Fermi National Accelerator Laboratory

FERMILAB-Pub-78/19-THY
February 1978

Determining the Fifth Quark's Charge:

The Role of Υ Leptonic Widths

JONATHAN L. ROSNER*
School of Physics and Astronomy
University of Minnesota
Minneapolis, Minnesota 55455

AND

C. QUIGG† and H. B. THACKER
Fermi National Accelerator Laboratory‡
Batavia, Illinois 60510

ABSTRACT

Lower bounds on the leptonic decay widths of Υ(9.4) and Υ'(10.0) are deduced from plausible general assumptions. It is shown that these may permit a distinction between the charge assignments $e_Q = (-1/3, +2/3)$ for the new quark.

*Supported in part by the Department of Energy under Contract No. E(11-1)-1764.

†Alfred P. Sloan Foundation Fellow; also at Enrico Fermi Institute, University of Chicago, Chicago, Illinois 60637

‡Operated by Universities Research Association Inc. under contract with the Energy Research and Development Administration

By far the most conservative interpretation of the recently discovered Υ particles[1] is that they are $^3S_1$ quarkonium bound states of a new heavy quark Q. The role of the new quark in the weak interactions is unknown, and has been the subject of considerable speculation.[2] Among the attributes of the new quark which remain to be determined is its electric charge e_Q. All the tentative inferences made until now[3] rely on necessarily vague models for the production of heavy particles in hadron-hadron collisions. It is usually expected that the increase ΔR in the ratio

$$R \equiv \frac{\sigma(e^+e^-\to\text{hadrons})}{\sigma(e^+e^-\to\mu^+\mu^-)} \tag{1}$$

above new-flavor threshold will yield a measurement of e_Q because of the correspondence

$$\Delta R = 3e_Q^{\,2} \quad . \tag{2}$$

However, early experiments might not exceed this threshold, and the production of more than one new charged fermion species could cloud the interpretation of ΔR. Such was the case for the coincident thresholds of charm and the heavy lepton τ near $E_{c.m.} = 4$ GeV. The issue of the charmed quark charge was not really settled until the discovery of charmed mesons.

In this note we draw attention to another source of information about e_Q. On rather general grounds we establish the lower bounds

$$\Gamma([\Upsilon,\Upsilon'] \to e^+e^-)/e_Q^2 \geq [2.6,\ 1.4]\ \text{keV}. \tag{3}$$

Moreover, we show that these bounds may provide a distinction between

$e_Q = 2/3$ and $e_Q = -1/3$.

We regard the $^3S_1$ levels of the $(c\bar{c})$ charmonium and $(Q\bar{Q})\Upsilon$ systems as bound by a flavor-independent nonrelativistic central potential V(r). The nonrelativistic connection[4]

$$\Gamma(V \to e^+e^-) = 16\pi\alpha^2 e_q^2 |\Psi_q(0)|^2 / M_V^2 \tag{4}$$

is assumed to hold. Here V represents a $^3S_1$ $(q\bar{q})$ meson with mass M_V, and $|\Psi_q(0)|$ is the magnitude of the $q\bar{q}$ wavefunction at the origin.

For any power-law potential

$$V(r) = ar^{\varepsilon} \quad , \tag{5}$$

it can be shown by elementary scaling arguments[5] that for a fixed principal quantum number

$$|\Psi_Q(0)|^2 = \left(\frac{m_Q}{m_c}\right)^{3/(2+\varepsilon)} |\Psi_c(0)|^2 \quad . \tag{6}$$

If the potential is concave downward,[F1]

$$d^2V/dr^2 \leq 0 \quad ,$$

i.e., for $\varepsilon \leq 1$, we may express (6) as a lower bound

$$|\Psi_Q(0)|^2 \geq \frac{m_Q}{m_c} |\Psi_c(0)|^2 \quad , \tag{7}$$

when $m_Q > m_c$. This is the principal ingredient in our subsequent discussion. We shall prove for the ground state that it holds for any monotonically-increasing potential which is concave downward.

