 4 \left\{ -\frac{2}{n} + \frac{6}{n+1} - \frac{6}{n+2} + \frac{2}{n^2} - \frac{4}{(n+1)^2} + \frac{4}{(n+2)^2} \right\} \qquad n \text{ even} \quad (6.21)$$

where $T(R) = f/2$, f being the number of flavors.

6.3. Calculation of $A_{nG}^{(2)\psi}$

The calculation of $A_{nG}^{(2)\psi}$ involves diagrams of Fig. 8. Using the minimal subtraction scheme we obtain

$$A_{nG}^{(2)\psi} = \frac{g^2}{16\pi^2} T(R)\ 4 \left\{ \frac{2}{n+1} - \frac{2}{n} - \frac{2}{n+2} + \frac{1}{n^2} - \frac{4}{(n+1)^2} \right.$$

$$\left. + \frac{4}{(n+2)^2} + \frac{n^2+n+2}{n(n+1)(n+2)} \sum_{j=1}^{n} \frac{1}{j} + \tilde{\gamma}_{\psi G}^{n} (\ln 4\pi - \gamma_E) \right\} \qquad n \text{ even} \qquad (6.22)$$

where

$$\tilde{\gamma}_{\psi G}^{n} = \frac{n^2+n+2}{n(n+1)(n+2)} = -\frac{1}{4f} \gamma_{\psi\ G}^{\ n} \quad . \qquad (6.23)$$

6.4. Final Result for $C_{L,n}^{G}(1, \bar{g}^2)$ and $C_{2,n}^{G}(1, \bar{g}^2)$

From equations (6.20 - 6.22) we finally obtain using (6.17) and (6.3) the following expressions for $C_{L,n}^{G}(1, \bar{g}^2)$ and $C_{2,n}^{G}(1, \bar{g}^2)$ to order $\bar{g}^2$ calculated in the minimal subtraction scheme

$$C_{L,n}^{G}(1, \bar{g}^2) = \frac{\bar{g}^2}{16\pi^2} T(R) \frac{16}{(n+1)(n+2)} \qquad n \text{ even} \qquad (6.24)$$

and

$$C_{2,n}^{G}(1, \bar{g}^2) = \frac{\bar{g}^2}{16\pi^2} T(R)\ 4 \left\{ \frac{4}{(n+1)} - \frac{4}{(n+2)} + \frac{1}{n^2} - \frac{n^2+n+2}{n(n+1)(n+2)} \sum_{j=1}^{n} \frac{1}{j} \right.$$

$$\left. - \tilde{\gamma}_{\psi\ G}^{\ n} (\ln 4\pi - \gamma_E) \right\} \qquad n \text{ even} \quad . \qquad (6.25)$$

The treatment of the $\ln 4\pi - \gamma_E$ term is exactly the same as in the non-singlet case.

6.5. Results for $\nu, \bar{\nu}$ Scattering

The results (6.18), (6.19) and (6.24) and (6.25) apply also to the structure function $F_2^{\nu+\bar{\nu}}$. On the other hand $F_3^{\nu\pm\bar{\nu}}$ does not receive contributions from gluon operators by the discussion of section 4. The result for $C_3^{\bar{\nu}\pm\bar{\nu}}(1, \bar{g}^2)$ is given in equation (4.9).

6.6. Discussion of Results for Singlet Parts

At present we are not ready to apply (6.25) to phenomenological analyses because the anomalous dimensions of singlet operators to order g^4 are not yet available. However, the longitudinal coefficient function (6.24) does not involve two-loop contributions and hence may be used directly in phenomenology. Such analyses have been performed in reference 14. It should be remarked that in (6.24) and (6.25) the g^2 corrections are small and vanish as $n \to \infty$. Therefore for large x the gluon contributions and their mixing with quark operators are of little importance.

To compare our results with those of other groups, we first note that the calculation of virtual photon-gluon scattering is equivalent to that of virtual photon-photon scattering if one replaces $g^2T(R)$ by e^4. Accordingly, the coefficients of the expansion in $1/x$, $T_{L,n}^G$ and $T_{2,n}^G$, are equal to moments of the imaginary part of the virtual photon-photon scattering amplitude. Our result for $T_{L,n}^G$ (6.20) agrees with the results of refs. 14,31 and 32, but disagrees in minor respects with refs. 11, 12, 13, 33,34.

Special care is needed in comparing our result for T_{2n}^G with others. In the calculation of T_{2n}^G there exists an infrared divergence coming from mass singularities while no ultraviolet divergence appears due to the gauge invariance. We have kept the gluon mass p^2 finite (space-like) to circumvent the mass singularities. Another way of avoiding the mass singularity is to introduce the quark mass m while $p^2 = 0$. This latter method has been used by other groups,[11,14,32,33,34] Naturally the results for T_{2n}^{G} are different in these two methods and should not be directly compared.[35] On the other hand the coefficient function C_{2n}^{G} should be insensitive to mass singularities which depend on the particular gluon or quark matrix element considered. Calculations of the coefficient function must agree in two methods up to the normalization scale. Our normalization scheme of the operator matrix element is different from that of other groups where the operator matrix element is normalized on the mass shell so that $T_{2n}^{G} = C_{2n}^{G}$. In order to translate our C_{2n}^{G} into theirs we must simply add to our C_{2n}^{G} the operator matrix element A_{2n}^{G} in the minimal subtraction scheme with $m \neq 0$ and $p^2 = 0$. We find

$$A_{2n}^{G} = \frac{g^2}{4\pi^2} T(R)\tilde{\gamma}^{n}_{\psi G}\left(\ln\frac{4\pi\mu^2}{m^2} - \gamma_E - 1\right) \quad . \tag{6.26}$$

We can now recast our result (6.25) into the form obtained by other groups:

$$\int_0^1 dx\, x^{n-2} F_2^{G}(x, Q^2) = \frac{g^2}{4\pi^2} T(R)\left[\tilde{\gamma}_{\psi G}^{n}\left(\ln\frac{Q^2}{m^2} - 1 - \sum_{j=1}^{n}\frac{1}{j}\right) + \frac{4}{n+1} - \frac{4}{n+2} + \frac{1}{n^2}\right] \quad , \tag{6.27}$$

or equivalently

$$F_2^{G}(x, Q^2) = \frac{g^2}{4\pi^2} T(R) x\left[(1 - 2x + 2x^2)\ln\frac{Q^2(1-x)}{m^2 x} - 1 + 8x(1-x)\right] . \tag{6.28}$$

This result agrees with that of Witten[34] (if we correct the factor 1/4) and Kingsley,[31] but disagrees with the result of Hinchliffe and Llewellyn-Smith.[14]

7. SUMMARY AND CONCLUSIONS

In this paper we have presented a general procedure (see sections 2 and 6) for the calculation of the Wilson coefficient functions $C_n(Q^2/\mu^2, g^2)$ to order g^2. An important step in this procedure, not previously discussed in the literature, involves the necessity of calculating both g^2 corrections to the virtual Compton amplitude and g^2 corrections to the matrix elements of local operators in order to find $C_n(Q^2/\mu^2, g^2)$ in a general renormalization scheme. Using this procedure we have calculated quark and gluon coefficient functions as predicted by asymptotically free gauge theories. This we have done for both non-singlet and singlet structure functions (F_2, F_3, F_L) relevant for electromagnetic and ν processes. Our results when combined with renormalization group equations give $\bar{g}^2$ corrections to the functions $C_n^{\,i}(1, \bar{g}^2)$ ($i = q, G$) which enter the solution of the equations in question. The results may be found in Eqs. (3.7)-(3.10), (4.7)-(4.9), (6.24), (6.25). We have emphasized following ref. 15 that the functions $C_n^i(1, \bar{g}^2)$ relevant to νW_2 and νW_3 are renormalization prescription dependent and that this renormalization prescription dependence is cancelled by that of two-loop anomalous dimensions γ_1^n when the full $\bar{g}^2$ corrections to the moments of various structure functions are computed. Of course in order for the cancellation to occur both $C_n(1, \bar{g}^2)$ and γ_1^n should be calculated in the same renormalization scheme. As a renormalization scheme we have chosen 't Hooft's minimal subtraction scheme since the only existing results for γ_1^n have been obtained in this scheme. The longitudinal structure functions to order $\bar{g}^2$ do not depend on two-loop anomalous dimensions and therefore $C_{n,L}(1, \bar{g}^2)$ is automatically renormalization prescription independent.

