DIFFRACTIVE HADRON DISSOCIATION

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ABSTRACT

Some simple general features of the diffractive hadron dissociation process are discussed with emphasis on its relation to elastic scattering and the total cross section of hadrons.

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In the past few years, the diffraction dissociation process
\[ h + A \rightarrow X + A \] at low \( t \) \hspace{1cm} (1)
has been studied for \( h, A = \bar{p}^\pm, \pi^\pm, K^\pm \) and also for \( pd \rightarrow Xd \). The differential production cross section
\[ \frac{d^2\sigma}{dt dM_x^2} = f_{hA}(s,t,M_x^2) \] \hspace{1cm} (2)
has been found to exhibit certain simple features. In this report, we review these features and examine some properties of hadron cross sections that can be derived from them. Specifically, we discuss:

(i) The \( s, t, \) and \( M_x^2 \) dependence of the diffractive cross sections,
(ii) the factorization of the diffractive vertex,
(iii) the first moment finite mass sum rule (FMSR) and the low mass enhancements of the diffractive cross section,
(iv) a comment on the \( s \)-dependence of the diffraction dissociation cross section,
(v) a relationship between the elastic and the total cross section of hadrons, and
(vi) the total cross section of the diffractive mass \( X \) with the nucleon -- and why the accepted method for extracting this cross section may be wrong!

With the exception of results from the Rockefeller experiment E-396 at Fermilab, all the experimental results referred to in this report are published. However, since this is not a summary of results but a review focusing on the elucidation of the points mentioned above, not all the published results on reaction (1) are quoted here.

High energies \((s \geq 100 \text{ GeV}^2)\) are essential for observing a diffraction dissociation signal above the non-diffractive "background" from central collisions. This is illustrated in Fig. 1 for \( pp \rightarrow Xp \). The cross section \( \frac{d\sigma}{dx} \) \( (x = p_{\text{min}}^*/p_{\text{max}}^* = 1 - M_x^2/s) \) which is approximately flat in the central \( x \) region increases dramatically with decreasing \( 1-x \) in the region \( 1-x \leq 0.1 \). This increase of the cross section is due to the diffraction dissociation of the proton which follows a \( 1/(1-x) \) law. For a given \( s \), as \( 1-x = M_x^2/s \) decreases, the value of \( M_x^2 \) enters the "resonance region" \( (M_x^2 \leq 5 \text{ GeV}^2) \) and finally reaches the pion production threshold where the cross section must come down to zero. The higher the \( s \), the lower the value of \( 1-x \) corresponding to a given mass \( M_x \), and the higher the invariant \( d\sigma/dx \) cross section for the diffractive production of this mass. Thus, large diffractive masses can be "seen" above background only at appropriately high values of \( s \). This point is discussed quantitatively in Part I below.
The invariant differential cross section \( \frac{d^2 \sigma}{dt d(M_x^2/s)} \) for \( pp \rightarrow Xp \) at \( t = -0.042 \) (GeV/c)^2 and \( s \) from 13 to 500 GeV^2.

I. The \( s, t, \) and \( M_x^2 \) dependence of the diffractive cross section

In the region \( M_x^2 > 5 \) GeV^2, \( |t| \leq 0.1 \) (GeV/c)^2 and \( 1-x \leq 0.1 \), the differential cross section for \( hA \rightarrow XA \) is found experimentally to be described by the simple formula \(^5,6,7\)

\[
\frac{d^2 \sigma}{dt dx} = \frac{A_1(1+B/s)}{1-x} e^{b_1 t} + A_2(1-x) e^{b_2 t}
\]

where the constants \( A_1, B, b_1, A_2 \) and \( b_2 \) depend on the particles \( h \) and \( A \). The first term on the right hand side is identified with the diffractive cross section while the second term is presumably due to non-diffractive processes. This view is supported by the fact that the first term follows simple factorization rules as discussed in Part II below, while the second
term does not follow these rules. The t-dependence is exponential as expected from a naive diffractive picture and the 1/1-x behavior is responsible for the increase of the cross section at small 1-x. For pp + Xp, the constants in Eq. (3) are approximately\(^5,6\) \(A_1 = 4 \text{ mb} \cdot (\text{GeV/c})^{-2}\), \(B = 65 \pm 25 \text{ GeV}^2\) (including estimated systematic errors), \(A_2/A_1 = 100\), and \(b_1 = b_2 = 7 \text{ (GeV/c)}^{-2}\). Thus, at 1-x = 0.1 the diffractive and non-diffractive terms are approximately equal while at 1-x = 0.03 the diffractive term is responsible for about 90% of the inclusive cross section.