The leptonic widths[7] of $\psi(3.095)$ and $\psi'(3.684)$,

$$\begin{aligned} \Gamma(\psi \to e^+e^-) &= 4.8 \pm 0.6 \text{ keV} \\ \Gamma(\psi' \to e^+e^-) &= 2.1 \pm 0.3 \text{ keV} \end{aligned} \tag{8}$$

measure the wavefunctions at the origin in the charmonium system. To make (7) useful for the Υ family, we must estimate (or bound) m_Q/m_c. In specific potential models,[8] we find

$$3 \lesssim \frac{m_Q}{m_c} \lesssim 4 \quad . \tag{9}$$

It is possible to argue more generally: as the quark mass increases, states become more tightly bound, so that[9]

$$M_\Upsilon - 2m_Q \leq M_\psi - 2m_c \quad , \tag{10}$$

and

$$M_{\Upsilon'} - 2m_Q \leq M_{\psi'} - 2m_c \quad . \tag{11}$$

These lead to nearly identical restrictions on m_Q/m_c, namely

$$m_Q/m_c \geq 1 + \frac{(M_\Upsilon - M_\psi)}{2m_c} \approx 1 + \frac{(M_{\Upsilon'} - M_{\psi'})}{2m_c} \approx 1 + \frac{3.15\ \mathrm{GeV}/c^2}{m_c} \quad . \tag{12}$$

All charmonium calculations known to us employ $m_c \leq 2\ \mathrm{GeV}/c^2$. This implies

$$m_Q/m_c \geq 2.6 \quad , \tag{13}$$

which we shall adopt hereafter. The lower bounds (3) follow from (7) and (13), together with the central values minus one standard deviation from (8).

The bounds (3) are shown in Fig. 1 together with a number of predictions derived from explicit potential models. We plot the bounds for $e_Q = -1/3$ and $e_Q = +2/3$, but the model calculations are shown only for $e_Q = -1/3$. It is noteworthy that many of the potentials imply values of $\Gamma(T \to e^+e^-)$ or $\Gamma(T' \to e^+e^-)$ which are too small to be interpreted in terms of $e_Q = +2/3$. Consequently, a timely measurement of the leptonic widths of T and T' could provide the first information on the charge of the fifth quark, if either of these proves incompatible with the lower bound for $e_Q = +2/3$. The key assumption which underlies this argument is that the same potential describes the ψ and T families.

The inequality (7) undoubtedly holds under more general conditions than we have assumed in motivating it. We now give a proof of its validity for the ground state. For excited states we have found neither a proof nor a counterexample. The reduced radial wavefunction satisfies the Schrödinger equation, which we write as

$$\frac{u''(r)}{u(r)} = m[V(r) - E] \quad . \tag{14}$$

We regard u as a function of both r and the mass m. Differentiating (14) with respect to the mass, we find

$$\frac{1}{u^2(r)}[u(r) \frac{\partial u''(r)}{\partial m} - u''(r) \frac{\partial u(r)}{\partial m}] = V(r) - \langle V\rangle \quad , \tag{15}$$

where the result[9]

$$\frac{\partial E}{\partial m} = - \frac{E-\langle V\rangle}{m} \tag{16}$$

has been used. Applying $\int_0^r dr'u^2(r')$ to both sides of (15) we have, after an integration by parts,[F2]

$$u(r) \frac{\partial u'(r)}{\partial m} - u'(r) \frac{\partial u(r)}{\partial m} = \int_0^r dr'[V(r') - \langle V\rangle]u^2(r') \tag{17}$$

$$\equiv H(r).$$

The solution to this differential equation for $\partial u/\partial m$ is

$$\frac{\partial u(r)}{\partial m} = u(r)\,[g(r) - \langle g\rangle] \quad , \tag{18}$$

where

$$g(r) = \int_0^r dr' H(r')/u^2(r') \quad . \tag{19}$$

If $dV/dr \geq 0$, it follows that $H(r) \leq 0$. Because the groundstate wavefunction has no nodes, the function $g(r)$ is a continuous, negative, monotonically decreasing function. We next define