In the course of the calculation of the quark coefficient function $C_n^{NS}(1, \bar{g}^2)$ we have found that the relevant Compton amplitude and the relevant matrix elements of local operators were separately gauge dependent. We have demonstrated however that in the renormalization scheme considered these gauge dependences cancel each other leaving $C_n(1, \bar{g}^2)$ gauge independent.

Another feature of the $\bar{g}^2$ corrections to $C_n(1, \bar{g}^2)$ concerns the ambiguity of these corrections in the term proportional to γ_0^n. This ambiguity and corresponding ambiguity in parameter Λ is related to the freedom of defining the effective coupling constant when solving renormalization group equations. We have discussed it in detail in section 3.4.

In section 5 we have combined our results for non-singlet structure functions with those for two-loop anomalous dimensions of ref. 15 and two-loop β function of ref. 21 to obtain the full $\bar{g}^2$ correction to the leading order formula for non-singlet structure functions. We have compared our results with the recently measured[10] moments of $\nu W_3^{\nu,\bar{\nu}}$. In order to demonstrate the ambiguity of $\bar{g}^2$ corrections mentioned above we have considered various schemes corresponding to various definitions of the effective coupling constant. All schemes gave a good agreement with experimental data although using one of them (Λ_n scheme) which is particularly suited for testing the n-dependence of $\bar{g}^2$ corrections, we have found that for $n > 5$ some indication of other effects (e.g. higher twist operators) not included in our analysis may be present in the data. It is important to check for these effects in experiments with high statistics.

Our results for singlet structure functions can be combined with two-loop singlet anomalous dimensions calculated in the minimal subtraction scheme once such a calculation is completed.[16]

Finally we have calculated $\bar{g}^2$ corrections to the Gross-Llewellyn-Smith and Bjorken sum rules. Our results for these sum rules agree with those of Calvo.[11] It turns out that the violations of the sum rules in question are of the order of 15% at presently available values of Q^2 and experiments with high statistics should detect them.

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APPENDIX

The details of our calculation of the coefficient functions in electroproduction and neutrino reactions are presented here. The results of the calculation are given for the minimal subtraction scheme in the dimensional regularization. The calculation is performed in an arbitrary covariant gauge, and the gauge term is given separately. The nonsinglet Born amplitude is normalized to 1. The following projection tensors are used to project out invariant amplitudes:

$$d_{\mu\nu} = -\frac{p_\mu p_\nu}{(p\cdot q)^2} q^2 + \frac{p_\mu q_\nu + p_\nu q_\mu}{p\cdot q} - g_{\mu\nu} \quad , \tag{A.1}$$

$$e_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \quad . \tag{A.2}$$

The constant γ_E appearing in the text is the Euler-Macheroni constant: $\gamma_E = 0.5772..$, and g is a dimensionless renormalized coupling constant in the minimal subtraction scheme.

A. Electroproduction-Nonsinglet

A1. Current correlation functions

Internal self-energy diagram (Fig. 2a):

$$\frac{g^2}{16\pi^2} C_2(R) \sum_{n=2,4...} \left(\frac{1}{x}\right)^n \left[d_{\mu\nu}\left(\gamma_E + \ln\frac{Q^2}{4\pi\mu^2} - 1 - \sum_{j=1}^{n}\frac{1}{j} + \frac{1}{n}\right) - \frac{p_\mu p_\nu}{(p\cdot q)^2} q^2 \frac{1}{n-1} \right] \quad . \tag{A.3}$$

Vertex-correction diagrams (Fig. 2b, c):

$$\frac{g^2}{16\pi^2} C_2(R) \sum_{n=2,4\ldots} (\tfrac{1}{x})^n \left[d_{\mu\nu}\left(-4 \sum_{j=2}^{n} \frac{1}{j} \ln \frac{Q^2}{-p^2} - 2\left(\gamma_E + \ln \frac{Q^2}{4\pi\mu^2} \right) \right.\right.$$

$$\left.\left. + \frac{4}{n} + 4 \sum_{j=1}^{n} \frac{1}{j} - 8 \sum_{j=1}^{n} \frac{1}{j^2} \right) + \frac{p_\mu p_\nu}{(p\cdot q)^2} q^2 \left(\frac{2}{n-1} - \frac{2}{n} \right) + g_{\mu\nu} \frac{2}{n} \right] \quad . \tag{A.4}$$

Box diagram (Fig. 2d):

$$\frac{g^2}{16\pi^2} C_2(R) \sum_{n=2,4\ldots} (\tfrac{1}{x})^n \left[e_{\mu\nu} \frac{4}{n+1} + d_{\mu\nu}\left(\frac{2}{n(n+1)} \ln \frac{Q^2}{-p^2} \right.\right.$$

$$\left.\left. - \frac{6}{n} + \frac{6}{n+1} + \frac{4}{n^2} - \frac{4}{(n+1)^2} \right) + \frac{p_\mu p_\nu}{(p\ q)^2} q^2 \left(\frac{2}{n} - \frac{1}{n-1} \right) - g_{\mu\nu} \frac{2}{n} \right] \quad . \tag{A.5}$$

Gauge term for the whole amplitude:

$$(1-\alpha) \frac{g^2}{16\ \ ^2} C_2(R) d_{\mu\nu} \sum_{n=2,4\ldots} (\tfrac{1}{x})^n \left(\frac{1}{n} + \sum_{j=1}^{n} \frac{1}{j} - 1 + \gamma_E + \ln \frac{-p^2}{4\pi\mu^2} \right) \quad . \tag{A.6}$$

Definition of the gauge parameter α is given in (3.3). The whole expression:

$$\frac{g^2}{16\pi^2} C_2(R) \sum_{n=2,4\ldots} (\tfrac{1}{x})^n \left[e_{\mu\nu} \frac{4}{n+1} + d_{\mu\nu} \left\{ - \tilde{\gamma}_0^{\,n} \ln \frac{Q^2}{-p^2} - \gamma_E - \ln \frac{-p^2}{4\pi\mu^2} \right.\right.$$

$$- 1 - \frac{1}{n} + \frac{6}{n+1} + \frac{4}{n^2} - \frac{4}{(n+1)^2} + 3 \sum_{j=1}^{n} \frac{1}{j} - 8 \sum_{j=1}^{n} \frac{1}{j^2}$$

$$\left.\left. + (1-\alpha)\left(\frac{1}{n} + \sum_{j=1}^{n} \frac{1}{j} - 1 + \gamma_E + \ln \frac{-p^2}{4\pi\mu^2} \right) \right\} \right] \quad , \tag{A.7}$$

where

$$\tilde{\gamma}_0^{\,n} = 1 - \frac{2}{n(n+1)} + 4 \sum_{j=2}^{n} \frac{1}{j} \quad , \tag{A.8}$$

A2. Operator matrix element

We present the product of the operator matrix element and the lowest order coefficient function. Triangle diagram (Fig. 3a):

$$\frac{g^2}{16\pi^2} C_2(R) d_{\mu\nu} \sum_{n=2,4\ldots} \left(\frac{1}{x}\right)^n \left[-\frac{2}{n(n+1)} \left(\gamma_E + \ln \frac{-p^2}{4\pi\mu^2} \right) + \frac{2}{n(n+1)} \sum_{j=1}^{n} \frac{1}{j} - \frac{4}{n} + \frac{2}{n+1} + \frac{2}{n^2} - \frac{4}{(n+1)^2} \right] . \tag{A.9}$$