As \(M_X^2\) decreases to values smaller than 5 GeV\(^2\), the slope b increases thus causing a low mass enhancement at low t values. However, it is important to observe that the integral of the cross section over t behaves very smoothly over the entire mass region, as illustrated\(^4\) in Fig. 2. This behavior is just what is necessary to satisfy the FMSR as discussed under Part III below.

**FIG. 2** - Values for pp + Xp vs \(M_X^2\), extracted from pd → Xd at 275 GeV/c. (a) The slope parameter, \(b(M_X^2)\). (b) \(d^2\sigma/dt dM_X^2\) multiplied by \(M_X^2\) and extrapolated to \(t = 0\) using \(b(M_X^2)\). (c) Values of (b) above, divided by values of (a): \(M_X^2(d\sigma/dM_X^2)\).
II. The factorization of the diffractive vertex -

It has been determined experimentally\(^5,7,9\) that at low \(t\) and small \(M_X^2/s\) the diffractive vertex factorizes as follows:

\[
\begin{align*}
&\begin{array}{c}
\text{g}_{hX}(t) \\
&h \\
\end{array} \\
&\begin{array}{c}
\text{g}_{AA}(t) \\
&A \\
\end{array} \\
\end{align*}
\]

As a consequence of this factorization rule, the cross sections for a hadron dissociating on different targets scale as the corresponding elastic cross sections\(^5,7\), while the cross sections for different hadrons dissociating on the same target scale as the corresponding total cross sections\(^9\).

a) Proton dissociation on different targets \(A\):

This has been studied for\(^7\) \(A = p^\pm, \pi^\pm, K^\pm\) (Single Arm Spectrometer) and for\(^5\) \(A = d\) (USA-USSR Collaboration). The diagram for elastic scattering analogous to (4) is

\[
\begin{align*}
&\begin{array}{c}
\text{g}_{hh}(t) \\
&h \\
\end{array} \\
&\begin{array}{c}
\text{g}_{AA}(t) \\
&A \\
\end{array} \\
\end{align*}
\]

Comparing (5) with (4), factorization implies that

\[
\frac{d^2\sigma/dtdx}{d\sigma/dt} = \frac{g_{hx}(t)\cdot g_{AA}(t)}{g_{hh}(t)\cdot g_{AA}(t)} = C_h(s,x,t) = C_h(s,x,t)
\]

i.e., at given \(s\), \(x\) and \(t\), the ratio of the diffractive to the elastic cross section of a hadron \(h\) interacting with a target particle \(A\) is a constant independent of the target particle. As mentioned in Part I, it is the first term in Eq. (3) that factorizes in this manner while the second term (non-diffractive) does not factorize\(^5\). Fig. 3 illustrates a test of this factorization rule for proton dissociation on \(p\) and \(d\) targets\(^5\). It is important to notice that the test is performed at the same \(s\)-value and therefore at different incident proton momenta in the laboratory.
b) Different hadrons \( h \) dissociating on the same target particle \( A \):

This has been checked recently for\(^9\) \( h = p^\pm, \pi^\pm, K^\pm \) and \( A = p \) (Rockefeller experiment E-396 at Fermilab). From the diagrams below

\[
\frac{d^2\sigma}{dt dx} = \left| \begin{array}{c} h \end{array} \right| X \left| \begin{array}{c} 2 \end{array} \right| \text{p} = \left| \begin{array}{c} \text{t} \end{array} \right| \text{p} = \left| \begin{array}{c} t=0 \end{array} \right| \text{p}
\]

\[
\sigma_{\text{tot}} = \left| \begin{array}{c} h \end{array} \right| X \left| \begin{array}{c} 2 \end{array} \right| \text{p} = \left| \begin{array}{c} \text{t} \end{array} \right| \text{p} = \left| \begin{array}{c} t=0 \end{array} \right| \text{p}
\]

one concludes that

\[
\frac{d^2\sigma}{dt dx} / \sigma_{\text{tot}} = c_p (s, x, t)
\]

