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Transverse Lattice Theory of Quantum Chromodynamics*

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A. INTRODUCTION

Quantum chromodynamics (QCD) has been proposed as a complete lagrangian field theory for the description strongly interacting particles. The theory consists of colored quarks coupled to a color octet of Yang-Mills vector bosons. This theory can be solved in the short distance region using renormalization group methods as the effective-coupling constant vanished in this limit. This "asymptotic freedon" at short distance provides a fundamental basis for understanding the success of the quark-parton phenomenology.

Various methods have been employed to study the large distance behavior of this theory in the attempt to interpret strong interaction physics in terms of confined quarks and gluons. Classes of perturbation theory diagrams have been analyzed in order to extract the leading infrared behavior of the theory. While renormalization group equations seem to be applicable,[1] no self consistant solution to the infrared problem appears to exist at weak coupling in a manner analogous the behavior at short distance. Another approach employs semiclassical methods[2] to include effects missing in a perturbation theory analysis but present at small but non zero effective coupling. Instantons and other semiclassical objects appear to have important consequences for the chiral structure of QCD[3] but their possible role in the confinement mechanism is not yet evident. A third approach involves the reformation of QCD in terms of lattice gauge theories. Wilson[4] has studied

QCD using a four dimensional euclidean lattice while Kogut and Susskind[5] have constructed a hamiltonium version of QCD on a three dimensional spacial lattice. The lattice formulations permit a study of the strong coupling limit where confinement is a manifest consequence of gauge invariance. Whether these lattice versions of QCD truly reflect the confinement aspects of continuum QCD remains an open question at this time. Also the qualitative success of these theories with respect to strong interaction physics is certainly not excessively successful quantitatively particularly with respect to aspects involving chiral symmetry.

In this lecture I will discuss yet another approach to QCD based on a reformulation of the theory using a transverse lattice and infinite momentum frame techniques. This formulation of QCD has the unique advantage (disadvantage) that neither the weak coupling limit nor the strong coupling limit are trivial in the theory. This theory was developed in conjunction with R. B. Pearson.[6] The strong coupling aspects of the theory were analyzed by R. B. Pearson and E. Rabinovici.[7]

In Section B, I will briefly review certain aspects of the axial gauge description of QCD relevant to confinement. The transverse lattice theory is formulated in Section C with particular reference to the strong coupling (confinement) phase of QCD. The methods developed for study of hadronic bound states are described in Section D and Section E. In Section F, the preliminary results of this program are discussed.

B. AXIAL GAUGE FORMULATION OF QUANTUM CHROMODYMANICS

The quantum mechanics of non-abelian gauge theories is complicated by the fact that only the transverse degrees of freedom are independent. The standard methods of quantization (convariant gauge, temporal gauge, or coulomb gauge) all involve the introduction of additional unphysical degrees of freedom or "ghost" contributions. Hence the physical aspects of the theory may be obsured. This is true for both continuum and lattice versions of the theory.

In the axial gauge only the two independent degrees of freedom associated with each vector potential are quantized with the other fields determined as dependent variables. The standard lagrangian for QCD is given by

$$\mathscr{L} = -\frac{1}{4}\vec{G}_{\mu\nu}\cdot\vec{G}_{\mu\nu} + \bar{q}(i\not{D} - m)q \,, \tag{1}$$

where $$\vec{G}_{\mu\nu} = \partial_\mu\vec{A}_\nu - \partial_\nu\vec{A}_\mu + g\vec{A}_\mu \times \vec{A}_\nu \,,$$

and $$D_\mu = \partial_\mu + ig\vec{T}\cdot\vec{A}_\mu \,.$$

The rest frame axial gauge is defined by $\vec{A}_z = 0$. Standard canonical quantiqation results in the hamiltonian of Eq. (2) where the field $\vec{A}_0$ has been eliminated in favor of a "coulomb" potential,

$$H = \int dx_\perp \int dz \left\{ \frac{1}{2} \vec{P}_\alpha^2 + \frac{1}{2} (\partial_z \vec{A}_\alpha)^2 + \frac{1}{4} \vec{G}_{\alpha\beta}^2 + q^+ \left[\alpha_z \frac{1}{i} \nabla_z + \alpha_\beta \frac{1}{i} D_\beta + \beta m \right] q \right\} \tag{2}$$

$$+ \int dx_\perp \int dz \int dz' \, g^2 \, \vec{J}_0(z, x_\perp) \frac{1}{4} |z - z'| \cdot \vec{J}_0(z', x_\perp),$$

where
$$\vec{J}_0 = \frac{1}{g} \partial_\alpha P_\alpha + \vec{A}_\alpha \times \vec{P}_\alpha - q^+ \vec{T} q .$$

The specification of $\vec{A}_z$ and $\vec{A}_0$ does not completely determine the gauge. The hamiltonian of Eq. (2) is invariant under the residual symmetry of transverse gauge transformatives which are local in $x_\perp$ but are global with respect to (z, t). The charges which generate these transformatives are given by

$$Q(\vec{\Lambda}) = \int dx \; \vec{P}_\alpha \cdot \overrightarrow{D_\alpha \Lambda} + \vec{\Lambda} \cdot q^+ \vec{T} q \; , \tag{3}$$

with $\vec{\Lambda} = \vec{\Lambda}(x_\perp)$ independent of z. These charges satisfy the local algebra $[Q(\vec{\Lambda}), Q(\vec{\Lambda}')] = i \, Q(\vec{\Lambda} \times \vec{\Lambda}')$. Each term in the hamiltonian of Eq. (2) is seperately invariant under this residual symmetry.