Vertex-correction diagram (Fig. 3b):

$$\frac{g^2}{16\pi^2} C_2(R) d_{\mu\nu} \sum_{n=2,4\ldots} \left(\frac{1}{x}\right)^n \left[4 \sum_{j=2}^{n} \frac{1}{j} \left(\gamma_E + \ln \frac{-p^2}{4\pi\mu^2} \right) + 8 - 4 \sum_{j=1}^{n} \frac{1}{j^2} - 4 \sum_{s=1}^{n} \frac{1}{s} \sum_{j=1}^{s} \frac{1}{j} \right] . \tag{A.10}$$

Gauge term for the whole amplitude:

$$(1-\alpha) \frac{g^2}{16\pi^2} C_2(R) d_{\mu\nu} \sum_{n=2,4\ldots} \left(\frac{1}{x}\right)^n \left(\frac{1}{n} + \sum_{j=1}^{n} \frac{1}{j} - 1 + \gamma_E + \ln \frac{-p^2}{4\pi\mu^2} \right) . \tag{A.11}$$

The whole expression:

$$\frac{g^2}{16\pi^2} C_2(R) d_{\mu\nu} \sum_{n=2,4\ldots} \left(\frac{1}{x}\right)^n \left[(\tilde{\gamma}_0{}^n - 1) \left(\gamma_E + \ln \frac{-p^2}{4\pi\mu^2} \right) + 8 - \frac{4}{n} + \frac{2}{n+1} + \frac{2}{n^2} - \frac{4}{(n+1)^2} - 4 \sum_{j=1}^{n} \frac{1}{j^2} + \frac{2}{n(n+1)} \sum_{j=1}^{n} \frac{1}{j} - 4 \sum_{s=1}^{n} \frac{1}{s} \sum_{j=1}^{s} \frac{1}{j} + (1-\alpha) \left(\frac{1}{n} + \sum_{j=1}^{n} \frac{1}{j} - 1 + \gamma_E + \ln \frac{-p^2}{4\pi\mu^2} \right) \right] . \tag{A.12}$$

A3. Coefficient functions

$$\frac{g^2}{16\pi^2} C_2(R) \sum_{n=2,4\ldots} \left(\frac{1}{x}\right)^n (e_{\mu\nu}\tilde{C}_{Ln} + d_{\mu\nu}\tilde{C}_{2n}) = (\text{A.7}) - (\text{A.12}) \quad ,$$

$$\tilde{C}_{Ln} = \frac{4}{n+1} \quad , \tag{A.13}$$

$$\tilde{C}_{2n} = -\tilde{\gamma}_0{}^n \left(\gamma_E + \ln\frac{Q^2}{4\pi\mu^2}\right) - 9 + \frac{3}{n} + \frac{4}{n+1}$$

$$+ \frac{2}{n^2} + 3\sum_{j=1}^{n}\frac{1}{j} - 4\sum_{j=1}^{n}\frac{1}{j^2} - \frac{2}{n(n+1)}\sum_{j=1}^{n}\frac{1}{j}$$

$$+ 4\sum_{s=1}^{n}\frac{1}{s}\sum_{j=1}^{s}\frac{1}{j} \quad , \tag{A.14}$$

where $\tilde{C}_{Ln}$ and $\tilde{C}_{2n}$ are related to $C_{Ln}{}^{NS}$ and $C_{2n}{}^{NS}$ defined in (3.7) and (3.8) such that $C_{Ln}{}^{NS} = (\bar{g}^2/16\pi^2)C_2(R)\tilde{C}_{Ln}$ and $C_{2n}{}^{NS} = 1 + (\bar{g}^2/16\pi^2)C_2(R)\tilde{C}_{2n}$ at $Q^2 = \mu^2$.

B. Electroproduction-Singlet

Since the quark contribution to the singlet part is exactly the same as that of the nonsinglet case, we discuss here only the gluon contributions.

B1. Current correlation functions

The contributions of diagrams Fig. 7d, e, f are equal to those of Fig. 7a, b, c respectively and Fig. 7b is a crossed diagram of Fig. 7a. Here we present the results corresponding to 2(a + b) and 2c.

Contribution of 2(a + b):

$$\frac{g^2}{4\pi^2} T(R) \sum_{n=2,4\ldots} \left(\frac{1}{x}\right)^n \left[d_{\mu\nu}\left(\frac{1}{n}\ln\frac{Q^2}{-p^2} - \frac{2}{n} + \frac{2}{n^2}\right)\right.$$

$$\left. - \frac{p_\mu p_\nu}{(p\cdot q)^2} q^2 \left(\frac{1}{n-1} - \frac{1}{n}\right)\right] + \frac{g^2}{4\pi^2} T(R) \left[-4e_{\mu\nu} + 2g_{\mu\nu}\left(1 - \gamma_E - \ln\frac{Q^2}{4\pi\mu^2}\right)\right] \tag{A.15}$$

Contribution of 2c:

$$
\frac{g^2}{4\pi^2} T(R) \sum_{n=2,4\ldots} \left(\frac{1}{x}\right)^n \left[e_{\mu\nu} \frac{4}{(n+1)(n+2)} + d_{\mu\nu} \left\{ -\frac{2}{(n+1)(n+2)} \ln \frac{Q^2}{-p^2} \right.\right.
$$

$$
\left.\left. + \frac{6}{n+1} - \frac{6}{n+2} - \frac{4}{(n+1)^2} + \frac{4}{(n+2)^2} \right\} + \frac{p_\mu p_\nu}{(p\cdot q)^2} q^2 \left(\frac{1}{n-1} - \frac{1}{n}\right) \right]
$$

$$
+ \frac{g^2}{4\pi^2} T(R) \left[4e_{\mu\nu} - 2g_{\mu\nu} \left(1 - \gamma_E - \ln \frac{Q^2}{4\pi\mu^2} \right) \right] \quad . \tag{A.16}
$$

The whole expression:

$$
\frac{g^2}{4\pi^2} T(R) \sum_{n=2,4\ldots} \left(\frac{1}{x}\right)^n \left[e_{\mu\nu} \frac{4}{(n+1)(n+2)} + d_{\mu\nu} \left\{ \frac{n^2+n+2}{n(n+1)(n+2)} \ln \frac{Q^2}{-p^2} \right.\right.
$$

$$
\left.\left. - \frac{2}{n} + \frac{6}{n+1} - \frac{6}{n+2} + \frac{2}{n^2} - \frac{4}{(n+1)^2} + \frac{4}{(n+2)^2} \right\} \right] \quad . \tag{A.17}
$$

B2. Operator matrix element

Contribution of Fig. 8a and b:

$$
\frac{g^2}{4\pi^2} T(R) d_{\mu\nu} \sum_{n=2,4\ldots} \left(\frac{1}{x}\right)^n \left[-\frac{1}{n} \left(\gamma_E + \ln \frac{-p^2}{4\pi\mu^2} \right) \right.
$$

$$
\left. - \frac{2}{n} + \frac{1}{n^2} + \frac{1}{n} \sum_{j=1}^{n} \frac{1}{j} \right] \quad . \tag{A.18}
$$

Contribution of Fig. 8c and d:

$$
\frac{g^2}{4\pi^2} T(R) d_{\mu\nu} \sum_{n=2,4\ldots} \left(\frac{1}{x}\right)^n \left[\frac{2}{(n+1)(n+2)} \left(\gamma_E + \ln \frac{-p^2}{4\pi\mu^2} \right) \right.
$$

$$
\left. + \frac{2}{n+1} - \frac{2}{n+2} - \frac{4}{(n+1)^2} + \frac{4}{(n+2)^2} - \frac{2}{(n+1)(n+2)} \sum_{j=1}^{n} \frac{1}{j} \right] \quad . \tag{A.19}
$$