i.e., the ratio of the diffractive to the total cross section of different hadrons dissociating on a proton is the same for all hadrons. Preliminary results of the Rockefeller experiment are given below:

\[
\begin{align*}
\frac{d^2\sigma}{dt dx} & \sim 308 \text{ GeV/c} \\
\sigma_{\text{tot}} & \sim 154 \text{ GeV/c}
\end{align*}
\]
Test of factorization
(h + p \rightarrow X + p at 100 \text{ GeV/c})

\begin{align*}
\begin{array}{c|c}
    h & R \text{ (arbitrary normalization)} \\
    \hline
    p & 1.00 \pm 0.04 \\
p^- & 0.92 \pm 0.11 \\
\pi^+ & 1.10 \pm 0.05 \\
\pi^- & 1.12 \pm 0.03 \\
K^+ & 0.86 \pm 0.21 \\
K^- & 1.15 \pm 0.14 \\
\end{array}
\end{align*}

R \equiv \frac{d^2\sigma/dtdx}{\sigma_{\text{tot}}}

\begin{align*}
\left\langle 0.02 < |t| < 0.1 \text{ (GeV/c)}^2 \right\rangle \\
\left\langle 0.02 < 1-x < 0.05 \right\rangle
\end{align*}

The agreement among the values of \( R \) for the various hadrons studied in this experiment is reasonably good.

Eq. (8) implies further that, for given \( s \) and \( x \), the \( t \) distribution of all hadrons dissociating on a proton should be the same. This aspect of the factorization rule has not been checked yet.

III. The first moment finite mass sum rule (FMSR) and the low mass enhancements of the diffractive cross sections

The FMSR is an extension of the finite energy sum rule for total cross sections. It derives from the hypothesis that the diffractive cross sections can be described either by \( s \)-channel resonance or by \( t \)-channel reggeon exchanges. Schematically,

\begin{align*}
\sum \frac{d^2\sigma}{dtd\Omega} \bigg|_{t=0} & = \sum \frac{d^2\sigma}{dtd\Omega} \bigg|_{t=\infty} \\
\Rightarrow R(M^2_x) & = \sum_i \alpha_i
\end{align*}

It is presumed that at high \( M^2_x \) overlapping resonances result in a smooth behavior of the cross section described by diagrams on the right hand side of Eq. (9). This behavior, extrapolated to low \( M^2_x \), should average over the non-smooth behavior caused by widely spaced resonances contributing to the left hand side of (9). Quantitatively, using analyticity and crossing symmetry, one derives the first moment FMSR

\begin{align*}
|t| \frac{d\sigma}{dt} + \int_{v_0}^{v} \int_{d\Omega \cdot d\Omega} \frac{d^2\sigma}{dtd\Omega} dv = & \int_{v_0}^{v} \int_{d\Omega \cdot d\Omega} \frac{d^2\sigma}{dtd\Omega} dv \bigg|_{t=\infty} \\
& \left\{ \text{function obtained from high } v \right\} \tag{10}
\end{align*}

where \( v = M^2_x - M^2_h = t \) is the cross-symmetric variable.
This rule has many far reaching consequences. Two of these consequences are described in Parts IV and V later on. Here we present the experimental tests of the rule and comment on its implications on the behavior of the low mass enhancements of the diffractive cross sections.

A very accurate test of the rule was first performed \(^4\) on pd + Xd. Fig. 4 shows this test at \(|t| = 0.035 \text{ (GeV/c)}^2\).

\[
\nu \frac{d^2\sigma}{dt \, dM_x^2} \bigg|_{|t|=0.035} = \frac{(p+d \rightarrow X+d)}{F_d}
\]

\((p_{\text{LAB}} = 275 \text{ GeV/c})\)

FIG. 4 - Test of the first-moment FMSR: Values of \(\nu(d^2\sigma/dtdM_x^2)\) vs \(M_x^2\) for \(p_{\text{LAB}} = 275 \text{ GeV/c}\) and \(|t| = 0.035 \text{ (GeV/c)}^2\).