$$\begin{aligned} G(r) &= \frac{\partial}{\partial m}\int_0^r dr' u^2(r') \\ &= 2\int_0^r dr' u(r')\,\frac{\partial u(r')}{\partial m} \quad , \end{aligned} \tag{20}$$

which measures the change with mass of the probability contained within a radius r. Substituting (18), we obtain

$$G(r) = \int_0^r dr' u^2(r')\,[g(r') - \langle g\rangle] \quad , \tag{21}$$

from which it follows that $G(r) \geq 0$.[F3] The inequality (7) can now be derived with the aid of the connection

$$|\Psi(0)|^2 = \frac{m}{4\pi}\left\langle\frac{dV}{dr}\right\rangle \quad . \tag{22}$$

We find

$$\begin{aligned} 4\pi\,\frac{\partial}{\partial m}\left(\frac{1}{m}|\Psi(0)|^2\right) &= 2\int_0^\infty dr u(r)\,\frac{\partial u(r)}{\partial m}\,V'(r) \\ &= \int_0^\infty dr\; G'(r)V'(r) = -\int_0^\infty dr\; G(r)V''(r)\,. \end{aligned} \tag{23}$$

For a potential which is concave downward, this implies that

$$\frac{\partial}{\partial m}\left(\frac{1}{m}|\Psi(0)|^2\right) \geq 0 \quad , \tag{24}$$

which is equivalent to (7).

In summary, lower bounds on the leptonic widths of Υ and Υ', which follow from rather general assumptions, may yield a strong indication of the new quark charge well before flavored particles are found.

FOOTNOTES

F1. The influence of d^2V/dr^2 on the relative sizes of ψ and ψ' leptonic widths is discussed in [6].

F2. It is convenient to adopt the normalization $\int_0^\infty dr\, u^2(r) = 1$.

F3. For a power-law potential (5), $G(r) = ru^2(r)/2(2 + \varepsilon) \geq 0$.

REFERENCES

[1] S.W. Herb, *et al.*, Phys. Rev.Lett. 39 (1977) 252; W.R. Innes, *et al.*, *ibid.* 39 (1977) 1240.

[2] J. Ellis, M.K. Gaillard, D.V. Nanopoulos, and S. Rudaz, Nucl. Phys. B131 (1977) 285; R.M. Barnett, invited paper at the 1977 European Conference on Particle Physics, Budapest, SLAC-Pub-1961 and addendum; C.H. Albright, R.E. Shrock, and J. Smith, Fermilab-Pub-77/81-THY.

[3] Ellis, *et al.*, Ref. 2: R.N. Cahn and S.D. Ellis, "Decays of the Τ' and the Structure of the Resonances at 10 GeV," University of Washington report (1977, unpublished); C.E. Carlson and R. Suaya, Phys. Rev. Lett. 39 (1977) 908; T. Hagiwara, Y. Kazama, and E. Takasugi, *ibid.* 40 (1978) 76.

[4] R. Van Royen and V.F. Weisskopf, Nuovo Cimento 50 (1967) 617, 51 (1967) 583.

[5] C. Quigg and Jonathan L. Rosner, "Scaling the Schrödinger Equation," Fermilab-Pub-77/90-THY, to appear in Comments on Nuclear and Particle Physics.

[6] A. Martin, Phys. Lett. 70B (1977) 192.

[7] P.A. Rapidis, *et al.*, Phys. Rev. Lett. 39 (1977) 526.

[8] H.B. Thacker, C. Quigg, and J.L. Rosner, "Inverse Scattering Approach to Quarkonium Potentials. II," Fermilab-Pub-77/109-THY.

[9] R.P. Feynman, Phys. Rev. 56 (1939) 340.

CAPTION

Fig. 1: Expectations for leptonic widths of Υ and Υ'. The lower bounds (3) are indicated for $e_Q = -1/3$ (solid lines) and $e_Q = +2/3$ (dashed lines). The shaded region shows the widths predicted for $e_Q = -1/3$ on the basis of twenty potentials from Ref. 8 which reproduce the ψ and ψ' positions and leptonic widths.

