Fermi National Accelerator Laboratory

FERMILAB-Conf-78/73-THY
September 1978

A Quasinuclear Colored Quark Model for Hadrons

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Presented at 1978 NATO Advanced Study Institute in Nuclear Theory:
Banff (Alberta) August 22 - September 2, 1978

Operated by Universities Research Association Inc. under contract with the Energy Research and Development Administration

1. INTRODUCTION

The Implications of Two New Fermilab Experiments

Once upon a time physicists believed that nucleons and pions were elementary like electrons and photons, and that Yukawa's theory of nuclear forces was the analog of QED for strong interactions. Then the Δ (3-3 resonance) was discovered, and then the ρ and other pion resonances, and it became apparent that neither the pion nor the nucleon was elementary and that both had a composite structure. Today pions and nucleons seem to be very similar objects, instead of being very different like the electron and photon, and made of the same basic building blocks: spin 1/2 quarks bound by colored gluons. But perhaps history will repeat itself. Maybe 25 years from now a lecture at the Banff summer school will begin with the statement "Once upon a time physicists believed that quarks and gluons were elementary, and that Quantrum Chromodynamics (QCD) was the analog of QED for strong interactions. Then ...... ?????"

But we shall not enter into such speculations, and examine the situation as it appears today. We have the new QXD model for everything, where X = A, B, C, D, E, F, G, etc. So far there are only models for X = C, E, F and G, but no doubt the others will eventually be discovered as well. However, it is amusing that in the great excitement about non-Abelian gauge theory, the original non-Abelian gauge model for hadron dynamics has faded away. This was the gauge theory of strong interactions mediated by the octet of vector mesons ρ, ω, and K^* coupled to conserved vector currects. The SU(3) group originally introduced by Gell-Mann and Ne'eman is now called flavor and dismissed as an irrelevant complication in the QCD description of strong interactions. Flavor is now discussed in QFD (Quantum Flavordymanics) the new fancy name for the kind of unified theory of weak and electromagnetic interactions discussed by Henry Primakoff in his lectures here. In this approach the mass differences between quarks of different flavors are assumed to come somehow from the weak and electromagnetic interactions and not from the strong interactions. The set of quarks with different flavors and masses is then given as input for the QCD description of strong interactions, and QCD does not attempt to explain flavor or mass differences. But the question of how many flavors there really are, and when and whether experimentalists will stop finding new and heavier bound states of new heavy quarks is still open. There is no theoretical clue to the answer yet.

I begin a review of the present status of the quark model by quoting some of the most recent results. These are motivated by two recent experiments at Fermilab. I take a "Galilean approach" which assumes that we learn about nature by making experimental observations like Galileo and trying to understand them, rather than by reading the words that great theorists like Aristotle have written. The quark model has grown out of such experimental observations, against the opposition of the Aristotelian establishment who have always found weighty theoretical reasons why quarks could not exist and why the regularities predicted from the quark model and found in experiment could not be significant.

The quark model is very ad hoc. It lacks a fundamental theoretical basis, yet it provides a very good description of much experimental data. Today theorists believe that the fundamental basis will eventually come from QCD, and there are may indications that this is indeed true. But there are many slips between cup and lip, and a measure of scepticism is always in place. We shall make free use of the approach and methods of QCD to guide our intuition in discussing the quark model, but we do not attempt any rigorous derivation, and are always looking at experimental data to see what nature is trying to tell us.

This discussion of quarks uses what might be called a "Newtonian approach." Newton was able to describe the motion of the earth around the sun with great precision without ever having heard of asymptotic freedom, while in fact, the gravitational field of the earth is indeed asymptotically free. At large distances the earth has a field given by Coulomb's law with a coupling constant proportional to the mass of the earth. But at short distances, less than the radius of the earth, the effective or "running" coupling constant decreases and goes to zero at the center of the earth. But this asymptotic freedom was irrelevant to Newton, who described the earth as a point mass with a pure Coulomb-like gravitational field. This is a drastic and unwarranted assumption for short distance phenomena. It ignores for example that fact that we exist on the earth. But it is certainly adequate for precise calculations of the earth's orbit.

In the same spirit we assume that the nucleon is made of three constituent quarks, which are treated as very simple objects. We know that they must in reality be much more complicated things including virtual gluons, quark-antiquark pairs, etc. But there seems to be a wide variety of phenomena successfully described by these simple constitutent quarks. Somewhere in the fundamental theory there must be an explanation, , just as there is an explanation justifying Newton's treatment of the earth as a simple point mass. But so far we do not have a good fundamental theory and do not have a satisfactory explanation. All we know is that the model works.

One interesting experimental result from Fermilab is the discovery[1] of the new fifth heavy quark in the bound states now called the ψ and ψ', with the surprising equality of the ψ-ψ' mass splitting to the mass splitting in the charmonium system, namely the ψ - ψ' splitting. Nature seems to be telling us that hadron mass splittings are simpler than expected: they do not depend very strongly on the flavor or mass of the quarks of which they are composed. This principle was incorporated formally in the logarithmic potential model of Quigg and Rosner[2] which has this mass scaling property. We can use the general philosophy of this model, without taking it too seriously in detail, and assume that flavor dependence of many mass splitting effects can be neglected. This might be called "Rolling off the Log."

A second interesting result from Fermilab is a new measurement[3] of the magnetic moment of the Λ to a precision of 1% by a Rutgers-Michigan-Wisconsin group. The number agrees with quark model predictions[4,5] of this moment to 1%. This agreement is completely unexpected, since the quark model is not expected to be that good. Let us review the simple-minded calculations of baryon magnetic moments in the quark model[6] to show what physics lies behind this surprising agreement.

The baryon octet is assumed to consist of three-quark states in a relative s-wave, with the total spin and magnetic moment given by simple vector addition of the quark spins and magnetic moments. The magnetic moment of a quark of flavor f is assumed to be the Dirac moment[7] which is

$$\mu_f = q_f(M_p/m_f) \text{ nuclear magnetons ,} \tag{1.1}$$

where q_f and m_f are the charge and mass of the quark of flavor f and M_p is the proton mass.

The magnetic moment of the baryon is then obtained by summing these quark moments in the SU(6) wave function which is totally symmetic in spin and flavor. For the Λ, the symmetry requirement means that the two nonstrange quarks which are coupled to isospin zero also have spin zero and do not contribute to the magnetic moment. The Λ magnetic moment is therefore given by the moment of the strange quark,

$$\mu_\Lambda = \mu_s = -(1/3)(M_p/m_s) . \tag{1.2}$$

The remaining baryons can be described as consisting of two quarks of flavor a and one of flavor b, where a and b can be u, d or s. The two quarks of flavor a are symmetric in flavor and coupled to spin 1, and these are then coupled with the spin of the third quark to give total spin 1/2. The general result for the magnetic moment of such a baryon is

$$\mu(aab) = (4/3)\mu_a - (1/3)\mu_b , \tag{1.3a}$$

where the coefficients come from the expectation values of S_z for the a and b quarks in this wave function. For the proton, where $a = u$, $b = d$, $q_a = +2/3$ and $q_b = -1/3$,

$$\mu_p = (8/9)(M_p/m_u) + (1/9)(M_p/m_d) = (M_p/m_u) , \tag{1.3b}$$

where we neglect the difference between m_u and m_d. Similarly for the neutron, where $a = d$ and $b = u$,

$$\mu_n = -(4/9)(M_p/m_d) - (2/9)(M_p/m_u) = -(2/3)(M_p/m_u). \tag{1.3c}$$

Combining Eqs. (3b) and (3c) gives the well known successful prediction for the neutron magnetic moment,

$$\mu_n = (-2/3)\mu_p = -1.86 \text{ nuclear magnetons} , \tag{1.4}$$

in excellent agreement with the experimental value of -1.91.

The Λ magnetic moment cannot be related directly to the nucleon magnetic moments without some assumption regarding the difference between m_s and m_u. Let us make the very drastic assumption that this quark mass difference is exactly equal to the hadron mass difference,

$$m_s - m_u = M_\Lambda - M_p . \qquad (1.5)$$

This ia a new ingredient leading to a very interesting prediction. *A priori* there is no reason to choose the Λ-N mass difference for the right hand side of (1.5) rather than Σ-N or Σ^*-Δ. The decuplet mass splitting has commonly been used because the equal mass spacing has been interpreted as indicating that decuplet mass splittings are simpler octet splittings. However, arguments based on QCD show that the decuplet splitting involves a complicated interplay of both the quark mass differences (1.5) and the spin splittings while use of the Λ-N mass difference eliminates effects of spin splittings.[8]

Comgining Eqs. (2), (3b) and (5) gives the prediction

$$\mu_\Lambda = (-1/3)\left[(1/\mu_p) + (M_\Lambda - M_p)/M_p\right]^{-1} = -0.61 \text{ n.m.} \qquad (1.6)$$

Another prediction is obtainable by assuming that the ratio of quark magnetic moments μ_u/μ_s is obtainable from hadron spin splittings like the ratio $(M_\Delta - M_N)/M_{\Sigma^*} - M_\Sigma)$. This ratio which is unity in the SU(3) limit is directly related to the ratio of quark magnetic moments under the assumption that the spin splittings come from a "color magnetic" interaction[4] proportional to the color magnetic moments of the quarks which are in turn proportional to electromagnetic moments. The result obtained is

$$\mu_\Lambda = -(\mu_P/3)(M_{\Sigma^{*+}} - M_{\Sigma^+})/(M_{\Delta^+} - M) = -0.61 \text{ n.m.} \tag{1.7}$$

Both predictions (1.6) and (1.7) are in remarkable agreement with the new experimental value $\mu_\Lambda = -0.6138 \pm 0.0047$ n.m. That they are also in remarkable agreement with one another suggests a new relation between hadron masses and the proton magnetic moment. Eliminating μ_Λ between (1.6) and (1.7) gives

$$\left[(M_{\Delta^+} - M_p)/(M_{\Sigma^{*+}} - M_{\Sigma^+})\right] - 1 = \mu_p(M_\Lambda - M_N)/M_N . \tag{1.8}$$

This peculiar relation is in excellent agreement with experiment. The left hand side is 0.523, the right hand side is 0.528. This unorthodox combination of hadron mass differences and the proton moment has a simple physical interpretation. The SU(3)-breaking quark mass parameter $(m_s - m_u)/m_u$ is computed in two ways. The LHS uses the quark mass <u>ratio</u> (m_s/m_u) obtained from hadron spin splittings. The RHS uses the quark mass <u>difference</u> $(m_s - m_u)$ obtained from hadron strangeness splittings, but needs the proton moment to provide a quark mass scale relating the mass difference to a mass ratio. Thus Eq. (1.8) says that the quark mass ratio and the quark mass difference determined in two different ways from hadron masses are consistent at the 1% level with the quark mass m_u determined from the proton mass and magnetic moment.