The linear potential in Eq. (2) represents the focusing of the "coulomb" electric fields along the z direction in the axial gauge. If this theory is studied perturbatively, the usual quark-gluon dynamics is recovered but in a manner which is reminiscent of the spontaneous breakdown symmetry in a Higgs theory in the coulomb gauge. We may see this effect by first ignoring the "coulomb" term in Eq. (2). To lowest order, the gluons have two independent modes. The mode with transverse polarization, $k_{\perp\alpha}\epsilon_{\perp\alpha} = 0$, has the usual dispersion $\omega^2_{\perp k} = k_z^2 + \vec{k}_\perp^2$. The "longitudinal" mode, $\epsilon_{1\alpha} = \hat{k}_{1\alpha}$, satisfies $\omega^2_{1k} = k_z^2$. When we include the "coulomb" term the linear term in $\vec{J}_0$ implies a mixing between the longitudinal mode and the "coulomb" potential which results in the usual dispersion for the longitudinal mode $\omega^2_{1k} = k_z^2 + k_\perp^2$ in perturbation theory. As in the Higgs theory, this effect may be interpreted as arising from the fact that there is a complete screening of the local color charge in the perturbation theory vacuum with only the global color charge as a residual symmetry. The local color screening may also be seen in the static coulomb potential where the linear (confining) "coulomb" potential of Eq. (2) is screened to become the usual $1/r$ potential.

At this point it is natural to speculate that color confinement would result in the true vacuum if the screening of the local color were incomplete. In this case the linear "coulomb" potential of Eq. (2) may reflect the true infrared structure of QCD. It is difficult to see how this "symmetrical" phase can naturally occur directly in the continuum theory. In the next section, we formulate a cutoff version of QCD where the possible realization of a symmetrical phase is more evident.

C. TRANSVERSE LATTICE FORMULATION OF QUANTUM CHROMODYNAMICS

A gauge theory may be seperated into potentials and the independent physical degrees of freedom. The precise nature of this seperation is of course gauge dependent. In the axial gauge, we have indicated the possibility of different realizations of the residual gauge symmetries than that indicated in perturbation theory. In order to discuss these symmetry properties, we introduce a cutoff version of QCD where all the gauge symmetries are preserved but where only the independent degrees of freedom are directly affected by the cutoff.

The cutoff procedure makes use of a discrete spacial lattice for the transverse directions ($\alpha, \beta = x, y$) while the longitudinal directions ($\mu, \nu = t, z$) remain continuous. The transverse gauge degrees of freedom are associated with links of the transverse lattice. The longitudinal gauge potentials and quark fields are to be associated with sites of the lattice. The gauge fields are defined by

$$\begin{aligned} \vec{B}_{\vec{n}\mu}(t,z) &= a\,\vec{A}_{\mu}(\vec{n}a,t,z)\;, & \mu &= t,z \\ M_{\vec{n}\hat{\alpha}}(t,z) &= \frac{1}{g}\,e^{ig\,\vec{T}\cdot\vec{A}_{\alpha}(\vec{n}a,t,z)}\;, & \alpha &= x,y\;, \end{aligned} \tag{4}$$

where a is the lattice spacing and $\{\vec{T}\}$ are a matrix representation for the color generators. The link variables $M_{\vec{n}\hat{\alpha}}(t,z)$ are analogous to the U variables introduced by Wilson[4] except for their normalization and the fact that they are fields in the longitudinal variables.

Using these lattice fields, the lattice analogue of the action of Eq. (1) for the pure gauge theory becomes

$$S = \int dtdz \sum_{\vec{n}\mu\nu} \left\{ -\frac{1}{4} \vec{G}^{\,2}_{\vec{n}\mu\nu} \right\}$$

$$+ \int dtdz \sum_{\vec{n}\mu\alpha} \mathrm{tr} \left\{ D_\mu M_{\vec{n}\hat{\alpha}} D_\mu M^+_{\vec{n}\hat{\alpha}} \right\} \tag{5}$$

$$+ \int dtdz \sum_{\vec{n}\alpha\beta} \frac{H}{a^2} \mathrm{tr} \left\{ M_{\vec{n}\hat{\alpha}} M_{\vec{n}+\hat{\alpha}\hat{\beta}} M^+_{\vec{n}+\hat{\beta}\,\hat{\alpha}} M^+_{\vec{n}\hat{\beta}} \right\},$$

where
$$\vec{G}_{\vec{n}\mu\nu} = \partial_\mu \vec{B}_{\vec{n}\nu} - \partial_\nu \vec{B}_{\vec{n}\mu} + \frac{g}{a} \vec{B}_{\vec{n}\mu} \times \vec{B}_{\vec{n}\nu}$$

and
$$D_\mu M_{\vec{n}\hat{\alpha}} = \partial_\mu M_{\vec{n}\hat{\alpha}} + i \frac{g}{a} \vec{T} \cdot B_{n\mu} M_{n\hat{\alpha}} - i \frac{g}{a} M_{n\hat{\alpha}} \vec{T} \cdot \vec{B}_{n\mu}.$$