The rule was tested for other \(t\)-values in the range \(|t| < 0.1 \text{ (GeV/c)}^2\) and was found to hold equally well (to a few % accuracy). As \(t \rightarrow 0\), the term \(|t| \frac{d\sigma}{dt} \rightarrow 0\) and therefore the low \(M_x^2\) region must have a large \(b\)-slope in order for the rule to continue to hold at small \(t\)-values \(^{11}\). This is what is actually happening (see Fig. 2). In fact, as was mentioned before, the integral over \(t\) behaves approximately as \(1/M_x^2\) even in the "resonance" region where it gradually drops with decreasing mass to become zero at the
pion production threshold. In this low mass region the b-slope increases
with decreasing $M_X^2$ in such a manner as to satisfy the FMSR. This behavior
of continuously increasing b-slope as one enters the low mass region has
been observed also in pion and kaon dissociation\textsuperscript{8}). In all cases, the slope
of the low mass enhancement is about twice as large as the slope of the
corresponding elastic scattering. These enhancements are the $N^*(1400)$ for
$pp \rightarrow Xp$, the $A_1(1100)$ for $\pi p \rightarrow Xp$, and the $Q(1300)$ for $Kp \rightarrow Xp$. None is
established as a resonance. Their behavior suggests that their production
is of the same nature as the high mass diffractive dissociation and there­
fore it should have the same s-dependence and follow the same scaling rules
discussed in Part II above.

Preliminary results from the Rockefeller experiment E-396 at Fermilab
testing the FMSR are shown on the Table below:

<table>
<thead>
<tr>
<th>$h$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>$1.04 \pm 0.05$</td>
</tr>
<tr>
<td>$\bar{p}$</td>
<td>$0.74 \pm 0.14$</td>
</tr>
<tr>
<td>$\pi^+$</td>
<td>$1.05 \pm 0.06$</td>
</tr>
<tr>
<td>$\pi^-$</td>
<td>$0.98 \pm 0.03$</td>
</tr>
<tr>
<td>$K^+$</td>
<td>$1.62 \pm 0.49$</td>
</tr>
<tr>
<td>$K^-$</td>
<td>$0.97 \pm 0.15$</td>
</tr>
</tbody>
</table>

The values of $R$ are compatible with unity
as predicted by the FMSR.

IV. A comment on the s-dependence of the diffraction dissociation cross
section -

The s-dependence of the high mass diffractive cross section has the form
given in Eq. (3), $1 + B/s$, with $B = 65 \pm 25$ GeV$^2$ for $pp \rightarrow Xp$. It was argued
in Part III that the integral over $t$ of the low mass diffractive production
should have the same s-dependence as that of the high mass production if the
result displayed in Fig. 3 i.e., the validity of the $1/M_X^2$ law all the way
down into the "resonance region" were to hold at all s-values. From the FMSR,
Eq. (10), one then obtains the result that the s-dependence of the diffractive
cross section is the same as that of the integral $\int_0^\infty |t| \frac{d\sigma}{dt} dt$. For

\[
pp \rightarrow Xp, \frac{d\sigma}{dt} = (184/s) e^{2.8t} + 51 e^{9.2t} + 23 e^{19t} \text{ ref.12,}
\]

which multiplied by $|t|$ and integrated over $t$ yields $0.67 (1 + 35/s)$. This s-dependence
is not statistically very different from the measured $1 + (65 \pm 25)/s$. 
V. A relationship between the elastic and the total cross section of hadrons.

In Part IIb it was shown that the high mass diffractive cross section for \( hp \rightarrow Xp \) scales as the total cross section, \( hp \rightarrow \text{anything} \). Furthermore, it was argued in Part III that the integral over \( t \) of the low mass diffractive production should scale as that of the high mass production if the \( 1/M^2 \) law was to be true for every hadron \( h \) dissociating on a proton. It then follows that the integral over \( t \) of the high mass term minus that of the low mass term in the FMSR Eq. (10) should be proportional to the total cross section, \( \sigma_{\text{tot}} \).

Thus, \( \sigma_{\text{tot}} \sim \int |t| (d\sigma_{\text{el}}/dt) dt = \sigma_{\text{el}}/b_{\text{el}} \), where in deriving the last step we assumed an \( e^{b_{\text{el}}t} \) form for \( d\sigma_{\text{el}}/dt \). Using the optical theorem at high energies where the ratio of the real to the imaginary part of the forward nuclear scattering amplitude (\( p \)-value) is close to zero, 

\[
d\sigma_{\text{el}}/dt = (\sigma_{\text{tot}}^2/16\pi) e^{b_{\text{el}}t},
\]