The whole expression:

$$\frac{g^2}{4\pi^2} T(R) d_{\mu\nu} \sum_{n=2,4...} \left(\frac{1}{x}\right)^n \left[-\frac{n^2+n+2}{n(n+1)(n+2)} \left(\gamma_E + \ln\frac{-p^2}{4\pi\mu^2}\right) \right.$$

$$\left. -\frac{2}{n} + \frac{2}{n+1} - \frac{2}{n+2} + \frac{1}{n^2} - \frac{4}{(n+1)^2} + \frac{4}{(n+2)^2} + \frac{n^2+n+2}{n(n+1)(n+2)} \sum_{j=1}^{n} \frac{1}{j} \right] . \qquad \text{(A.20)}$$

B3. Coefficient functions

$$\frac{g^2}{4\pi^2} T(R) \sum_{n=2,4...} \left(\frac{1}{x}\right)^n (e_{\mu\nu}\tilde{C}_{Ln} + d_{\mu\nu}\tilde{C}_{2n}) = \text{(A.17)} - \text{(A.20)} \quad ,$$

$$\tilde{C}_{Ln} = \frac{4}{(n+1)(n+2)} \quad , \qquad \text{(A.21)}$$

$$\tilde{C}_{2n} = \frac{n^2+n+2}{n(n+1)(n+2)} \left(\gamma_E + \ln\frac{Q^2}{4\pi\mu^2}\right) + \frac{4}{n+1} - \frac{4}{n+2} + \frac{1}{n^2}$$

$$- \frac{n^2+n+2}{n(n+1)(n+2)} \sum_{j=1}^{n} \frac{1}{j} \quad . \qquad \text{(A.22)}$$

where $\tilde{C}_{Ln}$ and $\tilde{C}_{2n}$ are related to $C_{Ln}{}^G$ and $C_{2n}{}^G$ in (6.24) and (6.25) such that $C_{Ln}{}^G = (\bar{g}^2/4\pi^2)T(R)\tilde{C}_{Ln}$ and $C_{2n}{}^G = (\bar{g}^2/4\pi^2)T(R)\tilde{C}_{2n}$ at $Q^2 = \mu^2$.

C. Neutrino Reactions

For the combinations $T_2^{\bar{\nu}} \pm T_2^{\nu}$ and $T_L^{\bar{\nu}} \pm T_L^{\nu}$ defined in section (4.1) the results are exactly the same as in the case of electroproduction with even n for $T_{2,L}^{\bar{\nu}} + T_{2,L}^{\nu}$ and with odd n for $T_{2,L}^{\bar{\nu}} - T_{2,L}^{\nu}$. The operator matrix elements are also the same as (A.12) with n even and odd, respectively.

For $T_3^{\bar{\nu}} \pm T_3^{\nu}$ which correspond to the VA intereference term, we obtain

$$\frac{g^2}{16\pi^2}C_2(R) \sum_{\substack{n=1,3\ldots\\2,4\ldots}} \left(\frac{1}{x}\right)^n \left[-\tilde{\gamma}_0^{\,n} \ln\frac{Q^2}{-p^2} - \gamma_E - \ln\frac{-p^2}{4\pi\mu^2} \right.$$

$$- 1 - \frac{3}{n} + \frac{4}{n+1} + \frac{4}{n^2} - \frac{4}{(n+1)^2} + 3\sum_{j=1}^{n}\frac{1}{j}$$

$$- 8\sum_{j=1}^{n}\frac{1}{j^2} + (1-\alpha)\left(\frac{1}{n} + \sum_{j=1}^{n}\frac{1}{j} - 1\right.$$

$$\left.\left. + \gamma_E + \ln\frac{-p^2}{4\pi\mu^2}\right)\right] \quad . \tag{A.23}$$

The corresponding operator matrix elements are identical to (A.12) with n odd or even. Consequently the coefficient function is given by

$$C_{3n}^{\bar{\nu}\pm\nu} = 1 + \frac{g^2}{16\pi^2}C_2(R)\left[-\tilde{\gamma}_0^{\,n}\left(\gamma_E + \ln\frac{Q^2}{4\pi\mu^2}\right)\right.$$

$$- 9 + \frac{1}{n} + \frac{2}{n+1} + \frac{2}{n^2} + 3\sum_{j=1}^{n}\frac{1}{j} - 4\sum_{j=1}^{n}\frac{1}{j^2}$$

$$\left. - \frac{2}{n(n+1)}\sum_{j=1}^{n}\frac{1}{j} + 4\sum_{s=1}^{n}\frac{1}{s}\sum_{j=1}^{s}\frac{1}{j}\right] \quad , \tag{A.24}$$

with n odd for $\bar{\nu}+\nu$ and n even for $\bar{\nu}-\nu$.

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TABLE CAPTION

Table I: The values of various quantities which enter equation (2.29) for $f = 4$. $\bar{B}_{2,n}^{NS} = B_{2,n}^{NS} - \frac{1}{2}\gamma_0^n(\ln 4\pi - \gamma_E)$. The values for odd n contribute to $B_{2,n}^{NS}$ in $\nu, \bar{\nu}$ reactions.

Table 1

n	$B_{2,n}^{NS}$	$\bar{B}_{2,n}^{NS}$	$\frac{\gamma_1^n}{2\beta_0}$	$-\frac{\gamma_0^n\beta_1}{2\beta_0^2}$
1	0	0	0	0
2	7.39	0.44	4.28	-2.63
3	14.08	3.22	6.05	-4.11
4	19.70	6.07	7.21	-5.16
5	24.53	8.73	8.09	-5.98
6	28.77	11.18	8.82	-6.66
7	32.55	13.44	9.44	-7.23
8	35.96	15.53	9.99	-7.73
9	39.08	17.48	10.46	-8.17
10	41.96	19.30	10.91	-8.57
11	44.63	21.01	11.31	-8.93
12	47.12	22.63	11.68	-9.27

FIGURE CAPTIONS

Fig. 1: The Born term for virtual Compton scattering. Inclusion of the crossed diagram is understood.

Fig. 2: Diagrams contributing in order g^2 to the virtual Compton scattering. Inclusion of the crossed diagrams is understood.

Fig. 3: Diagrams contributing in order g^2 to the matrix elements of the non-singlet operator between quark states.

Fig. 4: Order $\bar{g}^2$ deviations from the Gross-Llewellyn-Smith and Bjorken sum rules. The dashed lines (---) are parton model predictions. The solid (—) lines follow from Eqs. (4.11) and (4.12).

Fig. 5: Nachtmann moments of $xF_3(x, Q^2)$ vs. Q^2. The data are from ref. 10. The solid (—) lines represent the LO, MS, and $\overline{MS}$ schemes; the dashed (---) lines represent the Λ_n scheme.

Fig. 6: The n-dependence of Λ_n in the Λ_n scheme. The "data" points come from fitting each n separately. The curves are the predictions of Eq. (5.7).

Fig. 7: Diagrams contributing in order g^2 to the virtual photon-gluon scattering.

Fig. 8: Diagrams contributing in order g^2 to the matrix elements of the quark operator between gluon states.