The action of Eq. (5) is the simplest lattice action which reduces to continuum action in the naive continuum limit $a \to 0$ with g, $H = g^2$, and $\{\vec{A}_\mu(x_\mu)\}$ fixed. This action is not unique and other terms may be needed to obtain physical results for large lattice spacing.

This lattice action was constructed to be invariant under a full set of local gauge transformations. These transformations are associated with local color rotations at individual sites on the transverse lattice and may depend continuously on the longitudinal variables. The link fields, $M_{\vec{n}\hat{\alpha}}(z,t)$, are matrices which transform on the left according to color rotations at $(\vec{n}, z, t)$ and on the right according to color rotations at $(\vec{n} + \hat{\alpha}, z, t)$.

We may make use of this gauge freedom to study the theory in a longitudinal axial gauge. For the purposes of this lecture, we will use an infinite momentum axial gauge defined by $\vec{A}_{n-}(z,t) = 0$ where $\vec{A}_{\vec{n}\pm} = (\vec{A}_{\vec{n}t} \pm \vec{A}_{\vec{n}z})/\sqrt{2}$. The longitudinal potential $\vec{A}_{n+}$ may be eliminated in terms of the independent transverse fields, $M_{\vec{n}\hat{\alpha}}$, using the equations of motion. In this light cone gauge, the action of Eq. (5) becomes

$$\begin{aligned} S = &\int dx_+ dx_- \sum_{n\mu\alpha} \mathrm{tr}\,(\partial_\mu M_{\vec{n}\hat{\alpha}}\, \partial_\mu M^+_{\vec{n}\hat{\alpha}}) \\ &+ \int dx_+ dx_- \sum_{n\alpha\beta} \frac{H}{a^2}\, \mathrm{tr}\,(M_{\vec{n}\hat{\alpha}}\, M_{\vec{n}+\hat{\alpha}}\, M^+_{\vec{n}+\hat{\beta}\,\hat{\alpha}}\, M^+_{\vec{n}\hat{\beta}}) \\ &+ \int dx_- dx_+ dx'_+ \sum_n \frac{g^2}{4a^2}\, \vec{J}_{\vec{n}-}(x_- x_+)\, |x_+ - x'_+|\, \vec{J}_{\vec{n}-}(x_-, x'_+)\,, \end{aligned} \tag{6}$$

where the longitudinal charge density $\vec{J}_{\vec{n}-}(x_-, x_+)$ is given by

$$\vec{J}_{\vec{n}-}(x_-, x_+) = \sum_\alpha \mathrm{tr}\Big[\vec{T}\,(M_{\vec{n}\hat{\alpha}}\, i\, \overleftrightarrow{\partial}_-\, M^+_{\vec{n}\hat{\alpha}}) + \vec{T}\,(M^+_{\vec{n}-\hat{\alpha},\hat{\alpha}}\, i\, \overleftrightarrow{\partial}_-\, M_{\vec{n}-\hat{\alpha},\hat{\alpha}})\Big].$$

As was the case in Section B, the axial gauge leaves a residual gauge symmetry. The action of Eq. (6) is invariant color rotations at each site of the transverse lattice which are independent of t and z.

The conserved charge is given by

$$\vec{Q}_{\vec{n}} = \int dx_+ \ \vec{J}_{\vec{n}-} (x_-, x_+).$$

A perturbative treatment of the gauge theory assumes that this residual symmetry is spontaneously broken as the link fields are expanded as $M_{\vec{n}\hat{\alpha}} = \frac{1}{g} \, I + a \, \vec{T} \cdot \vec{A}_{\hat{\alpha}}(\vec{n}a, z, t) + \cdots$. With this formulation, the possibility that this residual gauge symmetry is not dynamically broken can be investigated.

If we focus only on the first term in the action of Eq. (6), we recognize SU(N) x SU(N) sigma model in two dimensions in the case where the full theory is a SU(N) gauge theory. The nonlinear sigma model is asymptotically free and renormalizable in two dimensions. The large N analysis of similar theories[8] indicates that only the disordered, symmetrical-phase is expected to exist contrary to the result of perturbation theory.