one obtains further the result \( \int |t|(d\sigma_{\text{el}}/dt) dt \sim \sigma_{\text{tot}}^2/b_{\text{el}}^2 \). Thus, \( \sigma_{\text{tot}} \sim \sigma_{\text{el}}/b_{\text{el}} \sim \sigma_{\text{tot}}^2/b_{\text{el}}^2 \), which yields the relationships

\[
\frac{b_{\text{el}}}{\sqrt[3]{\sigma_{\text{tot}}}} = C(s) \tag{11}
\]

\[
\frac{\sigma_{\text{el}}}{\sigma_{\text{tot}}^{3/2}} = C'(s) \tag{12}
\]

where \( C(s) \) and \( C'(s) \) are universal functions of \( s \) which are the same for all hadrons interacting with a proton.\(^\text{13}\)

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FIG. 5 - (a) The elastic cross section versus \( \sigma_{\text{tot}}^{3/2} \) for various hadrons interacting with a proton at 100 GeV/c. (b) The ratio \( \sigma_{\text{el}}/\sigma_{\text{tot}}^{3/2} \) versus \( \sigma_{\text{tot}}^{3/2} \) for the same data as in (a).
Fig. 5 shows $\sigma_{el}$ as a function of $\sigma_{tot}^{3/2}$ for various hadrons interacting with a proton at 100 GeV/c. The ratio $\sigma_{el}/\sigma_{tot}^{3/2}$ is remarkably constant for all hadrons in agreement with our result (12). Result (11) follows directly from (12) using the optical theorem. From Fig. 5, one now understands why the $\psi N$ elastic cross section is so small (about 30 μb for a 1 mb $\psi N$ total cross section).

VI. The total cross section of the diffractive mass $X$ with the nucleon. In proton dissociation on a deuterium target, the possibility exists for the dissociation to occur on one nucleon producing the particle $X$ which then scatters elastically from the second nucleon, as follows:

$$
\begin{array}{c}
\text{p} \\
\text{d} \\
\hline
\text{X} \\
\text{d} \\
\text{d}
\end{array}
$$

The forward elastic scattering of $X$ is proportional to the $X$-nucleon total cross section, $\sigma_{XN}$. Using the Glauber theory, one then calculates the ratio of diffractive to elastic scattering for $pp$ to that for $pd$ to be

$$
R = \frac{\left(\frac{d^2\sigma}{dtdx}\right)_{pp}}{\left(\frac{d^2\sigma}{dtdx}\right)_{pd}} = 1 + 0.2 \frac{\sigma_{XN}}{\sigma_{NN}}
$$

(14)

where the $pp$ and $pd$ cross sections are compared at the same incident proton momentum in the laboratory. Fig. 6 shows this ratio as a function of $1-x$ for two values of incident proton momentum. In the coherence region (small $1-x$), $R$ tends to a constant corresponding to $\sigma_{XN} = 28 \pm 10$ mb.
We would like to argue here that this long accepted method for determining the total cross section of an intermediate state $X$ with the nucleon may not be valid. The argument goes as follows: We have shown that at the same $s \approx 2m_{\text{target}} p_{\text{lab}}$ the ratio of the diffractive to the elastic cross section is the same for $pp$ and $pd$, i.e., $R = 1$ (see Fig. 3). This is a consequence of the factorization rule discussed in Part IIa. If it were not for the $s$-dependence of the diffractive cross section (see Eq. 3), $R$ would still be equal to unity when evaluated at the same incident proton momentum. But in Part IV we argued that the $s$-dependence of the diffractive cross section is tied up to that of elastic scattering through the FMSR. Thus, factorization of the diffractive vertex and the validity of the FMSR completely determine the value of $R$ and consequently the value of $\sigma^{XN}$. At high energies, as the $s$-dependence of the diffractive cross section dies out (except for possible $\ln s$ terms), $\sigma^{XN}$ as calculated from (14) goes to zero. One sees already in Fig. 6 the trend of decreasing $\sigma^{XN}$ as the energy increases. This behavior leads us to question the interpretation that $R$ is related to $\sigma^{XN}$ through Eq. (14).

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Results from the Single Arm Spectrometer at FERMILAB:


Results from Serpukov:


Other References:

9) Rockefeller Experiment E-396 at FERMILAB: Cool, Goulianos, Segler, Snow, Sticker and White.
13) Result (11) was previously observed by Quigg and Rosner, Phys. Rev. D14, 160 (1976).