The success of these relations suggests a review of the underlying physics and its implications for hadron models. The prediction (1.7) is equivalent to a similar prediction obtained by DGG using explicit expressions involving quark mass ratios. Our derivation shows that explicit reference to quark masses is unnecessary and that all that is needed is proportionality between electromagnetic and color magnetic moments. The prediction (1.6) and the relation (1.8) require the explicit assumption that quark magnetic moments depend upon masses like Dirac moments, and that the relevant quark mass difference is given by Eq. (1.5). This is a much more serious assumption which is generally not valid in conventional models. In the DGG model[4] Eq. (1.5) does not hold because the hadron masses include additional terms like kinetic energies which are inversely proportional to quark masses and do not cancel in the difference (1.5). The model of Ref. [8] avoids these terms by the use of scaling properties of the Quigg-Rosner[2] logarithmic potential model. In this model kinetic energies and mass splittings in the hadron spectrum are independent of the quark mass and cancel out of mass differences like (1.5). This can be seen explicitly by the use of the virial theorem.

Consider a three-body system with the non-relativistic Hamiltonian

$$H = \sum_{i=1} t_i + \sum_{i>j} v(r_{ij}) , \qquad (1.9a)$$

where

$$t_i = p_i^2/2m_i , \qquad (1.9b)$$

and

$$v(r_{ij}) = V \log(r_{ij}/r_o) . \qquad (1.9c)$$

The virial theorem then states that for any eigenfunction of H,

$$\left\langle \sum_i t_i \right\rangle = (1/2) \sum_{i>j} \left\langle r_{ij}\, dv(r_{ij})/dr_{ij} \right\rangle = (3/2)V . \qquad (1.10)$$

For the particular case of the log potential (1.9c) the right hand side of the virial theorem is a c-number, rather than the expectation value of an operator in the specific wave function, and is independent of both the wave function and the masses of the particles.

We now consider the flavor dependence of the spin splittings in more detail. These were first considered by Federman, Rubinstein and Talmi[9] in a nuclear shell model approach to baryon masses in 1966. In this model the low-lying baryon octet and decuplet were considered to all have the same "shell-model" wave functions and have their mass degeneracy split only by the strange quark mass difference $m_s - m_u$ and by a residual two-body interaction. If we denote the effective matrix elements for this interaction between two quarks of flavors a and b by V_0^{ab} and V_1^{ab} for the spin singlet and triplet states respectively, the following baryon mass differences are easily expressed in terms of these effective matrix elements.

$$M_\Delta - M_N = (3/2)(V_1^{ud} - V_0^{ud}) \ , \tag{1.11a}$$

$$M_{\Sigma^*} - M_\Lambda = (V_1^{ud} - V_0^{ud}) + (1/2)(V_1^{us} - V_0^{us}) \ , \tag{1.11b}$$

$$M_{\Sigma^*} - M_\Sigma = (3/2)(V_1^{us} - V_0^{us}) \ . \tag{1.11c}$$

Combining these relations gives a relation between hadron masses,

$$M_\Delta - M_N = (1/2)(2M_{\Sigma^*} + M_\Sigma - 3M_\Lambda) \ . \tag{1.12}$$

This relation was found to be in good agreement with experiment, the LHS is 307 MeV; the RHS is 294 MeV, thus providing support for the assumption that baryon mass splittings are described by two-body forces.

The next development in the description of spin splittings was the assumption by DeRujula, Georgi and Glashow (DGG)[4] that these are due to a color-magnetic hyperfine interaction having the form

$$V_{hf} = \mu_i^c \mu_j^c \vec{\sigma}_i \cdot \vec{\sigma}_j \ v(r_{ij}) \ , \tag{1.13}$$

where μ_i^c denotes the color magnetic moment of quark i. The particular form (1.13) immediately relates the triplet and singlet effective matrix elements by the eigenvalues of the operator $\vec{\sigma}_i \cdot \vec{\sigma}_j$.

$$V_0^{ab} = -3\, V_1^{ab} \quad . \tag{1.14}$$

This interaction (1.13) does not contribute to the difference $M_\Lambda - M_N$. If we assume that meson spin splittings are due to a similar interaction but with a different strength, we can construct linear combinations of meson masses to which an interaction satisfying the relation (1.14) does not contribute. Thus we can obtain the relation[7]

$$M_\Lambda - M_N = m_s - m_u = (3/4)(M_{K^*} - M_\rho) + (1/4)(M_K - M_\pi) \quad . \tag{1.15}$$

This relation is also in surprising agreement with experiment, the LHS is 177 MeV and the RHS is 180 MeV. Thus there seems to be experimental justification of the use of the hadron mass difference (1.15) as a quark mass difference in deriving the prediction (1.16).

We have now obtained three independent relations between hadron masses and moments which are experimentally confirmed at the 1% level, namely relations (1.6), (1.7) and (1.15). We can ask what we have put in to get these results.

1. We have identified hadron mass difference with quark mass differences in a very simple way, neglecting flavor dependence of binding effects and kinetic energies.

2. We have identified the quark masses in (1.15) with the masses appearing in the electromagnetic and color magnetic moments of the quarks.

3. We have assumed a hyperfine interaction having the form (1.13).

However, it is still a big step further to use the quark mass difference of Eq. (1.5) as the mass parameter in the magnetic moments and to obtain results valid to a few per cent. The success of the relations (1.6)-(1.8) at this level indicate that the "quasinuclear colored quark model" of Ref. [3] and the three basic assumptions above should be taken more seriously than indicated by their crude derivations. The underlying physics is that the same quark mass parameter appears in the simplest possible way in the electromagnetic moments, the color magnetic moments and the hadron mass splittings. That electromagnetic and color moments should depend upon the same mass parameter is not surprising. But the value of the magnetic moment is not expected to be determined to 1% by the mass parameter which enters hadron mass splittings and includes binding energies as well as quark masses.

The magnetic moment of a Dirac particle bound in an external potential depends upon a mass parameter which is a function of the Lorentz character of the potential [6]. For a Lorentz scalar potential this mass parameter is indeed the total energy of the bound state, including the binding energy. But for a Lorentz vector potential the magnetic moment is not affected by the binding (the magnetic moment of an electron strongly bound in the electrostatic field of a Van-de-Graaf accelerator is the same as that of a free electron). The results (1.6) and (1.7) suggest that the dominant binding potential for quarks in hadrons is Lorentz scalar rather than the Lorentz four-vector of a Coulomb or a one-gluon-exchange potential. But such an argument is not expected to hold to 1%. Note that Lorentz scalar confinement is implicit in bag models [11] which use Lorentz scalar bags as the principal confining mechanism and have only weak effects due to gluon exchange.

All the above leads to a deeper questioning of what indeed is the meaning of the quark mass. This mass appears as a parameter in many quark model calculations of observable hadron properties, but very different values are used in different calculations, varying from zero to infinity. There are "current quarks" which have nearly zero mass, bound "constituent quarks" whose mass is of the order of hadron masses and free quarks, which have a very heavy mass or an infinite mass if quarks are permanently confined.

An intuitive picture of quark masses motivated by QCD shows that an isolated quark has a strong color field at large distances and strong long range forces if there are no other quarks nearby to cut off the color lines of force and confine color. The mass of an isolated quark must include all the energy in the associated color field at large distances, since this field must move with the quark and contribute to its inertial mass. In models with quark confinement, the energy in the field of an isolated quark is infinite and quarks have infinite mass and are unobservable.

Quarks bound in color singlet hadrons do not have the large color field at large distances and therefore do not have a large inertial mass. The mass parameter associated with the motion of these bound quarks inside hadrons and with their magnetic moments must be simply related to the energy in the color field which moves with each quark. This may determine the value of the quark mass successfully used in constituent quark models and in the relations (1.1)-(1.5) of this paper. In scattering processes the mass to be used for the quark should depend upon how much of the associated color field recoils with the quark. At very high momentum transfers the quark may have received a kick which moves it so fast that its color field does not move with it. This would account for the small quark masses used for current quarks or quark partons, and the necessity to treat the color field separately as a "gluon component" in the hadron wave function for deep inelastic processes.

Within this continuum of quark mass values from zero to infinity used for different processes there seems to be an intermediate region relevant to hadron spectroscopy where each valence quark has an inertia roughly given by its share of the hadron mass and only valence quarks need by considered [12]. These "constituent quark masses" determine the scales of mass splittings in the hadron spectrum and of hadron magnetic moments. There is no rigorous derivation as yet of these properties of constituent quarks from QCD, but the remarkable success and precision of nonrelativistic quark model predictions in describing the experimental spectrum suggest that a more fundamental derivation must exist.

In the remainder of these lectures we consider a quasinuclear constituent quark model in which constituent quarks are assumed to be made of constitutent quarks interacting with a two-body color-exchange logarithmic potential. In Section II we discuss the color degree of freedom in detail. In Section III we consider some properties of the logarithmic potential. In Section IV we define the quasinuclear model and discuss its validity and compare some of its predictions with experiment.

To conclude this introduction we consider the validity of the nonrelativistic approximation used in constituent quark models and show that it cannot be valid for the nucleon. We consider this approximation in the spirit of similar approximations used elsewhere in physics. It is an expansion in powers of a "small" parameter, v/c, which however is manifestly not small. We cannot justify this expansion, but since it gives results that agree remarkably well with experiment, we continue to use it with the hope that some new idea like "relativistic freedom" or "asymptotic nonrelativity" will eventually come along to explain it.

To see that v/c is not small, we note that the velocity of a particle in an orbit is the product of the radius r and the angular frequency ω. Thus

$$v/c = \omega r/c = r/\lambda \;. \tag{1.16}$$

where λ is the wave length of a wave with velocity c and angular frequency ω; i.e. the wave length of a photon with energy $\hbar\omega$.