We may also study another piece of the gauge theory if we also include terms from the coulomb potential and focus on a single link of the lattice. An analysis of a similar theory[9] indicated that there could exist a broken symmetry, perturbation phase for weak coupling while the symmetrical phase exists for strong coupling. By studying these elements of the full gauge theory we conclude that the longitudinal dynamics of the lattice theory is nontrivial with respect to the possible realizations of the residual transverse gauge symmetries. Which phase actually is realized in the transverse lattice gauge theory would appear to be a detailed dynamical question. However it should also be clear that confinement of color would result in the disordered, symmetrical phase. For the remainder of this lecture we shall assume this to be true.

The link fields, $M_{\vec{n}\alpha}(z,t)$, used to define the action in Eq. (6) transform linearly under the transverse color rotations but are restricted by the nonlinear constraints, $MM^+ = M^+M = I/g^2$ and $\det\{Mg\} = 1$. The independent degrees of freedom have nonlinear transformation properties. <u>If</u> the gauge theory results in a symmetrical, disordered phase, then we expect linear realizations of the symmetry to provide a more appropriable discription of the physics especially for large lattice spacing.

We may obtain a linear realization of the symmetry by relaxing the nonlinear constraints on the link fields. Each component of the matrix, $M_{\vec{n}\hat{\alpha}}$, becomes an independent field. The physics of the nonlinear theory may be preserved through the addition of a local potential function of $M_{\vec{n}\alpha}$. The potential may be constructed such that the nonlinear theory results as a strong coupling limit of the linear theory. It may also be the case that the effective interactions in a large lattice approximation need not be strong. The nonlinear limit may then be combined with the continuum limit of the lattice theory.

In the linearized theory the full action may be written as the sum of the action of Eq. (5) and a local potential, V. V contains terms of the form

$$V = \int dtdz \sum_{n\alpha} \{ m \,\mathrm{tr}\, M_{\vec{n}\hat{\alpha}} M^{+}_{\vec{n}\hat{\alpha}} + \lambda_1 \,\mathrm{tr}\, (M_{\vec{n}\hat{\alpha}} M^{+}_{\vec{n}\hat{\alpha}})^2 + \lambda_2 [\det M_{\vec{n}\hat{\alpha}} + \det M^{+}_{\vec{n}\hat{\alpha}}] + \ldots \} , \tag{7}$$

where we have explicitly indicated those terms necessary to achieve the nonlinear theory as a strong coupling limit. In the light cone axial gauge, the linearized theory is obtained by adding the potential of Eq. (7) to the nonlocal terms in Eq. (6).

Since we will be interested primarily in the construction of the color singlet hadronic bound states of the theory, it is useful to consider the hamiltonian formulation of the theory. The infinite momentum frame hamiltonian is simply obtained from Eq. (6) and Eq. (7) with the result

$$H = -\int dx_+ \sum_{n\alpha\beta} \frac{H}{a^2} \,\mathrm{tr}\, (M_{\vec{n}\hat{\alpha}} M_{\vec{n}+\hat{\alpha}\,\hat{\beta}} M^{+}_{\vec{n}+\hat{\beta}\hat{\alpha}} M^{+}_{\vec{n}\hat{\beta}}) - \int dx_+ dx'_+ \sum_{n} \frac{g^2}{4a^2} \vec{J}_{n^-}(x_+) |x_+ - x'_+| \vec{J}_{n^-}(x'_+) + \int dx_+ \sum_{n\alpha} \{ m \,\mathrm{tr}\, (M_{\vec{n}\hat{\alpha}} M^{+}_{\vec{n}\hat{\alpha}}) + \lambda \,\mathrm{tr}\, (M_{\vec{n}\hat{\alpha}} M^{+}_{\vec{n}\hat{\alpha}})^2 + \ldots \} .$$

In the symmetrical vacuum, terms arising from normal ordering the hamiltonian may be absorbed as a mass renormalization. In the following sections we show how this hamiltonian may be used to determine what might be considered the strong coupling limit for the hadronic bound states of the non abelian gauge theory on the transverse lattice.

D. LONGITUDINAL DYNAMICS

In the previous section, a linearized hamiltonian formulation of QCD was defined on a transverse lattice. The physical degrees of freedom are associated the transverse link fields, $M_{\vec{n}\hat{\alpha}}$, which create and destroy the transverse gluons, link mesons in the linearized theory. The link mesons are generally expected acquire mass as indicated in the effective local potential, Eq. (7). On the lattice, the linear coulomb potential associated with each site of the lattice implies that finite energy states are color singlet bound states of link mesons (and quarks). In addition to the coulomb interaction the magnetic term (nonlocal box) in Eq. (8) results in a nonlocal interaction between the link mesons.

In this section, we focus on the longitudinal dynamics responsible for the color confinement of this theory. For a given configuration of link mesons in a color singlet state, we may indentify the direct coulomb interaction which does not change the link meson configuration. This part of the dynamics must be treated nonperturbatively as it is responsible for the binding of the link mesons.