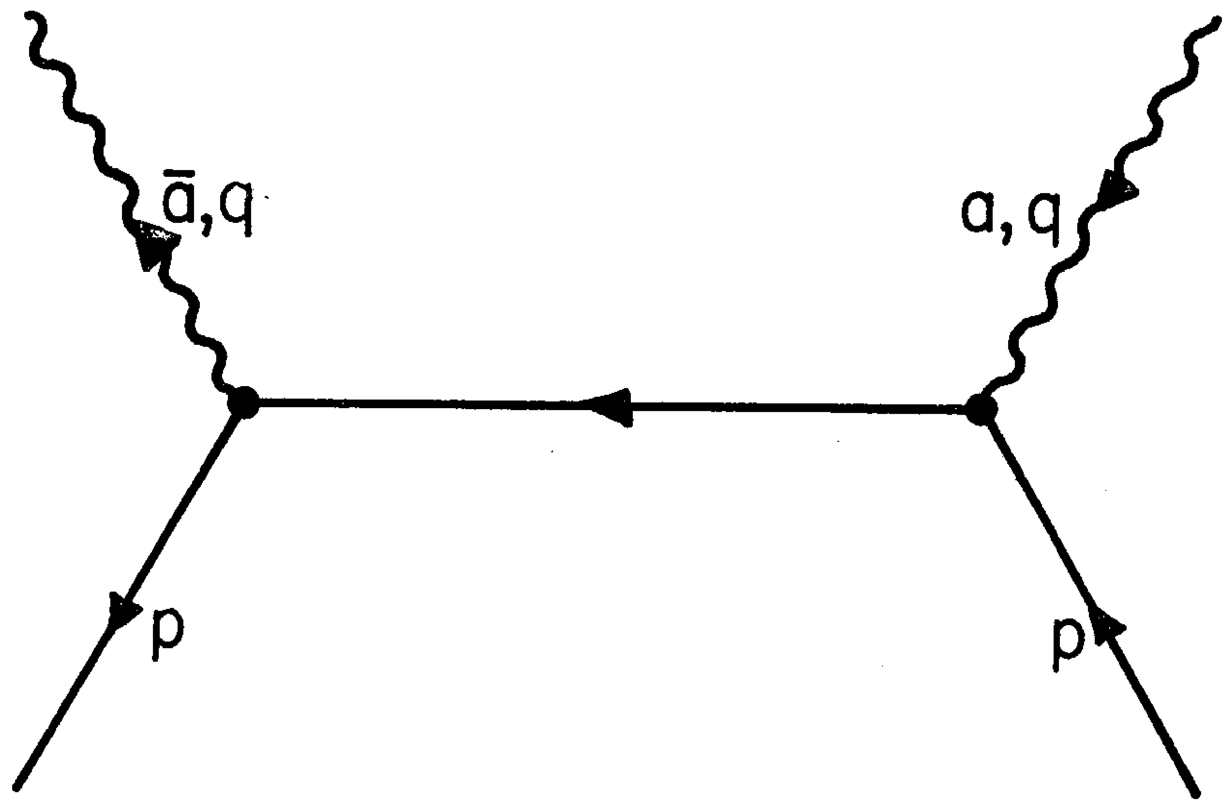


Fig. 1

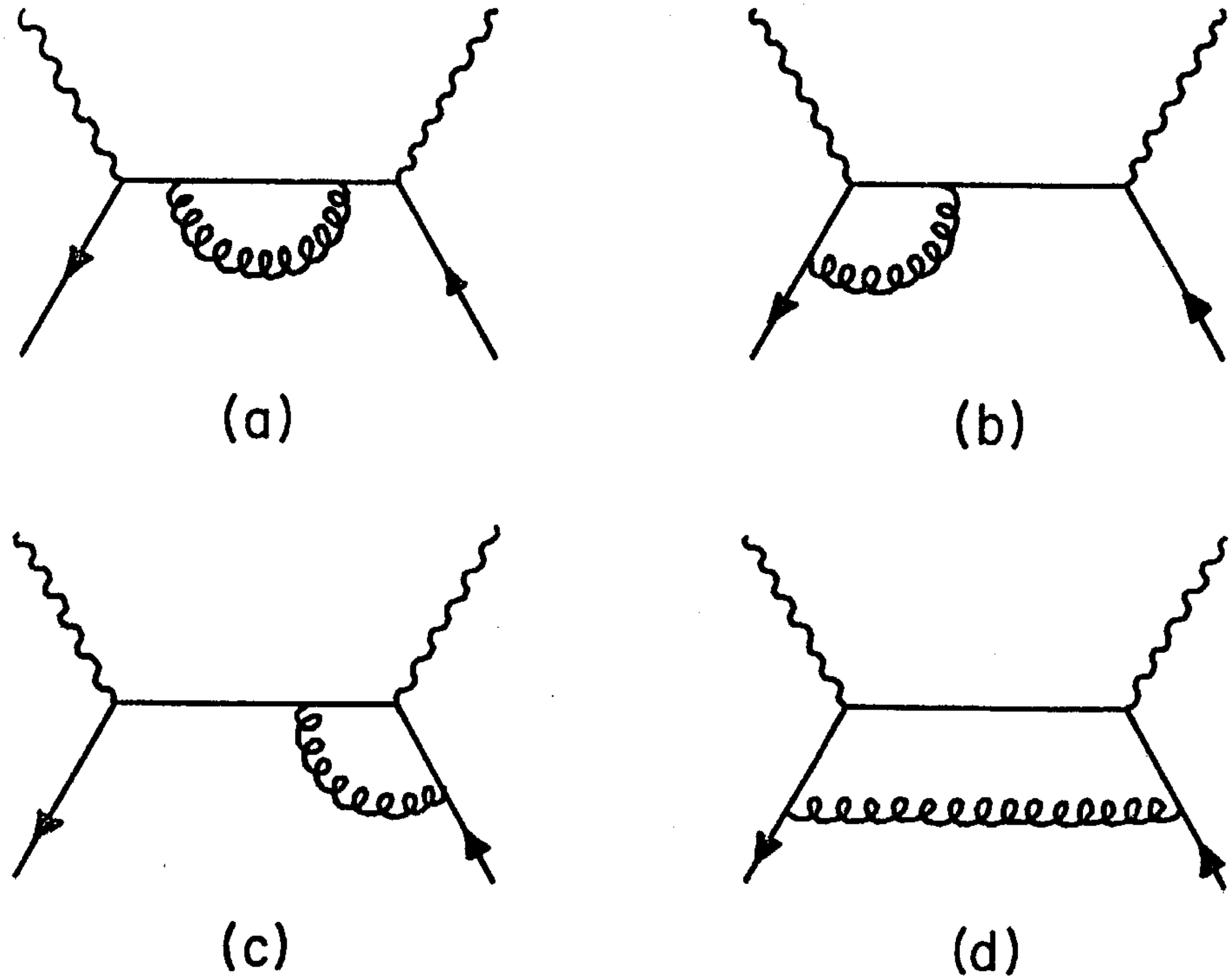


Fig. 2