The same result can be obtained more rigorously by use of the Heisenberg equation of motion for the quark co-ordinate x whose time derivative is the velocity v,

$$\frac{v^2}{c^2} = \langle \dot{x}^2\rangle/c^2 = -\langle [H,x]^2\rangle/\hbar^2c^2 = \sum_i (E_i - E_o)^2 \langle o|x|i\rangle\langle i|x|o\rangle/\hbar^2c^2, \tag{1.17a}$$

where E_i and E_o are the energies of the states i and o. Since the operator x changes parity, the minimum value of $E_i - E_o$ is the excitation energy ΔE_- of the lowest lying negative parity state of the three-quark system. We thus obtain the inequality

$$v^2/c^2 \geq (\Delta E_-)^2 \langle x^2\rangle /\hbar^2c^2 \;. \tag{1.17b}$$

The relation between the rigorous result (1.17) and the handwaving intuitive result (1.16) is now clear. The angular frequency ω to be used in the relation (1.16) is some average excitation energy for orbital excitation, and λ is just the wave length of a photon emitted in the transition from these excited states to the ground state. Thus Eq. (1.16) shows that the condition for the nonrelativistic approximation to be valid $(v/c << 1)$ is the same as that for the validity of the multipole expansion for the radiative transitions from orbitally excited states to the ground state. $(r << \lambda\!\!\!^{-})$.

For the case of the nucleon, where the measurement of the mean square radius by electron scattering shows a value of the order of one fermi, while the excitation energy of the first odd parity resonances is about 600 MeV and corresponds to a wave length of about 1/3 fermi, v/c is manifestly not small compared to unity.

Note that for a coulomb-like potential, the excitation energies are proportional to g^2/r, where g is the coupling constant, $\Delta E_- x$ is just g^2, and an expansion in v/c is equivalent to an expansion in powers of the coupling constant, or ordinary perturbation theory.

For charmonium, the upsilon system and heavier quarks, Eq. (1.17) show that the nonrelativistic approximation is probably all right. $<x^2>$ is presumably much smaller, of the order of the Compton wave length of heavy quarks having masses of 1.5 GeV, 4.5 GeV and higher, while ΔE_- for these systems seems to be around 300 MeV.

Note that the results (1.16) and (1.17) are model independent, as they use only the experimentally measured size of the system and excitation energies. Thus <u>any model</u> which fits the size of the proton and the excitation spectrum of low-lying negative parity states cannot have only nonrelativistic velocities.

1. COLOR

We now consider in detail the color degree of freedom and the properties of the color exchange force which seems to be present in bound states of constituent quarks. Much of this treatment is given in Refs. 13 and 14.

2.1 The Deuteron World

Some insight into the colored quark models is given by the analogy of a world in which all low-lying nuclear states are made of deuterons and have isospin zero, free nucleons have not yet been seen and experiment has not yet attained energies higher than the deuteron binding energy or the symmetry energy required to excite the first $I = 1$ states. In this isoscalar world where all observed states have isospin zero the isovector component of the electromagnetic current would not be observed since it has vanishing matrix elements between isoscalar states. The deuteron energy level spectrum (something like that of a diatomic molecule) would indicate that the deuteron was a two-body system, but there would be no way to distinguish between the neutron and the proton. The deuteron would thus appear to be composed of two identical objects which might be called nucleons. Since the deuteron has electric charge +1, the nucleon would be assumed to have electric charge +1/2. Furthermore, the nucleon would be observed to have spin 1/2 and be expected to satisfy Fermi statistics. However, the ground state of the deuteron and all other observed states would be found to be symmetric in space and spin. Thus, the nucleon would appear to be a spin 1/2 particle with fractional electric charge and peculiar statistics.

Some daring theorists might propose the existence of a hidden degree of freedom expressed by having nucleons of two different colors. There would be a hidden SU(2) symmetry (which might be called isospin) to transform between the two nucleon states of different colors. All the observed low-lying states would be singlets in this new color (or isospin) SU(2). Since the color singlet state of the two-particle system is antisymmetric in the color degree of freedom, the Pauli principle requires the wave function to be symmetric in space and spin, thus solving the statistics problem.

The direct analog of this deuteron problem in hadron quark models is the quark model for the Ω^-. In the conventional quark model, the Ω^- consists of three identical strange quarks (called λ-quarks by some people and s-quarks by others), with their spins of 1/2 coupled symmetrically to spin 3/2. Since the electric charge of the Ω^- is -1, the strange quark is required to have charge -1/3, and it is also required to have peculiar statistics because the system of three identical particles has a symmetric wave function in all known degrees of freedom. Some daring theorists have therefore proposed the existence of a hidden degree of freedom expressed by having strange quarks of three different colors,[5] and a hidden SU(3) symmetry to transform between the three strange quark states of different colors. All the observed low-lying states are singlets in this $SU(3)_{color}$ group. Since the color-singlet state of the three-particle system is antisymmetric in the color degree of freedom, the Pauli principle requires the wave function to be symmetric in the other degrees of freedom, in agreement with experiment and ordinary Fermi statistics. It is also possible to give these colored strange quarks different integral electric charges, one with charge -1 and two neutrals, by analogy with the nucleons in the deuteron. However, as we are concerned primarily with strong interactions, we need not choose between models having different electric charges for colored quarks.

We have chosen the example of the Ω^- for this discussion to simplify the treatment of the flavor degree of freedom by considering only strange quarks. When all flavors are considered, there are three colors for each flavor, and $3n_f$ quarks altogether. There are two SU(n) groups, the flavor $SU(n)_f$ and the color SU(3), which are combined into the direct product $SU(n)_f \times SU(3)_{color}$.

2.2 The Puzzles of Quark Model Predictions of the Hadron Spectrum

Let us now consider some puzzles posed by one of the outstanding "successes" of the quark model, the prediction of the hadron spectrum. The empirical rule that all observed hadron bound states and resonances have the quantum numbers found in the three-quark and quark-antiquark systems is in remarkable agreement with experiment. Since no alternative explanation or description has been given for this striking regularity in the hadron spectrum, this rule may constitute evidence for taking quarks seriously. The quark model also predicts the energy level spectrum of the states constructed from the three-quark and quark-antiquark systems and observed experimentally as hadron resonances. These predictions also seem to be in reasonable agreement with experiment, but pose additional questions.

Why is the observed baryon spectrum fit only by the symmetric quark model[4] which restricts the allowed states of the three-quark system to those being totally symmetric under permutations in the known degrees of freedom rather than totally antisymmetric, as one expects for fermions? This can be explained by assuming that quarks obey peculiar statistics, or that there is a hidden degree of freedom sometimes called "color." But this requires the additional ansatz that all observed hadrons are color singlets. Why and why only 3q and $q\bar{q}$? Why not other configurations? Why does the low-lying meson spectrum show all the states "predicted by the quark model" without any supplementary conditions and with no allowed states conspicuously absent?

There is an inconsistency between the observation of bound states in all channels for $q\bar{q}$ scattering and the absence of bound states with quantum numbers of $2q\bar{q}$ and $3q\bar{q}$. If the quark-antiquark interaction is attractive in all possible channels, as indicated by the presence of bound states, an antiquark should be attracted by any composite state containing only quarks, like a diquark or a baryon, to make a bound state with peculiar quantum numbers that have not been observed.

In our discussion, we assume that free quarks are very heavy, and we consider only effects on the mass scale of the quark mass. All observed particles have zero mass on this scale. The observed hadron spectrum is a "fine structure" which we are unable to resolve in this approximation. This is a reasonable approach, since as long as we are not treating spin in detail, we are unable to distinguish between a pion and a ρ meson, and are neglecting mass splittings of the order of the $\rho - \pi$ mass difference. We therefore are only able to discuss whether a particle has "zero mass" and appears as an observed hadron, or whether it has a mass of the order of the quark mass and should not have been observed.

The question why only $3q$ and $q\bar{q}$ can be stated more precisely in terms of the following three whys:

1. The triality why. With attractive interactions between quarks and antiquarks, why are three quarks and an antiquark not bound more strongly than a baryon or two quarks and an antiquark bound more strongly than a meson? Note that we are not asking about four quarks vs.

three quarks. Symmetry restrictions such as the Pauli principle with colored quarks can prevent the construction of a four quark state which is totally symmetric in space, spin and unitary spin. But there is no Pauli principle which prevents an antiquark from being added to a system of three quarks in all possible states. Thus if each quark in the baryon attracts the antiquark, some additional mechanism must be found to prevent it from being bound to the quark system.

2. The exotics why. Even assuming some mysterious symmetry principle which prevents fractionally charged states from being seen, why are there no strongly bound states of zero triality, like those of two quarks and two antiquarks or four quarks and one antiquark? The question of whether or not such bound four-quark states exist can be posed as follows: There are two analogs for the bound quark-antiquark meson state, the deuteron and positronium. If the meson is like the deuteron, then two mesons should form a bound four-quark system just as two deuterons bind together to form a much more strongly bound α particle. If the deuteron is like positronium, the forces saturate and the residual force between the two neutral systems is very small and does not produce a state more strongly bound than the original two particle states. From the experimental observation that there is no strongly bound doubly charged state of two positive pions, we conclude that the pion is more like positronium than like the deuteron.

However the positronium analogy is misleading because there is no bound state of three electrons while three quarks bind to make a baryon. The force between two positronium atoms is nearly zero because the repulsion between the electron pairs exactly cancels the attraction of the electron-positron pairs in the two positronium atoms. But in two positive pions the quark-quark force cannot be completely repulsive because the same quarks must have attractive forces to make baryons. If the quarks and antiquarks in two pions attract one another, why is there no net attraction between two positive pions to produce an $I = 2$ dipion resonance or bound state with a mass near the mass of two pions?