In order to study the bound states we first expand the link meson fields in creation and destruction operators appropriate to the infinite momentum frame

$$M(x_+) = \int_0^\infty \frac{dk}{k} \{A_x f_k(x_+) + B_k^+ f_k^*(x_+)\}$$
$$[A_k, A_{k'}^+] = 2k\,\delta(k-k')\ ,\ [B_k, B_{k'}^+] = 2k\,\delta(k-k')\ , \tag{9}$$

where the color indices have been suppressed. A color singlet two gluon state is easily constructed using these operators.[6]

$$|M\overline{M}\ P> = \int_0^1 dx\ \Phi(x)\ (2x(1-x))^{-1/2}\ A_{xp}^+\ B_{(1-x)p}^+ |\,0>$$
$$<M\overline{M}\ P'\,|\,M\overline{M}\ P> = 2P\,\delta(P-P'),\int_0^1 dx\,|\Phi(x)|^2 = 1\ . \tag{10}$$

If we include only the mass term and the direct coulomb interaction in the hamiltonian, the equation of motion may be obtain simply by applying H to the state of Eq. (10). In this approximation, the two-body wavefunction, $\Phi(x)$, satisfies the wave equation

$$2P^+P^-\ \Phi(x) = m^2\left(\frac{1}{x} + \frac{1}{1-x}\right)\Phi(x)$$
$$-\frac{2g^2}{\pi a^2}\ C_N \int_0^1 \frac{dy}{(x-y)^2}\ \Phi(y)\ \frac{(x+y)(2-x-y)}{4[y(1-y)x(1-x)]^{1/2}}\ , \tag{11}$$

where C_N is the SU(N) Casimir in the singlet representation and we have used a principle value definition for the coulomb potential. This wave equation is analogous to the one considered by 't Hooft for the meson bound states in two dimensional QCD. It differs from his result by the "spin" factor which follows $\Phi(y)$ in Eq. (11). This wave equation can be solved numerically to obtain an approximately linear trajectory of states.

In a similar fashion, multimeson bound states may be studied by simply constructing the appropriate color singlet state parametrized by a wavefunction $\Phi(y_1 -- y_N)$ as in Eq. (10). The wave equation for Φ is simply derived by evaluating the matrix element of the hamiltonian in such a state. As an example the wave equation for a string which consists of a chain of mesons on different links bound together by the coulomb interactions at the connecting sites is given by

$$|P\ \Phi> = \sqrt{2}\int dy_1 \ldots \int dy_N\ \Phi\,(y_1 \ldots y_N)\,(2y_1 \ldots 2y_N)^{-1/2}\,\delta(1-y_1 \ldots -y_N)$$

$$A^+_{y_1 p} \ldots A^+_{y_N p}\ |\ 0>$$

$$2P_+P_-\ \Phi\,(y_1 \ldots y_N) = m^2\left(\frac{1}{y_1} + \ldots + \frac{1}{y_N}\right)\Phi(y_1 \ldots y_N) \tag{12}$$

$$-\ \frac{g^2}{\pi a^2}\, C_N \int_{y_1}^{y_2} \frac{dz}{z^2}\ \frac{(2y_1+z)\,(2y_2-z)}{\sqrt{2(y_1+z)2y_1 2(y_2-z)2y_2}}\ \Phi(y_1+z, y_2-z, y_3 \cdots y_N)$$

+ sum on cyclic permutations.

A WKB solution to this wave equation leads to a linear spacing of eigenvalues for the energy $E_\perp^2 = 2P_+P_-$. Complicated configurations with more than one meson associated with each link lead to modified coulomb interactions due to the more complex color structure of the bound states.

While the wave equation such as exhibited in Eq. (12) seem somewhat complex, a systematic approximation scheme can be developed.[7] It is easy to check that the amplitude Φ must vanish at the endpoints $y_k = 0$, with limiting behavior $\Phi(y_1 \ldots y_k \ldots Y_N) \to y_k^\beta$ as $y_k \to 0$. The parameter β is determined from a self-consistancy relation between the mass operator and the coulomb potential, $m^2 = 2\pi\beta \tan \pi\beta$. This condition differs from a similar condition for the quark bound states discussed by 't Hooft.[10] Contrary to the quark case, the "renormalized" meson mass can not be that of a tachyon. Since all binding energies are positive, this result implies that the gluon bound states should be heavier than the corresponding quark bound states.

Except for the end point behavior determined by β, the wave function Φ is expected to be a smooth function of $\{y_k\}$. We have found that good approximations to the wave functions can be obtained using the form $\Phi(y_1 \ldots y_N) = (y_1 \ldots y_N)^\beta$ x Polynomials. This form for the wavefunctions is also convenient for evaluating matrix elements of hamiltonian as all of the integrals can be computed analytically for polynomial wavefunctions. A systematic truncation of the longitudinal dynamics results if only independent polynomials below a certain degree are retained. The truncation reduces the hamiltonian to matrix form. Since the matrix elements can be computed analyticly in this basis, the numerical problem is greatly simplified.

As a result of these calculations, good numerical results for the spectrum of these bare hadron states are obtained as shown in Fig. 1. The bare hadrons are color singlet bound states of gluons (and quarks) and are identified with a specific configuration of link mesons on the transverse lattice. In the following section, transverse motion of these states is considered.