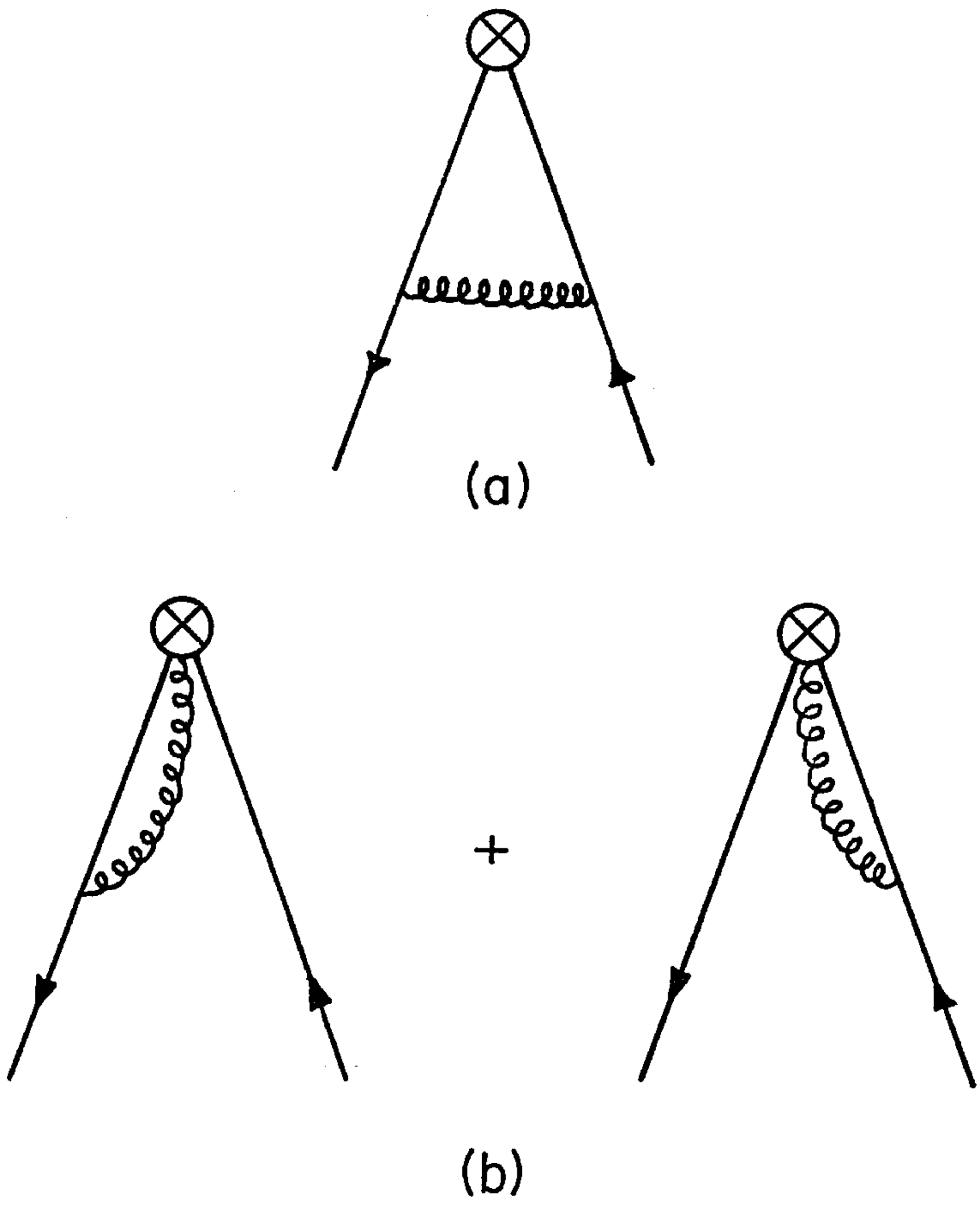
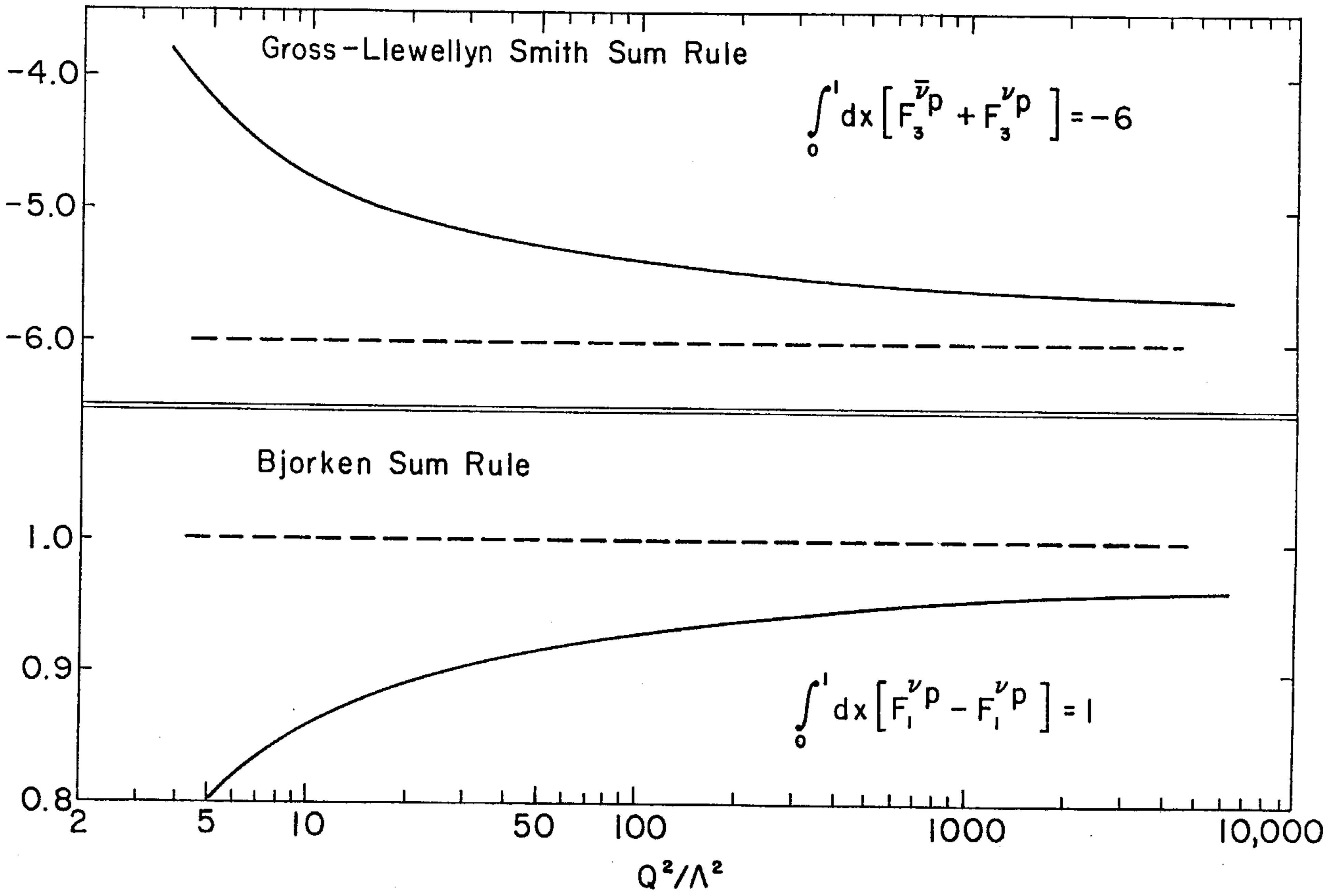
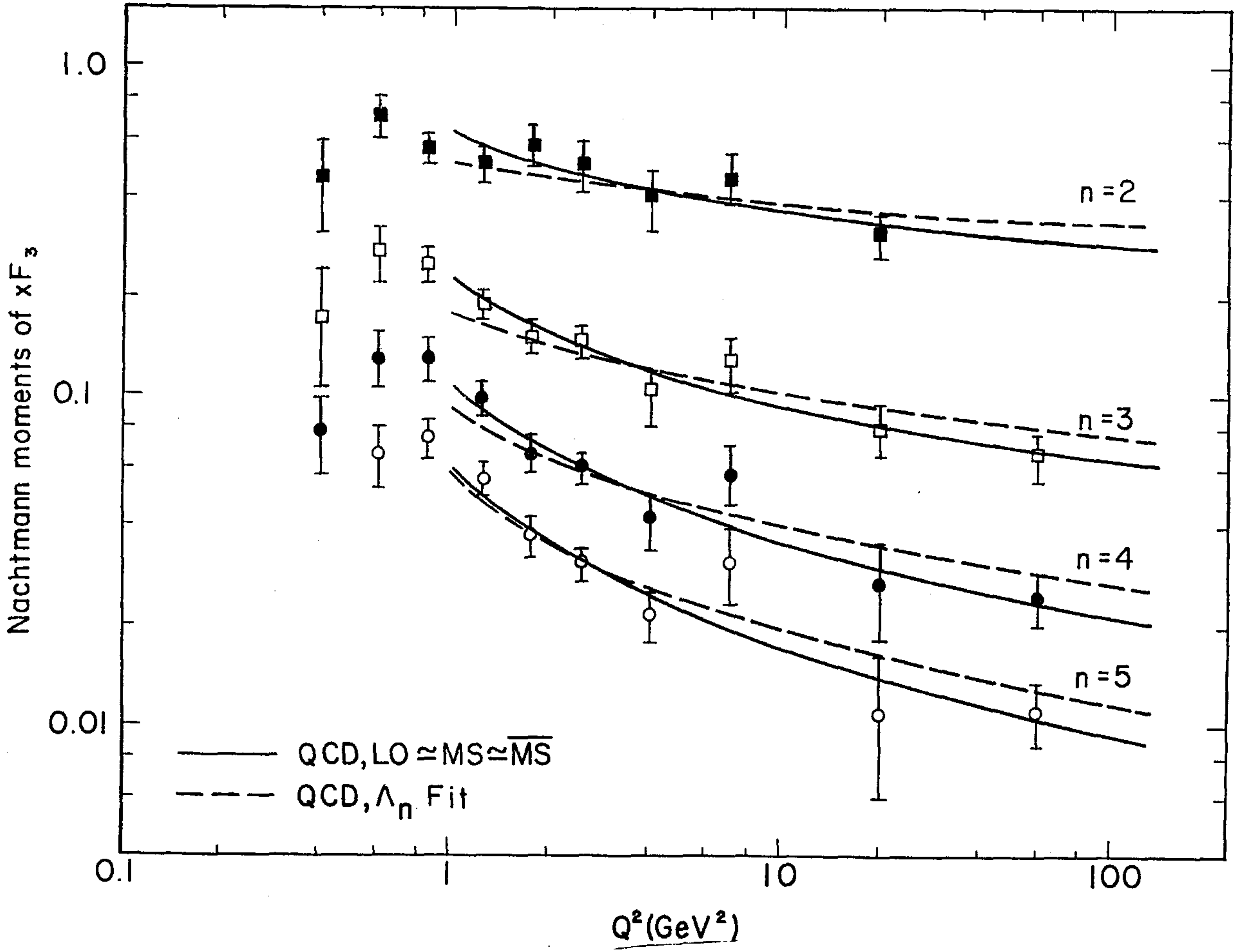