3. The diquark or meson-baryon why. Why is the quark-quark interaction just enough weaker than the quark-antiquark interaction so that diquarks near the meson mass are not observed, but three-quark systems have masses comparable to those of mesons? Vector gluons which are popular these days would bind the quark-antiquark system, but the force they provide between identical quarks is repulsive. Scalar or other gluons which are even under charge conjugation bind both the quark-antiquark and diquark systems equally. If the quark mass is very heavy, the single quark-antiquark interaction in a meson must cancel two quark masses, while the three quark-quark interactions in the baryon must cancel three quark masses. This suggests that the quark-quark interaction is exactly half the strength of the quark-antiquark

interaction.[17] Such a result can be achieved by a suitable mixture of vector and scalar interactions, but it is not very satisfying to obtain such a simple fundamental property of hadrons by a model which fits it with an adjustable parameter.

In all of this discussion, we are considering one-particle states, with the assumption that multiparticle states exist which contain separated particles each having the properties we are trying to explain. Multiparticle states pose additional problems. The allowed spectrum for multiparticle states is not specified by a set of allowed quantum numbers, but by the condition that their constituent particles individually have allowed quantum numbers. Thus the whys cannot be answered by general symmetry principles which apply to all states. The triality why is not answered by a symmetry principle forbidding all states which do not have zero triality, because multiparticle states of zero triality must also be forbidden if they are made of particles which individually have nonzero triality. Similarly, the exotics why is not answered by a symmetry principle forbidding all states with exotic quantum numbers because multiparticle exotic states made from nonexotic particles are allowed. Thus any treatment which attempts to answer these whys must discuss both single-particle and multiparticle states, and must consider the space-time properties which distinguish between them. Algebraic arguments involving only internal symmetry groups cannot be sufficient.

Our three whys involve only the strong interactions which do not depend upon the couplings of quarks to the electromagnetic and weak currents. The following discussion thus applies to both fractionally charged and integrally charged models.

2.3 The Colored Gluon Model

We now examine the three whys. In the colored quark description of hadrons the restriction that only color singlet states are observed immediately solves the triality why since only states of zero triality can be color singlets. But requiring all low-lying states to be color singlets is thus equivalent to requiring all low-lying states to have zero triality; it merely replaces one ad hoc assumption with another. What is needed is some dynamical description in which the color singlets turn out to be the low-lying states in a natural way. To attack this problem we return to the fictitious deuteron world where all low-lying states are isoscalar and which is the analog of the colored quark description of hadrons. We follow the treatment of ref. 44.

At first this isoscalar deuteron world seems very artificial. Why should all states with $I = 0$ be pushed down and all states with $I \neq 0$ be pushed up out of sight? But there turns out to be a very natural nuclear interaction which creates exactly this isoscalar deuteron world; namely nuclear two-body forces dominated by a very strong Yukawa interaction provided by ρ exchange. This interaction is attractive for isoscalar states and repulsive for isovector states, in both nucleon-nucleon and nucleon-antinucleon systems. It thus binds only isoscalar states. The ρ-exchange interaction between particles i and j can be expressed in the form

$$v_{ij} = V \vec{t}_i \cdot \vec{t}_j \quad , \tag{2.1a}$$

where $\vec{t}_i$ is the isospin of particle i and V contains the dependence on all other degrees of freedom except isospin. If we neglect these other degrees of freedom we can write for any n-particle system containing antinucleons and nucleons,

$$V(n) = \frac{1}{2}\sum_{i \neq j} v_{ij} = \frac{V}{2}\left[\sum_{\substack{\text{all}\\ ij}} \vec{t}_i \cdot \vec{t}_j - \sum_{i} \vec{t}_i \cdot \vec{t}_i\right] = \frac{V}{2}[I(I+1) - nt(t+1)] \tag{2.1b}$$

where I is the total isospin of the system and t is the isospin of one particle; i.e., 1/2 for a nucleon.

The interaction (2.1b) is seen to be repulsive for the two-body system with I = 1 and attractive for all isoscalar states. A pair of particles bound in the I=0 state is thus seen to behave like a neutral atom; it does not attract additional particles. Since the pair is "spherically symmetric" in isospace, a third particle brought near the pair sees each of the other particles with random isospin orientation, and its interaction with any member of the pair is described by the average of (2.1a) over a statistical mixture which is 3/4 isovector and 1/2 isoscalar. This average is exactly zero.

The neutral atom analogy is very appropriate for the description of the observed properties of hadrons. The forces between neutral atoms are not exactly zero, but are much weaker than the forces which bind the atom itself. These interatomic forces produce molecules which are much more weakly bound than atoms. Similarly the forces between hadrons do not vanish but are much weaker than the forces which bind the hadron itself. These interhadronic forces produce complex nuclei which are much more weakly bound than hadrons. In the approximation where we neglect energies much smaller than the quark mass these "molecular" effects are safely neglected.

We now generalize this picture for the colored quark description of hadrons. If there are n colors, the interaction (2.1) must be generalized from SU(2) to SU(n). The quark-antiquark system then still saturates at one pair, but the multiquark system can be seen to saturate at n quarks. A quark-antiquark system which is a singlet in SU(n) exists for all values of n. However, the existence of a singlet in the two-quark system is an accident which occurs only in SU(2) and is not generalizable to SU(n). However the $I = 0$ two-quark state is also characterized as antisymmetric under permutation of the two particles. This antisymmetry is generalized easily to SU(n) where totally antisymmetric states exist for a maximum of n particles, and the n particle antisymmetric state is a singlet in SU(n).

We now construct the analog of the interaction (2.1b) for a model with three triplets of different colors. Then the Yukawa interaction produced by the exchange of an octet of "colored gluons" has the form analogous to (2.1). For an n-particle system containing both quarks and antiquarks,

$$U(n) = \frac{1}{8} \sum_{i \neq j} u_{ij} \sum_{\sigma} \lambda_{i\sigma} \lambda_{j\sigma} \tag{2.2}$$

where u_{ij} depends on all the noncolor variables of particles i and j and $\lambda_{i\sigma}$ ($\sigma = 1, \ldots, 8$) denote the eight generators of $SU(3)_{color}$ acting on a single quark or antiquark i.

If the dependence of u_{ij} on the individual particles i and j is neglected, the interaction energy of an n-particle system can be calculated by the same trick used in Eq. (2.1b) to give

$$V(n) = \frac{u}{2}(C - nc) \tag{2.3a}$$

where u is the expectation value of u_{ij}, integrated over the noncolor variables, C is the eigenvalue of the Casimir operator for $SU(3)_{color}$

for the n-particle system and $c = 4/3$ is the eigenvalue for a single quark or antiquark. These eigenvalues are directly analogous to the SU(2) Casimir operator eigenvalues $I(I+1)$ and $t(t+1)$ in Eq. (2.1b).

In the approximation where all energies small compared to the quark mass M_q are neglected, the interaction (5.3a) gives the mass formula

$$M(n) = nM_q + V(n) = n(M_q - \frac{cu}{2}) + Cu/2 \quad . \qquad (2.3b)$$

The interaction (2.2) and the mass formula (2.3b) were first proposed by Nambu,[17] and the saturation properties of the interaction were considered by Greenberg and Zwanziger.[18] However, the remarkable properties of this interaction as demonstrated above in the simplified example of the analogous deuteron world have received little attention.

2.4 Answers to the Triality and Meson-Baryon Whys

The formula (2.3b) can test the triality why or the meson-baryon why by showing whether observable "zero mass" hadron states exist for a given number of quarks and antiquarks. However, it cannot test the exotics why, since it gives no information about the spatial properties of the states. It cannot distinguish between one-particle states and multiparticle scattering states and all zero-triality exotic states are allowed as multiparticle states.

Since C is positive definite and has the eigenvalue zero only for a singlet[45] in $SU(3)_{color}$, and $u \geq 0$ as is evident from the two-body system, the state of the n-particle system with the strongest attractive interaction is a color singlet. Since the interaction is a linear function of n all such singlet states have zero mass if $cu/2 = M_q$. For this case

$$M(n) = (C/c)M_q \text{ if } cu/2 = M_q \quad . \tag{2.3c}$$

The model thus gives observable hadron states for all quark and antiquark configurations for which C = 0 states exist. Since C = 0 states exist only for configurations of triality zero, this answers the triality why.

The meson-baryon why is also answered by this interaction, since zero mass is attained both in two-body and three-body systems. To obtain C = 0, the two-body system must be a quark-antiquark pair, while the three-body system must be a three quark state, totally antisymmetric in color space. The approximation of neglecting the dependence of u_{ij} on i and j is justified in these two cases since there is only one pair in the two-body system, and a totally antisymmetric function has the same wave function for all pairs. The values[18] of the interaction parameter C-nc and the mass parameter C/c are listed in Table 2.1 for all states of the two-body system. These show that the quark-quark interaction in the baryon is exactly half of the quark-antiquark interaction in the meson, as required for the meson-baryon puzzle. The diquark mass is thus equal to one quark mass, since its interaction only cancels the mass of one of the two quarks.

Table 2.1 Values of the Interaction and Mass Parameters C-nc and C/c

System	$SU(3)_{color}$	Representation	C	C-nc	C/c
quark-quark	triplet	(antisymmetric)	4/3	-4/3	1
quark-quark	sextet	(symmetric)	10/3	+2/3	5/2
quark-antiquark	singlet		0	-8/3	0
quark-antiquark	octet		3	+1/3	9/4

The interaction averaged over all quark-quark states is seen to be zero and similarly for all quark-antiquark states. An antiquark or quark added to a meson or baryon thus has a zero net interaction, as there can be no color correlations between particles in a singlet state and an external particle, and each pair feels the average interaction over all color states. This suggests that the exotics puzzle is also answered, and that the states of zero mass obtained from the interaction (2.2) for exotic quantum numbers are multiparticle continuum states rather than bound states or resonances.

2.5 The Exotics Why--Spatial Properties of Wave Functions

To examine the exotics why in more detail we consider the spatial dependence of the interaction (2.2) for the specific case of the two-quark-two-antiquark system, with an interaction u_{ij} depending only on the positions of the particles and not on momenta, spin and unitary spin.