E. TRANSVERSE LATTICE DYNAMICS

The bare hadron states are static configurations on the transverse lattice. The remaining operators in the full hamiltonian generate the transverse dynamics for the bare hadrons. The magnetic, coulomb production, etc. interactions couple neighboring bare hadron states. These interactions cause a mixing of static configurations and result in transverse motion when diagonalized. The physical hadrons consist of those linear combinations of the bare hadrons which move together on the transverse lattice.

For the low mass spectrum of states the few body bound states are expected to provide a good approximation to the physical states in the large lattice approximation. If only a few bare hadron states are kept the lattice problem becomes tractable. In Fig. 2, we show the complete set of two and four link meson bound state configurations on the lattice. Each configuration can be placed anywhere on the transverse lattice. The hamiltonian induces nearest neighbor type interactions between configurations.

The transverse motion is more easily discussed if the transverse momentum is diagonalized. For each value of transverse momentum, the energy, $E_{\perp}^2(k_{\perp}) = 2P_+P_-$, acts as a matrix operator on bare hadron states labelled by the specific type of lattice configuration and by the state of longitudinal excitation. For a truncated set of states, this matrix may be diagonalized for each value of $k_{\perp}$. The results may be improved either by incorporating more complicated lattice configurations or more longitudinal states for each configuration.

F. PRELIMINARY RESULTS AND CONCLUSIONS

The calculational method described in Section D and E was implemented by R. B. Pearson and E. Rabinovici[7] to compute the low mass gluonic states of QCD on the transverse lattice. For these calculations only the two and four link meson bound states shown in Fig. 2 were included. The longitudinal dynamics was approximated by wavefunctions involving low degree polynomials but consistant with the permutation symmetries of the two and four body wavefunctions. In this approximation eighty-two bare hadron states were used as the basis for the calculation. For each value of the transverse momentum, the eighty-two by eighty-two matrix for the transverse energy, $E_\perp^2(k_\perp)$, was constructed. The diagonalization of this matrix results in transverse energies for the eighty-two hadronic states.

The masses of the hadronic bound states are obtained by evaluating $E_\perp$ at zero transverse momentum. The mass spectrum is shown for the lightest states in Fig. 3. The labels, A1, B2, E, etc.,refer to states having definite rotational symmetry on the lattice at $k_\perp^2 = 0$. A1 is totally symmetric, E is a helicity one doublet, and so forth.

We may also consider states with having nonzero transverse momentum. The transverse energy should depend on the transverse momentum through the usual relation, $E_\perp^2 = m^2 + k_\perp^2$, for each bound state. On the lattice, the relation is modified but it should be a good approximation at least within the first brillouin zone. In Fig. 4, we show the dispersion relation for $E_\perp(k_\perp)$ for the lowest states of symmetry type A1, B2, and E. The two lowest states clearly behave as expected. These two states are largely two body states with sufficient mixing with four body states to have appropriate transverse motion. However the E states and the other excited states do not appear to mix properly and have unphysical dependence on the transverse momentum. Whether these results reflect intrinsic problems or merely reflect the fact that the higher states have energies comparable to the gound state energies at the brillouin zone boundary is uncertain at this time.

In this lecture I have presented a systematic analysis of the bound state structure of a transverse lattice version of Quantum Chromodynamics. In particular, the equivalent of the strong coupling expansions of the other lattice approachs[4,5] requires a nontrivial but tractable analysis of the longitudinal and transverse dynamics. We obtain encouraging results for the behavior of the ground state hadrons but the behavior of the excited states indicates that we are far from the continuum limit for these states. Although only bound states of the pure Yang-Mills theory have been considered, quarks can be included in the lattice formulation in a number of ways.

The transverse lattice, infinite momentum frame version of QCD represents the attempt to focus attention directly on the independent physical degrees of freedom of QCD while maintaining the full symmetry structure of the theory. The analysis of the longitudinal and transverse dynamics in this lecture represents the beginning to an understanding of the hadronic aspects of this theory.

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FIGURE CAPTIONS

Fig. 1: Two and Four Body Bare Hadron Masses.

Fig. 2: Two and Four Body Lattice Configurations.

Fig. 3: Low Mass Hadron Spectrum.

Fig. 4: Transverse Energy vs. Transverse Momentum.