Fig. 3



Gross-Llewellyn Smith Sum Rule
$\int_0^1 dx \left[F_3^{\bar{\nu}p} + F_3^{\nu p} \right] = -6$
Bjorken Sum Rule
$\int_0^1 dx \left[F_1^{\bar{\nu}p} - F_1^{\nu p} \right] = 1$
-4.0
-5.0
-6.0
1.0
0.9
0.8
2
5
10
50
100
1000
10,000
Q^2/Λ^2



1.0
0.1
0.01
0.1
1
10
100
Nachtmann moments of xF₃
n = 2
n = 3
n = 4
n = 5
QCD, LO ≃ MS ≃ MS
QCD, Λn Fit
Q²(GeV²)

Fig. 5

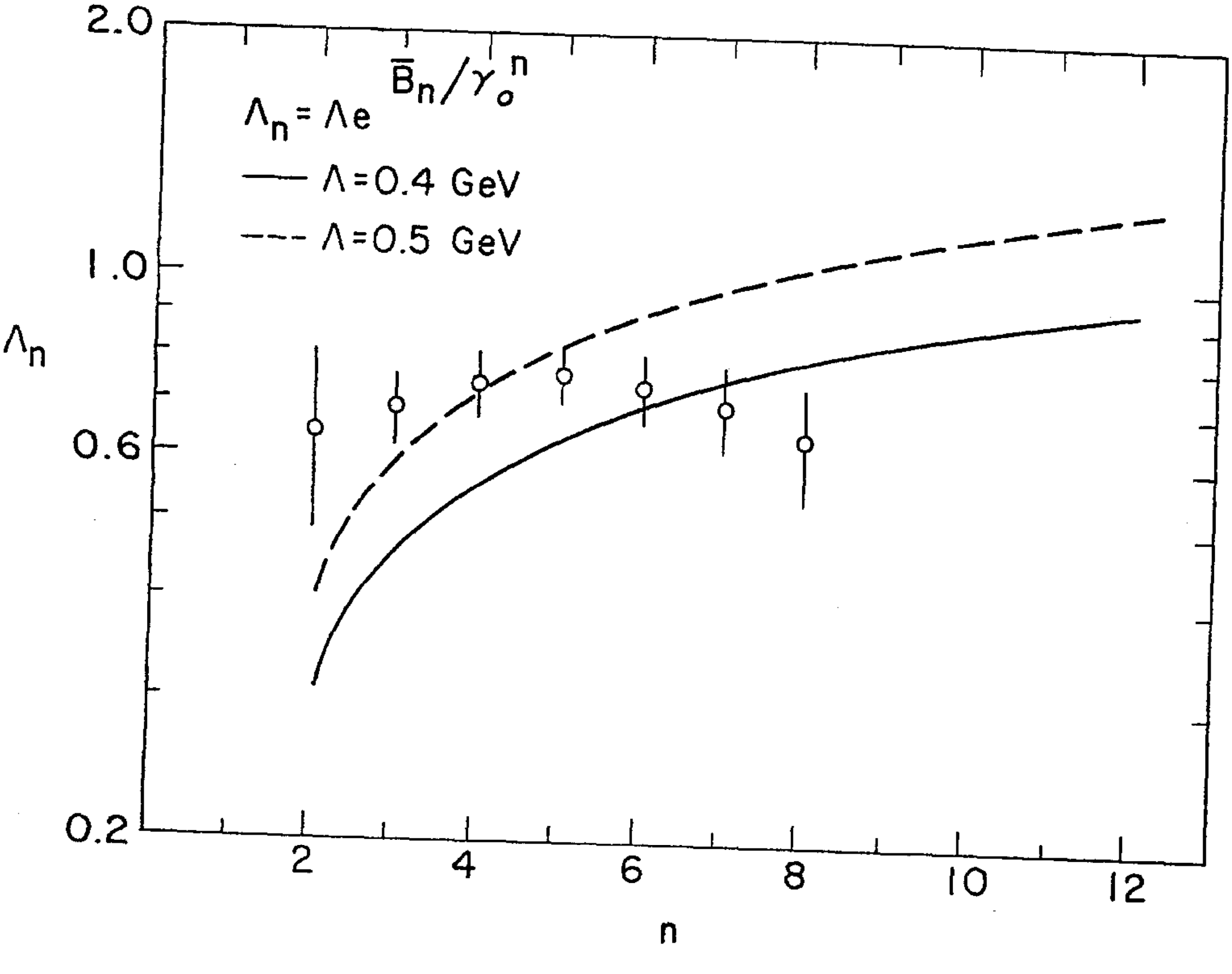

$\Lambda_n = \Lambda e^{\bar{B}_n/\gamma_o^n}$
Λ=0.4 GeV
Λ=0.5 GeV
2.0
1.0
0.6
0.2
Λ_n
2
4
6
8
10
12
n