We first note that the color exchange force of the form (2.1) or (2.2) gives no bound α-particle-like states of two quarks and two antiquarks. We consider a wave function which is totally symmetric in space for the four particles and is a color singlet,

$$\psi = \phi\,(r_1, r_2, r_3, r_4)\,\chi_0 \quad , \tag{2.4a}$$

where χ_0 depends upon the color variables of the four particles and couples them to an overall color singlet. Spin is disregarded. The expectation value of the interaction (2.2) with this wave function is given by Eq. (2.3a) with $C = 0$, $n = 4$ and

$$u = \langle \phi | u_{12} | \phi \rangle \quad . \tag{2.4b}$$

We can use u_{12} in Eq. (2.4b) since the wave function (2.4a) is symmetric in all pairs. Thus

$$\langle \psi | U(n) | \psi \rangle = -2uc \, . \tag{2.4c}$$

Let us now consider a wave function

$$\psi' = \phi\,(r_1, r_2, r_3 + X, r_4 + X)\chi_0(12)\chi_0(34) \; , \tag{2.5}$$

where X is a very large distance like 1 kilometer, and $\chi_0(12)$ and $\chi_0(34)$ couple the pairs (12) and (34) separately to a color singlet. The expectation value of the interaction (2.2) with this wave function involves only two terms in the interaction, those for $i, j = 1,2$ and $3, 4$ since all other pairs are separated by the large distance x. For each pair the interaction is given by Eq. (2.3a) with $C = 0$ and $n = 2$, and with u still given by Eq. (2.4b) because the dependence of the wave functions (2.4a) and (2.5) on r_{12} and r_{34} are identical. Thus

$$<\psi' | U(n) | \psi'> = -uc + (-uc) = -2uc = <\psi | U(n) | \psi> . \tag{2.6}$$

Thus any "α-particle-like" wave function can be broken up into two color singlet quark-antiquark pairs separated by a large distance for which the interaction energy is the same. There is always a gain in kinetic energy by allowing such a breakup, namely the kinetic energy required by the uncertainty principle to keep the separation of the pairs very small. Thus any "α-particle-like" state will be unstable against immediate breakup into two pairs. Since the results (2.5) and (2.6) hold for any value of X, the breakup can occur continuously with no change in potential energy, and there can be no barrier hindering the breakup.

Let us now consider the case of two separated pairs and possible long range "Van-der-Waals" type forces. For simplicity we consider the deuteron world model (2.1a). The generalization to SU(3) is straightforward. Let d be the distance between the centers of mass of pairs (12) and (34) and choose the x axis in the direction of d. It is then convenient to write the interaction operator (2.1) for the four particle system in the following form:

$$V(4) = \frac{1}{2} \sum_{i \neq j} v_{ij} = (\vec{t}_1 \cdot \vec{t}_2)\, v_{12} + (\vec{t}_3 \cdot \vec{t}_4)\, v_{34}$$
$$+ (\vec{t}_1 + \vec{t}_2) \cdot (\vec{t}_3 + \vec{t}_4)(v_{13} + v_{14} + v_{23} + v_{24})/4$$
$$+ (\vec{t}_1 - \vec{t}_2) \cdot (\vec{t}_3 + \vec{t}_4)(v_{13} + v_{14} - v_{23} - v_{24})/4$$
$$+ (\vec{t}_1 + \vec{t}_2) \cdot (\vec{t}_3 - \vec{t}_4)(v_{13} - v_{14} + v_{23} - v_{24})/4$$
$$+ (\vec{t}_1 - \vec{t}_2) \cdot (\vec{t}_3 - \vec{t}_4)(v_{13} - v_{14} - v_{23} + v_{24})/4 \,. \quad (2.7)$$

We consider color singlet wave functions for the four particle system. There are two independent couplings to an overall color singlet. We choose the basis in which the colors of the pairs (12) and (34) are diagonal. Both pairs can either be singlets or triplets (octets in SU(3)). For the wave function in which both pairs are singlets, the first two terms on the right hand side of Eq. (2.7) give the binding energies of the two pairs, the next three terms give zero, since either $\vec{t}_1 + \vec{t}_2$ or $\vec{t}_3 + \vec{t}_4$ annihilate the wave function with 12 and 34 individually coupled to singlets. The last term gives an off-diagonal matrix element which can produce a "polarization force." We rewrite this term by expanding the potentials around $r = d$,

$$V_{pol} = (\vec{t}_1 - t_2) \cdot (\vec{t}_3 - \vec{t}_4)\, x_{12}\, x_{34} (d^2 v / dr^2)_{r=d} \,. \quad (2.8)$$

This interaction is seen to connect the ground state configurations of the two bound pairs with excited states which are color octets and are p-wave excitations in configuration space. This interaction can produce an energy shift in second order perturbation theory which depends upon the distance d and could give a long range Van-der Waals force. The magnitude of this energy shift is given by the square of the matrix element of the interaction (2.8) divided by an energy denominator. Since the intermediate state has pairs (12) and (34) in color triplets, the excitation energy of this state and the corresponding energy denominator is dominated by the third term on the right hand side of Eq. (2.7) which does not vanish for color triplet states and depends upon V(d). Thus the energy shift is given approximately by

$$\Delta E = \left[(d^2V/dr^2)_{r=d}\right]^2 \cdot \left\langle x_{12}^2\right\rangle\left\langle x_{34}^2\right\rangle / 8V(d) \, . \tag{2.9a}$$

It is amising that for a linear potential, the expression (2.9a) vanishes because the second derivative of the potential is zero. For a logarithmic potential, or for a harmonic oscillator potential,

$$E = V_0\left\langle x_{12}^2\right\rangle\left\langle x_{34}^2\right\rangle / 8\, d^4 \left\langle \log(d/r_{12})\right\rangle \quad \text{for } V = V_0 \log(r) \tag{2.9b}$$

$$E = k \left\langle x_{12}^2\right\rangle\left\langle x_{34}^2\right\rangle / 8\, d^2 \qquad \text{for } V = kr^2 \, . \tag{2.9c}$$

Thus we see that even for confining potentials which increase to infinity at large distances, the residual force between two color singlet hadrons decreases rapily at large distances. Note that these estimates are large overestimates of the force because effects of retardation have been neglected and should add additional damping factors. The interaction (2.8) involves instantaneous color correlations between the two pairs, with each jumping simultaneously from color singlet to color triplet. The color-correlated triplet-triplet state involves instantaneous correlations which certainly cannot be maintained over large distances.

We have seen that exotic four quark states cannot be bound either in α-particle-like symmetric wave functions or in molecular type separated pairs. We now investigate a third possibility, a correlated four-particle state, which is neither an α particle nor a molecule. We consider in more detail the potential for any given spatial configuration which a 2×2 matrix in color space is explicitly constructed in Ref. 19 by evaluation of the λ-matrices in Eq. (2.2) and then diagonalized. The color degree of freedom was eliminated by use of a static approximation, analogous to the static Coulomb approximation in QED, which assumes that particle motion is slow in comparison with photon or colored gluon exchanges. This approximation is implied in all charmonium potential calculations, where a static potential is assumed to hold for a color singlet state of the two-body system, even though the colors of the individual constituents must be changing rapidly in time to make a color singlet state. The condition for validity of this approximation; namely that the time scale of color changes is much more rapid than the time scale of quark motion, is equivalent to the requirement that excitation of the color degree of freedom requires a much greater energy than excitations in space-time.[14,20]

Diagonalization of the potential matrix in color space gives the following "eigenpotentials" in the static approximation for the two color couplings.[14,19]

$$U' = (7/16)(u_\alpha + u_\beta) + (1/8)u_q \pm (3/16)\sqrt{8(u_\alpha - u_\beta)^2 + (u_\alpha + u_\beta - 2u_q)^2}\ , \quad (2.10a)$$

where

$$u_\alpha = u_{13} + u_{24};\ u_\beta = u_{14} + u_{23};\ u_q = u_{12} + u_{34}\ . \quad (2.10b)$$

To test the exotics puzzle we look for coordinate configurations where four-particle correlations may give stronger binding than in two noninteracting clusters. Since u_α and u_β appear symmetrically in (5.8) we need only consider values of $u_\beta \leq u_\alpha$. For any value of u_α the value of $u_\beta \leq u_\alpha$ which minimizes the interaction (2.10) is $u_\beta = u_\alpha$ with the negative sign for the square root. This gives

$$U' = -(8/3)u_\alpha - (2/3)(u_\alpha - u_q)\ . \quad (2.11)$$

This expression is minimized by choosing the minimum values of u_q consistent with a given value of u_α. For monotonically decreasing potentials this is achieved by placing the four particles at the corners of a square with the like particles at opposite diagonals.

For a square well potential the particles can be arranged in a square with the diagonal greater than the range of the forces and the sides less than the range. This configuration has $u_q = 0$ and forms a stable four-particle state with a binding 25% greater than that of two quark-antiquark pairs. However, the sharp edge of the square well is essential for this binding and does not seem reasonable physically. For smooth potentials without sharp edges such as Coulomb, linear, Gaussian, Yukawa or harmonic oscillator potentials Eq. (2.11) shows that such a four-particle cluster is less strongly bound than two noninteracting quark-antiquark pairs, and the system simply breaks up into two clusters. This leads to a description in which all states having exotic quantum numbers are just scattering states of particles which individually have nonexotic quantum numbers, and answers the exotics puzzle.

The eigenpotentials (2.10) can also be used to examine configurations described to a good approximation as a diquark and an antidiquark separated by a distance large compared with the diquark size. The quark-antiquark interaction should then be the same for all four quark-antiquark pairs; i.e. we neglect correlations between the motion of one particular quark in the diquark and one particular antiquark in the antidiquark. Then $u_\alpha = u_\beta$ and Eq. (2.10a) simplifies to give the two solutions

$$U' = \frac{1}{2}\, u_\alpha + \frac{1}{2}\, u_q \ , \tag{2.12a}$$

$$U' = \frac{5}{4}\, u_\alpha - \frac{1}{4}\, u_q \ . \tag{2.12b}$$

The eigenfunction corresponding to Eq. (2.12a) has both diquarks in the color triplet state. For Eq. (2.12b) both are in color sextet states.

Another case of interest is that of two separated quark-antiquark pairs. Here the neglect of correlations between particles gives $u_\beta = u_q$, and Eq. (2.12a) simplifies to

$$U' = u_\alpha \ , \tag{2.13a}$$

$$U' = -\frac{1}{8} u_\alpha + \frac{9}{8} u_q \ . \tag{2.13b}$$

The corresponding eigenfunctions have separated color singlet pairs for Eq. (2.13a) and separated octet paris for Eq. (2.13b).