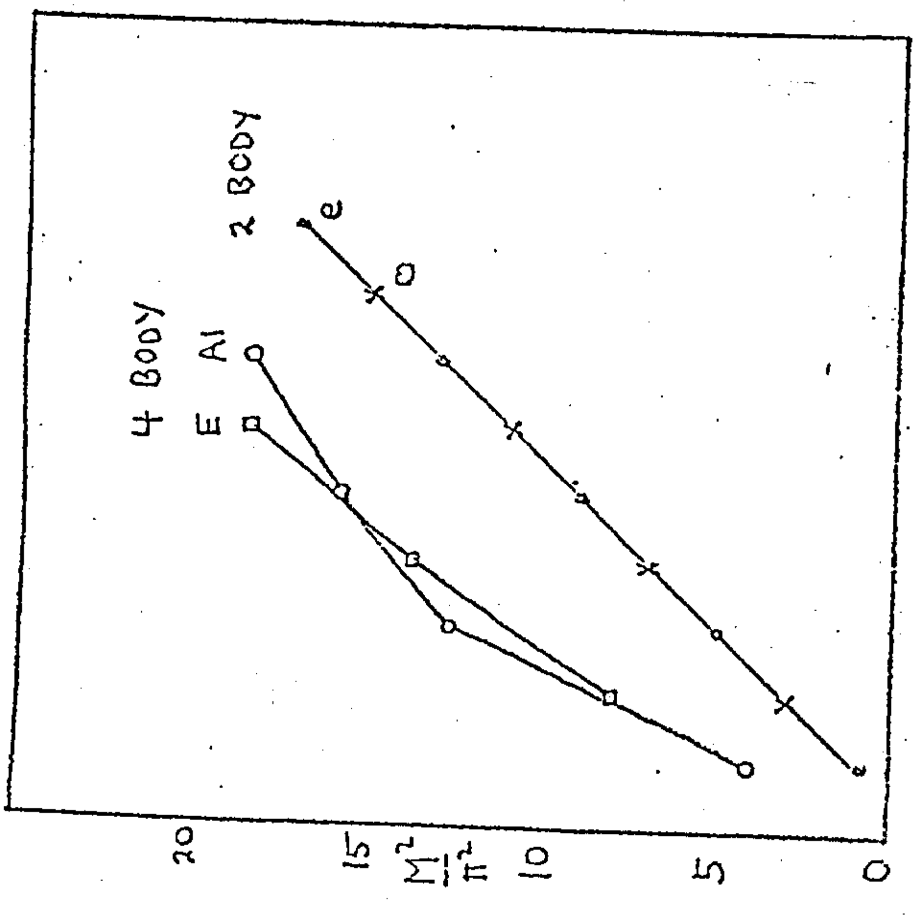

FIGURE 1

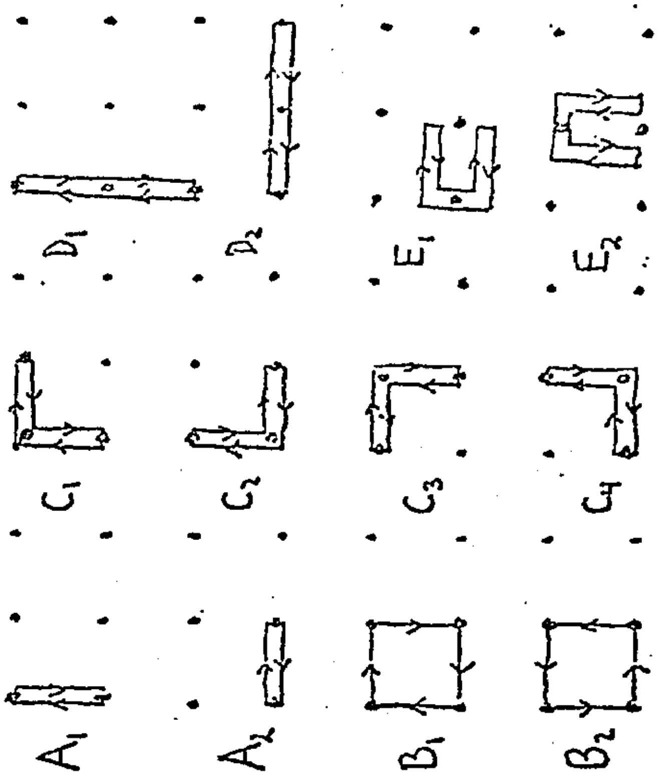

FIGURE 2