Fig. 6

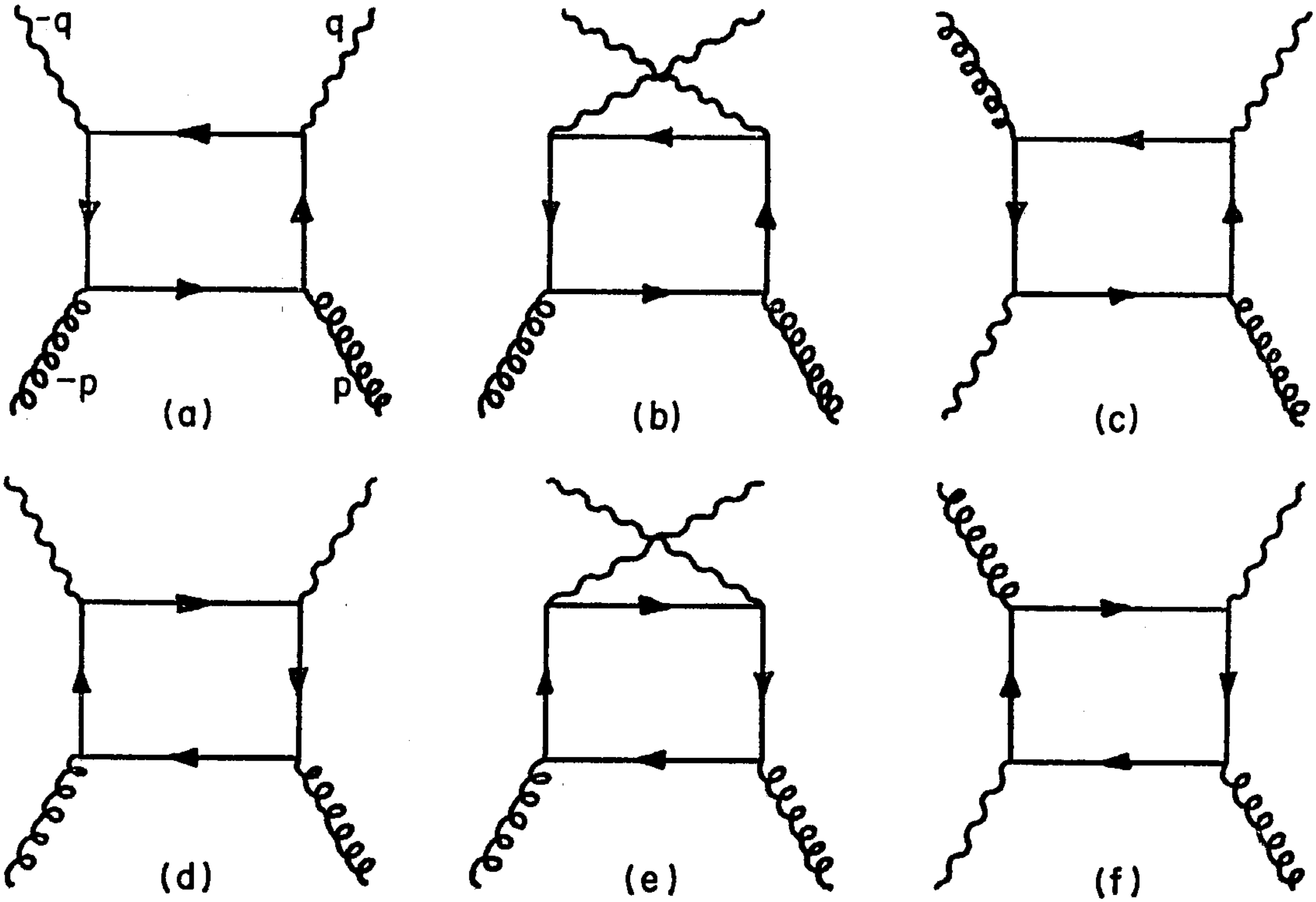


Fig. 7

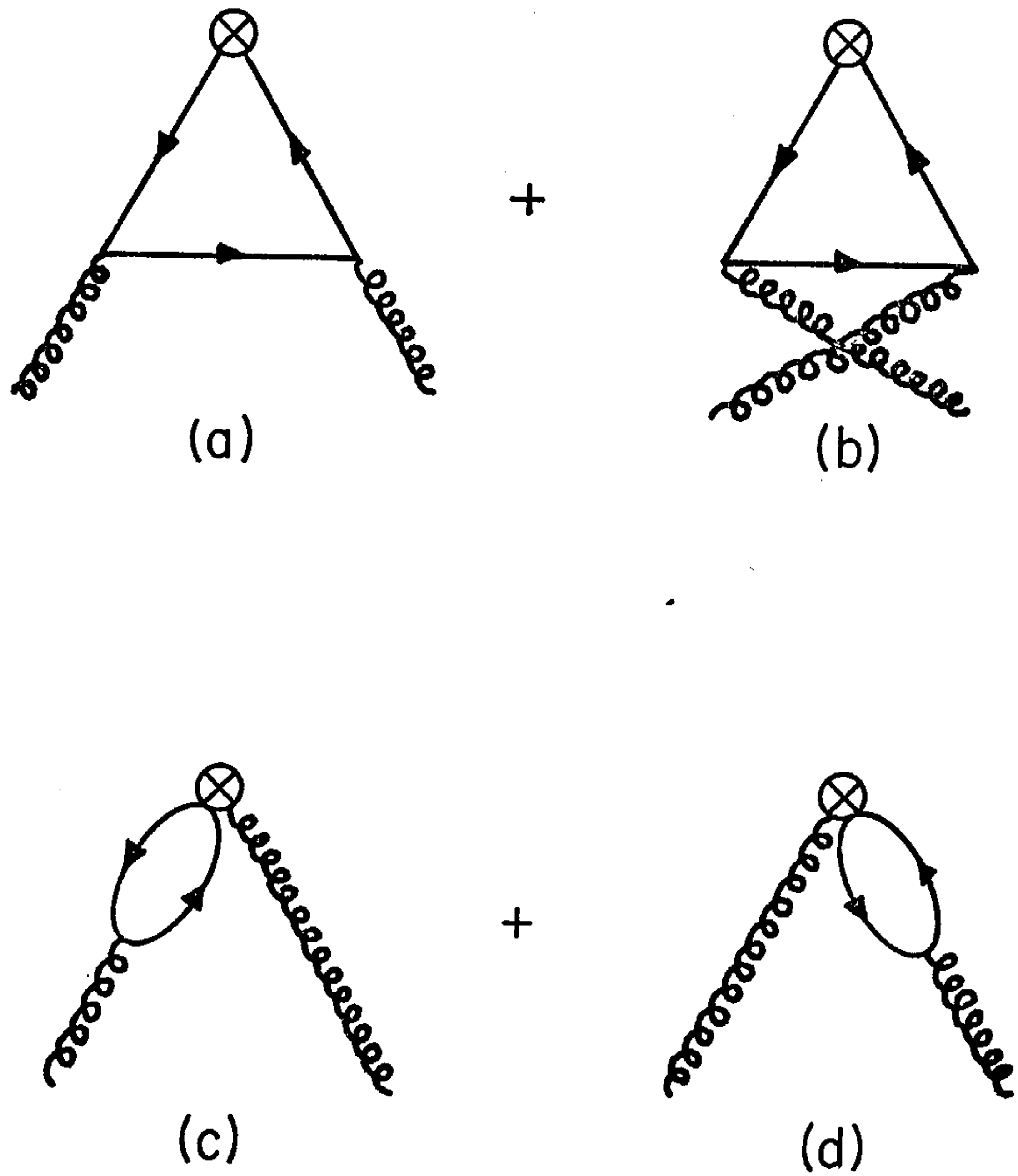


Fig. 8

ERRATUM

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Deep Inelastic Scattering Beyond the Leading Order in Asymptotically Free Gauge Theories

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We would like to correct one error and a few misprints which occurred in our paper. ALL corrections will be made in the published version of our paper (Phys. Rev. D).

Our procedure for calculating the gluon coefficient functions (eqs. 6.25 and A.22) does not exactly correspond to the minimal subtraction scheme used by Floratos, Ross and Sachrajda (Nucl. Phys. B129, (1977) 66) in their calculation of the two-loop anomalous dimensions. We have applied the minimal subtraction scheme to the spin averaged matrix elements of the local operators whereas conventionally and in the calculation of Floratos, et al. the minimal subtraction scheme is applied to the operators. Both schemes are valid normalization procedures but lead to slightly different results. In order to be consistent with the calculation of the two-loop anomalous dimensions of Floratos, et al., we should rather use the conventional minimal subtraction scheme.

The proper result for the gluon coefficient funciton is obtained by replacing γ_E by γ_E-1 in equations (6.22), (6.25), (6.26), (A.20) and (A.22). Correspondingly a term

$$\frac{g^2}{4\pi^2} T(R) d_{\mu\nu} \sum_{2,4,\ldots} \left(\frac{1}{x}\right)^n \frac{N^2+N+2}{(N+1)(N+2)N}$$

should be added on the r.h.s. of eqs. (A.18). Our results for the quark coefficient functions are unchanged.

Our paper contains the following misprints:

1. On the r.h.s. of equation (4.11) 6 should be replaced by -6.

2. In ref. 31 Kinglsey should be replaced by Kingsley.

We are grateful to E.G. Floratos, D.A. Ross and C.T. Sachrajda for informing us about the discrepancy between their results for the gluon coefficient functions and ours.