Equation (2.13a) shows that the lowest state for two separated quark-antiquark pairs has an interaction which depends only on the spatial separations within each pair and is independent of the distance between pairs. There is no long range residual force between two separated color singlet states, as expected from the saturation property of the color charge force.[14,19]

3. THE LOG POTENTIAL

We now consider some of the properties of the log potential and argue that it is a reasonable one to use for a quasinuclear model for hadtons. We first note the characteristic scaling property[5,21] which motivated its introduction by Quigg and Rosner. Consider the Hamiltonian for a particle in a log potential,

$$H = (p^2/2m) + V \log(r/r_0)$$

$$= (p^2/2m_0)(m_0/m) + V \log(r/r_0) \ , \qquad (3.1)$$

where m is the mass of the particle and m_0 is some standard mass. Let us now introduce the scale transformation,

$$p' = p(m_0/m)^{1/2} \ , \qquad (3.2a)$$

$$r' = r(m/m_0)^{1/2} \ . \qquad (3.2b)$$

Then

$$H = (p'^2/2m_0) + V \log(r'/r_0) + \frac{1}{2} V \log(m_0/m) \ . \qquad (3.3)$$

The scale transformation thus reduces the Hamiltonian (3.1) for any mass m to the Hamiltonian (3.3) which has the standard mass m_0 and the same potential and only an added constant term. Thus the energy spectrum of the Hamiltonian H depends upon the mass m only by the additive constant $1/2\, V \log(m_0/m)$; all energy aplittings are independent of the mass m.

However, there is no theoretical justification from first principles or QCD for this log potential. It was only introduced because it is the potential which gives equality for the $\gamma - \gamma'$ and $\psi - \psi'$ splittings. The potential previously used with at least hand waving support from QCD was a combination of a Coulomb and a linear potential,[21] giving Coulomb behavious at short distances and linear confinement at large distances. The relation between this potential and the log potential is seen by considering the potential

$$V = (V_0/\chi)\,\sinh\left[\chi \log(r/r_0)\right]. \tag{3.4a}$$

For $\chi = 0$, this is just the simple log potential. For $\chi = 1$ it is the Coulomb + linear potential. For intermediate values of χ, it is convenient to rewrite the potential in the form

$$V = (V_0/2\chi)\left[(r/r_0)^{\chi} - (r_0/r)^{\chi}\right] . \tag{3.4b}$$

Equation (3.4) defines a family of potentials characterized by a parameter χ, which are all singular at the origin and confining at large distances. Both the singularity at the origin and the strength of the confinement at large distances become weaker when χ decreases, and at $\chi = 0$ these become the singularities at the origin and infinity of the log potential. In the vicinity of $r = r_0$, which defines the transition region between Coulomb and log for $\chi = 1$, the hyperbolic sine can be expanded to give the log potential. Thus the log potential is a good approximation to the potential (3.4) for all values of χ in the vicinity of $r = r_0$.

As long as the properties of the system being considered do not depend on the exact form of the potential at very small or at very large distances, the log potential may give a good approximation for any potential of the type (3.4) which is singular at short distances and confining at large distances. Thus the log may be very useful for calculations, even though it has no deep fundamental significance. We accept its use on this basis and do not attempt to justify it on any more serious grounds.

The log potential can also be placed in a hierarchy of power law potentials. Potentials which vary as a positive power of r, like the linear or harmonic oscillator potentials, are always confining at large distances and approach a constant at the origin which can be chosen as zero energy. The spectrum is discrete and there is no continuum. The splittings of the energy levels decreases with increasing mass of the particle. Potentials which vary as a negative power of r, like the Coulomb potential, are singular at the origin and go to zero at infinity. They are not confining and have a continuous spectrum, with the possibility of a discrete spectrum as well if the potential is attractive. The splittings of the energy levels increases with increasing particle mass.

The log potential, which is in some sense the limiting case of a power law potential with the power zero is intermediate between the two cases. It is singular both at the origin and at infinity, has a discrete spectrum and no continuum, and the splittings of energy levels remains constant with changing particle mass.

It is also instructive to note that the characteristics of the energy spectrum of the log potential are nearly midway between the Coulomb and harmonic oscillator cases. The harmonic oscillator has an equally spaced set of energy levels. The Coulomb potential has energy levels with a spacing that decreases very rapidly with increasing energy. The log potential has a spectrum of energy levels with a spacing that decreases with increasing energy but not as rapidly as the Coulomb case. A convenient quantitative measure of this feature of the spectrum is the ratio (D-P)/(P-S) of the spacing between the lowest D state and the lowest P state to the spacing between the lowest P state and the ground state.[8] For the harmonic oscillator this is 1.0. For the Coulomb potential it is 0.2. For the log it is 0.6, just midway between the two.

We also recall the result from Eq. (1.10) that the expectation value of the kinetic energy in a multiquark system with two-body logarithmic potentials is given by a c-number rather than the expectation value of an operator. It depends only upon the strength parameter of the logarithmic potential and is independent of particle masses or the degree of excitation of the wave function. Here again it is in between the negative and positive power law potentials. In positive power potentials like the harmonic oscillator, the kinetic energy decreases with increasing mass of the particle and increases with the degree of radial excitation. In negative power law potentials, like the Coulomb potential the kinetic energy increases with increasing particle mass and decreases with higher radial excitation. In the log potential the kinetic energy is constant with both particle mass and radial excitation.

4. THE QUASINUCLEAR COLORED QUARK MODEL

The discovery of the first pion-nucleon and pion-pion resonances as low-lying p-wave resonances with equal widths suggested that mesons and baryons were composite objects with very similar structures, rather than elementary objects as different from one another as photons and electrons. The non-relativistic quark model described these first resonances and their photoexcitation as magnetic dipole excitations of quark spin flip for both mesons and baryons. The quark model also succeeded in describing other similar properties of mesons and baryons, including high energy scattering and reaction processes and strong, electromagnetic and weak decay processes. The introduction of the color degree of freedom[19] explained the difference between quark-antiquark interactions which made the low-lying states of the multiquark system appear as three-body states while the lowest states containing both quarks and antiquarks were two-body states.

Although meson and baryon spectra were seen to be qualitatively similar, quantitative relations between the mass splittings were difficult to obtain. Two basic physical assumptions are necessary to relate meson and baryon spectra: (1) a relation between the quark-antiquark forces binding mesons and the quark-quark forces binding baryons; (2) radial scaling properties of these interactions and of the baryon and meson wave functions. The assumption of color exchange forces has successfully related the gross features of meson and baryon spectra, but has been inadequate to describe the finer details. Quantitative estimates of mass splittings are sensitive to the difference in sizes of meson and baryon wave functions, and to flavor-dependent size effects arising from mass differences between quarks of different flavors. Since these size effects are model-dependent, it has been difficult to obtain significant predictions without introducing too many free parameters.

The successful description of the charmonium spectrum using the nonrelativistic quark model[23,24] suggests that it is reasonable to describe heavy quark systems by a Schroedinger equation for colored quarks interacting with a confining two-body color-exchange potential and no additional bag. We extend this model to the three-quark system and assume that baryons are described by the same Schroedinger equation with the same two-body forces, and that the effective matrix elements of the two-body interaction in the three-body system are related to those for the two-body system by simple scaling laws obtained by analysis of heavy quarkonium spectroscopy. In this way we construct a "Quasinuclear Colored Quark Model" for hadrons with the same spproach as that of nuclear physics, to determine the properties of the n-body system from the known properties of the two-body system. In the hadron case, where free quark scattering is not observed, the only input comes from effective matrix elements of the two-body interaction in bound quark-antiquark states, and we must use this information for the quark-quark interactions in three-body systems. We also use a nonrelativistic formulation for light quarks which are certainly relativistic in light hadrons. However, the results for the magnetic moments shown in the introduction above indicate that this approach works, even though we do not yet understand why. We therefore continue to use it wherever we can, to see where it continues to work and where it might break down.

The recent discovery that the $\psi - \psi'$ and $\psi - \psi'$ mass splittings are equal has motivated the introduction of a model with a flavor-independent logarithmic potential[2] whose mass splittings depend only on the strength of the potential and are independent of reduced mass. We extend this model to the conventional hadron spectrum and compare its predictions for orbital, spin and strangeness mass splittings with experiment. This model makes possible quantitative predictions relating meson and baryon spectra in which baryon mass splittings are predicted with only meson mass splittings used as inputs. The log potential provides a unique prescription without free parameters. The success of this prescription should be considered as further evidence for the common structure and interactions of mesons and baryons, rather than as a test for the details of the potential. Any potential with similar scaling properties in the relevant radial domain would presumably make similar predictions.

We first consider orbital excitations. Since clearly defined radially excited states are not easily seen experimentally, in contrast with the heavy quarkonium systems, we choose for comparison of theory and experiment the S, P and D states with "stretched" angular momenta; namely those with the highest values of spin and J for a given configuration. These are the 1-, 2+ and 3- mesons and the 3/2+, 5/2- and 7/2+ baryons. The results are shown in Table 4.1. The masses of the isovector mesons ρ, A2 and g are taken as input along with the lowest states in the K^*, ϕ and nonstrange baryon families. The masses of the remaining excited states are then predicted with no free parameters. Predictions from harmonic oscillator and Coulomb potentials are presented for comparison.

Three types of predictions were considered.

1. The ratio of the P-D and S-P splittings predicted to be 0.6 by the log potential model, 1.0 by a harmonic oscillator potential and 0.2 by a Coulomb potential. The values 0.68, 0.65 and 0.58 obtained for the ρ, K^* and baryon systems respectively support the log potential model.

2. Flavor-independent mass splittings predicted for the ρ, K^* and ϕ systems. The results in Table 4.1 show reasonable agreement.

3. Extension to baryons. This requires relating quark-quark and quark-antiquark interactions and treating the three-body problem. A

straightforward analysis discussed in detail below leads to the prediction that baryon splittings are reduced by a factor 3/4 relative to meson splittings. The results in Table 4.1 show surprising agreement.

For the spin splittings we assume that the scaling property of the spin dependent part of the two-body interaction is that suggested by simple arguments based on the logarithmic potential. This leads immediately to predictions for baryon spin splittings with meson spin splittings used as input and no parameters, listed in Table 4.2.