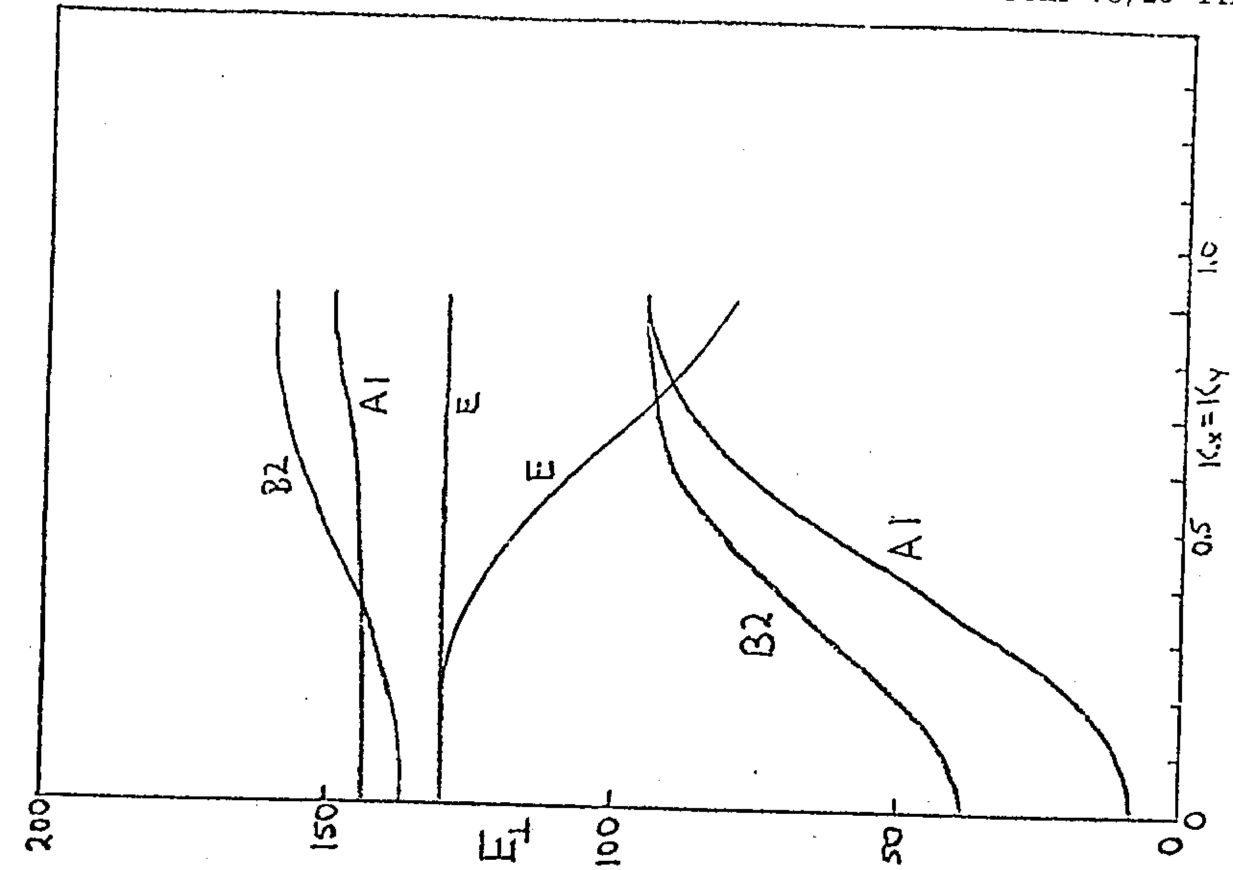

FIGURE 4

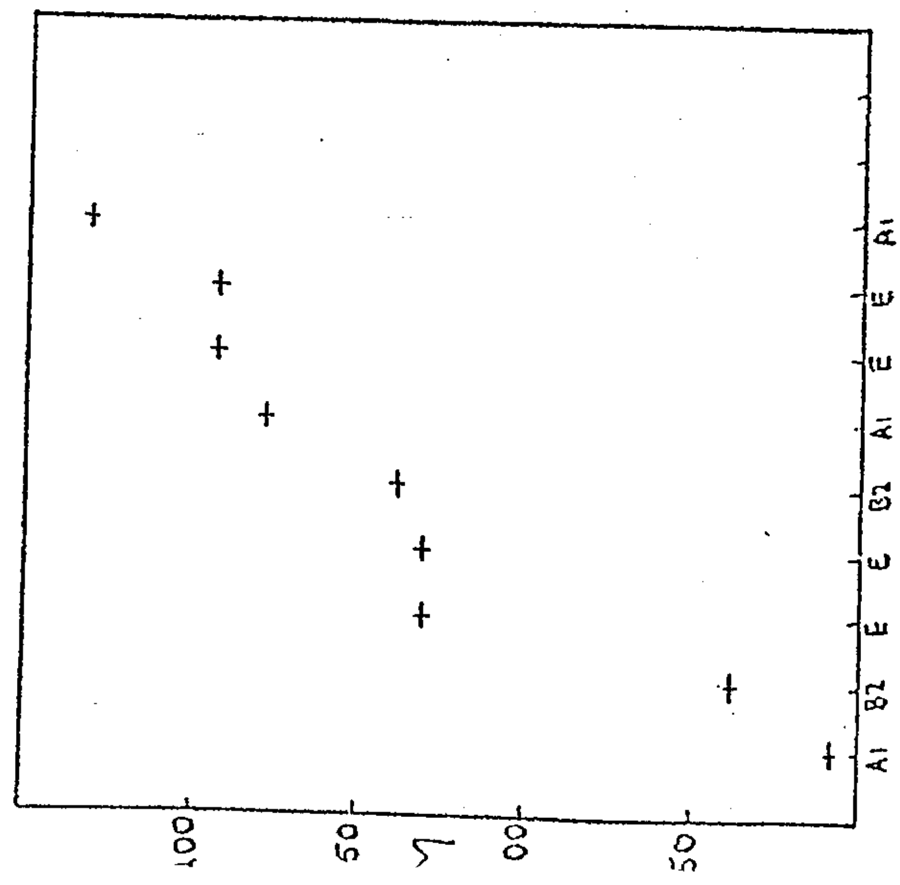

FIGURE 3