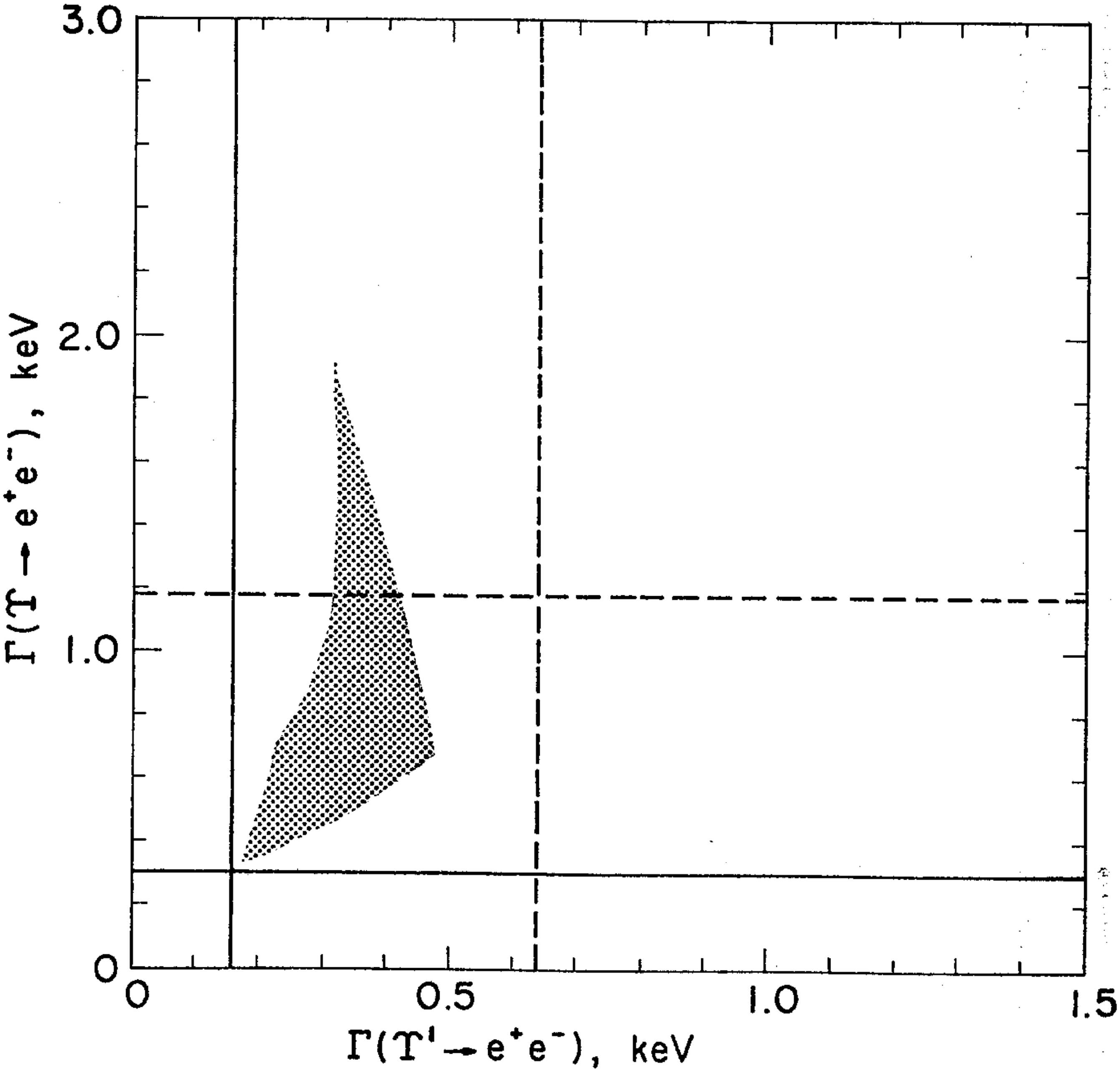

Fig. 1