For the strangeness splittings we assume two sources of SU(3) symmetry breaking: (1) a constant mass difference between strange and nonstrange quarks over the whole spectrum; (2) the dependence upon quark masses of the spin dependent interaction already used for spin splittings. This again gives predictions for baryon spin splittings with meson spin splittings used as input and no parameters, also listed in Table 4.2.

The details of the model which lead to these predictions are discussed below. The essential numerical factors which appear in relations between meson and baryon spectra are: (1) a factor 1/2 between strengths of quark-quark and quark-antiquark potentials which comes from color couplings and (2) a factor 3/4 which comes from scaling of baryon and meson wave functions. It is the factor 3/4 which is new and which is responsible for the success of the predictions listed in Tables 4.1 and 4.2. It is clear from Table 4.2 that reducing this factor 3/4 would lead to better agreement for spin splittings, but this is parameter juggling, which obscures the physics if it has no strong theoretical motivations.

Table 4.1 Orbital Splittings from Logarithmic Potential Model
Theoretical and Experimental Values of Masses in MeV

	I = 1 Mesons	Kaon Family		Family		Baryon Family		Harmonic Oscillator Model
	Expt	Theory	Expt	Theory	Extp	Theory	Expt	
L = 0	770*	*	892*	*	1020*	*	1232*	*
L = 1	1310*	1432	1421	1560	1516	1637	1670	1700
L = 2	1680*	1802	1765	1930		1914	1925	2020
	Ratio of P-D and S-P Splittings					(Coulomb Potential Gives 0.2)		
	0.68	0.6	0.65	0.6		0.6	0.58	1.0

* Denotes Input

Table 4.2 Spin and Strangeness Splittings from Logarithmic Potential Model
Theoretical and Experimental Mass Differences in MeV

Spin Splittings			Strangeness Splittings		
Mass Difference	Theory	Expt	Mass Difference	Theory	Expt
$M(\rho) - M(\pi)$	*	630	$M(K^*) - M(\rho)$	*	122
$M(K^*) - M(K)$	*	396	$M(\Sigma^*) - M(\Delta)$	137	153
$M(\Delta) - M(N)$	354	292	$M(\Xi^*) - M(\Sigma^*)$	149	148
$M(\Sigma^*) - M(\Sigma)$	223	192	$M(\Xi) - M(\Sigma)$	149	125
$M(\Xi^*) - M(\Xi)$	223	216	$M(\Omega) - M(\Xi^*)$	160	139
$M(\Sigma) - M(\Lambda)$	88	77	$M(\Lambda) - M(N)$	180	177

* Denotes Input

The overall agreement with predictions of the quasinuclear colored model with the logarithmic potential is impressive and suggests further investigation, both theoretical and experimental.

We now consider multiquark systems in detail: The origin of the baryon factor 3/4 will be seen explicitly, and the formulation will be sufficiently general to be applicable to systems with more than three quarks. Our phenomenological "quasinuclear nonrelativistic colored quark model" is motivated by ideas similar to QCD but not justified by any rigorous argument. The two basic assumptions of the model are: (1) the quasinuclear approximation of n constituents interacting with two-body forces without any additional parton-antiparton pairs, gluons or bag; (2) a color-exchange force with a color dependence given by Eq. (2.2), and no additional Wigner force.

The quasinuclear assumption is implicit in all conventional spectroscopy as well as in the successful charmonium calculations, where all the confinement effects are in the two-body potential and there is no bag. The pure color exchange assumption is required to obtain a long range force which confines quarks within hadrons, but leaves no residual long range forces between physical hadrons. Any other long range force at the quark level gives long range hadron-hadron forces. Arguments from QCD which base this color exchange force on the color properties of the one gluon exchange contribution are misleading because the experimental evidence supporting color exchange is in the long range (infra red) part of the force which is definitely not one gluon exchange. We accept color exchange on phenomenological grounds as necessary to fit the experimental spectrum with no rigorous justification at this stage. We do not need to choose between models with quark confinement and those with heavy liberated quarks, but can include both cases by appropriate choices of the radial dependence of the interaction.

The model hamiltonian for a system of n particles which can be any combination of quarks and antiquarks is (3, 4):

$$H = \sum_{i=1}^{n} \frac{p_i^2}{2m} - \frac{3}{16} \sum_{\alpha=1}^{8} \sum_{i>j} \lambda_i^\alpha \lambda_j^\alpha u_{ij}, \tag{4.1}$$

where λ_i^α and λ_j^α are the λ-matrices for the color SU(3) group for particles i and j, u_{ij} depends on all the noncolor variables of particles i and j, and the normalization factor (-3/16) is chosen so that the potential is exactly equal to u_{ij} for the case of a quark-antiquark pair in the color singlet state. Thus for the physical mesons

$$H_{mes} = \sum_{i=1}^{2} \frac{p_i^2}{2m} + u_{12} . \tag{4.2}$$

The interaction (4.1) was first introduced by Nambu[17] who obtained it from one gluon exchange and considered only the color degree of freedom to show that it gave a spectrum where all low-lying states were color singlets. The dynamics of a similar phenomenological interaction were considered in a series of previous papers[25,19,14] which introduced the spatial dependence and analyzed the simplest non-trivial case where color and space do not factorize, namely the two-quark-two-antiquark system. Calculations showed that the interaction (4.1) did not bind four-particle states and gave only two-meson scattering states if a reasonable radial dependence was assumed for the interaction and spin dependence was neglected. Spin dependence was later introduced by De Rujula et al,[4] who showed that the sign of one gluon exchange predicted the right sign for the N-Δ and Σ-Λ mass splittings. Jaffe[26] has shown that the spin-dependent interaction responsible for these mass splittings can also lead to the binding of exotic multiquark configurations.

For a system of quarks which is totally antisymmetric in color, as in the baryon case, the summation of the λ-matrices can be evaluated explicitly in Eq. (4.1) and space and color factorize to give the result

$$H_{bar} = \sum_{i=1} \frac{p_i^2}{2m} + \frac{1}{2} \sum_{i>j} u_{ij} \; . \tag{4.2b}$$

For the three particle system it is convenient to express the hamiltonian (2) in terms of the center-of-mass momentum P and two independent relative co-ordinates and their canonically conjugate momenta,

$$\vec{x} = \vec{r}_1 - \vec{r}_2 \ , \tag{4.3a}$$

$$\vec{y} = (\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)/\sqrt{3} \ , \tag{4.3b}$$

$$\vec{p}_x = (\vec{p}_1 - \vec{p}_2)/2 \ , \tag{4.4a}$$

$$\vec{p}_y = (\vec{p}_1 + \vec{p}_2 - 2\vec{p}_3)/2\sqrt{3} \ , \tag{4.4b}$$

$$H = \frac{P^2}{6m} + \frac{p_x^2}{m} + \frac{p_y^2}{m} + \frac{1}{2}\left[u(\vec{x}) + u\left(\frac{\vec{x}}{2} + \frac{\sqrt{3}\vec{y}}{2}\right) + u\left(\frac{\vec{x}}{2} - \frac{\sqrt{3}\vec{y}}{2}\right)\right] \ . \tag{4.5}$$

Two special cases of interest are the harmonic oscillator and logarithmic potentials,

$$u_{osc} = k \ r \ , \tag{4.6a}$$

$$u_{log} = V \log \ r \ . \tag{4.6b}$$

For these potentials the Hamiltonian (4.5) becomes

$$H_{osc} = \frac{P^2}{6m} + \frac{p_x^2}{m} + \frac{p_y^2}{m} + \frac{3}{4} \ k \, [x^2 + y^2] \ , \tag{4.7}$$

$$H_{log} = \frac{P^2}{6m} + \frac{p_x^2}{m} + \frac{p_y^2}{m} + \frac{1}{2} \ V\left[\log x + \log\left(\frac{\vec{x}}{2} + \frac{\sqrt{3}\vec{y}}{2}\right) + \log\left(\frac{\vec{x}}{2} - \frac{\sqrt{3}\vec{y}}{2}\right)\right]. \tag{4.7b}$$

Equation (4.7a) shows the factor (3/4) relating the strengths of the potentials in the baryon and meson cases. However, the harmonic oscillator model does not have the desired scaling property, as shown in Table 4.1. Its level splittings change with mass and are proportional to the square root of the potential strength for constant mass.

For the logarithmic potential (4.7b) the two internal degrees of freedom are not separable. The factor 3/4 does not appear explicitly in the Hamiltonian but can be seen from application of the virial theorem and simple sum rules. The "scale" of the energy spectrum can be defined by the quantity

$$\overline{E}(x) = \frac{\sum_i \langle 0|x|i\rangle\langle i|x|0\rangle (E_i - E_0)^2}{\sum_i \langle 0|x|i\rangle\langle i|x|0\rangle (E_i - E_0)} \quad . \tag{4.8}$$

This is the ratio of the mean square energy to the mean energy of the states excited from the ground state by the operator x, with a weighting factor proportional to the square of the transition matrix element. For the case where all the transition strength is dominated by a single excited state (as is exactly true for the harmonic oscillator) the quantity $\overline{E}$ is just the excitation energy of this state. For the case of the Hamiltonian (4.5) and the operators x and y, $\overline{E}$ can be evaluated exactly by using commutators with H and closure to give

$$\overline{E}(x) = \frac{\sum_i \langle 0|[x,H]|i\rangle\langle i|[H,x]|0\rangle}{\frac{1}{2}\sum_i \langle 0|[x,H]|i\rangle\langle i|x|0\rangle - \langle 0|x|i\rangle\langle i|[x,H]|0\rangle} = \frac{\hbar^2\langle p_x^2\rangle/m^2}{\hbar^2/2m} =$$

$$= \frac{2\langle p_x^2\rangle}{m} \quad , \tag{4.8b}$$

and similarly for $\overline{E}(y)$.

The virial theorem gives for the general Hamiltonian (4.1)

$$\langle T\rangle = (-3/32) \sum_{\alpha=1} \sum_{i>j} \langle r_{ij}(du_{ij}/dr_{ij})\lambda_i^{\alpha}\lambda_j^{\alpha}\rangle . \qquad (4.9a)$$

For the case where the space and color factorize in this expression, the summation over color can be evaluated explicitly using the trick of Eq.(2.3a) to give

$$\langle T\rangle = (-3/16)(C-nc)\langle r_{ij}\, du_{ij}/dr_{ij}\rangle = \frac{n}{4} \langle r_{ij}\, du_{ij}/dr_{ij}\rangle , \qquad (4.9b)$$

where $C=0$ for a color singlet state and $c=4/3$. Note that space and color factorize trivially for the case of a logarithmic potential, where $r_{ij}\, du_{ij}/dr_{ij}$ is a c-number and is independent of i and j. For this case Eq. (4.9b) shows that the kinetic energy of any color singlet state is proportional to the number of particles n and is the same for all color singlet states with the same value of n. This tells us immediately that the mean kinetic energy of a baryon is 3/2 the mean kinetic energy of a meson. This can be seen explicitly for the case of the logarithmic baryon Hamiltonian (4.7b).

$$\frac{1}{2}\left[\frac{\langle p_x^2\rangle}{m} + \frac{\langle p_y^2\rangle}{m}\right] = \frac{3}{4}\left(\frac{1}{2}\ V\right). \qquad (4.9c)$$

Thus

$$\frac{1}{2}\left[\overline{E}(x) + \overline{E}(y)\right] = \frac{3}{4}\ V . \qquad (4.9d)$$

For the meson case, with only one relative co-ordinate $\vec{r}$ and the log potential, $\overline{\overline{E}}(r) = V$. Thus Eqs. (4.9c) and (4.9d) show that the mean kinetic energy and the mean excitation energy for each degree of freedom is 3/4 of the value in the corresponding meson case.

Encouraged by the success of the scaling potential shown in Table 4.1, we attempt to extend this approach to the spin dependent part of the interaction u(r). We assume that u_{ij} contains a term u^s_{ij} proportional to $\vec{\sigma}_i \cdot \vec{\sigma}_j$ which can be treated as a perturbation. The spin splittings are therefore given by the expectation values of the spin dependent interaction in the eigenfunctions of the log potential. To determine the scaling properties of the interaction, we make the standard assumption[4,26] that it is a "color-magnetic" interaction between two quarks and is therefore inversely proportional to the product of the quark masses.

$$u^s_{ij} = V^s \, \frac{\vec{\sigma}_i \cdot \vec{\sigma}_j}{m_i \cdot m_j} \, f(r) \ , \tag{4.10a}$$

where V^s is the strength of the spin-dependent potential and includes the color dependent factors in Eq. (4.1), and f(r) is some function of the radial distance between the quarks. If we are only interested in the scaling property, and assume that the spin-independent color charge force is scale invariant; i.e. a log, then dimensional considerations force f(r) to have the form $(1/r^2)$ to compensate for the additional mass factors. For scaling properties this is equivalent to a factor p^2, where p is the relative momentum of the pair. The scaling properties of the spin-dependent interaction are therefore given by the expectation value of the modified interaction

$$w_{ij} = V^s \frac{\vec{\sigma}_i \cdot \vec{\sigma}_j}{(m_i+m_j)} \frac{p^2}{m_r} = V^s \frac{2\,\vec{\sigma}_i \cdot \vec{\sigma}_j}{m_i+m_j} t_{ij} \quad , \tag{4.10b}$$

where $m_r = m_i \cdot m_j/(m_i+m_j)$ is the reduced mass of the pair and t_{ij} is the kinetic energy of the relative motion.

From the Hamiltonian (4.1), the ansatz (4.10b) for the spin-dependent part of the potential, and the assumption that hadrons containing strange quarks have an additional contribution to the mass proportional to the number of strange quarks, we obtain a simple unified formula for the description of meson and baryon masses,

$$M = A(n) + Bn_s + \sum_{i>j} K_{ij} \left\{ [\langle u_{ij}\rangle/(n-1)] + \left[nK_{ij}\, \vec{\sigma}_i \cdot \vec{\sigma}_j \langle v_{ij}\rangle / 2(m_i+m_j) \right] \right\} , \tag{4.11a}$$

where the color coupling factor

$$K_{ij} = (3/32) \sum_{\alpha} \lambda_i^{\alpha} \lambda_j^{\alpha} \quad , \tag{4.11b}$$

n is the total number of quarks and antiquarks in the system, n_s is the total number of strange quarks and antiquarks, λ_i^{α} are the λ-matrices for the color SU(3) group for particle i, $\langle u_{ij}\rangle$ and $\langle v_{ij}\rangle$ are reduced matrix elements for the two-body spin independent and spin dependent interactions respectively, m_i is the mass of quark i, and A and B are parameters. The n-dependence of A is unknown. Thus we only relate mass splittings and not absolute mass values. The parameter B and the reduced matrix elements $<u_{ij}>$ and $<v_{ij}>$ are universal for all n. They may depend upon the angular momentum quantum numbers of particles i and j but not on spin, flavor or radial wave functions.

The spin dependent term is reduced to the form (4.11a) by noting that $<1/r^2>$ scales like $<p^2>$ and using the virial theorem,

$$<t_{ij}> = <p_{ij}^2> (m_i + m_j)/2m_i m_j = nK_{ij}\, U/2 \ , \tag{4.11c}$$

where t_{ij} and p_{ij} are the relative kinetic energy and the relative momentum of particles i and j and U is the strength of the log potential. The n-dependence of the overall scaling factor is needed to relate meson and baryon spin splittings, but specific flavor dependence of the quark-mass factors has no significant effect on our results.

For the meson and baryon cases, the color factors can be evaluated explicitly to give the relations

$$M(\text{meson}) = A(\text{mes}) + Bn_s + <u_{ij}> + \vec{\sigma}_i \cdot \vec{\sigma}_j <v_{ij}> / (m_i + m_j) \ , \tag{4.12a}$$

$$M(\text{baryons}) = A(\text{bar}) + Bn_s + \sum_{i>j} \left[(1/4)<u_{ij}> + (3/8)\, \vec{\sigma}_i \cdot \vec{\sigma}_j <v_{ij}> / (m_i + m_j) \right] . \tag{4.12b}$$

These differ from similar formulas in Ref. (4) by one essential new ingredient, the scaling factors (1/4) and (3/8) obtained from the log model which removes ambiguities of wave functions and matrix elements. These enable the quantitative predictions of baryon mass splittings from meson mass splittings (with no free parameters) displayed in Tables 4.1 and 4.2. The remarkable agreement shows that mesons and baryons do have a similar structure and should have a unified description, even though the simple nonrelativistic quark model may not be valid.

The most interesting prediction is the Λ-N mass difference which follows from very general grounds independent of the log potential and scaling factors. It holds in any model where all the flavor dependence appears in linear quark mass and spin-spin interaction terms proportional to n_s and $\vec{\sigma}_i \cdot \vec{\sigma}_j$ respectively. A $\vec{\sigma}_i \cdot \vec{\sigma}_j$ term does not contribute to the Λ-N mass difference because the interactions of the two strange-nonstrange pairs in the Λ cancel exactly,

$$\vec{\sigma}_u \cdot \vec{\sigma}_s + \vec{\sigma}_d \cdot \vec{\sigma}_s = (\vec{\sigma}_u + \vec{\sigma}_d) \cdot \vec{\sigma}_s = 0 \ . \qquad (4.13a)$$

We construct a linear combination of meson masses in which the contribution of the $\vec{\sigma}_i \cdot \vec{\sigma}_j$ term vanishes and obtain

$$M(\Lambda) - M(N) = (3/4)[M(K^*) - M(\rho)] + (1/4)[M(K) - M(\pi)] = 180 \text{ MeV}. \qquad (4.13b)$$

The result (4.13b) is in remarkable agreement with the experimental value of 177 MeV.

The remaining strangeness splitting predictions are dominated by this 180 MeV splitting due to the strange quark mass difference. Other model-dependent terms are down in the noise. Predictions for all baryons with strangeness 0 and -1 are given by combining Eq. (4.13) and spin splitting predictions listed in Table 4.2 and discussed below. The remaining three baryon masses must satisfy two general constraints from Ref. (9) and would be determined completely by the additional assumption of the equal-spacing rule for the decuplet. The specific model (4.1) used for Table 4.2 predicts insignificantly small deviations from the equal-spacing rule.*

The remaining predictions in Tables 4.1 and 4.2 depend upon the scaling factors (1/4) and (3/8) in Eq. (4.12b). The results for orbital excitations in Table 4.1 show reasonable agreement for the flavor-independent mass splittings predicted for the ρ, K^* and ϕ systems. Surprising agreement is shown for the predicted baryon splittings which depend upon the scaling factor (1/4) in Eq. (4.12b) but not on the spin dependent term.

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*Rubinstein[27] calculated strangeness splittings with the ad hoc assumption neglecting SU(3) breaking in the spin triplet state. This is now seen as a rough approximation for the $\vec{\sigma}_i \cdot \vec{\sigma}_j$ force whose triplet interaction is weaker than the singlet by a factor of three.

For the spin splittings the results in Table 4.2 arise from the scaling factor (3/8) in Eq. (4.12b) which gives the following new predictions:

$$[M(\Delta)-M(N)]/[M(\rho)-M(\pi)] = [M(\Sigma^*)-M(\Sigma)]/[M(K^*)-M(K)] = 9/16\,, \quad (4.14a)$$

$$M(\Sigma)-M(\Lambda) = (3/8)\{[M(\rho)-M(\pi)]-[M(K^*)-M(K)]\} = 88\ \text{MeV}\,. \quad (4.14b)$$

The equality of the $\Sigma^* - \Sigma$ and $\Xi^* - \Xi$ splittings predicted in Ref. (9) holds for the most general two-body interaction. The 24 MeV discrepancy indicates the inherent error in the baryon sector in any model with only two-body interactions. Thus the 15-20% agreement of the new predictions (4.14) with experiment is as good as can be expected.

The same approach applied to charmed particles predicts the mass values 2305, 2435 and 2570 for the charmed baryons C_0, C_1 and C^* from the masses of the charmed mesons by the analogs of Eqs. (4.14) and (4.14). The spin splitting of the charmed-strange mesons $M(F^*) - M(F)$ is predicted by Eq. (4.12b) to be 122 MeV with the D^* and D masses used as input in addition to the ρ, π, K and K^*. This prediction is sensitive to the flavor dependent quark mass factor. If the sum of the quark masses is replaced by the product as in Ref. (4) the corresponding prediction is 87 MeV.

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